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Assessing Environmental Pollution: Quality Control Using CRITIC-WASPAS Methods under Intuitionistic Fuzzy Z-Numbers

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Abstract. Industrial pollution is still a major worldwide issue. Strong decision-making frameworks are required to evaluate and reduce its detrimental effects on the environment. An extensive case study assessing different industrial pollution produced by the energy industry, textile industry, chemical industry, manufacturing industry, agriculture industry, and construction industry and its environmental impact is presented in this paper. We present a structured evaluation of environmental degradation by analyzing key environmental criteria, including waste generation, noise pollution, soil contamination, air pollution, and water pollution. In order to accomplish a systematic and objective evaluation, we provide an advanced multi-criteria decision-making (MCDM) framework that combines the CRITIC-WASPAS and CRITIC-EDAS approaches with intuitionistic fuzzy Z-number (IFZN). This integration ensures a more reliable, flexible, and data-driven assessment of industrial pollution sources by addressing uncertainty, imprecision, and inconsistency in the data effectively. In order to determine the objective weights of environmental criteria , the CRITIC (Criteria Importance Through Intercriteria Correlation) method is applied here. These weights undergo additional processing using the WASPAS and EDAS approaches, which combine aggregated sum product weighting (WASPAS) and positive and negative distance-based evaluation (EDAS) to check the ranking of given criteria. Additionally, sensitivity analysis is used to assess the robustness and dependability of the suggested fuzzy model by altering input parameters, including weights, membership values, and decision-maker preferences. Moreover, a comparison with earlier approaches is carried out to evaluate the consistency and accuracy of the proposed model's ranking.

AMS (MOS) Subject Classification Codes: 03E72; 90C70; 68T37
Key Words: Intuitionistic fuzzy Z-numbers; weighted arithmetic averaging operator; weighted geometric averaging operator; MCDM.

1. INTRODUCTION

In the real world, uncertainty in data is prevalent, and a lot of information necessary for decision-making is often ambiguous, inaccurate, or incomplete. Although humans are remarkably adept at making logical conclusions in these kinds of situations but formalizing this capacity is still quite difficult. Multi-Criteria Decision-Making (MCDM) is a well-known approach in fields like management, engineering, and economics, where solutions are evaluated based on predefined criteria and weights, often using precise numerical values. However, ambiguity often affects real-world data, making decision-making more challenging as systems become more complex. This highlights the need for advanced strategies, such as fuzzy systems, to effectively manage uncertainty and provide more accurate and flexible solutions. In order to deal with the ambiguity in real-world scenarios fuzzy sets (FS) were first proposed by Lotfi A. Zadeh in 1965 [27]. He indicate the degree of uncertainty about an event by giving a membership degree between 0 and 1. In 1970, Bellman and Zadeh [4] extends this work into decision-making and allowing more realistic handling of uncertainty in complex system. Additionally, the concept of intuitionistic fuzzy sets (IFS) a generalization of FS by adding opposition and support within the range of $[0, 1]$ was proposed by Atanassov [3], to improve the modeling of uncertainty even more. For describing imprecision, especially when there is inadequate data, the idea of IFS offers an alternate framework to conventional FS. The IFS has garnered more attention ever since it was first proposed because of its exceptional capacity to manage ambiguity. Since its inception it has been widely used in a number of fields, such as engineering, optimization and also been used in a variety of domains by numerous researchers such as Deschrijver and Kerre [7], who investigated intuitionistic fuzzy relations composition. Szmidt and Kacprzyk [17] defined the four basic distances between IFS: the Euclidean distance, the Hamming distance, the normalized Hamming distance, and the normalized Euclidean distance. Szmidt and Kacprzyk [18] suggested an entropy measure for IFS that is not of the probabilistic type based on the distance measure of a double sequence of a confined variation. Khan, Lohani, and Ieee [10] presented a novel similarity measure about IFS. IVIFS (interval-values intuitionistic fuzzy sets) were used by Wu et al. [22] to present the VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje) method for financing risk evaluation of rural tourism projects. In the setting of intuitionistic fuzzy logic, Chen et al. [5] created a unique MCDM method based on similarity measurements and the TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) approach. Wu et al. [23] proposed some interval-valued intuitionistic fuzzy dombi heronian mean operators for evaluating the ecological value of forest ecological tourism demonstration sites and t enhance the modeling of dynamic and uncertain environments, Kamacı et al. [11] proposed a novel decision-making approach by integrating generalized temporal intuitionistic fuzzy

sets with soft sets. Further, to improve decision-making under uncertainty, Saeed et al. [16] proposed an extension of the classical TOPSIS method using linguistic terms within a triangular intuitionistic fuzzy framework.
Moreover, based on previous research, Zadeh developed a new theory named Z-numbers that is the combination of uncertainty and its reliability to create a pair of hybrid fuzzy numbers that are used extensively in many different domains. This method allows for more precise decision-making. For instance, this method has been utilized in risk analysis in conjunction [14] with the ranking method. Z-numbers are also used in the modified TOPSIS approach (Z-TOPSIS) for stock selection [25].Aboutorab et al. [1] used the Best-Worst Method to address supplier development issues and suggested a Z-number extension. A Z-number-based group decision-making framework based on the TOPSIS approach combined with power aggregation operators was presented by Wang and Mao [21]. Alam et al. [9] created an intuitive multiple centroid defuzzification method for intuitionistic Z-numbers in the area of fuzzy logic and defuzzification. In keeping with the design and assessment area, Qi et al. [15] evaluated conceptual designs under uncertainty using interval intuitionistic fuzzy Z-numbers. Additionally, Ashraf et al. [2] demonstrated the suitability of Z-number-based models in uncertain and sustainable decision-making contexts by utilizing Pythagorean fuzzy Z-numbers in a green supplier selection model through an expanded EDAS technique. These applications demonstrate the versatility of Z-numbers in managing uncertainty and reliability across various contexts.
By considering intuitionistic fuzziness in terms of both the support and opposition degrees along with their associated reliabilities, this study introduces the concept of intuitionistic fuzzy Z-numbers (IFZN). To support decision-making, fundamental properties and aggregation operators of IFZN such as IFZNWA (intuitionistic fuzzy Z-number weighted averaging) and IFZNWG (intuitionistic fuzzy Z-number weighted geometric) are defined. A novel MCDM approach is proposed to reduce the overall degree of ambiguity in the decision matrix and enhance the credibility of the findings. The effectiveness of the proposed model is demonstrated through a practical example. Moreover, to evaluate the robustness and stability of the suggested method under various circumstances, a sensitivity analysis is carried out by changing the values of certain parameters and also a comparative analysis with current approaches is conducted to confirm the model's efficacy and superiority in terms of accuracy and consistency. This paper is organized as follows: Section I provides a comprehensive review of existing literature on IFS and MCDM approaches. Section II discusses the definitions and related concepts of IFZN and aggregation operators. Section III illustrates the framework of proposed methods under the IFZN. Section IV presents a detailed explanation of the case study. Section V shows the empirical example of the case study. A sensitivity and comparative analysis are presented in sections VI and VII, respectively. The conclusion of the paper is presented in Section VIII.

1.1. **Literature review.** The literature on fuzzy data handling strategies, such as CRITIC, WASPAS, and EDAS are thoroughly reviewed in this section.

1.1.1. *A brief overview of CRITIC method.* A popular MCDM technique for establishing objective criterion weights is the CRITIC method, which was first presented by Diakoulaki et al. (1995) [8]. By examining the interdependencies and variability of criteria, it assesses their relative importance, guaranteeing that criteria that are more independent and informative are given greater weight. This approach has been widely used in a number of disciplines, such as economics, engineering, management, and environmental research, where weighing several criteria is essential for making decisions [26]. When assigning weights, the CRITIC technique takes into consideration two important factors: each criterion's standard deviation, which represents its discriminative ability, and its correlation with other criteria, which takes redundancy into account. This eliminates the need for subjective judgment and guarantees a weighting procedure that is both balanced and data-driven. Because of its efficacy, CRITIC has been used in mini-grid business models, software selection, supply chain risk assessment, and construction equipment selection.CRITIC is a dependable tool for complicated decision-making scenarios because it integrates statistical variability and inter-criteria interactions to create an objective and systematic weighing process.

1.1.2. *A brief overview of WASPAS method.* The WASPAS method, introduced by Zavadskas et al. (2012) [28], is a hybrid MCDM technique that combines the weighted sum model (WSM) and the weighted product model (WPM).This combination improves decision-making consistency and accuracy, making WASPAS a robust and adaptable approach in a variety of domains. Initially, WASPAS gained prominence in engineering applications, specifically in civil and mechanical engineering, where it was used for material selection, infrastructure assessment, and mechanical system optimization. Later on, due to its reliability and computational efficiency, the method was extended to environmental sustainability, helping with pollution control, renewable energy ranking, and waste management. Additionally, it found applications in healthcare, where it was used for pharmaceutical evaluation and surgical procedure optimization. Fuzzy WASPAS and intuitionistic fuzzy WASPAS are two developments that arose between 2015 and 2018 to handle imprecision and uncertainty in production planning, manufacturing procedures, and supplier selection. This approach is also applicable in the context of website evaluation [19].
WASPAS became more efficient in complicated decision contexts as a result of these improvements, which allowed it to manage language factors and subjective assessments.Z-numbers, interval-valued numbers, and gray systems have also been added recently to improve WASPAS's processing of ambiguous or partial data. These changes have increased its applicability to Industry 4.0 technologies, industrial risk assessment, and transportation systems. Furthermore, WASPAS has been used in mathematical fluid mechanics to analyze non-Newtonian fluids, in education to assess AI-based learning resources and online courses, and in agriculture to choose crops and manage water resources. Fuzzy logic and entropy weighting have been used in studies to optimize the WASPAS framework, guaranteeing increased resilience in multi-criteria decision situations. These developments have further cemented WASPAS position as a flexible and effective decision support tool across numerous disciplines by making it a competitive alternative to well-known MCDM techniques like TOPSIS, VIKOR, and PROMETHEE.

1.1.3. *A brief overview of EDAS method.* Keshavarz et al. [12] established the EDAS approach, which was a game changer in the field of MCDM approaches. This ranking algorithm emphasized other possibilities based on multiple criteria, which made it ideal for complex decision-making scenarios. EDAS stood out from other approaches due to its unique norm-establishing methodology. Established techniques like TOPSIS and VIKOR aimed to determine the desirable alternatives by evaluating both ideal and anti-ideal alternatives. However, in contemporary decision-making scenarios, preventing proximity to the ideal alternative and optimizing the separation from the anti-ideal alternative might not always have led to the most suitable choice. The primary goal of the EDAS approach was to select the best option by utilizing the average solution-based norm-establishing methodology. The EDAS approach evaluated alternative performance by calculating both positive and negative variances from an average solution and ranking them accordingly. Torkayesh et al. [20] conducted a considerable review of the literature to determine the current level of EDAS. The investigators identified potential limitations of EDAS in a total of nine sectors for implementation: business management, crop cultivation, mineral fibers and electrical power resources, strategic planning, healthcare supervision, manufacturing, logistics management, advanced technology, and accountability in transportation. The results of this investigation helped decision-makers handle unclear and unreliable data during the decision-making stages. Darko & Liang [6] introduced some q-rung orthopair fuzzy Hamacher aggregation operators and their application. The EDAS approach was used to evaluate commercial airline services and renewable energy technology for manufacturing and automation.

TABLE 1. Summary of previous studies on Z-numbers

Authors	Year	Applications
Sahrom and Dom [14]	2015	A Z-Number-Based Decision Making Procedure for Risk Analysis
Yaakob and Gegov [25]	2016	Modified TOPSIS (Z-TOPSIS) for group decision-making in Stock Selection
Aboutorab et al. [1]	2018	Z-number extension of Best Worst Method and its application for Supplier Development
Wang and Mao [21]	2019	Group decision making with Z-numbers based on TOPSIS and power aggregation operators
Alam et al. [9]	2022	Intuitive multiple centroid defuzzification of intuitionistic Z-numbers
Qi et al. [15]	2023	Conceptual design evaluation using interval intuitionistic fuzzy Z-numbers under uncertainty
Ashraf et al. [2]	2023	Green supplier selection using extended EDAS approach under Pythagorean fuzzy Z-numbers

1.2. **Research Gap.** Despite comprehensive research on MCDM methods for assessing environmental pollution, significant gaps still exist, especially in integrating advanced fuzzy models with objective weighting techniques for evaluating industrial pollution.

1. The limitations of traditional fuzzy models include the inability of intuitionistic and z-number fuzzy techniques to adequately capture the reliability and hesitation elements of decision-makers when assessing industrial pollution. Due to their inability to manage uncertainty and divergent expert viewpoints, conventional models frequently produce less accurate rankings of polluting businesses. A more sophisticated method is provided by IFZNs, which incorporate support, opposition, and reliability degrees to ensure a more accurate depiction of uncertainty.
2. The majority of studies assessing industrial pollution do not integrate weight determination methods and instead rely on stand-alone MCDM techniques like AHP, TOPSIS, or

VIKOR. By utilizing CRITIC for objective weighting, WASPAS for enhanced ranking stability, and EDAS for alternative evaluation under uncertainty, a hybrid approach combining CRITIC-WASPAS and CRITIC-EDAS can offer a more comprehensive ranking of industries.
3. The current approaches for assessing pollution are frequently inconsistent when used in various industrial sectors and geographical areas. The link between environmental criteria is often overlooked by frameworks, which results in inconsistent identification of the most polluting industries. The use of CRITIC for weight determination guarantees that the interdependencies among pollution criteria are appropriately taken into account.
4. Future laws, emission limitations, and sustainability goals must all be able to be included in decision-making models as environmental policies change. The effectiveness of current MCDM models for long-term policy recommendations is diminished by their inability to adjust to shifting environmental requirements. CRITIC-WASPAS and CRITIC-EDAS in conjunction with IFZN give policymakers a flexible and dynamic decision-support tool.
5. Numerous studies on pollution assessment rely on theoretical models that have not been thoroughly empirically validated using data from actual industries. To confirm the efficacy of hybrid MCDM methodologies in ranking sectors according to their environmental impact, comprehensive case studies and sensitivity analysis are required.

1.3. **Motivation.** This study is motivated by the urgent need to develop more reliable and efficient frameworks for industrial pollution decision-making using intuitionistic fuzzy Z-numbers (IFZN). A strong data-driven method for identifying the most polluting industry is ensured by the integration of IFZN with CRITIC-WASPAS and CRITIC-EDAS techniques.
1. Determine the main industrial pollutants with accuracy in order to support environmental sustainability programs and legal frameworks.
2. Create a hybrid decision-making model that can manage conflicting, ambiguous, and imprecise pollution data.
3. Utilize IFZN to improve decision-maker preference representation and get beyond the drawbacks of conventional fuzzy and crisp approaches.
4. Use cutting-edge MCDM methods (CRITIC-WASPAS and CRITIC-EDAS) to increase ranking accuracy, stability, and objectivity.
5. Verify the suggested framework's practicality for industrial pollution assessment by conducting sensitivity studies and empirical research.

2. Preliminaries

Here we define some important concept related to our work.

Definition 1. An IFS in the universal set ψ is described by

$$\mathbb{Q} = \{\langle \zeta, \mu_{\mathbb{Q}}(\zeta), \nu_{\mathbb{Q}}(\zeta)\rangle \mid \zeta \in \psi\},$$

where $\mu_{\mathbb{Q}} : \psi \to [0,1]$ is the membership function and $\nu_{\mathbb{Q}} : \psi \to [0,1]$ is the non-membership function such that $0 \leq \mu_{\mathbb{Q}}(\zeta) + \nu_{\mathbb{Q}}(\zeta) \leq 1, \quad \forall \zeta \in \psi$.

The value of hesitation (indeterminacy) is given by $\pi_{\mathbb{Q}}(\zeta) = 1 - \mu_{\mathbb{Q}}(\zeta) - \nu_{\mathbb{Q}}(\zeta)$.

Definition 2. Z-numbers extend traditional fuzzy models by adding reliability to uncertain information. A Z-number is expressed in the form of a pair (S, R), where S denotes an uncertain information and R shows the confidence or reliability associated with that information. This makes Z-numbers useful for modeling real-world situations where both the estimated and reliable values are important.

Definition 3. An IFZN in the universal set ψ is defined as:

$$\mathbf{G_z} = \{(\zeta, \mu(S,R)(\zeta), \nu(S,R)(\zeta)) \mid \zeta \in \psi\} = \{(\zeta,\ (\mu_S(\zeta),\ \mu_R(\zeta)),\ (\nu_S(\zeta),\ \nu_R(\zeta))) \mid \zeta \in \psi\},$$

where $\{(\mu_S(\zeta),\ \nu_S(\zeta)) : \psi \to [0,1]\}$ indicates the uncertain value of membership and nonmembership function, while $\{(\mu_R(\zeta),\ \nu_R(\zeta)) : \psi \to [0,1]\}$ indicates the reliability of the membership and nonmembership function. These functions satisfy the following conditions for all $\zeta \in \psi$:

$$0 \le \mu_S(\zeta) + \nu_S(\zeta) \le 1, \quad 0 \le \mu_R(\zeta) + \nu_R(\zeta) \le 1.$$

Definition 4. Let
$\mathbf{G_{z_1}} = (\mu_1(S,R), \nu_1(S,R)) = (\mu_{S_1}, \mu_{R_1}), (\nu_{S_1}, \nu_{R_1})$ and $\mathbf{G_{z_2}} = (\mu_2(S,R), \nu_2(S,R)) = (\mu_{S_2}, \mu_{R_2}), (\nu_{S_2}, \nu_{R_2})$ are two IFZN and $\beth > 0$ which satisfies the following characteristics, then the operation defined on IFZN are given bellow:

(1) $\mathbf{G_{z_1}} \supseteq \mathbf{G_{z_2}}$ if and only if $\mu_{S_1} \ge \mu_{S_2},\ \mu_{R_1} \ge \mu_{R_2},\ \nu_{S_1} \le \nu_{S_2},\ \nu_{R_1} \le \nu_{R_2}$
(2) $\mathbf{G_{z_1}} = \mathbf{G_{z_2}}$ if and only if $G_{z_1} \supseteq G_{z_2}$ and $G_{z_1} \subseteq G_{z_2}$
(3) $\mathbf{G_{z_1}} \cup \mathbf{G_{z_2}} = \{(\mu_{S_1} \vee \mu_{S_2}, \mu_{R_1} \vee \mu_{R_2}), (\nu_{S_1} \wedge \nu_{S_2}, \nu_{R_1} \wedge \nu_{R_2})\}$
(4) $\mathbf{G_{z_1}} \cap \mathbf{G_{z_2}} = \{(\mu_{S_1} \wedge \mu_{S_2}, \mu_{R_1} \wedge \mu_{R_2}), (\nu_{S_1} \vee \nu_{S_2}, \nu_{R_1} \vee \nu_{R_2})\}$
(5) $(\mathbf{G_{z_1}})^{\mathbf{c}} = (\nu_{S_1}, \nu_{R_1}), (\mu_{S_1}, \mu_{R_1})$
(6) $\mathbf{G_{z_1}} \oplus \mathbf{G_{z_2}} = \{(\mu_{S_1} + \mu_{S_2} - \mu_{S_1}\mu_{S_2}, \mu_{R_1} + \mu_{R_2} - \mu_{R_1}\mu_{R_2}), (\nu_{S_1}\nu_{S_2}, \nu_{R_1}\nu_{R_2})\}$
(7) $\mathbf{G_{z_1}} \otimes \mathbf{G_{z_2}} = \{(\mu_{S_1}\mu_{S_2}, \mu_{R_1}\mu_{R_2}), (\nu_{S_1} + \nu_{S_2} - \nu_{S_1}\nu_{S_2}, \nu_{R_1} + \nu_{R_2} - \nu_{R_1}\nu_{R_2})\}$
(8) $\beth \odot \mathbf{G_{z_1}} = \left(1 - (1-\mu_{S_1})^{\beth}, 1 - (1-\mu_{R_1})^{\beth}\right), (\nu_{S_1}^{\beth}, \nu_{R_1}^{\beth})$
(9) $\mathbf{G_{z_1}}^{\beth} = (\mu_{S_1}^{\beth}, \mu_{R_1}^{\beth}), \left(1 - (1-\nu_{S_1})^{\beth}, 1 - (1-\nu_{R_1})^{\beth}\right)$

Definition 5. Let $\mathbf{G_{z_d}} = \{(\mu_{S_d}, \mu_{R_d}), (\nu_{S_d}, \nu_{R_d})\}$ be an IFZN, then the score function is calculated by given formula

$$\mathbb{L} = \frac{1 + \mu_{S_d}\mu_{R_d} - \nu_{S_d}\nu_{R_d}}{2} \tag{2. 1}$$

2.1. **Weighted arithmetic averaging operator of IFZN.** Under the IFZN category, this section presents a few aggregation operations, including IFZNWA and IFZNWG operators.

Definition 6. Let $\mathbf{G_{z_d}} = (\mu_{S_d}, \mu_{R_d}), (\nu_{S_d}, \nu_{R_d})$ $(d = 1, 2, \ldots, n)$ be a group of IFZN. Then, the IFZNWA operator is defined as:

$$\text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_n}}) = \bigoplus_{d=1}^{n} \beth_d \mathbf{G_{z_d}} \tag{2. 2}$$

where $\beth_d$ $(d = 1, 2, \ldots, n)$ is the weight vector of $(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_n}})$, such that

$$0 \le \beth_d \le 1, \quad \sum_{d=1}^{n} \beth_d = 1.$$

Then, the IFZNWA operator can be explicitly expressed as:

$$\text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_n}}) = \left(\left(1 - \prod_{d=1}^{n} (1-\mu_{S_d})^{\beth_d}, 1 - \prod_{d=1}^{n} (1-\mu_{R_d})^{\beth_d} \right), \left(\prod_{d=1}^{n} \nu_{S_d}^{\beth_d}, \prod_{d=1}^{n} \nu_{R_d}^{\beth_d} \right) \right). \tag{2. 3}$$

Theorem 2.2. *Let $\mathbf{G_{z_d}} (d = 1, 2, \ldots, n)$ be a collection of IFZN. Because of this, the IFZNWA operator's aggregated value is still an IFZN, as determined by the following formula:*

$$\begin{aligned} \textit{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_n}}) &= \bigoplus_{d=1}^{n} \beth_d \mathbf{G_{z_d}} \\ &= \beth_1 \mathbf{G_{z_1}} \bigoplus \beth_2 \mathbf{G_{z_2}} \bigoplus \cdots \bigoplus \beth_n \mathbf{G_{z_n}} \\ &= \left(\left(1 - \prod_{d=1}^{n} (1-\mu_{S_d})^{\beth_d}, 1 - \prod_{d=1}^{n} (1-\mu_{R_d})^{\beth_d} \right), \left(\prod_{d=1}^{n} \nu_{S_d}^{\beth_d}, \prod_{d=1}^{n} \nu_{R_d}^{\beth_d} \right) \right) \end{aligned}$$

where $\beth_d$ $(d = 1, 2, \ldots, n)$ is the weight vector of $(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_n}})$, such that

$$0 \leq \beth_d \leq 1, \quad \sum_{d=1}^{n} \beth_d = 1.$$

Proof. One can verify Eq. (2. 3) by means of mathematical induction.

(1) Assign $n = 2$: the outcome is as follows, based on operational laws (6) and (9):

$$\begin{aligned} \text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}) &= \beth_1 \mathbf{G_{z_1}} \bigoplus \beth_2 \mathbf{G_{z_2}} \\ &= \begin{pmatrix} 1-(1-\mu_{S_1})^{\beth_1} + 1 - (1-\mu_{S_2})^{\beth_2} \\ \quad - \left(1-(1-\mu_{S_1})^{\beth_1}\right)\left(1-(1-\mu_{S_2})^{\beth_2}\right), \\ 1-(1-\mu_{R_1})^{\beth_1} + 1 - (1-\mu_{R_2})^{\beth_2} \\ \quad - \left(1-(1-\mu_{R_1})^{\beth_1}\right)\left(1-(1-\mu_{R_2})^{\beth_2}\right), \\ (\nu_{S_1}^{\beth_1})(\nu_{S_2}^{\beth_2}),\ (\nu_{R_1}^{\beth_1})(\nu_{R_2}^{\beth_2}) \end{pmatrix} \end{aligned} \tag{2. 4}$$

$$\text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}) = \left(1 - \prod_{d=1}^{2} (1-\mu_{S_d})^{\beth_d},\ 1 - \prod_{d=1}^{2} (1-\mu_{R_d})^{\beth_d},\ \prod_{d=1}^{2} \nu_{S_d}^{\beth_d},\ \prod_{d=1}^{2} \nu_{R_d}^{\beth_d} \right). \tag{2. 5}$$

(2) **Set** $n = m$: Equation (2. 3) holds for $n = m$:

$$\begin{aligned} \text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_m}}) = \Bigg(&\left(1 - \prod_{d=1}^{m} (1-\mu_{S_d})^{\beth_d},\ 1 - \prod_{d=1}^{m} (1-\mu_{R_d})^{\beth_d} \right), \\ &\left(\prod_{d=1}^{m} \nu_{S_d}^{\beth_d},\ \prod_{d=1}^{m} \nu_{R_d}^{\beth_d} \right) \Bigg) \end{aligned} \tag{2. 6}$$

(3) **Set** $n = m + 1$: The outcome is as follows, based on operational laws (6) and (9):

$$\text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_{m+1}}}) = \bigoplus_{d=1}^{m+1} \beth_d \mathbf{G_{z_d}}$$

$$= \begin{pmatrix} 1 - \prod_{d=1}^{m}(1-\mu_{S_d})^{\beth_d} + 1 - (1-\mu_{S_{m+1}})^{\beth_{m+1}} \\ \quad - \left(1 - \prod_{d=1}^{m}(1-\mu_{S_d})^{\beth_d}\right)\left(1-(1-\mu_{S_{m+1}})^{\beth_{m+1}}\right), \\ 1 - \prod_{d=1}^{m}(1-\mu_{R_d})^{\beth_d} + 1 - (1-\mu_{R_{m+1}})^{\beth_{m+1}} \\ \quad - \left(1 - \prod_{d=1}^{m}(1-\mu_{R_d})^{\beth_d}\right)\left(1-(1-\mu_{R_{m+1}})^{\beth_{m+1}}\right), \\ \prod_{d=1}^{m+1}(\nu_{S_d})^{\beth_d}, \prod_{d=1}^{m+1}(\nu_{R_d})^{\beth_d} \end{pmatrix}. \tag{2. 7}$$

On simplifying, we get:

$$\text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_{m+1}}}) = \left(\left(1 - \prod_{d=1}^{m+1}(1-\mu_{S_d})^{\beth_d}, 1 - \prod_{d=1}^{m+1}(1-\mu_{R_d})^{\beth_d}\right), \left(\prod_{d=1}^{m+1}(\nu_{S_d})^{\beth_d}, \prod_{d=1}^{m+1}(\nu_{R_d})^{\beth_d}\right)\right) \tag{2. 8}$$

□

Theorem . (2.2) is verified. Also, IFZNWA operator of Eq. (2. 3) satisfies the following properties:

(1) **Idempotency:** Let $\mathbf{G_{z_d}}$ $(d = 1, 2, \ldots, n)$ be a group of IFZN such that

$$\mathbf{G_{z_d}} = \mathbf{G_z} = ((\mu_S, \mu_R), (\nu_S, \nu_R)).$$

Then, the IFZNWA operator satisfies:

$$\text{IFZNWA}(\mathbf{G_{z_1}}, \mathbf{G_{z_2}}, \ldots, \mathbf{G_{z_n}}) = \mathbf{G_z}.$$

(2) **Boundedness:** Let $\mathbf{G_{z_d}}$ $(d = 1, 2, \ldots, n)$ be a group of IFZN. Define:

$$\mathbf{G_z}_{\min} = \left(\min_d(\mu_{S_d}), \min_d(\mu_{R_d}), \min_d(\nu_{S_d}), \min_d(\nu_{R_d})\right),$$

$$\mathbf{G_z}_{\max} = \left(\max_d(\mu_{S_d}), \max_d(\mu_{R_d}), \max_d(\nu_{S_d}), \max_d(\nu_{R_d})\right).$$

as the minimum IFZN and the maximum IFZN, respectively. Then, the following inequality holds:

$$\mathbf{G}_{\mathbf{z}\min} \leq \text{IFZNWA}(\mathbf{G}_{\mathbf{z_1}}, \mathbf{G}_{\mathbf{z_2}}, \ldots, \mathbf{G}_{\mathbf{z_n}}) \leq \mathbf{G}_{\mathbf{z}\max}.$$

(3) **Monotonicity:** Let $\mathbf{G}_{\mathbf{z_d}}$ $(d = 1, 2, \ldots, n)$ and $\mathbf{G}_{\mathbf{z_d}}{}^*$ $(d = 1, 2, \ldots, n)$ be two groups of IFZN such that

$$\mathbf{G}_{\mathbf{z_d}} \leq \mathbf{G}_{\mathbf{z_d}}{}^*.$$

Then, the following holds:

$$\text{IFZNWA}(\mathbf{G}_{\mathbf{z_1}}, \mathbf{G}_{\mathbf{z_2}}, \ldots, \mathbf{G}_{\mathbf{z_n}}) \leq \text{IFZNWA}(\mathbf{G}_{\mathbf{z_1}}{}^*, \mathbf{G}_{\mathbf{z_2}}{}^*, \ldots, \mathbf{G}_{\mathbf{z_n}}{}^*).$$

Proof of the properties are omitted for conciseness, see [2] for its further detail.

2.3. **Weighted Geometric Averaging Operator of IFZN.** This subsection proposes the IFZNWG operator of IFZN corresponding to the operational laws (7) and (8) and introduces their properties.

Definition 7. Let $\mathbf{G}_{\mathbf{z_d}}$ $(d = 1, 2, \ldots, n)$ be a group of IFZN. Then, the IFZNWG operator is defined as:

$$\text{IFZNWG}(\mathbf{G}_{\mathbf{z_1}}, \mathbf{G}_{\mathbf{z_2}}, \ldots, \mathbf{G}_{\mathbf{z_n}}) = \bigotimes_{d=1}^{n} \mathbf{G}_{\mathbf{z_d}}{}^{\beth_d} \tag{2. 9}$$

where $\beth_d (d = 1, 2, \ldots, n)$ is the weight vector satisfying $0 \leq \beth_d \leq 1$ and $\sum_{d=1}^{n} \beth_d = 1$.

$$IFZNWG(\mathbf{G}_{\mathbf{z_1}}, \mathbf{G}_{\mathbf{z_2}}, \ldots, \mathbf{G}_{\mathbf{z_n}}) = \left(\left(\prod_{d=1}^{n} \mu_{S_d}^{\beth_d}, \prod_{d=1}^{n} \mu_{R_d}^{\beth_d} \right), \left(1 - \prod_{d=1}^{n} (1 - \nu_{S_d})^{\beth_d}, 1 - \prod_{d=1}^{n} (1 - \nu_{R_d})^{\beth_d} \right) \right). \tag{2. 10}$$

Theorem 2.4. *Let $\mathbf{G}_{\mathbf{z_d}}$ $(d = 1, 2, \ldots, n)$ be a group of IFZN. Then, the aggregated value of the IFZNWG operator is still an IFZN, given by:*

$$\textit{IFZNWG}(\mathbf{G}_{\mathbf{z_1}}, \mathbf{G}_{\mathbf{z_2}}, \ldots, \mathbf{G}_{\mathbf{z_n}}) = \left(\left(\prod_{d=1}^{n} \mu_{S_d}^{\beth_d}, \prod_{d=1}^{n} \mu_{R_d}^{\beth_d} \right), \left(1 - \prod_{d=1}^{n} (1 - \nu_{S_d})^{\beth_d}, 1 - \prod_{d=1}^{n} (1 - \nu_{R_d})^{\beth_d} \right) \right).$$

By following a similar proof process as Theorem. (2.2), one can easily verify Eq. (2. 10), and thus, proof of Theorem . (2.4) is omitted. Moreover, the IFZNWG operator also satisfies several important properties similar to those of the IFZNWA operator. The proofs of these properties follow a similar approach and are therefore omitted here. For detailed proofs, refer to [2].

3. Framework of CRITIC-WASPAS and CRITIC-EDAS approaches

In Table 2, we first provide a list of all the variables used in these approach.

TABLE 2. List of variables

Symbol	Description
ψ	Universal Set
ζ	Element of Universal set
$\mathbb{L}$	Score Function
D	Set of decision-makers
$\mathcal{P}_\tau$	Alternatives
$\mathcal{Q}_\gamma$	Criteria
Av_j	Average value(Av)
$\mathbb{Z}$	Positive distances from Av
$\mathbb{X}$	Negative distances from Av
$\mathbb{D}$	Weighted positive distances from Av
E	Weighted negative distances from Av
M	Weighted normalized positive distances from Av
N	Weighted normalized negative distances from Av
$\mathbb{J}$	Appraisal score

TABLE 3. Linguistic values for DMs and their corresponding IFZNs.

Linguistic Values	IFZNs
Extremely-insignificant (EIs)	$(\langle 0.050, 0.010\rangle, \langle 0.900, 0.900\rangle)$
Very-insignificant (VIs)	$(\langle 0.100, 0.100\rangle, \langle 0.800, 0.850\rangle)$
Insignificant (Is)	$(\langle 0.200, 0.250\rangle, \langle 0.700, 0.700\rangle)$
Moderate-insignificant (MIs)	$(\langle 0.300, 0.350\rangle, \langle 0.600, 0.550\rangle)$
Average (Av)	$(\langle 0.450, 0.500\rangle, \langle 0.450, 0.500\rangle)$
Moderate-significant (MS)	$(\langle 0.600, 0.550\rangle, \langle 0.300, 0.350\rangle)$
Significant (S)	$(\langle 0.700, 0.700\rangle, \langle 0.200, 0.250\rangle)$
Very-significant (VS)	$(\langle 0.800, 0.850\rangle, \langle 0.100, 0.100\rangle)$
Extremely-significant (ES)	$(\langle 0.900, 0.900\rangle, \langle 0.050, 0.010\rangle)$

Assume that n options for alternatives are provided as

$$\mathcal{P}_\tau = \{\mathcal{P}_{\tau_1}, \ldots, \mathcal{P}_{\tau_i}, \ldots, \mathcal{P}_{\tau_n}\}$$

and finite set of m criteria is given as

$$\mathcal{Q}_\gamma = \{\mathcal{Q}_{\gamma_1}, \ldots, \mathcal{Q}_{\gamma_j}, \ldots, \mathcal{Q}_{\gamma_m}\}$$

Moreover,

$$D = \{D_1, \ldots, D_t, \ldots, D_s\}$$

represent the group of decision-makers(DMs), whose linguistic evaluations are used to construct the decision matrices for analysis (See Table 4).

TABLE 4. DMs Evaluations

Experts	Alternatives	Criteria			
D_1		$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\cdots$	$\mathcal{Q}_{\gamma_t}$
	$\mathcal{P}_{\tau_1}$	$(\langle\mu^1_{S_{11}},\mu^1_{R_{11}}\rangle,\langle\nu^1_{S_{11}},\nu^1_{R_{11}}\rangle)$	$(\langle\mu^1_{S_{12}},\mu^1_{R_{12}}\rangle,\langle\nu^1_{S_{12}},\nu^1_{R_{12}}\rangle)$	$\cdots$	$(\langle\mu^1_{S_{1m}},\mu^1_{R_{1m}}\rangle,\langle\nu^1_{S_{1m}},\nu^1_{R_{1m}}\rangle)$
	$\mathcal{P}_{\tau_2}$	$(\langle\mu^1_{S_{21}},\mu^1_{R_{21}}\rangle,\langle\nu^1_{S_{21}},\nu^1_{R_{21}}\rangle)$	$(\langle\mu^1_{S_{22}},\mu^1_{R_{22}}\rangle,\langle\nu^1_{S_{22}},\nu^1_{R_{22}}\rangle)$	$\cdots$	$(\langle\mu^1_{S_{2m}},\mu^1_{R_{2m}}\rangle,\langle\nu^1_{S_{2m}},\nu^1_{R_{2m}}\rangle)$
		$\vdots$	$\ddots$		
	$\mathcal{P}_{\tau_n}$	$(\langle\mu^1_{S_{n1}},\mu^1_{R_{n1}}\rangle,\langle\nu^1_{S_{n1}},\nu^1_{R_{n1}}\rangle)$	$(\langle\mu^1_{S_{n2}},\mu^1_{R_{n2}}\rangle,\langle\nu^1_{S_{n2}},\nu^1_{R_{n2}}\rangle)$	$\cdots$	$(\langle\mu^1_{S_{nm}},\mu^1_{R_{nm}}\rangle,\langle\nu^1_{S_{nm}},\nu^1_{R_{nm}}\rangle)$
D_2					
	$\mathcal{P}_{\tau_1}$	$(\langle\mu^2_{S_{11}},\mu^2_{R_{11}}\rangle,\langle\nu^2_{S_{11}},\nu^2_{R_{11}}\rangle)$	$(\langle\mu^2_{S_{12}},\mu^2_{R_{12}}\rangle,\langle\nu^2_{S_{12}},\nu^2_{R_{12}}\rangle)$	$\cdots$	$(\langle\mu^2_{S_{1m}},\mu^2_{R_{1m}}\rangle,\langle\nu^2_{S_{1m}},\nu^2_{R_{1m}}\rangle)$
	$\mathcal{P}_{\tau_2}$	$(\langle\mu^2_{S_{21}},\mu^2_{R_{21}}\rangle,\langle\nu^2_{S_{21}},\nu^2_{R_{21}}\rangle)$	$(\langle\mu^2_{S_{22}},\mu^2_{R_{22}}\rangle,\langle\nu^2_{S_{22}},\nu^2_{R_{22}}\rangle)$	$\cdots$	$(\langle\mu^2_{S_{2m}},\mu^2_{R_{2m}}\rangle,\langle\nu^2_{S_{2m}},\nu^2_{R_{2m}}\rangle)$
		$\vdots$	$\ddots$		
	$\mathcal{P}_{\tau_n}$	$(\langle\mu^2_{S_{n1}},\mu^2_{R_{n1}}\rangle,\langle\nu^2_{S_{n1}},\nu^2_{R_{n1}}\rangle)$	$(\langle\mu^2_{S_{n2}},\mu^2_{R_{n2}}\rangle,\langle\nu^2_{S_{n2}},\nu^2_{R_{n2}}\rangle)$	$\cdots$	$(\langle\mu^2_{S_{nm}},\mu^2_{R_{nm}}\rangle,\langle\nu^2_{S_{nm}},\nu^2_{R_{nm}}\rangle)$
D_t					
	$\mathcal{P}_{\tau_1}$	$(\langle\mu^t_{S_{11}},\mu^t_{R_{11}}\rangle,\langle\nu^t_{S_{11}},\nu^t_{R_{11}}\rangle)$	$(\langle\mu^t_{S_{12}},\mu^t_{R_{12}}\rangle,\langle\nu^t_{S_{12}},\nu^t_{R_{12}}\rangle)$	$\cdots$	$(\langle\mu^t_{S_{1m}},\mu^t_{R_{1m}}\rangle,\langle\nu^t_{S_{1m}},\nu^t_{R_{1m}}\rangle)$
	$\mathcal{P}_{\tau_2}$	$(\langle\mu^t_{S_{21}},\mu^t_{R_{21}}\rangle,\langle\nu^t_{S_{21}},\nu^t_{R_{21}}\rangle)$	$(\langle\mu^t_{S_{22}},\mu^t_{R_{22}}\rangle,\langle\nu^t_{S_{22}},\nu^t_{R_{22}}\rangle)$	$\cdots$	$(\langle\mu^t_{S_{2m}},\mu^t_{R_{2m}}\rangle,\langle\nu^t_{S_{2m}},\nu^t_{R_{2m}}\rangle)$
		$\vdots$	$\ddots$		
	$\mathcal{P}_{\tau_n}$	$(\langle\mu^t_{S_{n1}},\mu^t_{R_{n1}}\rangle,\langle\nu^t_{S_{n1}},\nu^t_{R_{n1}}\rangle)$	$(\langle\mu^t_{S_{n2}},\mu^t_{R_{n2}}\rangle,\langle\nu^t_{S_{n2}},\nu^t_{R_{n2}}\rangle)$	$\cdots$	$(\langle\mu^t_{S_{nm}},\mu^t_{R_{nm}}\rangle,\langle\nu^t_{S_{nm}},\nu^t_{R_{nm}}\rangle)$

Step 1: Utilize the linguistic values presented in Table 3 to construct the linguistic decision matrices based on the evaluations provided by the decision makers.
Step 2: Determining the weights of DMs.
Step 3: Calculate the IFZN aggregated $n \times m$ matrix $\tilde{A}$ by using INZNWA or INFZWG.
Step 4: Normalize the overall IFZN aggregated matrix.

$$\tilde{A}_{ij}^{\;N} = \begin{cases} \big(\langle\mu_{S_{ij}},\mu_{R_{ij}}\rangle,\langle\nu_{S_{ij}},\nu_{R_{ij}}\rangle\big), & \text{if } \mathcal{Q}_\gamma \text{ is a benefit criterion,} \\ \big(\langle\nu_{S_{ij}},\nu_{R_{ij}}\rangle,\langle\mu_{S_{ij}},\mu_{R_{ij}}\rangle\big), & \text{if } \mathcal{Q}_\gamma \text{ is a cost criterion.} \end{cases}$$

The methodology adopted in this study is illustrated in Figure 1.

FIGURE 1. Methodology of the MCDM approach

Further, the methodologies of the CRITIC-WASPAS and CRITIC-EDAS approaches are presented in detail below.

3.1. **CRITIC-WASPAS. Step 5**: Apply the CRITIC method to compute the weights of criteria.

Let

$$\mathbf{G_{z_d}} = \Im(\delta_{ij}) = \big((\mu_{S_{ij}}, \mu_{R_{ij}}), (\nu_{S_{ij}}, \nu_{R_{ij}})\big)$$

1. The correlation $\mathbb{C}$ between characteristics can be computed as follows, depending on the normalized IFZN decision matrix.

$$\mathbb{C}_{jk} = \frac{\sum_{i=1}^{m}(\Im(\delta_{ij}) - \bar{y}_j)(\Im(\delta_{ik}) - \bar{y}_k)}{\sqrt{\sum_{i=1}^{m}(\Im(\delta_{ij}) - \bar{y}_j)^2} \cdot \sqrt{\sum_{i=1}^{m}(\Im(\delta_{ij}) - \bar{y}_k)^2}} \tag{3. 11}$$

where

$$\bar{y}_j = \frac{1}{m}\sum_{i=1}^{m}\Im(\delta_{ij}), \quad \text{and} \quad \bar{y}_k = \frac{1}{m}\sum_{i=1}^{m}\Im(\delta_{ik})$$

2. Calculate the standard deviation σ_j of each criterion.

$$\sigma_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\Im(\delta_{ij}) - \bar{y}_j)^2} \tag{3. 12}$$

3. Then, the index α_j is calculated using 3. 13 .

$$\alpha_j = \sigma_j \sum_{t=1}^{n} (1 - \mathbb{C}_{jk}), \quad j = 1, \ldots, n \tag{3. 13}$$

4. Finally, the criteria weights are given by equation 3. 14 ..

$$w_j = \frac{\alpha_j}{\sum_{j=1}^{n} \alpha_j} \tag{3. 14}$$

These criteria weights can now be used to rank the alternatives.
Step 6: In WASPAS approach, the Joint Generalized Criterion function Q_i for each alternative i is determined by combining the Weighted Sum Model (WSM) and the Weighted Product Model (WPM) approaches. It is calculated as:

$$Q_i = \beta \cdot C_i + (1 - \beta) \cdot P_i \tag{3. 15}$$

where β is a weight perimeter in the range $0 \leq \beta \leq 1$, and C_i is the additive relative importance of alternative i calculated using the WSM:

$$C_i = \sum_{j=1}^{m} (w_j^{\mathrm{N}})(\Im(\delta_{ij})), \tag{3. 16}$$

p_i is the multiplicative relative importance of alternative i calculated using the WPM:

$$P_i = \prod_{j=1}^{m} (\Im(\delta_{ij}))^{w_j^{\mathrm{N}}} \tag{3. 17}$$

Eventually, the alternatives $Q_1, Q_2, \ldots, Q_n$ were ranked and the alternatives with the highest Q_i was selected as the best.

3.2. **CRITIC-EDAS. Step 7:** For the EDAS approach, the same criteria weights that were calculated using the CRITIC method are employed, and the average solution Av_j for each criterion is computed as follows:

$$Av_j = \frac{1}{n} \sum_{i=1}^{n} \Im(\delta_{ij}) \quad \text{for each} \quad j = 1, 2, \ldots, m.$$

Step 8: Evaluate the score function $\mathbb{L}$ of the normalized aggregated matrix, by equation (2. 1) (See Table 5).

TABLE 5. Score function

	$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\cdots$	$\mathcal{Q}_{\gamma_m}$
$\mathcal{P}_{\tau_1}$	$S(\delta_{11})$	$S(\delta_{12})$	$\cdots$	$S(\delta_{1m})$
$\mathcal{P}_{\tau_2}$	$S(\delta_{21})$	$S(\delta_{22})$	$\cdots$	$S(\delta_{2m})$
$\mathcal{P}_{\tau_3}$	$S(\delta_{31})$	$S(\delta_{32})$	$\cdots$	$S(\delta_{3m})$
$\mathcal{P}_{\tau_n}$	$S(\delta_{n1})$	$S(\delta_{n2})$	$\cdots$	$S(\delta_{nm})$

Step 9: The positive distance from average (PDA) $\mathbb{Z}$ and negative distance from average (NDA) $\mathbb{X}$ can be computed as follows, depending on the average results:

$$\mathbb{Z}_{ij} = \max\left(0, \frac{S(\delta_{ij}) - \overline{Av}_j}{\overline{Av}_j}\right), \quad i = 1, \ldots, m, \quad j = 1, \ldots, n$$

$$\mathbb{X}_{ij} = \max\left(0, \frac{\overline{Av}_j - S(\delta_{ij})}{\overline{Av}_j}\right), \quad i = 1, \ldots, m, \quad j = 1, \ldots, n$$

Additionally, the following equations are used to compute the $\mathbb{Z}$ and $\mathbb{X}$ values of the negative attributes:

$$\mathbb{Z}_{ij} = \max\left(0, \frac{\overline{Av}_j - S(\delta_{ij})}{\overline{Av}_j}\right), \quad i = 1, \ldots, m, \quad j = 1, \ldots, n$$

$$\mathbb{X}_{ij} = \max\left(0, \frac{S(\delta_{ij}) - \overline{Av}_j}{\overline{Av}_j}\right), \quad i = 1, \ldots, m, \quad j = 1, \ldots, n$$

Step 10: The weighted positive distances from average (D_i) and the weighted negative distances from average (E_i) is calculated as:

$$D_i = \sum_{j=1}^{n} \mathbb{Z}_{ij} \cdot w_j, \quad i = 1, \ldots, m \tag{18}$$

$$E_i = \sum_{j=1}^{n} \mathbb{X}_{ij} \cdot w_j, \quad i = 1, \ldots, m \tag{19}$$

Step 11: Now, use equations (20) and (21) to calculate the weighted normalized positive distances from average (M_i) and weighted normalized negative distances from average (N_i), respectively.

$$M_i = \frac{D_i}{\max_i(D_i)}, \quad i = 1, \ldots, m \tag{20}$$

$$N_i = \frac{E_i}{\max_i(E_i)}, \quad i = 1, \ldots, m \tag{21}$$

Step 12: In the end, the appraisal score for each alternative is determined using equation (22), on the basis of which we rank the alternatives

$$\mathbb{J}_i = \frac{1}{2}\left(M_i + N_i\right), \quad i = 1, \ldots, m \tag{22}$$

Step 13: Furthermore, by altering the weight perimeter β, a sensitivity analysis was carried out to check the accuracy of our data.

Step 14: In order to verify the consistency of the alternatives and our best alternatives, a comparison analysis was conducted between our suggested approach and the preliminary research.

Figure 2 represents the step-by-step methodology of the CRITIC–WASPAS and CRITIC–EDAS approaches.

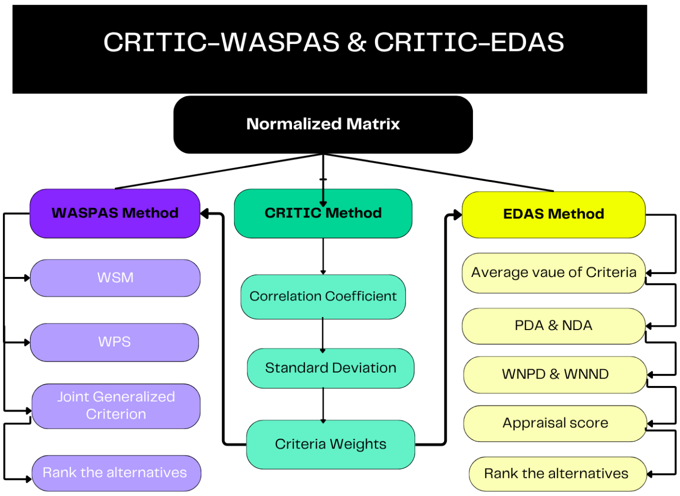

FIGURE 2. Weight and Ranking Methods

4. CASE STUDY: ENVIRONMENTAL IMPACT OF INDUSTRIAL POLLUTION FOR QUALITY CONTROL

Six important industries—energy, textiles, chemicals, manufacturing, agriculture, and construction are the subject of this case study. These industries have been selected because of their substantial contributions to environmental deterioration. Waste creation, soil contamination, water pollution, air pollution, and noise pollution are the five standard criteria that will be used in the study to assess their impact. Based on variables including waste management procedures and the utilization of raw materials, every industry pollutes the environment differently. Examining the main pollutants these industries create and their environmental impacts can help us understand how they contribute to different types of industrial pollution.

Energy Industry: The energy sector destroys land, contaminates water, and pollutes the air due to greenhouse gas emissions. It generates a lot of waste, including old machinery and coal byproducts. A lot of land and water resources are needed for energy generation. Furthermore, noise pollution is a serious issue, particularly when it comes to machinery, power plant operations, and the transfer of energy resources. The noise produced can have an adverse effect on wildlife behavior, local ecosystems, and the health and welfare of communities in the vicinity.

Textile industry: The textile business generates trash from synthetic fibers, uses energy and water, and contaminates water with chemicals and effluent. It releases volatile organic compounds and greenhouse gases, and nearby communities are impacted by noise pollution from vehicles and machinery.

Chemical industry: Through greenhouse gas emissions, dangerous chemical spills, and soil, water, and air pollution, the chemical industry has a major negative influence on the environment. The environmental impact is further increased by noise pollution from industry operations and machinery. These impacts are made worse by excessive energy and water use.

Manufacturing industry: The manufacturing industry considerably contributes to air, water, and soil pollution through greenhouse gas emissions, industrial waste, and chemical discharges. In addition to its significant energy and water use, it generates noise pollution from machinery and production operations, hurting adjacent populations.

Agriculture industry: Deforestation, soil erosion, water pollution, greenhouse gas emissions, synthetic fertilizers, and large-scale farming all contribute to the negative environmental effects of the agriculture sector. A major source of methane emissions is livestock farming. Additionally, nearby towns and the environment are impacted by noise pollution from machinery and equipment.

Construction industry: By polluting the air, water, soil, and trash, the building sector contributes to environmental deterioration. It creates dust and carbon emissions, contaminates soil with dangerous substances, and contaminates water sources through runoff. Overflowing landfills are caused by poor waste management and excessive water use.

In Fig. 3, we show the relationships between the alternatives and the criteria.

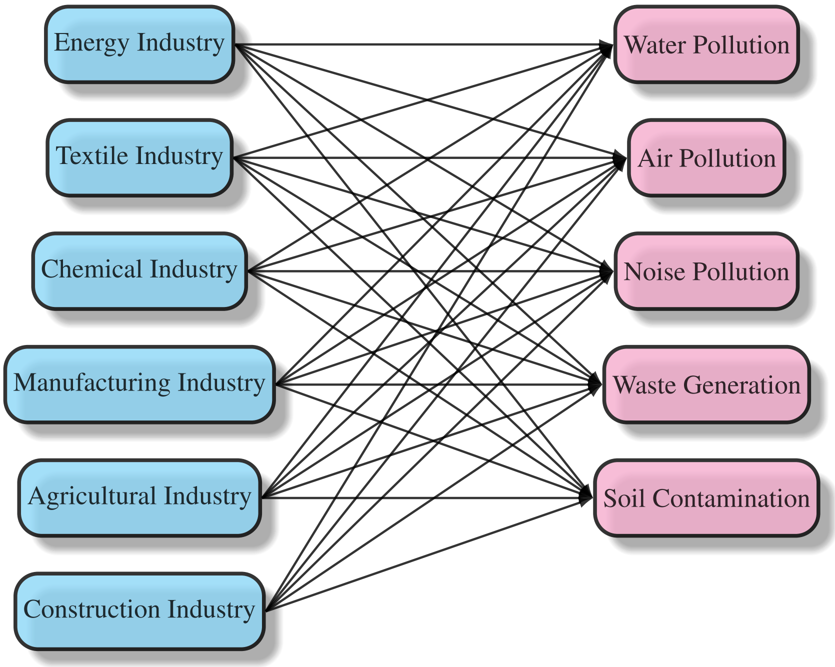

FIGURE 3. **Relationship between alternatives and criteria**

5. THE COMPARATIVE ANALYSIS AND EMPIRICAL EXAMPLE

Our practical approach involved inviting three decision-makers, $D = \{D_1, D_2, D_3\}$, to participate in order to analyze the impact of environmental contamination. Each expert gave their evaluation data based on one of six alternatives: $\mathcal{P}_{\tau_1}$, which represents the energy industry; $\mathcal{P}_{\tau_2}$, which represents the textile industry; $\mathcal{P}_{\tau_3}$, which represents the chemicals industry; $\mathcal{P}_{\tau_4}$, which represents the manufacturing industry; $\mathcal{P}_{\tau_5}$, which represents the agriculture industry; and $\mathcal{P}_{\tau_6}$, which represents the construction industry. These alternatives were assessed based on five criteria: $\mathcal{Q}_{\gamma_1}$, which takes into account air pollution; $\mathcal{Q}_{\gamma_2}$, which evaluates water pollution; $\mathcal{Q}_{\gamma_3}$, which measures soil contamination; $\mathcal{Q}_{\gamma_4}$, which evaluates waste creation; and $\mathcal{Q}_{\gamma_5}$, which takes noise pollution into account. All of the criteria are cost criteria.

The following calculations are made to determine the pollution impact of the aforementioned industries:

Step 1: The weightage of the three decision makers (D_1, D_2, D_3) are $(0.5, 0.3, 0.2)$.

Step 2: The linguistic variables (LVs) associated with IFZN, as detailed in Table 3, were used to construct the decision matrices presented in Table 6 and Table 7.

TABLE 6. Linguistic variables given by Decision makers

Experts	Alternatives	Criteria				
D_1		$\mathcal{Q}_{\gamma 1}$	$\mathcal{Q}_{\gamma 2}$	$\mathcal{Q}_{\gamma 3}$	$\mathcal{Q}_{\gamma 4}$	$\mathcal{Q}_{\gamma 5}$
	$\mathcal{P}_{\tau_1}$	VIS	VS	MS	AV	MIs
	$\mathcal{P}_{\tau_2}$	MIs	AV	VIs	Is	Is
	$\mathcal{P}_{\tau_3}$	AV	ES	VS	ES	Is
	$\mathcal{P}_{\tau_4}$	MS	EIs	MS	AV	ES
	$\mathcal{P}_{\tau_5}$	IS	VIs	VIs	EIs	MS
	$\mathcal{P}_{\tau_6}$	S	MIs	S	Is	MS
D_2						
	$\mathcal{P}_{\tau_1}$	VS	MS	EIs	VS	VIs
	$\mathcal{P}_{\tau_2}$	ES	VIs	ES	AV	AV
	$\mathcal{P}_{\tau_3}$	MS	ES	S	AV	MS
	$\mathcal{P}_{\tau_4}$	EIs	EIs	MIs	VIs	AV
	$\mathcal{P}_{\tau_5}$	AV	VS	MS	S	Is
	$\mathcal{P}_{\tau_6}$	VIs	S	Is	AV	Is
D_3						
	$\mathcal{P}_{\tau_1}$	MS	EIs	VIs	ES	VIs
	$\mathcal{P}_{\tau_2}$	VS	S	AV	VS	MS
	$\mathcal{P}_{\tau_3}$	VIs	VS	MS	EIs	AV
	$\mathcal{P}_{\tau_4}$	VIs	VIs	MS	MIs	Is
	$\mathcal{P}_{\tau_5}$	ES	AV	VIs	VIs	AV
	$\mathcal{P}_{\tau_6}$	Is	VIs	Is	AV	VIs

TABLE 7. IFZN's information by Decision makers

Experts	Alternatives	Criteria				
D_1		$\mathcal{Q}_{\gamma 1}$	$\mathcal{Q}_{\gamma 2}$	$\mathcal{Q}_{\gamma 3}$	$\mathcal{Q}_{\gamma 4}$	$\mathcal{Q}_{\gamma 5}$
	$\mathcal{P}_{\tau_1}$	(0.1, 0.1), (0.8, 0.85)	(0.8, 0.85), (0.1, 0.1)	(0.6, 0.55), (0.3, 0.35)	(0.45, 0.5), (0.45, 0.5)	(0.3, 0.35), (0.6, 0.55)
	$\mathcal{P}_{\tau_2}$	(0.3, 0.35), (0.6, 0.55)	(0.45, 0.5), (0.45, 0.5)	(0.1, 0.1), (0.8, 0.85)	(0.2, 0.25), (0.7, 0.7)	(0.2, 0.25), (0.7, 0.7)
	$\mathcal{P}_{\tau_3}$	(0.45, 0.5), (0.45, 0.5)	(0.9, 0.9), (0.05, 0.01)	(0.8, 0.85), (0.1, 0.1)	(0.9, 0.9), (0.05, 0.01)	(0.2, 0.25), (0.7, 0.7)
	$\mathcal{P}_{\tau_4}$	(0.6, 0.55), (0.3, 0.35)	(0.05, 0.01), (0.9, 0.9)	(0.6, 0.55), (0.3, 0.35)	(0.45, 0.5), (0.45, 0.5)	(0.9, 0.9), (0.05, 0.01)
	$\mathcal{P}_{\tau_5}$	(0.2, 0.25), (0.7, 0.7)	(0.1, 0.1), (0.8, 0.85)	(0.1, 0.1), (0.8, 0.85)	(0.05, 0.01), (0.9, 0.9)	(0.6, 0.55), (0.3, 0.35)
	$\mathcal{P}_{\tau_6}$	(0.7, 0.7), (0.2, 0.25)	(0.3, 0.35), (0.6, 0.55)	(0.7, 0.7), (0.2, 0.25)	(0.2, 0.25), (0.7, 0.7)	(0.6, 0.55), (0.3, 0.35)
D_2						
	$\mathcal{P}_{\tau_1}$	(0.8, 0.85), (0.1, 0.1)	(0.6, 0.55), (0.3, 0.35)	(0.05, 0.01), (0.9, 0.9)	(0.8, 0.85), (0.1, 0.1)	(0.1, 0.1), (0.8, 0.85)
	$\mathcal{P}_{\tau_2}$	(0.9, 0.9), (0.05, 0.01)	(0.1, 0.1), (0.8, 0.85)	(0.9, 0.9), (0.05, 0.01)	(0.45, 0.5), (0.45, 0.5)	(0.45, 0.5), (0.45, 0.5)
	$\mathcal{P}_{\tau_3}$	(0.6, 0.55), (0.3, 0.35)	(0.9, 0.9), (0.05, 0.01)	(0.7, 0.7), (0.2, 0.25)	(0.45, 0.5), (0.45, 0.5)	(0.6, 0.55), (0.3, 0.35)
	$\mathcal{P}_{\tau_4}$	(0.05, 0.01), (0.9, 0.9)	(0.05, 0.01), (0.9, 0.9)	(0.3, 0.35), (0.6, 0.55)	(0.1, 0.1), (0.8, 0.85)	(0.45, 0.5), (0.45, 0.5)
	$\mathcal{P}_{\tau_5}$	(0.45, 0.5), (0.45, 0.5)	(0.8, 0.85), (0.1, 0.1)	(0.6, 0.55), (0.3, 0.35)	(0.7, 0.7), (0.2, 0.25)	(0.2, 0.25), (0.7, 0.7)
	$\mathcal{P}_{\tau_6}$	(0.1, 0.1), (0.8, 0.85)	(0.7, 0.7), (0.2, 0.25)	(0.2, 0.25), (0.7, 0.7)	(0.45, 0.5), (0.45, 0.5)	(0.2, 0.25), (0.7, 0.7)
D_2						
	$\mathcal{P}_{\tau_1}$	(0.6, 0.55), (0.3, 0.35)	(0.05, 0.01), (0.9, 0.9)	(0.1, 0.1), (0.8, 0.85)	(0.9, 0.9), (0.05, 0.01)	(0.1, 0.1), (0.8, 0.85)
	$\mathcal{P}_{\tau_2}$	(0.8, 0.85), (0.1, 0.1)	(0.7, 0.7), (0.2, 0.25)	(0.45, 0.5), (0.45, 0.5)	(0.8, 0.85), (0.1, 0.1)	(0.6, 0.55), (0.3, 0.35)
	$\mathcal{P}_{\tau_3}$	(0.1, 0.1), (0.8, 0.85)	(0.8, 0.85), (0.1, 0.1)	(0.6, 0.55), (0.3, 0.35)	(0.05, 0.01), (0.9, 0.9)	(0.45, 0.5), (0.45, 0.5)
	$\mathcal{P}_{\tau_4}$	(0.1, 0.1), (0.8, 0.85)	(0.1, 0.1), (0.8, 0.85)	(0.6, 0.55), (0.3, 0.35)	(0.3, 0.35), (0.6, 0.55)	(0.2, 0.25), (0.7, 0.7)
	$\mathcal{P}_{\tau_5}$	(0.9, 0.9), (0.05, 0.01)	(0.45, 0.5), (0.45, 0.5)	(0.1, 0.1), (0.8, 0.85)	(0.1, 0.1), (0.8, 0.85)	(0.45, 0.5), (0.45, 0.5)
	$\mathcal{P}_{\tau_6}$	(0.2, 0.25), (0.7, 0.7)	(0.1, 0.1), (0.8, 0.85)	(0.2, 0.25), (0.7, 0.7)	(0.45, 0.5), (0.45, 0.5)	(0.1, 0.1), (0.8, 0.85)

Step 3: By using aggregation operators INZNWA Table 8 presents aggregated $n \times m$ matrix .

TABLE 8. Aggregated Decision Matrices by IFWA

Alternatives	Criteria				
	$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\mathcal{Q}_{\gamma_3}$	$\mathcal{Q}_{\gamma_4}$	$\mathcal{Q}_{\gamma_5}$
$\mathcal{P}_{\tau_1}$	(0.5126, 0.5423), (0.3523, 0.3746)	(0.6637, 0.6958), (0.2158, 0.226)	(0.3902, 0.3451), (0.5075, 0.5549)	(0.7113, 0.7475), (0.1847, 0.1411)	(0.2063, 0.2351), (0.6928, 0.6837)
$\mathcal{P}_{\tau_2}$	(0.6961, 0.7235), (0.0574, 0.1175)	(0.4352, 0.4615), (0.4547, 0.5104)	(0.5781, 0.5861), (0.3104, 0.2016)	(0.4582, 0.5187), (0.4154, 0.4288)	(0.3776, 0.4004), (0.5175, 0.5509)
$\mathcal{P}_{\tau_3}$	(0.4484, 0.4551), (0.4471, 0.4996)	(0.8851, 0.8916), (0.0574, 0.0158)	(0.7405, 0.7699), (0.1534, 0.1691)	(0.7384, 0.7437), (0.1723, 0.0795)	(0.3971, 0.4067), (0.497, 0.5316)
$\mathcal{P}_{\tau_4}$	(0.3902, 0.3451), (0.5075, 0.5549)	(0.0602, 0.0287), (0.879, 0.8898)	(0.5269, 0.4975), (0.3693, 0.4008)	(0.3309, 0.3714), (0.5665, 0.5976)	(0.7472, 0.7575), (0.1639, 0.0756)
$\mathcal{P}_{\tau_5}$	(0.5283, 0.5562), (0.3617, 0.2705)	(0.4806, 0.5325), (0.3821, 0.4023)	(0.2944, 0.269), (0.5961, 0.6513)	(0.335, 0.3211), (0.5598, 0.6059)	(0.4752, 0.4643), (0.4195, 0.4628)
$\mathcal{P}_{\tau_6}$	(0.4925, 0.499), (0.3895, 0.4434)	(0.4291, 0.4499), (0.4571, 0.4736)	(0.5101, 0.5257), (0.3742, 0.4183)	(0.3367, 0.3876), (0.5612, 0.5916)	(0.4208, 0.3975), (0.4707, 0.5146)

Step 4: The given aggregated matrix is normalized as shown in Table 9.

TABLE 9. Normalized Decision Matrix

Alternatives	$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\mathcal{Q}_{\gamma_3}$	$\mathcal{Q}_{\gamma_4}$	$\mathcal{Q}_{\gamma_5}$
$\mathcal{P}_{\tau_1}$	(0.3523, 0.3746), (0.5126, 0.5423)	(0.2158, 0.2260), (0.6637, 0.6958)	(0.5075, 0.5549), (0.3902, 0.3451)	(0.1847, 0.1411), (0.7113, 0.7475)	(0.6928, 0.6837), (0.2063, 0.2351)
$\mathcal{P}_{\tau_2}$	(0.0574, 0.1175), (0.6961, 0.7235)	(0.4547, 0.5104), (0.4352, 0.4615)	(0.3104, 0.2016), (0.5781, 0.5861)	(0.4154, 0.4288), (0.4582, 0.5187)	(0.5175, 0.5509), (0.3776, 0.4004)
$\mathcal{P}_{\tau_3}$	(0.4471, 0.4996), (0.4484, 0.4551)	(0.0574, 0.0158), (0.8851, 0.8916)	(0.1534, 0.1691), (0.7405, 0.7699)	(0.1723, 0.0795), (0.7384, 0.7437)	(0.4970, 0.5316), (0.3971, 0.4067)
$\mathcal{P}_{\tau_4}$	(0.5075, 0.5549), (0.3902, 0.3451)	(0.8790, 0.8898), (0.0602, 0.0287)	(0.3693, 0.4008), (0.5269, 0.4975)	(0.5665, 0.5976), (0.3309, 0.3714)	(0.1639, 0.0756), (0.7472, 0.7575)
$\mathcal{P}_{\tau_5}$	(0.3617, 0.2705), (0.5283, 0.5562)	(0.3821, 0.4023), (0.4806, 0.5325)	(0.5961, 0.6513), (0.2944, 0.2690)	(0.5598, 0.6059), (0.3350, 0.3211)	(0.4195, 0.4628), (0.4752, 0.4643)
$\mathcal{P}_{\tau_6}$	(0.3895, 0.4434), (0.4925, 0.4990)	(0.4571, 0.4736), (0.4291, 0.4499)	(0.3742, 0.4183), (0.5101, 0.5257)	(0.5612, 0.5916), (0.3367, 0.3876)	(0.4707, 0.5146), (0.4208, 0.3975)

5.1. **CRITIC-WASPAS.** **Step 5**: Apply The CRITIC method to find the criteria weight. The correlation coefficients and standard deviations of the normalized decision matrix are calculated, as shown in Table 10 and Table 11, respectively. The final IFZN criteria weights, obtained using the CRITIC method, are presented in Table 12.

TABLE 10. Correlation Coefficients

Criteria	$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\mathcal{Q}_{\gamma_3}$	$\mathcal{Q}_{\gamma_4}$	$\mathcal{Q}_{\gamma_5}$
$\mathcal{Q}_{\gamma_1}$	(1.0000, 1.0000, 1.0000, 1.0000)	(0.1277, 0.0696, 0.1287, 0.3078)	(-0.0066, -0.0170, -0.0979, -0.1068)	(0.0489, -0.1042, -0.0171, 0.0996)	(-0.4425, -0.4829, -0.4968, -0.5964)
$\mathcal{Q}_{\gamma_2}$	(0.1277, 0.0696, 0.1287, 0.3078)	(1.0000, 1.0000, 1.0000, 1.0000)	(0.1900, 0.1512, 0.2896, 0.2507)	(0.7798, 0.7975, 0.8243, 0.7370)	(-0.8053, -0.7951, -0.7625, -0.7987)
$\mathcal{Q}_{\gamma_3}$	(-0.0066, -0.0170, -0.0979, -0.1068)	(0.1900, 0.1512, 0.2896, 0.2507)	(1.0000, 1.0000, 1.0000, 1.0000)	(0.3757, 0.3669, 0.4000, 0.4016)	(0.0876, 0.0077, 0.0838, 0.0775)
$\mathcal{Q}_{\gamma_4}$	(0.0489, -0.1042, -0.0171, 0.0996)	(0.7798, 0.7975, 0.8243, 0.7370)	(0.3757, 0.3669, 0.4000, 0.4016)	(1.0000, 1.0000, 1.0000, 1.0000)	(-0.6893, -0.5786, -0.6588, -0.6134)
$\mathcal{Q}_{\gamma_5}$	(-0.4425, -0.4829, -0.4968, -0.5964)	(-0.8053, -0.7951, -0.7625, -0.7987)	(0.0876, 0.0077, 0.0838, 0.0775)	(-0.6893, -0.5786, -0.6588, -0.6134)	(1.0000, 1.0000, 1.0000, 1.0000)

TABLE 11. Standard Deviations

Criterion				
$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\mathcal{Q}_{\gamma_3}$	$\mathcal{Q}_{\gamma_4}$	$\mathcal{Q}_{\gamma_5}$
(0.1559, 0.1611) , (0.1034, 0.1253)	(0.2782, 0.2943) , (0.2749, 0.2892)	(0.1544, 0.1898) , (0.1542, 0.1779)	(0.1882, 0.2402) , (0.1920, 0.1901)	(0.1722, 0.2067) , (0.1768, 0.1719)

TABLE 12. CRITIC Weights

Criterion	w_1	w_2	w_3	w_4	w_5
Weights	(0.1717, 0.1602), (0.1274, 0.1411)	(0.2660, 0.2438), (0.2660, 0.2657)	(0.1335, 0.1454), (0.1409, 0.1576)	(0.1691, 0.1854), (0.1822, 0.1683)	(0.2598, 0.2652), (0.2836, 0.2674)

Step 6: Using the WASPAS technique, the Joint generalized criterion function is determined and presented in Table 13. Moreover, the score function is calculated using Equation (2. 1), and the results are presented in Table 14. Finally, the alternatives are ranked based on these scores.

TABLE 13. Joint Generalized Criterion (Q)

Alternative	Q_i
$\mathcal{P}_{\tau_1}$	$(0.3719, 0.3732), (0.4591, 0.4800)$
$\mathcal{P}_{\tau_2}$	$(0.3427, 0.3732), (0.4714, 0.5064)$
$\mathcal{P}_{\tau_3}$	$(0.2336, 0.1986), (0.6271, 0.6397)$
$\mathcal{P}_{\tau_4}$	$(0.4681, 0.4249), (0.3505, 0.3178)$
$\mathcal{P}_{\tau_5}$	$(0.4429, 0.4623), (0.4280, 0.4332)$
$\mathcal{P}_{\tau_6}$	$(0.4536, 0.4919), (0.4276, 0.4428)$

TABLE 14. WASPAS Scores Matrix

Alternative	Score Function	Rank
$\mathcal{P}_{\tau_1}$	0.4592	4
$\mathcal{P}_{\tau_2}$	0.4446	5
$\mathcal{P}_{\tau_3}$	0.3226	6
$\mathcal{P}_{\tau_4}$	0.5437	1
$\mathcal{P}_{\tau_5}$	0.5097	3
$\mathcal{P}_{\tau_6}$	0.5169	2

5.2. **CRITIC-EDAS. Step 7**: In this approach, the same criteria weights, as presented in Table 12, are applied. Based on these weights and the suggested criteria, the average value (AV) is then determined and shown in Table 15 .

TABLE 15. Average Scores of Criteria

Criteria				
Av_1	Av_2	Av_3	Av_4	Av_5
0.4334	0.4600	0.4505	0.4586	0.5111

Step 8: The score function of the normalized matrix is calculated and given in Table 16.

TABLE 16. Score Function

Alternatives	Criteria				
	$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\mathcal{Q}_{\gamma_3}$	$\mathcal{Q}_{\gamma_4}$	$\mathcal{Q}_{\gamma_5}$
$\mathcal{P}_{\tau_1}$	0.4270	0.2935	0.5735	0.2472	0.7126
$\mathcal{P}_{\tau_2}$	0.2516	0.5156	0.3619	0.4702	0.5670
$\mathcal{P}_{\tau_3}$	0.5096	0.1059	0.2279	0.2323	0.5513
$\mathcal{P}_{\tau_4}$	0.5735	0.8902	0.4430	0.6078	0.2232
$\mathcal{P}_{\tau_5}$	0.4020	0.4489	0.6545	0.6158	0.4868
$\mathcal{P}_{\tau_6}$	0.4635	0.5117	0.4442	0.6008	0.5375

TABLE 17. Score Weights

Criteria				
w_1	w_2	w_3	w_4	w_5
0.2021	0.1990	0.1997	0.2003	0.1988

Step 9, 10: In Table (18, 19) the weighted positive and negative distances from the average solution are determined in this stage.

TABLE 18. Weighted Positive Distance

Alternatives	Criteria					
	$\mathcal{Q}_{\gamma_1}$	$\mathcal{Q}_{\gamma_2}$	$\mathcal{Q}_{\gamma_3}$	$\mathcal{Q}_{\gamma_4}$	$\mathcal{Q}_{\gamma_5}$	$\mathbb{D}_i$
$\mathcal{P}_{\tau_1}$	0	0	0.0545	0	0.0784	0.1329
$\mathcal{P}_{\tau_2}$	0	0.0241	0	0.0051	0.0217	0.0509
$\mathcal{P}_{\tau_3}$	0.0355	0	0	0	0.0156	0.0512
$\mathcal{P}_{\tau_4}$	0.0653	0.1862	0	0.0652	0	0.3166
$\mathcal{P}_{\tau_5}$	0	0	0.0904	0.0687	0	0.1591
$\mathcal{P}_{\tau_6}$	0.0140	0.0224	0	0.0621	0.0102	0.1087

TABLE 19. Weighted Negative Distance

Alternatives	Criteria					
	Q_{γ_1}	Q_{γ_2}	Q_{γ_3}	Q_{γ_4}	Q_{γ_5}	E_i
$\mathcal{P}_{\tau_1}$	0.0030	0.0721	0	0.0924	0	0.1674
$\mathcal{P}_{\tau_2}$	0.0848	0	0.0393	0	0	0.1241
$\mathcal{P}_{\tau_3}$	0	0.1532	0.0987	0.0989	0	0.3508
$\mathcal{P}_{\tau_4}$	0	0	0.0033	0	0.1120	0.1154
$\mathcal{P}_{\tau_5}$	0.0146	0.0048	0	0	0.0095	0.0289
$\mathcal{P}_{\tau_6}$	0	0	0.0028	0	0	0.0028

Step 11: The results of weighted normalized positive and negative distances (WNPD and WNND) form the average solution are calculated by equation 20 and 21 and shown in Table 20.

TABLE 20. WNPD & WNND

	M_i	N_i
$\mathcal{P}_{\tau_1}$	0.4196	0.4773
$\mathcal{P}_{\tau_2}$	0.1607	0.3538
$\mathcal{P}_{\tau_3}$	0.1617	1.0000
$\mathcal{P}_{\tau_4}$	1.0000	0.3289
$\mathcal{P}_{\tau_5}$	0.5024	0.0825
$\mathcal{P}_{\tau_6}$	0.3434	0.0080

Step 12: Finally, Table 21 presents the appraisal scores computed using Equation (22), based on which the alternatives are ranked accordingly.

TABLE 21. Appraisal Score

	$\mathbb{J}_i$	Rank
$\mathcal{P}_{\tau_1}$	0.4485	3
$\mathcal{P}_{\tau_2}$	0.2572	5
$\mathcal{P}_{\tau_3}$	0.5808	2
$\mathcal{P}_{\tau_4}$	0.6644	1
$\mathcal{P}_{\tau_5}$	0.2925	4
$\mathcal{P}_{\tau_6}$	0.1757	6

6. SENSITIVITY ANALYSIS

In our research, the sensitivity analysis of the hybrid WASPAS method (a combination of WSM and WPM) in Table 22 modifies the weighting parameter β, which balances the contributions of the WSM and the WPM to assess the stability of decision-making outcomes. This parameter, β, which ranges from 0 to 1, determines which model predominates during the decision-making process. It evaluates the robustness of the rankings by recalculating scores for each alternative and systematically altering β. Stable rankings over a broad range of β values indicate a robust choice and show the stability of the model, whereas notable variations suggest sensitivity to the parameter selection. This study is particularly useful in identifying critical parameter values where rankings may shift and in understanding the impact of additive versus multiplicative aggregation procedures. Since, the manufacturing sector has a significant influence on environmental pollution, thus $\mathcal{P}_{\tau_4}$ emerges the most suitable alternative. Its consistent dominance is clearly illustrated in the flowchart in Fig. 4. Furthermore, the rankings remain stable across the entire range of β, indicating that the method is reliable and capable of producing stable decision outcomes.

TABLE 22. Sensitivity Analysis of WASPAS for varying parameter β

β	$\mathbf{Q_1}$	$\mathbf{Q_2}$	$\mathbf{Q_3}$	$\mathbf{Q_4}$	$\mathbf{Q_5}$	$\mathbf{Q_6}$	Ranking
0.0	0.4609	0.4368	0.3230	0.5418	0.5093	0.5168	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$
0.2	0.4602	0.4398	0.3226	0.5428	0.5094	0.5168	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$
0.4	0.4595	0.4430	0.3225	0.5435	0.5096	0.5169	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$
0.6	0.4589	0.4463	0.3228	0.5439	0.5097	0.5169	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$
0.8	0.4583	0.4497	0.3235	0.5439	0.5099	0.5170	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$
1.0	0.4577	0.4532	0.3244	0.5435	0.5101	0.5170	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$

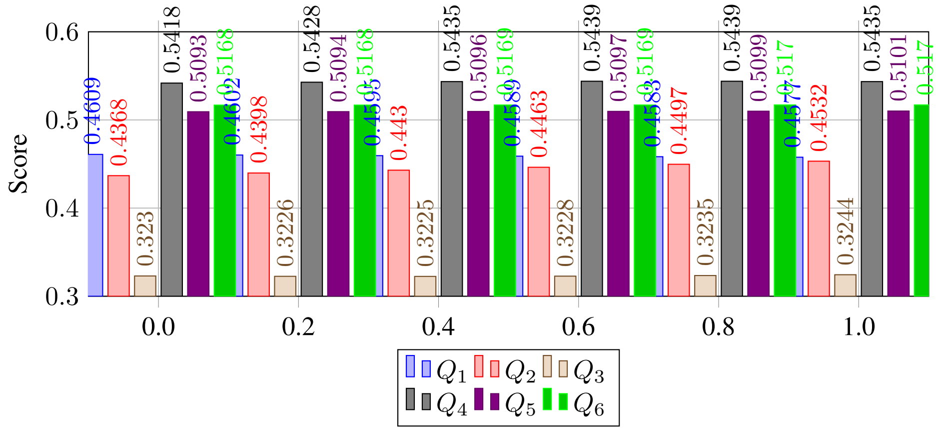

FIGURE 4. Sensitivity Analysis of WASPAS by varying parameter β

7. COMPARISON ANALYSIS

Through a comparison analysis with well-known IF-based aggregation operators, specifically with IFWG [24], intuitionistic fuzzy Dombi Bonferroni mean operators IFDBM [13], and intuitionistic fuzzy Einstein hybrid aggregation operators IFEHA [29], the efficacy of the suggested IFZN-based CRITIC-WASPAS and CRITIC-EDAS approach was evaluated. All mentioned aggregation operators, showed good performance by applying the above approaches and generating reliable ranking patterns. Among them, the CRITIC-WASPAS technique distinguished itself by producing the most dependable and coherent results throughout the evaluation. Although the initial results in Table 23 showed changes in the rankings, while the subsequent outcomes demonstrated accuracy and consistency, with $\mathcal{P}_{\tau_4}$ consistently ranked highest and $\mathcal{P}_{\tau_3}$ ranked lowest . It successfully handles the innate imprecision and uncertainty seen in MCDM problems by incorporating IFZN. This improves the decision-making process's overall accuracy and dependability. In conclusion, while the comparison approaches function well, the CRITIC-WASPAS approach is the most thorough and successful, providing the most assistance for precise and accurate decision-making in the face of uncertainty.

TABLE 23. Comparison of MCDM Method with Our Proposed Method

Method	**Final Value**	**Ranking Order**
IFWG [24]	$\mathcal{P}_{\tau_3} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_4}$	$\mathcal{P}_{\tau_3}$
IFDBM [13]	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$	$\mathcal{P}_{\tau_4}$
IFEHA [29]	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$	$\mathcal{P}_{\tau_4}$
Proposed Method	$\mathcal{P}_{\tau_4} > \mathcal{P}_{\tau_6} > \mathcal{P}_{\tau_5} > \mathcal{P}_{\tau_1} > \mathcal{P}_{\tau_2} > \mathcal{P}_{\tau_3}$	$\mathcal{P}_{\tau_4}$

8. Conclusion

This study presents a comprehensive framework for assessing the effects of six important sectors on environmental pollution utilizing IFZN in combination with the CRITIC-EDAS and CRITIC-WASPAS methodologies. By managing the uncertainty and imprecision present in real-world environmental data, the inclusion of IFZN greatly enhances the assessment process. The CRITIC technique was used to objectively evaluate the weights of important environmental criteria, including waste creation, air pollution, soil contamination, noise pollution, and water pollution. These weights were then added to the WASPAS and EDAS methodologies to provide a fair and comprehensive assessment. Furthermore, the accuracy and consistency of the industrial rankings were further improved by modifying parameters in the WASPAS approach. Additionally, in the comparative analysis of our proposed aggregation techniques, such as IFWG, IFDBM, and IFEHA, we found that those methods often fail to fully capture the complex and varied nature of environmental data. However, our proposed method provides more accurate and consistent data. Overall, the methodology contributes a robust decision-making tool for environmental impact analysis, particularly in sectors affected by multiple uncertain factors. The suggested framework was implemented and analyzed using MATLAB software, which facilitated the computation of fuzzy logic operations and decision-making processes.

This study has some limitations, though. It is limited to only six industries, and the results might not be applicable to other industrial sectors and the model relies on expert judgment and linguistic terms, which may introduce subjectivity. Furthermore, it only focused on determining which industries had the highest pollution impact; it made no recommendations for specific strategies to reduce pollution. In future work, the IFZN-based methodology will be expanded in further research to provide the most practicable and efficient methods for reducing pollution in the given sector. The findings may be made even more thorough and useful by incorporating more topic experts and a larger variety of criteria. The goal of this extended research is to assist industry policymakers and stakeholders in making well-informed, ecologically conscious decisions for a more sustainable future.

Conflict of Interests

The authors declare that there is no conflict of interest regarding the publication and funding that they have received.

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Authors' Contributions

Bushra Khalid: Conceptualization, Methodology, Writing–Original Draft, Formatting. Amara Mujeeb: Data Curation, Software Writing–Review & Editing. Asim Zafar: Supervision, Validation, Analysis, Writing–Review & Editing. All authors read and approved the final manuscript.

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