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Optimizing Cancer Treatment Decisions through Density based-Fuzzy Clustering and Multi-Criteria Decision Making Techniques

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*Uzma Ahmad
Institute of Mathematics,
University of the Punjab, New Campus, Lahore, Pakistan

Saira Hameed
Institute of Mathematics,
University of the Punjab, New Campus, Lahore, Pakistan

Hafiza Areeba Ashfaq
Institute of Mathematics,
University of the Punjab, New Campus, Lahore, Pakistan

Abstract. The increasing complexity and uncertainty in medical data, including electronic health records, imaging studies, and laboratory results, pose significant challenges for healthcare analytics. Traditional clustering techniques struggle to manage incomplete and noisy data, necessitating more robust methodologies. This study presents a hybrid decision-support framework that integrates fuzzy density-based clustering with fuzzy multi-criteria decision-making to improve healthcare decision-making under uncertainty. By using fuzzy cubic numbers, the proposed model enhances patient stratification, treatment ranking, and resource allocation. Applications include chronic disease management, pandemic response, and predictive analytics for early disease detection. The approach is particularly relevant for ICU prioritization, ventilator allocation, and real-time monitoring using wearable medical devices. Case study validation in oncology demonstrates that this hybrid framework significantly improves decision accuracy while reducing the cognitive burden on healthcare professionals. Sensitivity analysis confirms robustness across parameter variations, and comparative evaluation shows advantages over traditional approaches in handling clinical uncertainty.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: Fuzzy C-means clustering, Fuzzy density-based clustering, TOPSIS method, ARAS method, BORDA count method, Medical data uncertainty, Fuzzy cubic numbers.

**Corresponding Author*: uzma.math@pu.edu.pk

1. INTRODUCTION

Clustering is a fundamental unsupervised learning technique used to identify inherent structures in complex datasets by grouping data points according to similarity. Conventional clustering techniques, like K-means, employ a hard partitioning strategy, in which each data point is solely allocated to a single cluster. Real world data especially in medical and therapeutic decision making frequently exhibit ambiguity and noise. In cancer diagnosis and treatment priority, patients may concurrently display traits of various risk categories due to ambiguous diverse clinical profiles and insufficient observations. Consequently, inflexible clustering methodologies may yield untrustworthy conclusions.

The lack of clarity and ambiguity seen in real-world data can be effectively addressed using fuzzy logic. Fuzzy logic allows for degrees of truth, which indicate information that is partially true or partially false, in contrast to classical logic, which works with binary true or false values. Zadeh [31] introduced the concept of a fuzzy set which generalizes classical set theory by allowing partial membership, where each element has a degree of belonging between 0 and 1. Due to their adaptability, fuzzy systems can capture subtleties that traditional approaches often overlook. Numerous fields, including automated control systems, robotics, medical diagnosis, and decision support systems, have made extensive use of fuzzy logic. In industrial control, for example, fuzzy controllers are capable of interpreting noisy and imprecise sensor data, adjusting processes more efficiently than traditional binary systems [22].

Density based clustering strategy is very useful for complicated and noisy data since it can discover arbitrary formed clusters and differentiate outliers without knowing the number of clusters. Fuzzy density based clustering [5] improves on this capability by allowing data points to be assigned to several clusters with varying membership degrees which results in a more realistic representation of overlapping structures. Due to the prevalence of confusing patient profiles and borderline instances in health care analysis, this trait is particularly useful. However, when options need to be assessed and ranked based on several clinical criteria, clustering by itself does not offer a comprehensive answer for decision making

Multi-criteria decision making (MCDM) procedures provide a formal framework for evaluating and ranking alternatives when multiple frequently competing, criteria must be addressed at the same time. Traditional MCDM techniques assume exact and well defined data that is rarely met in real world clinical settings. Fuzzy MCDM techniques overcome this constraint by introducing fuzzy sets into the decision making process directly models uncertainty in both criteria values and expert opinions. Health care decision support systems have made extensive use of popular fuzzy MCDM techniques such as fuzzy AHP, fuzzy TOPSIS and fuzzy VIKOR [8], [23]. However, the richness of the fuzzy representation employed to capture uncertainty has a significant impact on how effective fuzzy MCDM is. Table 1 provides a summary of representative machine learning and MCDM strategies used in different domains. By combining membership, possibility and necessity functions at the same time, cubic fuzzy numbers (CFNs), first introduced in [13], offer a sophisticated and expressive representation of uncertain information. CFNs provide for a more thorough modeling of hesitation, risk and confidence than traditional fuzzy numbers. This is especially crucial in high stakes decision making situations like cancer diagnosis

and treatment planning [30]. Furthermore, CFNs are relatively useful for risk assessments and complex decision making processes. CFNs provide a major methodology for examining the likelihood and needs of an occurrence simultaneously, as opposed to standard fuzzy numbers. Despite their benefits integration of CFNs with density based clustering and fuzzy MCDM frameworks is relatively understudied in the existing literature.

1.1. **Research Gap.** Although prior studies have investigated fuzzy density-based clustering and fuzzy MCDM approaches independently, and in some cases in combination, existing frameworks largely rely on simpler fuzzy number representations and often lack rigorous justification of methodological choices and comprehensive empirical validation. Moreover, there is a lack of unified decision-support models that simultaneously address clustering, ranking, and uncertainty modeling using advanced fuzzy representations such as cubic fuzzy numbers, particularly in clinically relevant applications.

1.2. **Motivation and Contributions.** It is possible to solve empirical decision making problems requiring complicated and ambiguous information by combining fuzzy density based clustering with fuzzy MCDM approaches. This approach enables the identification of relevant clusters within a dataset while simultaneously evaluating alternatives with regard to several fuzzy criteria. In medical analytics, individuals can be divided into clinically significant risk groups based on criteria such as age, lifestyle and genetic predisposition allowing for more tailored treatment methods. The combination of clustering with MCDM increases not only interpretability, but also scalability and practicality of decision support systems.
The combination of cubic fuzzy numbers, fuzzy density based clustering and fuzzy MCDM improves decision making by providing a consistent representation of uncertainty throughout the analysis process. By including both requirement and possibility indicators, the proposed framework provides decision makers with a more complete understanding of opportunity and risk. This capacity is especially useful in fields like health care and environmental management when choices have long term effects and unpredictable results. For example, fuzzy density based clustering can be used in resource and environmental planning to classify regions based on pollution levels, bio-diversity risk and climate vulnerability While fuzzy MCDM can assess these regions based on social acceptability, ecological impact and economic cost. The fundamental goal of this research is to create a novel fuzzy MCDM framework that combines cubic fuzzy numbers with fuzzy density based clustering to produce a precise, adaptable and uncertainty aware decision support system. The primary contributions of this work are summarized as follows:

- A hybrid methodology that combines fuzzy density based clustering with CFN based MCDM to jointly handle patient stratification and treatment prioritization under uncertainty.
- A consistent use of cubic fuzzy numbers across clustering and decision making stages to enhance the modeling of vagueness and hesitation.
- A CFN based ranking strategy that integrates TOPSIS and ARAS results using the BORDA aggregation method to reduce ranking inconsistencies.
- A case study in cancer diagnosis and treatment prioritization complemented by sensitivity analysis to assess the robustness of the proposed approach.

All symbols used in this paper are defined in Table 2 for clarity and consistency. The remaining sections of this work are organized as follows: Section 2 provides the necessary preliminary and mathematical underpinnings. Section 3 introduces the suggested CFN based fuzzy density clustering and decision making system. Section 4 demonstrates its applicability in cancer detection. Section 5 analyzes the findings and robustness analysis. While Section 6 finishes the work with recommendations for further research.

2. PRELIMINARIES

Definition 2.1. [31] *In situations when the parameters of set membership are not precisely stated, a fuzzy set offers a mathematical method for addressing and describing vagueness or partial truth. Fuzzy set theory introduces a continuum of membership values ranging from 0 to 1, in contrast to conventional set theory, which uses binary logic to classify elements as either totally in or completely out of a set (values of 1 or 0). This strategy makes it possible to characterize vague notions and regular evolutions between membership and non-membership by signifying the degree to which an element belongs to a given set.*

A CFN is an upgraded mathematical concept that models uncertainty by combining the principles of classical fuzzy sets and interval valued fuzzy sets.

Definition 2.2. [13] *A Cubic Fuzzy Number (CFN) is defined as* $< [x^-, x^+], x^F$*, where* $[x^-, x^+] \subseteq [0, 1]$ *denotes the lower and upper bounds of the membership interval, and* $x \in [0, 1]$ *represents the fuzzy membership degree.*

The interval $[x^-, x^+]$ captures the range of uncertainty associated with the attribute, while the fuzzy value reflects the most plausible or representative membership level. CFNs are especially well-suited for medical decision-making situations where clinical measures and expert judgments are intrinsically ambiguous and uncertain because of their dual representation which enables them to simultaneously model imprecision and confidence. In this study, CFNs are generated by extending normalized crisp values into small symmetric intervals while keeping the original normalized value as the modal membership degree. The cubic fuzzy number operations combine interval-valued fuzzy arithmetic for the interval component with standard fuzzy operations.

Definition 2.3. *Following the interval-valued fuzzy arithmetic framework* [21] *extended to CFN,* $X = \langle [x, y], \lambda \rangle$ *and* $Y = \langle [m, n], \theta \rangle$*, where all components satisfy* $0 \leq x, y, \lambda, m, n, \theta \leq 1$*, the fundamental operations on* X *and* Y *are stated as:*

(1) *Addition of two CFNs is considered as*

$$X + Y = \langle [\min(1, x + m), \min(1, y + n)], \min(1, \lambda + \theta - \lambda\theta) \rangle,$$

where $(\lambda + \theta - \lambda\theta)$ *ensures the fuzzy membership remains within* $[0, 1]$ *while preserving the probabilistic union interpretation.*

(2) *Multiplication of* X *and* Y *is stated as*

$$X \times Y = \langle [\min(xm, xn, ym, yn), \max(xm, xn, ym, yn)], \lambda\theta \rangle = \langle [xm, yn], \lambda\theta \rangle.$$

Example 2.4. *Let* $A = \langle [0.1, 0.4], 0.6\rangle$ *and* $B = \langle [0.3, 0.5], 0.8\rangle$ *be two CFNs. Then, the operations are computed as follows:*

$$\begin{aligned} A + B &= \langle [\min(1, 0.1 + 0.3), \min(1, 0.4 + 0.5)], \min(1, 0.6 + 0.8 - 0.6 \times 0.8)\rangle \\ &= \langle [0.4, 0.9], 0.92\rangle \end{aligned}$$

$$\begin{aligned} A \times B &= \langle [\min(0.03, 0.05, 0.12, 0.20), \max(0.03, 0.05, 0.12, 0.20)], 0.6 \times 0.8\rangle \\ &= \langle [0.03, 0.20], 0.48\rangle \end{aligned}$$

Definition 2.5. *Following the interval-valued fuzzy arithmetic framework* [21] *extended to CFN* $A = \langle [\alpha_L(x_i), \alpha_U(x_i)], \alpha_M(x_i)\rangle$ *and* $B = \langle [\beta_L(x_i), \beta_U(x_i)], \beta_M(x_i)\rangle$*, the normalized Euclidean distance between* A *and* B *is defined as:*

$$d(A, B) = \left(\frac{1}{2m} \sum_{i=1}^{m} (\alpha(x_i) - \beta(x_i))^2 \right)^{\frac{1}{2}}$$

Alternatively, it can be expressed as:

$$\begin{aligned} & d(A, B) \\ = & \left(\frac{1}{2m} \sum_{i=1}^{m} \left((\alpha_L(x_i) - \beta_L(x_i))^2 + (\alpha_U(x_i) - \beta_U(x_i))^2 + (\alpha_M(x_i) - \beta_M(x_i))^2 \right) \right)^{\frac{1}{2}}, \end{aligned}$$

where: $\alpha_L(x_i)$ *and* $\beta_L(x_i)$ *are the lower membership functions,* $\alpha_U(x_i)$ *and* $\beta_U(x_i)$ *are the upper membership functions, and* $\alpha_M(x_i)$ *and* $\beta_M(x_i)$ *are the modal (exact) membership values of* A *and* B*, respectively, for each* x_i *with* $i = 1, 2, \ldots, m$*.*

Example 2.6. *Let* $P = \langle [0.1, 0.4], 0.6\rangle$ *and* $Q = \langle [0.3, 0.5], 0.8\rangle$ *be two CFNs. Then, the normalized Euclidean distance between* P *and* Q *is given by:*

$$dis(P, Q) = \sqrt{\frac{1}{2(2)} \left((0.1 - 0.3)^2 + (0.4 - 0.5)^2 + (0.6 - 0.8)^2 \right)} = 0.244.$$

Density-Based Clustering:

Density-based clustering [12] is a clustering technique that groups data points based on the density of their distribution in a given space. This technique is highly effective for identifying arbitrarily shaped clusters while effectively handling noise and outliers. Unlike partition-based clustering methods such as k-means, which require predefined cluster numbers, density-based approaches identify clusters as dense regions of data points separated by areas of lower density. This method is particularly beneficial in fields such as medical data analysis, where datasets often contain uncertainties due to incomplete or noisy data .
DBSCAN (Density-Based Spatial Clustering of Applications with Noise) [6] is a frequently used density based clustering algorithm, which utilizes:

- ϵ (epsilon): represents a neighborhood radius around a data point.
- minPts: identify the minimum number of points to form a dense region.

DBSCAN is prominent for its capacity to identify clusters of any shape. The algorithm doesn't require a fix number of clusters and runs in an unverified fashion. It finds noise points and categorizes data points according to their density [12].
In healthcare diagnostics, density based clustering is applied in various fields, including disease classification, patient stratification, and anomaly detection in imaging data, electronic health records, and yields from wearable devices [7]. By following fuzzy extensions of density based clustering, such as fuzzy DBSCAN, researchers can incorporate vagueness, which advances decision support in clinical settings [25].

Definition 2.7. [6]

- ***A point's ϵ-neighborhood:*** *Denoted by $N_\epsilon(p)$, ϵ-neighborhood of a point p is defined as:*

$$N_\epsilon(p) = \{q \in D \mid dist(p, q) \leq \epsilon\}$$

  , where D is a collection of points, and dist(p, q) is the distance function between points p and q.
- ***Core point:*** *A point p is called core point if the number of points in ϵ-neighborhood is $|N_\epsilon(p)| \geq$ minPts.*
- ***Direct density-reachable point:*** *A point p is said to be directly density-reachable from a point q with respect to the parameters ϵ and minPts if $p \in N_\epsilon(q)$ and q is a core point.*
- ***Border point:*** *A point p is called a border point if it is directly density-reachable from a core point but the neighborhood of p contains fewer than minPts neighbors.*
- ***Noise point:*** *Point p is called a noise point if it is neither a core point nor a border point.*

BORDA Count method:
The Borda Count method is a decision-making technique used to identify the most ideal option among a set of alternatives by assigning scores based on their rankings across multiple assessments. Each alternative obtains a score corresponding to its position in the ranking. The least preferred option is assigned zero points, while higher ranked options receive gradually larger scores. By aggregating these values, the method highlights the alternatives with the greatest overall preference and making it a powerful tool for synthesizing rankings derived from multiple decision-making criteria.
Borda Count method follows the following steps:
Step 1: Assign scores to the alternatives according to their ranking, the highest ranked alternative receives a score of $n - 1$, the second ranked receives $n - 2$ and so on with the lowest ranked alternative assigned a score of 0.
Step 2: Repeat this scoring technique for each MCDM method applied.
Step 3: Calculate the overall Borda score for each alternative by adding the scores obtained from all evaluation methods.
Step 4: Recognize the best alternative as the one with the highest overall Borda score.
By following these steps, the Borda Count method classifies the alternative that holds the strongest overall preference when evaluated across multiple decision-making criteria.

3. METHODOLOGY OF PROPOSED TECHNIQUE

The fuzzy density-based clustering algorithm is an unsupervised technique that clusters data points according to their density and detects clusters of different shapes and sizes. FDBSCAN assigns each point with a degree of membership to multiple clusters. By using CFNs, FDBSCAN can effectively depict imprecision and vagueness from data such as expert judgments or sensor readings. In decision-making, TOPSIS ranks alternatives based on their closeness to an ideal solution. Using CFNs, TOPSIS turns out to be more dynamic in handling unclear or vague input data including partial capacities. This dual methodology reinforces its capability to address complex decision-making states and making it particularly applicable in fields where uncertainty plays a central role as financial analysis and healthcare.

It is crucial to remember that the basic clinical data does not directly witness CFN. Rather, as part of the framework for making decisions the initial patient data are first standardized and then converted into CFNs. The CFN interval bounds represent measurement error and clinical variability but the modal membership values reflect decision makers most representative evaluations. This fuzzification approach enables the proposed model to incorporate expert reasoning and uncertainty handling while maintaining the original data structure. We propose the following algorithm for applying TOPSIS and ARAS with CFNs.

In this section, MCDM technique built on fuzzy density based clustering, TOPSIS, ARAS and BORDA using CFNs.

Step 1.To Apply the fuzzy density based clustering to given data.:

Step 1.1: Define Key Parameters:: DBSCAN suggest two parameters: ϵ(epsilon) represents the radius of a neighborhood around a data point, whereas minPts represents the minimum number of points required to form a dense region. The DBSCAN parameters e (neighborhood radius) and minPts are selected based on the size and characteristics of the dataset. Given the relatively small number of patient cases and low-dimensional clinical features, minPts is chosen to ensure that meaningful local density structures are detected while avoiding excessive noise classification. The e value is determined empirically by examining inter-point distances, allowing the algorithm to distinguish dense patient subgroups with similar diagnostic profiles. For the case study presented in Section 4, we used $\epsilon = 0.15$ and minPts $= 3$, determined through k-distance plot analysis and consideration of clinical interpretability.

Step 1.2: Compute the Distance Between Points.: Let $p = \mu(x_i)$ and $q = \nu(x_i)$be two CFNs. To measure similarity between points, DBSCAN typically uses the normalized Euclidean distance:

$$\text{dis}(p, q) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}(p_i - q_i)^2}.$$

Moreover,

$$\text{dis}(p, q) = \sqrt{\frac{1}{2n}\left(\mu(x_i) - \nu(x_i)\right)^2}.$$

Step 1.3: Classify Each Data Point.: For each point P, classify it based on its ε-neighborhood :

- **Core point:** If at least minPts points (including itself) exists in the ε-neighborhood

$$|N_\epsilon(p)| \geq \text{minPts}$$

- **Border point:** Lies inside the ε-neighborhood of a core point but has fewer than minPts neighbors.
- **Noise (outliers) point:** Not a core or border point. It is an isolated point.

Step 1.4: Expand Clusters.: For every core point, form a cluster. Start with an unvisited core point. Retrieve all its ε-neighbors. Expand the cluster by checking all directly density-reachable points. A point q is density-reachable from a core point p if dis$(p, q) \leq \varepsilon$ and q has at least minPts neighbours.This process continues recursively until no more density-reachable points are found.

Step 1.5: Handle Border and Noise Points.: Border points belong to a cluster but do not expand the cluster. Noise points remain unclustered (label = -1). After DBSCAN completes, the dataset will be labeled as: Cluster 0, Cluster 1, Cluster 2, etc. (Each cluster gets a unique number).Noise points labeled as -1.

Step 2. To apply TOPSIS Method with Cubic Fuzzy Numbers: The steps for applying the TOPSIS method, extended with CFNs, are outlined as follows:

Step 2.1: Define the Decision Matrix.: The decision matrix $X = (x_{ij})$ is created where each row represents an alternative, and each column represents a criterion. Let $X = (x_{ij})$ be the decision matrix, where:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{in} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}$$

Step 2.2: Normalize the Fuzzy Decision Matrix.: The decision matrix elements are all normalized to fuzzy values. Each matrix element is normalized by dividing it by the square root of the sum of the squares of the elements in the corresponding columns. Compute the normalize fuzzy matrix $\tilde{R}$ as

$$\tilde{r}_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{m} (\tilde{x}_{ij})^2}} \tag{3. 1}$$

where, m is the number of alternatives.

Step 2.3 Fuzzification of Clinical Data Using Cubic Fuzzy Numbers: To incorporate uncertainty and decision-maker perception, $\tilde{R}$ is transformed into CFN using matrix $\tilde{C} = \tilde{c}_{ij}$ such that

$$\tilde{c}_{ij} =< [\tilde{r}_{ij} - \delta_{ij}, \tilde{r}_{ij} + \delta_j], \tilde{r}_{ij} >, \tag{3. 2}$$

where $\delta_j \in (0, 1)$ represents a small uncertainty parameter associated with the j-th criterion. The interval $[\tilde{r}_{ij} - \delta_{ij}, \tilde{r}_{ij} + \delta_j]$ captures possible variation due to measurement error and clinical uncertainty, while the fuzzy membership value

reflects the most representative assessment used by decision-makers. Boundary conditions are enforced to ensure that all CFN components remain within the unit interval$[0, 1]$.This fuzzification process does not change the original ranking structure of the data rather it allows for the integration of uncertainty modeling into the subsequent fuzzy density-based clustering and MCDM procedures. In our experiments, we used a fixed value of $\delta_j = 0.05$ for all criteria, corresponding to a $\pm 5\%$ perturbation around the normalized values. This choice reflects moderate measurement uncertainty commonly encountered in clinical settings, where diagnostic measurements typically have $\pm 5\%$ to $\pm 10\%$ variability due to instrument precision, biological fluctuations, and inter-observer differences. The symmetric interval $[\tilde{r}_{ij} - 0.05, \tilde{r}_{ij} + 0.05]$ ensures all CFN components remain within the unit interval $[0, 1]$ after transformation.

Step 2.4: Determine the Weighted Normalized Decision Matrix.: Each criterion is assigned a weight w_j based on its importance. For the MCDM analysis, equal weights ($w_{ij} = 1/n$) can be assigned to all criteria due to the absence of prior clinical evidence favoring one criterion over another. This assumption avoids subjective bias and ensures neutrality in the evaluation process. Equal weighting is a commonly adopted strategy in medical decision-making studies when expert consensus on criterion importance is unavailable, and it allows the comparative performance of TOPSIS and ARAS methods to be evaluated objectively. Compute the fuzzy weighted normalized decision matrix $\tilde{V} = (\tilde{v}_{ij})$ as:

$$\tilde{v}_{ij} = w_j \cdot < [\tilde{r}_{ij} - \delta_{ij}, \tilde{r}_{ij} + \delta_j], \tilde{r}_{ij} >$$

Step 2.5: Determine the Ideal and Negative Ideal Solutions.: The ideal solution A^+ and the negative ideal solution A^- are determined based on the maximum and minimum fuzzy values for each criterion across all alternatives.

- **Positive Ideal Solution** A^+: For each criterion j,

$$A_j^+ = \begin{cases} \max\left\{\left([\tilde{v}_{ij}^-, \tilde{v}_{ij}^+], \tilde{v}_{ij}^F\right)\right\} & \text{if it's a benefit criterion} \\ \min\left\{\left([\tilde{v}_{ij}^-, \tilde{v}_{ij}^+], \tilde{v}_{ij}^F\right)\right\} & \text{if it's a cost criterion} \end{cases}$$

- **Negative Ideal Solution** A^-: For each criterion j,

$$A_j^- = \begin{cases} \min\left\{\left([\tilde{v}_{ij}^-, \tilde{v}_{ij}^+], \tilde{v}_{ij}^F\right)\right\} & \text{if it's a benefit criterion} \\ \max\left\{\left([\tilde{v}_{ij}^-, \tilde{v}_{ij}^+], \tilde{v}_{ij}^F\right)\right\} & \text{if it's a cost criterion} \end{cases}$$

Step 2.6: Calculate the Distance from the Ideal and Negative Ideal Solutions.: The distances from each alternative to the ideal and negative ideal solutions are computed as follows:

- **Distance from Positive Ideal Solution** D_i^+: Use the Euclidean distance formula for fuzzy numbers

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(\left([\tilde{v}_{ij}^-, \tilde{v}_{ij}^+], \tilde{v}_{ij}^F\right) - A_j^+\right)^2}$$

- **Distance from Negative Ideal Solution** D_i^-: Use the Euclidean distance formula for fuzzy numbers.

$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(\left([\tilde{v}_{ij}^-, \tilde{v}_{ij}^+], \tilde{v}_{ij}^F \right) - A_j^- \right)^2}$$

Step 2.7: Calculate the Relative Closeness to the Ideal Solution.: The relative closeness C_i of each alternative to the ideal solution is calculated using the following formula:

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}$$

Step 2.8: Rank the Alternatives.: Finally, alternatives are ranked based on the relative closeness values C_i. The alternative with the highest value of C_i is considered the best choice.

Step 3: To apply ARAS Method Based on Cubic Fuzzy Numbers: The ARAS method evaluates alternatives by calculating their total utility. When fuzzy evaluations are involved, CFNs are employed to manage uncertainty. The ARAS steps using CFNs are as follows:

Step 3.1: Define the Decision Matrix.: Consider the matrix $\tilde{C} = \tilde{c}_{ij}$ given in Equation (3.1). We construct a ARAS decision matrix which has $m + 1$ rows and n columns representing the optimal value of each attribute. Equations (3.3) and (3.4) yield the optimal values of the qualities.

$$x_{0j} = \max \left\{ \left([x_{ij}^-, x_{ij}^+], x_{ij}^F \right) \right\} \text{ (for benefit attributes)} \tag{3. 3}$$

$$x_{0j} = \min \left\{ \left([x_{ij}^-, x_{ij}^+], x_{ij}^F \right) \right\} \text{ (for cost attributes),} \tag{3. 4}$$

where x_{0j} denotes the optimal value under the j-th attribute, $\left([x_{ij}^-, x_{ij}^+], x_{ij}^F \right)$ denotes the value of the i-th alternative under the j-th attribute in the form of a CFN. The formula (1) corresponds to the situation that the j-th attribute is a benefit attribute, while the formula (2) corresponds to the situation that the j-th attribute is a cost attribute. The ARAS decision matrix X is given as follows:

$$X = \begin{bmatrix} x_{01} & \cdots & x_{0j} & \cdots & x_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix},$$

where $i = 0, 1, 2, \ldots, m$ and $j = 1, 2, 3, \ldots, n$.

Step 3.2: Normalize the Decision Matrix.: The normalized data are denoted by $\tilde{x}_{FCN}$, and all the values are in the form of CFNs. The normalized decision-making matrix

can be denoted as follows:

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{01} & \cdots & \tilde{x}_{0j} & \cdots & \tilde{x}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{i1} & \cdots & \tilde{x}_{ij} & \cdots & \tilde{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \cdots & \tilde{x}_{mj} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

Where,

$$\tilde{x}_{ij} = \frac{\left[x_{ij}^{-}, x_{ij}^{+}\right], x_{ij}^{F}}{\sum_{i=1}^{m}\left[x_{ij}^{-}, x_{ij}^{+}\right], x_{ij}^{F}}$$

if the jth attribute is benefit attribute

$$\tilde{x}_{ij} = \frac{\sum_{i=1}^{m}\left[x_{ij}^{-}, x_{ij}^{+}\right], x_{ij}^{F}}{\left[x_{ij}^{-}, x_{ij}^{+}\right], x_{ij}^{F}}$$

if the jth attribute is cost attribute

Step 3.3: Weighted Normalized Matrix.: The normalized decision matrix can be converted into a weighted normalized matrix by criterion weights that are represented by w_j, with the rule that the sum of the criteria weights is 1, given by:

$$\sum_{j=1}^{n} w_j = 1$$

The formula below is applied to all criteria for the calculation of $[\hat{x}_{ij}^{-}, \hat{x}_{ij}^{+}], \hat{x}_{ij}^{F}$ values:

$$\hat{x}_{ij} = \left[\tilde{x}_{ij}^{-}, \tilde{x}_{ij}^{+}\right], \tilde{x}_{ij}^{F} \cdot w_j, \quad i = 0, \ldots, m$$

Step 3.4: Compute the Optimality Function.: The values of the optimality function are determined by the following equation:

$$S_i = \sum_{j=1}^{n}\left(\left[\hat{x}_{ij}^{-}, \hat{x}_{ij}^{+}\right], \hat{x}_{ij}^{F}\right), \quad i = 0, 1, 2, \ldots, m$$

Step 3.5: Compute the Relative Efficiency Score.: The high S_i value brings the alternative to the upper ranks, while the low S_i value harms the preferability of the alternative. For this order, Q_i values for each alternative should be calculated by the following equation:

$$Q_i = \frac{S_i}{S_0}$$

where S_i and S_0 are the optimal function values of alternatives.

Step 3.6: Rank the Alternatives.: The alternatives are ranked in descending order based on their relative efficiency scores Q_i.

Step 4: Apply the BORDA count method.: By applying BORDA count method on the rankings derived from Step 2 and Step 3 we obtained the final ranking of the suggested integrated technique.

Following this framework, our study ensures that the decision-making processes characterized by uncertainty and imprecision using CFNs, providing more accurate and realistic outcomes in complex decision making scenarios. The algorithmic representation of fuzzy density-based clustering and fuzzy cubic MCDM is shown in Figure 1.

3.1. **Methodological Validation and Comparative Advantages.** To conceptually validate the proposed framework, we contrast its operation with standard non-fuzzy approaches using our synthetic dataset. Traditional DBSCAN would assign each patient to exactly one cluster (e.g., Patient 9 to Cluster 2) or label them as noise, failing to represent the partial membership that FDBSCAN provides a critical feature for patients with borderline characteristics. Similarly, applying classic TOPSIS to the raw data in Table 6 yields a single deterministic ranking, ignoring the interval-valued imprecision inherent in medical measurements. Our use of Cubic Fuzzy Numbers explicitly models this uncertainty, as seen in Table 7, where each criterion is represented as a range with a modal value. This allows the subsequent MCDM steps to process not just values, but confidence in those values, leading to more nuanced rankings that better reflect clinical reality.

4. Case Study: Cancer Diagnosis and Treatment Using DBSCAN and Hybrid MCDM.

In cancer care, doctors work with many different types of patient information - from genetic test results and medical scans to electronic health records. All this data helps in understanding and treating cancer, but a computer based methods are needed to extract significant patterns from those datasets. During the treatment of cancer, various elements are involved including treatment response variability, biomarker level or tumor size. DBSCAN enables the classification of oncological cases based on clinical characteristics, supporting more precise and adapted decision-making in diagnosis and therapy planning through finer-grained analysis of cancer data. Researchers and doctors identify infrequent alteration profiles by using density based clustering.
One of the biggest challenges in cancer treatment is balancing different goals that sometimes conflict. For example, the most effective treatment might also have more side effects. Methods like ARAS and TOPSIS help doctors navigate these tough decisions by systematically comparing options and finding the best balance between different priorities.
These techniques enable a more detailed assessment of treatment by integrating patient preferences, clinical recommendations and expert judgment. ARAS estimates alternatives in terms of their utility function while TOPSIS ranks them built on closeness to an ideal solution. Both techniques support clinical environments characterized by vagueness. They help a lot in optimizing complex systems like cancer care.

Ten cases of different patients with identified tumor growth ranging from 2 to 3 cm were chosen for this investigation of a prominent cancer treatment center. The functional coordinates of each tumor's location and distribution form are obtained using medical imaging instruments. Different cases with comparable tumor sizes are preferred in order to minimize clinical inconsistency and ensure the study's assumptions may be easily interpreted. Due to low clinical risk, cases with very tiny tumors might not need intense risk monitoring or sophisticated treatment planning.

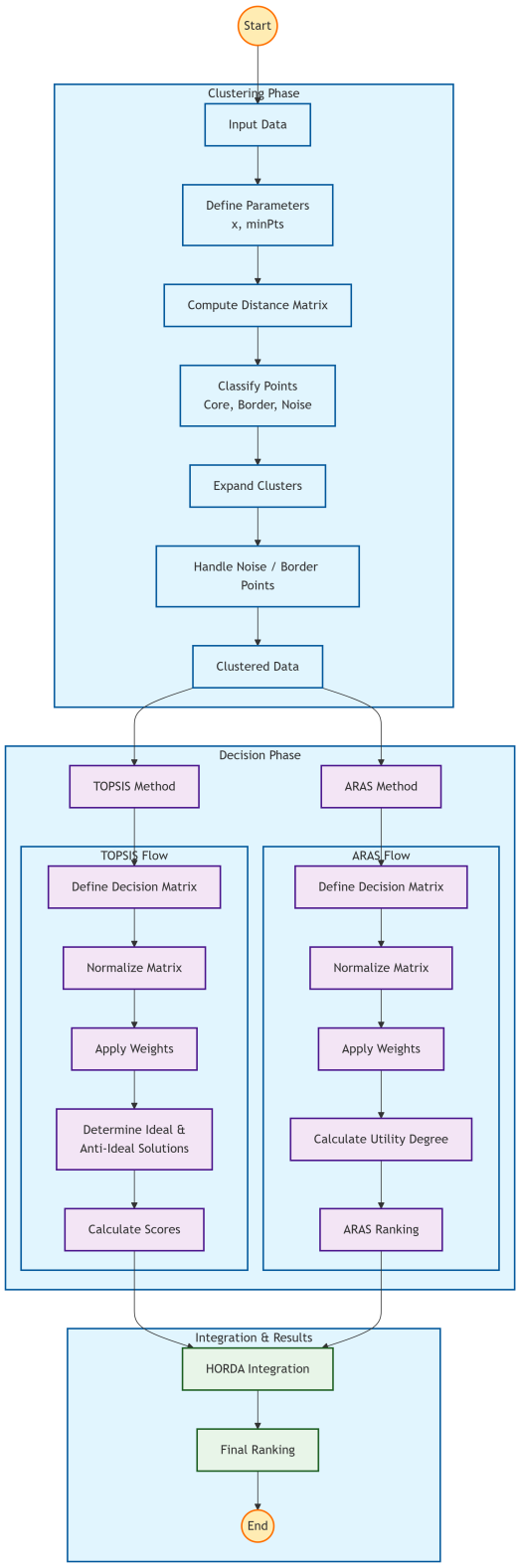

FIGURE 1. Flowchart of the proposed hybrid framework integrating fuzzy density-based clustering with MCDM techniques.

To authenticate the reliability and the assortment of the suggested cases, anatomical position data for each tumor size and spread region along with their respective measurements. This approach ensures that the results of the risk evaluation and clustering analysis are meaningful and broadly applicable to real-world clinical oncology scenarios. The anatomical coordinates of each tumor in relation to each diagnostic and treatment criterion are provided in Table 3. A robust technique for classifying cancer patients based on their diagnostic and pathological profiles is Density-Based Spatial Clustering of Applications with Noise, which considers multiple clinical factors in this case, tumor size and biomarker level. Unlike conventional partition-based clustering algorithms, DBSCAN identifies clusters based on data density, enabling the discovery of arbitrary-shaped clusters and the detection of outliers (i.e., atypical or rare cases). This characteristic is particularly advantageous in oncology, where patient cases often exhibit heterogeneous and overlapping clinical features.

By relying on local density thresholds, DBSCAN allows for the differentiation of patient subgroups with distinct risk patterns, while simultaneously flagging anomalous cases that may require individualized diagnostic attention or novel treatment approaches.

DBSCAN Implementation Details: For the clustering analysis, DBSCAN was implemented with parameters $\epsilon = 0.15$ and minPts $= 3$. These values were selected based on the k-distance plot analysis (with $k = 4$) and consideration of the dataset characteristics. The ϵ value of 0.15 corresponds to approximately 15% of the normalized feature space range, effectively distinguishing dense patient subgroups while maintaining interpretable cluster granularity. The choice of minPts $= 3$ (as minPts $\geq dimension + 1$) ensures that clusters represent meaningful local density patterns while avoiding excessive fragmentation given the small sample size ($n = 10$).

Uncertainty Parameter Selection: The transformation to cubic fuzzy numbers employed a uniform uncertainty parameter $\delta = 0.05$ across all clinical criteria. This value was chosen to represent typical measurement uncertainties in oncology diagnostics, where tumor size measurements can vary by ± 0.5 cm (approximately $5 - 10\%$ relative error for 2-3 cm tumors) and biomarker assays typically exhibit $\pm 5\%$ analytical variability. The symmetric $\pm 5\%$ perturbation creates realistic interval representations while maintaining computational simplicity and interpretability. The diagnostic and latitudinal coordinate information of different cases are calculated using the DBSCAN clustering algorithm, shows patient with their alike characteristic presented in Table 4.

Table 5 presents the decision matrix used in TOPSIS and ARAS Methods. The weights of the criteria are taken into account when applying the TOPSIS and ARAS Techniques are assumed as equal. Next, to ensure consistency and comparability among heterogeneous clinical variables, the original data points are first normalized into a standard range between 0 and 1 by uing the following formula shown in Table 6,

$$\frac{x - x_{min}}{x_{max} - x_{min}}$$

where x is the data points in the given columns of Table 5. This standardization step ensures that variables with different units or magnitudes are made dimensionless, allowing for fair and unbiased integration during subsequent analysis. Post-normalization, the data

is transformed into CFNs shown in Table 7 to effectively capture the multifaceted uncertainty inherent in medical datasets by adding subtracting values in interval and placing fuzzy membership degrees as its original value. CFNs, characterized by the combination of interval-valued membership functions and fuzzy membership degrees, offer a more comprehensive mathematical structure to model imprecise, vague, and sometimes conflicting information. This representation is particularly suitable for oncology applications, where patient responses and disease characteristics often exhibit ambiguity and variability. By utilizing CFNs, the model preserves both the range of possible values and the degree of confidence associated with them, thus enhancing the accuracy and resilience of downstream processes such as density-based clustering and MCDM through TOPSIS and ARAS methods. Table 8 displays the outcomes of the decision-making techniques. While comparable rankings are derived from some of the techniques. For certain strategies, the choices are not arranged in the same sequence as for others. The TOPSIS reference point is the best option (Patient 6), also for ARAS, the best option is the (Patient 6). But for more conveniece and modified result we use borda count method. Therefore, the obtained rankings must be merged according to dominance in order to examine the combined effect of the approaches on the alternative rankings.
Table 9 displays the outcomes of the Borda Count approach, which is applied to this combination based on the number of alternatives in the decision problem.

The differences in intermediate rankings between TOPSIS and ARAS Table 8) are not limitations but rather reflect their complementary methodological philosophies. TOPSIS, being distance based favors alternatives that are geometrically closest to the ideal solution while being farthest from the negative ideal. This explains why Patient 7 ranks highly (2nd) in TOPSIS: this patient has balanced scores across all parameters with no extreme values in any dimension indicating a ”safe” pick. In contrast, ARAS, which is utility based, estimates the relative efficiency of each alternative in comparison to the ideal reference. This technique recognizes outstanding achievement on certain criteria. Patient 3 unusually high bio marker level (0.97) which significantly contributes to the utility score despite middling performance on side effects accounts for its high ranking (2nd) in ARAS. These differences draw attention to a key advantage of our hybrid strategy. Certain MCDM systems, for example analyze criteria and alternatives in pairs whilst others use distance based metrics to compare criteria and alternatives to an ideal solution. Some systems are better suited to quantitative criteria but others may work with fuzzy data or qualitative criteria. Therefore, depending on the situation and the kind of decision issue various MCDM techniques may have distinct advantages and disadvantages. Choose the MCDM approach that best suits your decision making objectives and the features of the situation. To solve the specific shortcomings of each MCDM technique or enhance its advantages. Researchers are considering using a hybrid strategy that combines two or more MCDM approaches. As a result, integrated and hybrid techniques reduce subjectivity in decision making processes while producing more comprehensive and exact results. TOPSIS offers comprehensive possibilities whereas ARAS recognizes experts. The BORDA approach is utilized in this study to combine several MCDM methodologies and reach an agreement on the findings (Table 9). The Borda count synthesis makes advantage of these complementary viewpoints

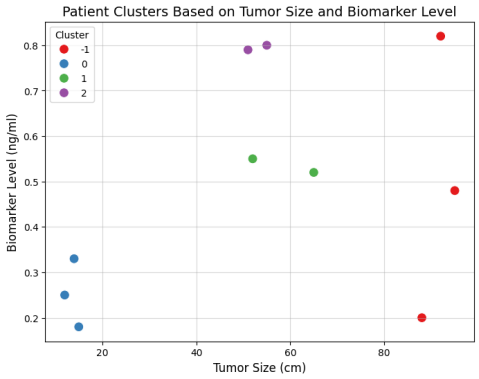

FIGURE 2. Comparative representation of the results of the DBSCAN clustering.

and Patient 9 is the consensus top priority because it strikes a balance between these factors, demonstrating both excellent overall performance (like Patient 7) and brilliance in crucial areas (like Patient 3). The representation of the graph of DBSCAN clustering result is shown in Figure 2.

5. RESULTS AND DISCUSSION

Prioritization of patients for treatment is a critical consideration in cancer care management, as it can significantly impact clinical outcomes, resource allocation, and system efficiency. Several influential factors must be taken into account when determining treatment order, including: Efficiency of density-based clustering: The application of density-based clustering algorithms enabled a colordata-driven understanding of patient similarities and risk stratifications. This clustering approach allowed for the proactive identification of high-risk patient groups, ensuring that treatment is prioritized for those most in need. Value of MCDM techniques in treatment prioritization: The use of MCDM methods, specifically TOPSIS and ARAS, proved effective in evaluating complex clinical scenarios with multiple, and often conflicting, criteria. By incorporating variables such as disease severity, prognosis, response to treatment, and patient comorbidities, the methods provided a comprehensive and balanced foundation to prioritize patients for treatment. Operational and strategic benefits: reducing mortality and morbidity: By prioritizing patients at higher risk, early intervention can prevent disease progression and improve survival rates. Efficient resource utilization: prioritization ensures that critical healthcare resources (e.g., ICU beds, specialized oncology treatments) are allocated where they are most needed. Enhancing patient satisfaction and outcomes: Timely and appropriate treatment improves clinical outcomes and patient trust in healthcare services. The TOPSIS and ARAS techniques were first used in this study to rank patients according to a variety of operational and clinical factors. The consequences demonstrated that using dual methodologies Patient 6 should receive treatment first. Patient 9 was acknowledged with the highest treatment priority when the rankings from ARAS and TOPSIS were joined using the BORDA count method.

These outcomes indicates that while depending only on MCDM technique a more comprehensive and integrated valuation approach combining MCDM techniques can produce more consistent and clinically substantial prioritization. Additionally, the density-based clustering technique demonstrated that Patient 9 belongs to a particular cluster distinguished by higher level of clinical insistence. Due to comparable risk and clinical needs, it stated that Patient 9 along the remaining patients in the same cluster should be given preference for early intervention. Combining the BORDA count method with density-based clustering and MCDM techniques provided a strong and well-structured methodology for patient priority in cancer treatment. This methodology reduces the risk of adversative outcomes, guarantees the effective therapy and improves the general resilience of cancer care systems.

5.1. **Empirical Validation Sensitivity Analysis of Proposed Technique.** We examine the robustness of the proposed hybrid framework, a sensitivity analysis was carried out focusing on two essential components of the methodology: the parameters of the fuzzy density-based clustering algorithm and the criterion weights used in the multi-criteria decision-making stage.

Parameter Sensitivity in FDBSCAN: The influence of clustering parameters was evaluated by systematically varying the neighborhood radius ϵ and the minimum number of points (minPts). When the value of ϵ was increased by 20%, Patients 2 and 8 shifted from Cluster 1 to the boundary region of Cluster 2, reflecting their intermediate clinical characteristics. In contrast, decreasing ϵ by 20% resulted in Patient 4 being classified as noise. Despite these local changes, the core cluster structurecomprising Patients 1, 3, 5, 6, 9, and 10remained unchanged. This observation indicates that the algorithm reliably identifies well-defined patient subgroups and is not overly sensitive to moderate parameter perturbations.

Weight Sensitivity in MCDM: To evaluate the stability of patient rankings, three clinically motivated weighting scenarios were considered. The first scenario employed balanced weights, assigning equal importance to all criteria. The second scenario prioritized patient safety by assigning a higher weight to treatment side effects, while the third emphasized treatment efficacy. The corresponding weight vectors are summarized in Table 10. Here balanced Weights: [0.25, 0.25, 0.25, 0.25] for [Tumor Size, Biomarker, treatment efficacy, Side Effects], safety-first: [0.15, 0.15, 0.30, 0.40] (emphasizing minimal side effects),efficacy-first: [0.20, 0.20, 0.40, 0.20] (emphasizing treatment effectiveness) and stability score: Rank based on average position across all scenarios (1 = most stable). The analysis reveals that while middle rankings show expected variability, Patients 6 and 9 consistently appear in the top tier across all scenarios. This stability in identifying high-priority patients confirms the model's reliability under different clinical prioritization philosophies. Across these scenarios, some variability was observed in the middle-ranked patients, which is expected when decision priorities shift. However, Patients 6 and 9 consistently appeared among the highest-ranked cases under all weighting configurations. This consistency suggests that the framework reliably identifies high-priority patients regardless of the specific clinical emphasis. The stability score further indicates that Patient 6 exhibits the most consistent ranking across all scenarios.

From a clinical perspective, the model's outputs align well with expert reasoning. Patient

6, consistently ranked as the highest priority, exhibits both a large tumor size and maximal biomarker expression, corresponding to a high-risk clinical profile requiring urgent intervention. Patient 9, while belonging to a different cluster, is characterized by moderate tumor size located in clinically sensitive regions, justifying elevated treatment priority. Conversely, Patient 1, associated with low tumor size and weak biomarker expression, was consistently ranked as a low-priority case. These observations provide face validity for the proposed framework. Although this sensitivity analysis is not exhaustive, it demonstrates that the proposed methodology maintains stable clustering and ranking behavior under clinically meaningful variations in parameters and decision preferences.

5.2. **Comparative Advantages.** Table 11 presents a conceptual comparison between the proposed framework and commonly used clustering decision-making combinations. Traditional DBSCAN integrated with classical TOPSIS is effective in identifying clusters of arbitrary shapes and explicitly handling noise; however, it relies on hard assignments and lacks a mechanism to represent uncertainty in the data. On the other hand, fuzzy C-means combined with fuzzy MCDM incorporates partial memberships but assumes spherical cluster structures and forces all data points into clusters, which can be problematic in the presence of noise or outliers. The proposed approach bridges these limitations by integrating density-based clustering with cubic fuzzy number representations. By preserving the ability of DBSCAN to detect arbitrarily shaped clusters while introducing cubic fuzzy memberships, the framework simultaneously captures spatial density, overlap, and uncertainty. The inclusion of interval-valued and membership-based information enables a richer description of vagueness than single-valued fuzzy methods. As a result, the proposed methodology offers improved noise handling, enhanced uncertainty modeling, and higher interpretability, which are particularly important in medical decision-making contexts where data ambiguity and borderline cases are common. From a computational perspective, the hybrid structure maintains a complexity comparable to density-based clustering while benefiting from a reduced decision space in the subsequent MCDM stage. This balance between expressiveness and efficiency makes the framework suitable for complex, real-world decision problems involving uncertain and heterogeneous clinical data.

5.3. **Limitations.** While the proposed framework demonstrates strong performance and robustness, certain limitations should be acknowledged. The effectiveness of the method depends on appropriate parameter selection in both the FDBSCAN clustering stage and the MCDM weighting process. Although the sensitivity analysis indicates stability within reasonable ranges, parameter calibration in real clinical settings would benefit from expert input or adaptive optimization strategies.

In addition, the framework assumes that clinical data can be reliably transformed into cubic fuzzy numbers. This transformation may be challenging when uncertainty is poorly quantified or expert assessments are inconsistent. Finally, the empirical validation presented in this study is based on synthetic data designed to illustrate the methodology. Validation using real-world clinical datasets is necessary to fully assess scalability, interpretability, and practical utility in operational healthcare environments.

6. CONCLUSION

This study demonstrates that integrating density-based clustering with multi-criteria decision-making (MCDM) techniques significantly enhances cancer treatment prioritization accuracy and supports more informed therapy selection. The proposed framework provides physicians with validated tools to address uncertainties inherent in oncology, thereby improving diagnostic precision and decision confidence. By reducing computational complexity and processing time, the methodology facilitates practical implementation in clinical settings.

The model offers a reliable decision-support mechanism for selecting optimal treatment courses through balanced consideration of multiple clinical factors, including treatment efficacy, side effects, and quality of life implications. As analytical approaches become increasingly vital in modern healthcare, this framework empowers clinicians to make more balanced, adaptable, and patient-centered decisions. The proposed approach ensures comprehensive and flexible treatment planning for cancer care through integrated analysis of both quantitative and qualitative clinical data. The natural progression of this work is application to real-world oncology datasets. We are preparing to apply this framework to the METABRIC breast cancer dataset [11], which contains comprehensive treatment records and outcomes for over 2,000 patients. This will provide the large-scale validation and comparison with clinical ground truth that represents the logical next step for this methodological framework. For practical clinical integration, we propose a three-phase implementation pathway. Initially, the framework would be deployed as a standalone decision support module for multidisciplinary tumor boards, allowing clinicians to compare its recommendations with conventional assessments. Subsequently, integration with Electronic Health Record (EHR) systems would enable automated extraction and formatting of patient data. Finally, adaptation for real-time settings such as intensive care units could support dynamic prioritization. Validation should follow a staged roadmap, beginning with retrospective analysis of established datasets like METABRIC [11] to benchmark performance against historical outcomes. This should be followed by a prospective pilot study in a single institution to evaluate usability and concordance with clinical decisions, culminating in a multi-center trial assessing impact on objective treatment outcomes and workflow efficiency. Successful translation will depend on addressing key requirements: developing clinician-friendly interfaces that intuitively visualize uncertainty, creating mechanisms to calibrate model parameters using institutional data, and incorporating explainability features to foster trust. This structured approach ensures the framework evolves from a methodological proof-of-concept to a tool with tangible clinical utility.

CREDIT AUTHORSHIP CONTRIBUTION'S STATEMENT

Saira Hameed: Concept, Design, Analysis and Writing of the manuscript. Uzma Ahmad: Concept, Design, Analysis and Writing of the manuscript. Hafiza Areeba Ashfaq: Concept, Design, Analysis and Writing of the manuscript.

DECLARATIONS

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TABLE 1. Summary of MCDM and ML Techniques from Literature

Authors	Methodologies	Application Area
[1]	Fuzzy Logic Methods	Disease Diagnosis
[2]	MCDM on Pythagorean CFN	investment management
[3]	Fuzzy MCDM	COVID-19 Severity Prediction
[4]	Hybrid Group MCDM, Clustering Method	Evaluation of Clustering
[9]	Fuzzy Time Series	Alzheimer Forecasting
[10]	Genetic Algorithm, Fuzzy Clustering	Bankruptcy Prediction
[14]	MCDM	Surgical Management
[15]	Fuzzy Logic, ML Integration	Decision Support
[16]	MCDM with m-Polar Neutrosophic Topology	Medical Diagnosis
[17]	Fuzzy Density-Based Clustering	Feature Clustering
[18]	Fuzzy logic-based health monitoring system	COVID-19 patients
[19]	Fuzzy Clustering Algorithms	Cancer Data
[20]	Intuitionistic Fuzzy MCDM	Environmental Management
[23]	Fuzzy Sets, Decision-Making	Medical Evaluation
[24]	Fuzzy- based classifier model	clinical decision support systems
[25]	Fuzzy classifier and modified DBSCAN algorithm	High Risk Prediction of Diabetes Mellitus
[26]	Fuzzy MCDM, Hybrid Methods	Transplant Risk
[27]	Fuzzy Logic Systems	Healthcare Applications
[28]	cluster-based fuzzy hybrid decision support model	sustainable healthcare landfill location selection
[30]	Fuzzy Logic, Uncertainty Quantification	Patient Survival Modeling
[29]	Fuzzy C-Means, Hybrid Decision-Making	Health and Safety Risk

TABLE 2. Key Symbols and Notation

Symbol	Meaning
MCDM	Multi-criteria decision making
DBSCAN	Density-Based Spatial Clustering of Applications with Noise
FDBSCAN	Fuzzy Density-Based Spatial Clustering of Applications with Noise
CFN	Cubic fuzzy number
$\langle [a^-, a^+], a^F \rangle$	CFN with interval bounds $[a^-, a^+]$ and fuzzy value a^F
ϵ, minPts	Neighborhood radius and minimum points
$N_\epsilon(p)$, $d(p, q)$	ϵ-neighborhood and distance
$X = (x_{ij})_{m \times n}$	$m \times n$ decision matrix
w_j	Weight of criterion j ($\sum w_j = 1$)
A^+, A^-	Positive and negative ideal solutions
D_i^+, D_i^-	Distances from ideal solutions
C_i	Relative closeness coefficient
S_i, Q_i	Optimality function and relative efficiency
B_i	Final Borda score for alternative i

TABLE 3. Tumor data of Patients

Patient ID	Tumor Size	Biomarker Level
1	12	0.25
2	52	0.55
3	55	0.80
4	15	0.18
5	95	0.48
6	92	0.82
7	88	0.20
8	65	0.52
9	14	0.33
10	51	0.79

TABLE 4. DBSCAN output

Patient ID	Tumor Size	Biomarker Level	Clusters
1	12	0.25	0
2	52	0.55	1
3	55	0.80	2
4	15	0.18	0
5	95	0.48	-1
6	92	0.82	-1
7	88	0.20	-1
8	65	0.52	1
9	14	0.33	0
10	51	0.79	2

TABLE 5. Decision matrix

Patient ID	Tumor Size	Biomarker Level	Treatment Efficacy	Side Effect
1	12	0.25	90	3
2	52	0.55	78	6
3	55	0.80	85	4
4	15	0.18	88	8
5	95	0.48	60	5
6	92	0.82	70	7
7	88	0.20	85	9
8	65	0.52	80	2
9	14	0.33	82	5
10	51	0.79	87	4

TABLE 6. Fuzzy decision matrix for patient evaluation

Patient ID	Tumor Size	Biomarker Level	Treatment Efficacy	Side Effect
1	0.00	0.11	1.00	0.14
2	0.48	0.58	0.60	0.57
3	0.52	0.97	0.83	0.29
4	0.04	0.00	0.93	0.86
5	1.00	0.47	0.00	0.43
6	0.96	1.00	0.33	0.71
7	0.92	0.03	0.83	1.00
8	0.64	0.53	0.67	0.00
9	0.02	0.23	0.73	0.43
10	0.47	0.95	0.90	0.29

TABLE 7. Decision matrix with cubic fuzzy numbers for TOPSIS

Patient ID	Tumor Size	Biomarker	Treatment	Side Effect
1	$\langle[0.00, 0.05], 0.00\rangle$	$\langle[0.06, 0.16], 0.11\rangle$	$\langle[0.95, 1.00], 1.00\rangle$	$\langle[0.09, 0.19], 0.14\rangle$
2	$\langle[0.43, 0.53], 0.48\rangle$	$\langle[0.53, 0.63], 0.58\rangle$	$\langle[0.55, 0.65], 0.60\rangle$	$\langle[0.52, 0.62], 0.57\rangle$
3	$\langle[0.47, 0.57], 0.52\rangle$	$\langle[0.92, 1.00], 0.97\rangle$	$\langle[0.78, 0.88], 0.83\rangle$	$\langle[0.24, 0.34], 0.29\rangle$
4	$\langle[0.00, 0.09], 0.04\rangle$	$\langle[0.00, 0.05], 0.00\rangle$	$\langle[0.88, 0.98], 0.93\rangle$	$\langle[0.81, 0.91], 0.86\rangle$
5	$\langle[0.95, 1.00], 1.00\rangle$	$\langle[0.42, 0.52], 0.47\rangle$	$\langle[0.00, 0.05], 0.00\rangle$	$\langle[0.38, 0.48], 0.43\rangle$
6	$\langle[0.91, 1.00], 0.96\rangle$	$\langle[0.95, 1.00], 1.00\rangle$	$\langle[0.28, 0.38], 0.33\rangle$	$\langle[0.66, 0.76], 0.71\rangle$
7	$\langle[0.87, 0.97], 0.92\rangle$	$\langle[0.00, 0.08], 0.03\rangle$	$\langle[0.78, 0.88], 0.83\rangle$	$\langle[0.95, 1.00], 1.00\rangle$
8	$\langle[0.59, 0.69], 0.64\rangle$	$\langle[0.48, 0.58], 0.53\rangle$	$\langle[0.62, 0.72], 0.67\rangle$	$\langle[0.00, 0.05], 0.00\rangle$
9	$\langle[0.00, 0.07], 0.02\rangle$	$\langle[0.18, 0.28], 0.23\rangle$	$\langle[0.68, 0.78], 0.73\rangle$	$\langle[0.38, 0.48], 0.43\rangle$
10	$\langle[0.42, 0.52], 0.47\rangle$	$\langle[0.90, 1.00], 0.95\rangle$	$\langle[0.85, 0.95], 0.90\rangle$	$\langle[0.24, 0.34], 0.29\rangle$

TABLE 8. Ranking through TOPSIS and ARAS

Patient ID	TOPSIS	Ranking	ARAS	Ranking
1	$< [0.4, 0.4], 0.4 >$	8	$< [0.3, 0.3], 0.3 >$	9
2	$< [0.5, 0.5], 0.5 >$	5	$< [0.5, 0.6], 0.7 >$	5
3	$< [0.6, 0.6], 0.6 >$	3	$< [0.6, 0.7], 0.7 >$	2
4	$< [0.5, 0.5], 0.4 >$	7	$< [0.5, 0.5], 0.5 >$	6
5	$< [0.5, 0.5], 0.5 >$	6	$< [0.5, 0.5], 0.5 >$	7
6	$< [0.7, 0.7], 0.7 >$	1	$< [0.8, 0.8], 0.8 >$	1
7	$< [0.7, 0.7], 0.7 >$	2	$< [0.6, 0.7], 0.7 >$	3
8	$< [0.4, 0.4], 0.4 >$	9	$< [0.4, 0.5], 0.5 >$	8
9	$< [0.4, 0.4], 0.4 >$	10	$< [0.3, 0.3], 0.3 >$	10
10	$< [0.6, 0.6], 0.6 >$	4	$< [0.6, 0.7], 0.7 >$	4

TABLE 9. The results of BORDA Count technique.

Patient	**TOPSIS**		**ARAS**		**BORDA**	
ID	Ranking	Score	Ranking	Score	Score	Ranking
1	8	2	9	1	17	2
2	5	5	5	5	10	6
3	3	7	2	8	5	8
4	7	3	6	4	13	4
5	6	4	7	3	13	5
6	1	9	1	9	2	10
7	2	8	3	7	5	9
8	9	1	8	2	17	3
9	10	0	10	0	20	1
10	4	6	4	6	8	7

TABLE 10. Borda rankings under different weighting scenarios

Patient ID	**Balanced Weights**	**Safety-First**	**Efficacy-First**	**Stability Score**
1	10	10	9	1
2	5	6	5	4
3	3	4	2	3
4	7	8	8	8
5	6	7	7	7
6	1	2	1	2
7	4	3	5	4
8	9	9	10	9
9	2	1	3	2
10	5	5	4	5

TABLE 11. Conceptual Comparison of Approaches

Aspect	**Traditional DBSCAN + TOPSIS**	**Fuzzy C-Means + Fuzzy MCDM**	**Our Approach**
Cluster Shape Flexibility	Arbitrary shapes	Spherical only	Arbitrary shapes
Membership Representation	Binary (0/1)	Single value [0,1]	Cubic (interval + value)
Noise Handling	Explicit detection	Forces clustering	Fuzzy detection
Uncertainty Modeling	None	One-dimensional	Three-dimensional
Clinical Interpretability	Low	Medium	High
Parameter Sensitivity	High (ϵ, minPts)	Moderate (cluster count)	Moderate (robust)
Computational Cost	$O(n \log n)$	$O(nkt)$	$O(n \log n + nm)$