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Improved Intelligent Decision Model for Climate Change Adaptation Strategies Using q-Rung Orthopair Fuzzy Dombi Prioritized Z-Information

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Abstract. Climate change effects are becoming progressively more region-specific, which places the development of effective climate change adaptation strategies at urban and regional scales at a premium. These strategies are crucial for protecting natural ecosystems, improving resource management, and strengthening the resilience of vulnerable communities. But the identification and selection of climate change adaptation strategies involves complex multi-attribute decision-making (MADM) with multiple attributes, conflicting criteria, environmental trade-offs and uncertainty associated with expert opinions. Traditional MADM and current fuzzy-based methodologies are often inadequate to consider simultaneously higher uncertainty and information reliability, which may result in poor decision-making performance. This paper presents a new decision-making model incorporating q-rung orthopair fuzzy Z-numbers (qROFZNs) and Dombis parametric aggregation operators and prioritized aggregation approaches. The method is able to effectively account for uncertainty and information reliability of expert assessments and offers adaptive aggregation of multi-criteria data. A case study is presented to illustrate the use of the model for determining the best climate change adaptation strategy. In addition, comparisons with existing EDAS method and sensitivity analysis are carried out to ensure the reliability, stability and consistency of the proposed model. The findings suggest that the proposed approach provides improved discrimination and reliability, and is thus a robust and effective decision-making tool to tackle climate adaptation problems.

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1. INTRODUCTION

Changing climate adaptation strategies must be assumed top priority in order to effectively counteract the escalating consequences of increasing in global warming, which include increasing sea levels, more recurrent and severe extreme climate events like storms, droughts, significant disturbances to systems and biodiversity. Through vigilant selection and implementation of adaptation policies, communities can improve their resilience, protection vulnerable groups and protection critical infrastructure from the many intimidations posed by a changing climate.A robust framework for this prioritization is providing by the Multi-Attribute Decision-Making (MADM) approach, which systematically assesses numerous adaption alternatives in the light of several factors, including cost, feasibility, ecological effect and social equality. After elaborating and analyzing these factors , MADM assurances that strategies are both realistic and link with long-term sustainability objectives to solve climate-related problem.

1.1. Literature Review.

Literature Review. In 1965, Zadeh [33] pioneered to introduce Fuzzy set (FS) theory which extended classical set theory to model partial memberships grade and vagueness, revolutionizing uncertainty and ambiguity modelling in mathematics. The classical mathematics gives only exact solution by using binary number 0 and 1 e.i, if element exist then answer will be 1 and if element does not exist then answer will be 0, which is not able to solve real life problems. The membership grade minimize the limitation of classical mathematics by solving the real life problem according to human nature, where membership grade belongs to [0, 1]. In 1986, Atanassov [2] introduced Intuitionistic Fuzzy Set (IFS) by made a revolutionary amendment in FS theory and add non-membership grade. In IFS the membership and non-membership grade are complement of each others. Membership grade and non-membership grade range separately from 0 to 1 but sum of these grade less than or equal to 1 i.e., if $\wp$, $\overline{\wp}$ denote the membership and non-membership grade respectively then $\wp+\overline{\wp} \leq 1$ where $\wp \in [0,1]$ and $\overline{\wp} \in [0,1]$. The IFS [29] is also used for medical diagnosis, risk management, ML, AI and pattern recognition etc. The IFS recently used in solving decision support (DS) problems with the collaboration of hesitant fuzzy distance [22].

To minimize the limitations in IFS the Yager [30] introduce Pythagorean fuzzy set (PFS) which is more flexible and adaptable approach to solve MADM problem than IFS [35, 20]. In PFS the sum of square of membership grade and non-membership grade less than or equal to 1 i.e., $\wp^2 + \overline{\wp}^2 \leq 1$ where $\wp \in [0,1]$, $\overline{\wp} \in [0,1]$. This condition of PFS covers more data, minimize uncertainty, which make it more generalized to solve MADM prolems [16, 17]. PFS has also been used in decision-support systems to solve MADM with Dombi aggregation operators [10] and further hesitant/probabilistic PFS comparison with EDAS ranking [3]. In 2020 Ashraf et. al [1] used PFS to select green supplier in muti-attribute decision making MADM process and proven its suitability related to sustainability problems.

In 2017 Yager [32] proposed the Q-rung orthopair fuzzy set (qROFS) provided a generalised form that overcome modelling uncertainty minimize limitations of IFS and PFS.A more detailed representation of uncertainty is possible with a parameter q where $q \geq 1$ that modifies the restriction on membership degree and non-membership degree. In qROFS [5], the sum of q^{th} power of degree of membership grade and degree of non-membership grade less or equal to 1 i.e., $\vartheta^q + \overline{\vartheta}^q \leq 1$ where $\vartheta \in [0, 1]$, $\overline{\vartheta} \in [0, 1]$. The qROFS has been commonly employed in real-life applications for MADM because it effectively synthesizes a range of expert opinions over aggregation operators [13], yager operators [19, 18]. With the collaboration of time series the qROFS used in application like occurrence of natural disaster or analysis of university admissions [21]. In selection of sustainable energy problems the qROFS play a vital rule in context of hyper soft set [23]. The investment evaluation set by improved partition algorithm which based on qROFS in linguistic decision making process [37].

Zadeh [34] the pioneer of FS again made a revolutionary amendment in FS theory by introducing Z-number to discuss the credibility of degree of membership grade. Here he combined discuss the degree of membership grade and its credibility at the same time i.e., an ordered pair $Z = (\wp, A_\wp)$ is known as a Z-number, in which $\wp$ is a degree of membership grade for fuzzy number and $A_\wp$ is again a fuzzy number that shows the credibility of degree of membership grade $\wp$. Usually in FS theory the truthness of data is not discussed [28]. The Z-number enables the criteria to communicate both the ambiguity of the data and the imprecise surrounding its dependability in the face of complex uncertainty, offering a more comprehensive approach to decision-making. In 2024, Kumar and Gupta [11] integrate the q-rung orthopair fuzzy set with Z-number and developed a novel set "q-rung orthopair Z-number" in fuzzy set theory to make TOPSIS algorithm which helped in decision making process. Here they used ν, μ degree of membership and degree of non-membership respectively, where $(Z(\nu), Z(\mu))$ their reliability respectively such that $(Z(\nu))^q + (Z(\mu))^q \leq 1$, $q \geq 1$.

In 1982 L. Dombi [4] introduce Dombi aggregation operator which is more flexible in mathematical modelling for decision making tool based on Dombi t-norm and t-conorm that aggregate uncertain fuzzy information to find out optimum solution in complex decision making problems. Their primary feature is the addition of an operational parameter, λ, that may be adjusted, allowing decision researcher to modify the relative importance of different criteria. This adaptable parameter makes Dombi operators much more reliable and adaptable than standard algebraic operators. For MADM problems involving uncertainty, ambiguity and imprecision in a range of fuzzy environments, such as Dombi aggregation operators [8] used in spherical fuzzy sets to find out optimum solution in MADM, intuitionistic fuzzy set also used its t-norm and t-conorm for MADM [25], [36] used it in C-IFS scenario, these operators minimize the limitation because they can represent a extensive range of aggregation behaviors. The adaptableness of Dombi aggregation operators has ensued in a variety of uses, including the appraisal of emerging technology corporations, the selection of green suppliers, and the assessment of solar sites [15].

In decision making problems a revolutionary step taken by K. Ghorabaee et al. [6] in 2015. , this method is most valueable when the decision criteria is conflicted in MADM then EDAS method is very useful [25].Usually the running method like WASPAS and TOPSIS evaluate alternatives by measuring distances from negative and positive optimum solution but in EDAS method the alternative evaluate by measuring distance from its average solution. Its involve measuring both positive distance from average (PDAS) and negative distance from average (NDAS), where a optimum alternative selected in a higher PDAS and a lower NDAS [9]. The EDAS framework has been extensively stretched and integrated with numerous fuzzy environments, such as intuitionistic, neutrosophic, and Pythagorean fuzzy sets, to handle uncertain, ambiguous and imprecise data. Its application includes solving real-world problems including identifying the optimal sites for solid waste disposal, measuring situations involving fuzzy decision-making, and assessing new technologies [8].

1.2. **Motivations of the Study.** The evaluation of climate change adaptation measures is inherently a MADM problem under uncertain and imprecise information with reliability concerns. Conventional MADM methods, based on crisp numerical assessments [7, 24], fail to represent the vagueness and uncertainty inherent in expert opinions. Even the current extensions of fuzzy sets, such as intuitionistic and Pythagorean fuzzy sets, are limited in terms of the membership-non-membership spaces, which prevent them from representing greater uncertainty. To overcome these shortcomings, this research uses q-rung orthopair fuzzy Z-numbers (q-ROFZNs), which offer a more expressive framework by removing the constraint of orthopair fuzzy sets, and incorporating a new confidence level for evaluating a set of fuzzy information based on the theory of Z-numbers. This allows the representation of the evaluation and its corresponding reliability, thus increasing decision-making reliability. Additionally, by incorporating Dombis parametric operators, flexible aggregation with variable compensation can be achieved, along with prioritized aggregation rules to accommodate hierarchical importance of criteria. Taken together, these attributes provide a more thorough and robust decision-making approach to manage the uncertainty and complexity of selecting climate adaptation strategies. Here some points written below that motivate us to work on it:

- qROFS gives the more flexibility and adaptability in uncertainty modeling than IFS and PyFS by allowing generalized constraint ($1 \leq q$) to degree of membership and non-membership grade, qROFS demonstrate wider range of imprecision data and tackle higher level of uncertainty than IFS and PyFS.
- The T-norm and conorm of Dombi aggregation operators compensate and facilitate the complex modeling required to solve MADM scenarios, also offers flexible, adoptable and generalized method to analysis the sets of fuzzy information than Algebraic, Hamacher and Sxhwiezer Sklar aggregation operatotrs.
- The Z-numbers combine with q-rung orthopair fuzzy sets (qROFS) bids a vigorous framework to tackel data uncertainty, vagueness, with truthness and reliability of data in complex MADM scenarios.
- The EDAS method is better than TOPSIS, VIKOR and PROMETHEE methods by offering average solution evaluation minimize the risk of rank reversal, sensitivity of parameter, computationally simple but effecient and effective, enhance the flexibility, reliability in MADM problems.

- It is quite possible to develop a novel, efficient and effective decision-making assistance tool that integrates qROFZN with Dombi aggregation and EDAS separately to enhance MADM processing in terms of flexibility, precision, adaptability and practical solutions.

We are motivated by the above described elements to combine the Dombi t-norm and t-conorm with qROFZN in order to build aggregation operators that include priority weights. This combination will provide decision makers the best answer while sinking data imprecision, ambiguity, and uncertainty.

1.3. **Objectives of the Study.** While fuzzy Multi-Attribute Decision-Making (MADM) methods are increasingly used for environmental decision-making, there are still key gaps. Current methods have relied on conventional fuzzy structures that provide limited modelling capacity of uncertainty because of restricted membership-non-membership domains and lack of reliability assessment in expert judgements. Furthermore, the aggregation techniques often lack adaptability to represent different decision-makers' attitudes and priority relationships between decision criteria, which are crucial for sustainable decision-making. To overcome these limitations, this research seeks to provide a sound and flexible decision-making model for the uncertainty-based selection of climate change adaptation strategies. This research aims to: (i) represent uncertain and reliability-sensitive data using q-rung orthopair fuzzy Z-numbers, (ii) integrate flexible aggregation through the use of Dombis parametric operators, (iii) consider prioritized relationships among decision criteria, and (iv) rank alternative adaptation strategies in a MADM context. This research not only strengthens the theoretical and practical underpinnings of fuzzy MADM in climate decision-making but also improves the reliability of the decision-making process. Following are main and core objectives which will be achieved during the research:

- To construct a reliable multi-attribute decision-making (MADM) model using q-rung orthopair fuzzy Z-numbers (qROFZNs) to deal with uncertainty and reliability of expert assessments.
- To embed Dombis parametric T-norm and T-conorm operators in the qROFZN framework for adaptive aggregation of the decision-makers' information.
- To develop Dombi prioritized aggregation operators to consider prioritized criteria for improved decision-making.
- To use the proposed method in the assessment and selection of the best climate change adaptation strategies at urban/regional scale.
- To assess the efficiency, consistency and robustness of the proposed method with comparative analysis with EDAS method and sensitivity analysis.

1.4. **Structure of the Study.** The research seeks to provide a novel method to the Dombi aggregation operator with the framework of qROFZNs, efficiently and effectively addressing the challenges of imprecision and reliability with greater efficiency, validity and accuracy. The research structured as follows: Section 2 will provide the necessary preliminaries i.e, basic and core definitions of qROFZN and Dombi norms with their operations. In section 3, we provide the definitions and proof of the Dombi aggregation operators with the framework of qROFZN and conversely as well . The section 4 provides algorithm for solving MADM problems with uses of qROFZN with the framework of Dombi method. The section 5 provides solution of numerical example for selection the optimum strategy of from available Climate change adaptive strategies (urban/regional). The section 6 provides the detail of EDAS method and numerical example solved by it. In section 8 provide the comparison of Dombi aggregation operators in context of qROFZN with EDAS methods, the sensitivity analysis and practical implication is also the part of this section. In section 9 provide the conclusion of the research, limitations and important suggestions for enhancement in future research.

2. Preliminaries

This section gives an overview of basic definitions and fundamental operations that provide help in proposed research work. This section revisits key fuzzy set theory mathematical notions and notations that help to grasp the ideas used in the next sections.

Definition 2.1. [33] *The FS $\mathfrak{A}$ on a universal set $\mathfrak{U}$ is defined as*

$$\mathfrak{A} = \{(\mathfrak{u}, \mathfrak{f}(\mathfrak{u})) \mid \mathfrak{u} \in \mathfrak{U}\} \tag{2. 1}$$

where $\mathfrak{f} : \mathfrak{U} \longrightarrow [0, 1]$ and $\mathfrak{f}(\mathfrak{u})$ for every $\mathfrak{u} \in \mathfrak{U}$ is the degree of membership of $\mathfrak{u}$ in the set $\mathfrak{A}$.

Definition 2.2. [34] *A $\breve{Z}$ number is defined as an ordered pair of fuzzy number incorporated $Z = (C, D)$ where L is degree of membership grade and C is the reliability of D.*

Definition 2.3. [31] *Let $\mathfrak{U}$ be a universal set then q-rung orthopair fuzzy set is define as:*

$$\breve{\zeta} = \{(\mathfrak{u}, (\varrho(\mathfrak{u}), \tau(\mathfrak{u})))|\mathfrak{u} \in \mathfrak{U}\} \tag{2. 2}$$

where $\varrho : \mathfrak{U} \longrightarrow [0, 1]$, $\tau : \mathfrak{U} \longrightarrow [0, 1]$ are the degree of membership and non-membership grade respectively with the condition:

$$0 \leq (\varrho(\mathfrak{u}))^q + (\tau(\mathfrak{u}))^q \leq 1, \forall\, q \geq 1$$

Definition 2.4. [11] *Let $\mathfrak{U}$ be a universal set then q-rung orthopair fuzzy Z-number (qROFZN) to be $\daleth_Q$ is define as:*

$$\daleth_Q = \{\mathfrak{u}, \varrho(C, D)(\mathfrak{u}), \varpi(C, D)(\mathfrak{u})|\mathfrak{u} \in \mathfrak{U}\}$$

The mapping $\varrho(C, D)(\mathfrak{u}) : \mathfrak{U} \longrightarrow [0, 1]$ and $\varpi(C, D)(\mathfrak{u}) : \mathfrak{U} \longrightarrow [0, 1]$ are the degree of membership and non-membership respectively with their reliability are constructed as below:

$$\daleth_Q = \{\varrho(C, D), \varpi(C, D)\} = \{(\varrho_C, \varrho_D), (\varpi_C, \varpi_D)\} \tag{2. 3}$$

with the following condition:

$$0 \leq \varrho_C^q(\mathfrak{u}) + \varpi_C^q(\mathfrak{u}) \leq 1,\;\; 0 \leq \varrho_D^q(\mathfrak{u}) + \varpi_D^q(\mathfrak{u}) \leq 1.\;\; where\ q \geq 1$$

Definition 2.5. [11] *Take any two qROFZN $\daleth_{Q1} = \{(\varrho_{C1}, \varrho_{D1}), (\varpi_{C1}, \varpi_{D1})\}$ and $\daleth_{Q2} = \{(\varrho_{C2}, \varrho_{D2}), (\varpi_{C2}, \varpi_{D2})\}$ and $\lambda \geq 0$ then following operation defined as:*

(1) $\daleth_{Q2} \supseteq \daleth_{Q1} \iff \varrho_{C2} \geq \varrho_{C1}, \varrho_{D2} \geq \varrho_{D1}, \varpi_{C2} \leq \varpi_{C1}, \varpi_{D2} \leq \varpi_{D1}$.
(2) $\daleth_{Q1} = \daleth_{Q2} \iff \daleth_{Q1} \supseteq \daleth_{Q2} and \daleth_{Q1} \subseteq \daleth_{Q2}$.
(3) $\daleth_{Q1} \cup \daleth_{Q2} = \{(\varrho_{C1} \vee \varrho_{C2}, \varrho_{D1} \vee \varrho_{D2}), (\varpi_{C1} \vee \varpi_{C2}, \varpi_{D1} \vee \varpi_{D2})\}$.
(4) $\daleth_{Q1} \cap \daleth_{Q2} = \{(\varrho_{C1} \wedge \varrho_{C2}, \varrho_{D1} \wedge \varrho_{D2}), (\varpi_{C1} \wedge \varpi_{C2}, \varpi_{D1} \wedge \varpi_{D2})\}$.
(5) $(\daleth_{Q1})^c = \{(\varrho_{C1}, \varrho_{D1}), (\varpi_{C1}, \varpi_{D1})\}^c = \{(\varpi_{C1}, \varpi_{D1}), (\varrho_{C1}, \varrho_{D1})\}$

$$(6)\ \gimel_{Q1} \bigoplus \gimel_{Q2} = \left\{ \begin{array}{l} \left(\sqrt[q]{\varrho_{C1}^q + \varrho_{C2}^q - \varrho_{C1}^q \varrho_{C2}^q}, \sqrt[q]{\varrho_{D1}^q + \varrho_{D2}^q - \varrho_{D1}^q \varrho_{D2}^q} \right), \\ (\varpi_{C1}\varpi_{C2}, \varpi_{D1}\varpi_{D2}) \end{array} \right\}$$

$$(7)\ \gimel_{Q1} \bigotimes \gimel_{Q2} = \left\{ \begin{array}{l} (\varrho_{C1}\varrho_{C2}, \varrho_{D1}\varrho_{D2}), \\ \left(\sqrt[q]{\varpi_{C1}^q + \varpi_{C2}^q - \varpi_{C1}^q \varpi_{C2}^q}, \sqrt[q]{\varpi_{D1}^q + \varpi_{D2}^q - \varpi_{D1}^q \varpi_{D2}^q} \right) \end{array} \right\}$$

$$(8)\ \lambda\gimel_{Q1} = \{ (\sqrt[q]{(1-(1-\varrho_{C1}^q)^\lambda)}, \sqrt[q]{(1-(1-\varrho_{D1}^q)^\lambda)}), (\varpi_{C1}^\lambda, \varpi_{D1}^\lambda) \}$$

$$(9)\ (\gimel_{Q1})^\lambda = \{ (\varrho_{C1}^\lambda, \varrho_{D1}^\lambda), (\sqrt[q]{(1-(1-\varpi_{C1}^q)^\lambda)}, \sqrt[q]{(1-(1-\varpi_{D1}^q)^\lambda)}) \}$$

Definition 2.6. [11] *To compare $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i}), (\varpi_{C_i}, \varpi_{D_i})\}$ the following score function is defined:*

$$\digamma(\gimel_{Q_i}) = \frac{1 + (\varrho_{C_i}\varrho_{D_i}) - (\varpi_{C_i}\varpi_{D_i})}{2} \tag{2. 4}$$

Here if $\digamma(\gimel_{Q_1}) \geq \digamma(\gimel_{Q_2})$ then $\gimel_{Q_1} \geq \gimel_{Q_2}$, where $\digamma(\gimel_{Q_i}) \in [0,1]$.
If $\digamma(\gimel_{Q_i}) = \digamma(\acute{\gimel}_{Q_i})$ then following accuracy defined as:

$$\Re(\gimel_{Q_i}) = (\varrho_{C_i}\varrho_{D_i})^q + (\varpi_{C_i}\varpi_{D_i})^q \tag{2. 5}$$

Definition 2.7. [11] *The distance between two qROFZN $\gimel_{Q_1} = \{(\varrho_{C_1}, \varrho_{D_1}), (\varpi_{C_1}, \varpi_{D_1})\}$ and $\gimel_{Q_2} = \{(\varrho_{C_2}, \varrho_{D_2}), (\varpi_{C_2}, \varpi_{D_2})\}$, is defined as:*

$$\Delta(\gimel_{Q_1}, \gimel_{Q_2}) = \sqrt{(\varrho_{C_1}\varrho_{D_1} - \varrho_{C_2}\varrho_{D_2})^2 + (\varpi_{C_1}\varpi_{D_1} - \varpi_{C_2}\varpi_{D_2})^2} \tag{2. 6}$$

Definition 2.8. [4] *The Dombi T-Norm and T-Conorm are respectively defined as:*

$$D_r(\alpha, \beta) = \frac{1}{1 + \{(\frac{1-\alpha}{\alpha})^p + (\frac{1-\beta}{\beta})^p\}^{\frac{1}{p}}} \tag{2. 7}$$

$$D_r^*(\alpha, \beta) = 1 - \frac{1}{1 + \{(\frac{\alpha}{1-\alpha})^p + (\frac{\beta}{1-\beta})^p\}^{\frac{1}{p}}} \tag{2. 8}$$

where $p \geq 1$ and $\alpha \in [0,1]$, $\beta \in [0,1]$.

3. PROPOSED AGGREGATION OPERATORS

In this section we discussed Dombi prioritized aggregation operators and notions with some basic operators in context of qROFZN. These operators provide a flexible approach for merging imprecise and uncertain data, given different levels of data reliability and decision-maker confidence.

3.1. **Dombi Prioritized Aggregation Operators based on q-rung Orthopair Fuzzy Z-Numbers.** The Dombi AOs are the key components in the proposed decision-making approach as they enable dynamic aggregation of multi-attribute information in the qROFZN environment. Unlike traditional AOs, Dombi AOs have a parametric form that enables the control of the level of compensation between criteria. This is a critical characteristic in multi-attribute decision making, given the potential for non-linearity and interdependency between attributes. Through the adjustment of the Dombi parameter, different aggregation strategies can be adopted to reflect different decision-making attitudes, from compensatory to non-compensatory.

Furthermore, when combined with prioritized aggregation, Dombi AOs can incorporate prioritization of attributes, such that more important attributes carry greater weight in the aggregation process. It should be noted that the focus of Dombi AOs is not feature selection, but rather improved information aggregation of uncertain and reliability-sensitive data. This results in enhancing the discrimination, consistency, and robustness in ranking alternatives, and thus facilitating more powerful decision making.

Definition 3.2. *Consider two qROFZN as* $\gimel_{Q_1} = \{(\varrho_{C_1}, \varrho_{D_1}), (\varpi_{C_1}, \varpi_{D_1})\}$ *and* $\gimel_{Q_2} = \{(\varrho_{C_2}, \varrho_{D_2}), (\varpi_{C_2}, \varpi_{D_2})\}$*, then*

$$(1)\ \gimel_{Q_1} \bigoplus \gimel_{Q_2} = \left\{ \begin{array}{c} \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1+\left\{\left(\frac{\varrho_{C_1}}{1-\varrho_{C_1}}\right)^p + \left(\frac{\varrho_{C_2}}{1-\varrho_{C_2}}\right)^p\right\}^{\frac{1}{p}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\left\{\left(\frac{\varrho_{D_1}}{1-\varrho_{D_1}}\right)^p + \left(\frac{\varrho_{D_2}}{1-\varrho_{D_2}}\right)^p\right\}^{\frac{1}{p}}}} \end{array} \right), \\ \left(\begin{array}{c} \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-\varpi_{C_1}}{\varpi_{C_1}}\right)^p + \left(\frac{1-\varpi_{C_2}}{\varpi_{C_2}}\right)^p\right\}^{\frac{1}{p}}}}, \\ \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-\varpi_{D_1}}{\varpi_{D_1}}\right)^p + \left(\frac{1-\varpi_{D_2}}{\varpi_{D_2}}\right)^p\right\}^{\frac{1}{p}}}} \end{array} \right) \end{array} \right\}.$$

$$(2)\ \gimel_{Q_1} \bigotimes \gimel_{Q_1} = \left\{ \begin{array}{c} \left(\begin{array}{c} \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-\varrho_{C_1}}{\varrho_{C_1}}\right)^p + \left(\frac{1-\varrho_{C_2}}{\varrho_{C_2}}\right)^p\right\}^{\frac{1}{p}}}}, \\ \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-\varrho_{D_1}}{\varrho_{D_1}}\right)^p + \left(\frac{1-\varrho_{D_2}}{\varrho_{D_2}}\right)^p\right\}^{\frac{1}{p}}}} \end{array} \right), \\ \left(\begin{array}{c} \sqrt[q]{1 - \frac{1}{1+\left\{\left(\frac{\varpi_{C_1}}{1-\varpi_{C_1}}\right)^p + \left(\frac{\varpi_{C_2}}{1-\varpi_{C_2}}\right)^p\right\}^{\frac{1}{p}}}}, \\ \sqrt[q]{1 - \frac{1}{1+\left\{\left(\frac{\varpi_{D_1}}{1-\varpi_{D_1}}\right)^p + \left(\frac{\varpi_{D_2}}{1-\varpi_{D_2}}\right)^p\right\}^{\frac{1}{p}}}} \end{array} \right) \end{array} \right\}$$

$$(3)\ \lambda \bigodot \gimel_{Q_1} = \left\{ \begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left\{\lambda\left(\frac{\varrho_{C_1}}{1-\varrho_{C_1}}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1 - \frac{1}{1+\left\{\lambda\left(\frac{\varrho_{D_1}}{1-\varrho_{D_1}}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{\frac{1}{1+\left\{\lambda\left(\frac{1-\varpi_{C_1}}{\varpi_{C_1}}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\lambda\left(\frac{1-\varpi_{D_1}}{\varpi_{D_1}}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\}$$

$$(4)\ \gimel_{Q_1}^{\lambda} = \left\{ \begin{array}{c} \left(\sqrt[q]{\frac{1}{1+\left\{\lambda\left(\frac{1-\varrho_{C_1}}{\varrho_{C_1}}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\lambda\left(\frac{1-\varrho_{D_1}}{\varrho_{D_1}}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{1 - \frac{1}{1+\left\{\lambda\left(\frac{\varpi_{C_1}}{1-\varpi_{C_1}}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1 - \frac{1}{1+\left\{\lambda\left(\frac{\varpi_{D_1}}{1-\varpi_{D_1}}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\}$$

3.3. **Dombi Prioritized Aggregation Operators of qROFZNs.** In this section we provide number of novel prioritized aggregation operators that deal with imprecision of data, priority and relationship of criteria in MADM problems. This framework is presented in multiple ordered, weighted and prioritized aggregation operators that minimize uncertainty, enhance the reliability and adaptability of decision-making structures, owing the expanding capability of qROFZNs and flexibility of Dombi t-norms and t-conorms.

3.3.1. *q-rung Orthopair fuzzy Z numbers Dombi Prioritized Averaging Aggregation Operator(qROFZNPA$_D$).* In this subsection, the prioritized averaging aggregation operators are presented which combines the expansion of qROPFZN to minimize uncertainty with the adoptability of Dombi framework to improve decision making in the environment of uncertainty.

Definition 3.4. *Let qROFZN's are $\daleth_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$ where $i = 1, 2, 3, \ldots \xi$ then qROFZNPA$_D$ is as follows:*

$$qROFZNPA_D(\daleth_{Q_1}, \daleth_{Q_2}, \daleth_{Q_3}, \daleth_{Q_4} \ldots \daleth_{Q_\xi}) = \sum_{i=1}^{\xi} \widetilde{\Psi}_i \daleth_{Q_i} \tag{3. 9}$$

Note that $\widetilde{\Psi_i} = \frac{\Psi_i}{\sum_{i=1}^{\xi} \Psi_i}$ and $\Psi_1 = 1$, where $\Psi = \sum_{i=1}^{k-1} \aleph(\daleth_{Q_k})$, $k = 2, 3, 4, \ldots \xi$

Theorem 3.5. *Let qROFZN's are $\daleth_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$, $i = 1, 2, 3, \ldots \xi$ then qROFZNPA$_D$ is as follows:*

$$qROFZNPA_D(\daleth_{Q_1}, \daleth_{Q_2}, \daleth_{Q_3}, \daleth_{Q_4} \ldots \daleth_{Q_\xi}) = \sum_{i=1}^{\xi} \widetilde{\Psi}_i \daleth_{Q_i}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{\varrho_{C_i}^q}{1-\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1 - \frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{\varrho_{D_i}^q}{1-\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{1-\varpi_{C_i}^q}{\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{1-\varpi_{D_i}^q}{\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\} \tag{3. 10}$$

Proof. The prove of this theorem is by mathematical induction by using Definition 3.2. □

Property 3.6. ***Idempotency:*** *Let $\daleth_Q = \daleth_{Q_i} = \{(\varrho_L, \varrho_M), (\varpi_L, \varpi_M)\}$ and $i = 1, 2, 3, \ldots \xi$ are identical then*

$$qROFZNPA_D(\daleth_{Q_1}, \daleth_{Q_2}, \daleth_{Q_3}, \daleth_{Q_4} \ldots \daleth_{Q_\xi}) = \daleth_Q \tag{3. 11}$$

Proof. For idempotency suppose $\daleth_{Q_i}$ are identical then

$$qROFZNPA_D(\daleth_{Q_1},\daleth_{Q_2},\daleth_{Q_3},\daleth_{Q_4}\ldots\daleth_{Q_\xi}) = \sum_{i=1}^{\xi}\widetilde{\Psi_i}\daleth_{Q_i}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{1-\frac{1}{1+\left\{\sum_{i=1}^{\xi}\widetilde{\Psi}_i\left(\frac{\varrho_{C_i}^q}{1-\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{i=1}^{\xi}\widetilde{\Psi}_\xi\left(\frac{\varrho_{D_i}^q}{1-\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}\widetilde{\Psi}_i\left(\frac{1-\varpi_{C_i}^q}{\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}\widetilde{\Psi}_i\left(1-\frac{\varpi_{D_i}^q}{\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{1-\frac{1}{1+\left\{\left(\frac{\varrho_{C}^q}{1-\varrho_{C}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\left(\frac{\varrho_{D}^q}{1-\varrho_{D}^q}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-\varpi_{C}^q}{\varpi_{C}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\left(\frac{1-\varpi_{D}^q}{\varpi_{D}^q}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{1-\frac{1}{1+\left(\frac{\varrho_{C}^q}{1-\varrho_{C}^q}\right)}}, \sqrt[q]{1-\frac{1}{1+\left(\frac{\varrho_{D}^q}{1-\varrho_{D}^q}\right)}} \right), \\ \left(\sqrt[q]{\frac{1}{1+\left(\frac{1-\varpi_{C}^q}{\varpi_{C}^q}\right)}}, \sqrt[q]{\frac{1}{1+\left(\frac{1-\varpi_{D}^q}{\varpi_{D}^q}\right)}} \right) \end{array} \right\}$$

$$= \left\{ \left(\sqrt[q]{1-(1-\varrho_C^q)}, \sqrt[q]{1-(1-\varrho_D^q)} \right), \left(\sqrt[q]{\varpi_C^q}, \sqrt[q]{\varpi_D^q} \right) \right\}$$

$$= \{(\varrho_C,\varrho_D),(\varpi_C,\varpi_D)\}$$

$$= \daleth_Q$$

□

Property 3.7. ***Monotonicity:*** *Let two sets of qROFZN such that* $\daleth_{Q_i} = \{(\varrho_{C_i},\varrho_{D_i})(\varpi_{C_i},\varpi_{D_i})\}$ *and* $\daleth_{Q_{i'}} = \left\{(\varrho_{C_{i'}},\varrho_{D_{i'}})(\varpi_{C_{i'}},\varpi_{D_{i'}})\right\}$ *and* $\daleth_{Q_i} \subseteq \daleth_{Q_{i'}}$ *then*

$$qROFZNPA_D(\daleth_{Q_1},\daleth_{Q_2}\daleth_{Q_3}\ldots\daleth_{Q_\xi}) \leq qROFZNPA_{SS}(\daleth_{Q_{1'}},\daleth_{Q_{2'}}\daleth_{Q_{3'}}\ldots\daleth_{Q_{\xi'}}) \tag{3. 12}$$

Proof. As $\daleth_{Q_i} \subseteq \daleth_{Q_{i'}}$ thus $(\varrho_{C_i}, \varrho_{D_i}) \leq (\varrho_{C_{i'}}, \varrho_{D_{i'}})$ and $(\varpi_{C_i}, \varpi_{D_i}) \geq (\varpi_{C_{i'}}, \varpi_{D_{i'}})$
This implies

$$\sqrt[q]{1-\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{\varrho_{C_i}^q}{1-\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \leq \sqrt[q]{1-\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{\varrho_{C_{i'}}^q}{1-\varrho_{C_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}$$

$$\sqrt[q]{1-\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{\varrho_{D_i}^q}{1-\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \leq \sqrt[q]{1-\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{\varrho_{D_{i'}}^q}{1-\varrho_{D_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}$$

$$\sqrt[q]{\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{C_i}^q}{\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \geq \sqrt[q]{\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{C_{i'}}^q}{\varpi_{C_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}$$

$$\sqrt[q]{\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{D_i}^q}{\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \geq \sqrt[q]{\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{D_{i'}}^q}{\varpi_{D_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}$$

Implies that

$$\left\{\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{\varrho_{C_i}^q}{1-\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{\varrho_{D_i}^q}{1-\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}}\right), \\ \left(\sqrt[q]{\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{C_i}^q}{\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_i^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{D_i}^q}{\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}}\right)\end{array}\right\} \leq$$

$$\left\{\begin{array}{c}\left(\sqrt[q]{1-\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{\varrho_{C_{i'}}^q}{1-\varrho_{C_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{\varrho_{D_{i'}}^q}{1-\varrho_{D_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}\right), \\ \left(\sqrt[q]{\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{C_{i'}}^q}{\varpi_{C_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i'}^\xi \widetilde{\Psi}\left(\frac{1-\varpi_{D_{i'}}^q}{\varpi_{D_{i'}}^q}\right)^p\right\}^{\frac{1}{p}}}}\right)\end{array}\right\}$$

Hence

$$qROFZNPA_D(\daleth_{Q_1}, \daleth_{Q_2}\daleth_{Q_3} \ldots \daleth_{Q_\xi}) \leq qROFZNPA_D(\daleth_{Q_{1'}}, \daleth_{Q_{2'}}\daleth_{Q_{3'}} \ldots \daleth_{Q_{\xi'}})$$

□

Property 3.8. ***Boundedness:*** *For any* $\daleth_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i}), (\varpi_{C_i}, \varpi_{D_i})\}$, $i = 1, 2, 3, \ldots, \xi$ *be any set of qROFZN such that* $\daleth_{Q_{DX}} = max\daleth_{Q_i}$ *and* $\daleth_{Q_{DN}} = min\daleth_{Q_i}$ *then*

$$\daleth_{Q_{DN}} \leq qROFZNPA_D(\daleth_{Q_1}, \daleth_{Q_2}\daleth_{Q_3} \ldots \daleth_{Q_\xi}) \leq \daleth_{Q_{DX}} \tag{3. 13}$$

Proof. It can be proved as above. □

3.8.1. *q-rung orthopair fuzzy Z numbers Dombi weighted prioritized averaging aggregation operators qROFZNWPA$_D$.* In this subsection, the weighted prioritized averaging aggregation operators are presented which combines the expansion of qROPFZN to minimize uncertainty with the adoptability of Dombi framework to improve decision making in the environment of uncertainty.

Definition 3.9. *Let qROFZN's are* $\daleth_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$ *where* $i = 1, 2, 3, \ldots \xi$ *then qROFZNWPA$_D$ is as follows:*

$$qROFZNWPA_D(\daleth_{Q_1}, \daleth_{Q_2}, \daleth_{Q_3}, \daleth_{Q_4} \ldots \daleth_{Q_\xi}) = \sum_{i=1}^{\xi} \widetilde{\Psi}_i \daleth_{Q_i}$$

Note that $\widetilde{\Psi_i} = \frac{\widetilde{\omega_i}\Psi_i}{\sum_{i=1}^{\xi} \widetilde{\omega_i}\Psi_i}$ *and* $\Psi_1 = 1$, *where* $\Psi = \sum_{i=1}^{k-1} \aleph(\daleth_{Q_k})$, $k = 2, 3, 4, \ldots \xi$ *and the weight is given as* $\sum_{i=1}^{\xi} \widetilde{\omega}_i = 1$, $\widetilde{\omega}_i \in [0, 1]$.

Theorem 3.10. *Let qROFZN's are* $\daleth_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$ *where* $i = 1, 2, 3, \ldots \xi$ *then qROFZNWPA$_D$ is as follows:*

$$qROFZNOPA_D(\daleth_{Q_1}, \daleth_{Q_2}, \daleth_{Q_3}, \daleth_{Q_4} \ldots \daleth_{Q_\xi}) = \sum_{i=1}^{\xi} \widetilde{\Psi}_i \daleth_{Q_i}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{1 - \frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi}_i)\left(\frac{\varrho_{C_i}^q}{1-\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1 - \frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi}_i)\left(\frac{\varrho_{D_i}^q}{1-\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi}_i)\left(\frac{1-\varpi_{C_i}^q}{\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi}_i)\left(\frac{1-\varpi_{D_i}^q}{\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\} \tag{3. 14}$$

Proof. This proof is similar as theorem 3.5. □

Property 3.11. ***Idempotency:*** *Let* $\daleth_Q = \daleth_{Q_i} = \{(\varrho_L, \varrho_M), (\varpi_L, \varpi_M)\}$ *and* $i = 1, 2, 3, \ldots \xi$ *are identical then*

$$qROFZNWPA_D(\daleth_{Q_1}, \daleth_{Q_2}, \daleth_{Q_3}, \daleth_{Q_4} \ldots \daleth_{Q_\xi}) = \daleth_Q \tag{3. 15}$$

Property 3.12. ***Monotonicity:*** *Let* $\daleth_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})(\varpi_{C_i}, \varpi_{D_i})\}$ *and* $\daleth_{Q_{i'}} = \left\{(\varrho_{C_{i'}}, \varrho_{D_{i'}})(\varpi_{C_{i'}}, \varpi_{D_{i'}})\right\}$ *be two sets of qROFZN such that* $\daleth_{Q_i} \subseteq \daleth_{Q_{i'}}$ *then*

$$qROFZNWPA_D(\daleth_{Q_1}, \daleth_{Q_2}\daleth_{Q_3} \ldots \daleth_{Q_\xi}) \leq qROFZNWPA_D(\daleth_{Q_{1'}}, \daleth_{Q_{2'}}\daleth_{Q_{3'}} \ldots \daleth_{Q_{\xi'}})$$

Property 3.13. ***Boundedness:*** *For any* $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i}), (\varpi_{C_i}, \varpi_{D_i})\}$, $i = 1, 2, 3, \ldots, \xi$ *be any set of qROFZN such that* $\gimel_{Q_{DX}} = max\gimel_{Q_i}$ *and* $\gimel_{Q_{DN}} = min\gimel_{Q_i}$ *then*

$$\gimel_{Q_{DN}} \leq qROFZNWPA_D(\gimel_{Q_1}, \gimel_{Q_2}\gimel_{Q_3} \ldots \gimel_{Q_\xi}) \leq \gimel_{Q_{DX}}$$

3.13.1. *q-rung Orthopair fuzzy Z numbers Dombi prioritized Geometric Aggregation Operator(qROFZNPG$_D$).* In this subsection, the prioritized geometric aggregation operators are presented which combines the expansion of qROPFZN to minimize uncertainty with the adoptability of Dombi framework to improve decision making in the environment of uncertainty.

Definition 3.14. *Let qROFZN's are* $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$ *where* $i = 1, 2, 3, \ldots \xi$ *then qROFZNPG$_D$ is as follows:*

$$qROFZNPG_D(\gimel_{Q_1}, \gimel_{Q_2}, \gimel_{Q_3}, \gimel_{Q_4} \ldots \gimel_{Q_\xi}) = \prod_{i=1}^{\xi} \gimel_{Q_i}^{\widetilde{\Psi_i}} \tag{3. 16}$$

Note that $\widetilde{\Psi_i} = \frac{\Psi_i}{\sum_{i=1}^{\xi} \Psi_i}$ *and* $\Psi_1 = 1$, *where* $\Psi = \sum_{i=1}^{k-1} \aleph(\gimel_{Q_k})$, $k = 2, 3, 4, \ldots \xi$

Theorem 3.15. *Let qROFZN's are* $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$ *where* $i = 1, 2, 3, \ldots \xi$ *then qROFZNPG$_D$ is as follows:*

$$qROFZNPG_D(\gimel_{Q_1}, \gimel_{Q_2}, \gimel_{Q_3}, \gimel_{Q_4} \ldots \gimel_{Q_\xi}) = \prod_{i=1}^{\xi} \gimel_{Q_i}^{\widetilde{\Psi}_i}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{\dfrac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{1-\varrho_{C_i}^q}{\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\dfrac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{1-\varrho_{D_i}^q}{\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{1-\dfrac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{\varpi_{C_i}^q}{1-\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1-\dfrac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{\varpi_{D_i}^q}{1-\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\} \tag{3. 17}$$

Proof. This proof is similar as theorem 3.5. □

Property 3.16. ***Idempotency:*** *Let* $\gimel_Q = \gimel_{Q_i} = \{(\varrho_L, \varrho_M), (\varpi_L, \varpi_M)\}$ *and* $i = 1, 2, 3, \ldots \xi$ *are identical then*

$$qROFZNPG_D(\gimel_{Q_1}, \gimel_{Q_2}, \gimel_{Q_3}, \gimel_{Q_4} \ldots \gimel_{Q_\xi}) = \gimel_Q \tag{3. 18}$$

Property 3.17. ***Monotonicity:*** *Let* $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})(\varpi_{C_i}, \varpi_{D_i})\}$ *and* $\gimel_{Q_{i'}} = \left\{(\varrho_{C_{i'}}, \varrho_{D_{i'}})(\varpi_{C_{i'}}, \varpi_{D_{i'}})\right\}$ *be two sets of qROFZN such that* $\gimel_{Q_i} \subseteq \gimel_{Q_{i'}}$ *then*

$$qROFZNPG_D(\gimel_{Q_1}, \gimel_{Q_2}\gimel_{Q_3} \ldots \gimel_{Q_\xi}) \leq qROFZNPG_D(\gimel_{Q_{1'}}, \gimel_{Q_{2'}}\gimel_{Q_{3'}} \ldots \gimel_{Q_{\xi'}}) \tag{3. 19}$$

Property 3.18. ***Boundedness:*** *For any* $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i}), (\varpi_{C_i}, \varpi_{D_i})\}$, $i = 1, 2, 3, \ldots, \xi$ *be any set of qROFZN such that* $\gimel_{Q_{DX}} = max\gimel_{Q_i}$ *and* $\gimel_{Q_{DN}} = min\gimel_{Q_i}$ *then*

$$\gimel_{Q_{DN}} \leq qROFZNPG_D(\gimel_{Q_1}, \gimel_{Q_2}\gimel_{Q_3} \ldots \gimel_{Q_\xi}) \leq \gimel_{Q_{DX}} \tag{3. 20}$$

3.18.1. *q-rung Orthopair fuzzy Z numbers Dombi weighted prioritized Geometric Aggregation Operator(qROFZNWPG$_D$).* In this subsection, the weighted prioritized geometric aggregation operators are presented which combines the expansion of qROPFZN to minimize uncertainty with the adoptability of Dombi framework to improve decision making in the environment of uncertainty.

Definition 3.19. *Let qROFZN's are $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$ where $i = 1, 2, 3, \ldots \xi$ then qROFZNWPG$_D$ is as follows:*

$$qROFZNWPG_D(\gimel_{Q_1}, \gimel_{Q_2}, \gimel_{Q_3}, \gimel_{Q_4} \ldots \gimel_{Q_\xi}) = \prod_{i=1}^{\xi} \gimel_{Q_i}^{\widetilde{\Psi_i}} \tag{3. 21}$$

Note that $\widetilde{\Psi_i} = \frac{\widetilde{\omega_i}\Psi_i}{\sum_{i=1}^{\xi}\widetilde{\omega_i}\Psi_i}$ and $\Psi_1 = 1$, where $\Psi = \sum_{i=1}^{k-1} \aleph(\gimel_{Q_k})$, $k = 2, 3, 4, \ldots \xi$ and the weight is given as $\prod_{i=1}^{\xi} \widetilde{\omega}_i = 1$, $\widetilde{\omega}_i \in [0, 1]$.

Theorem 3.20. *Let qROFZN's are $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})\}$ where $i = 1, 2, 3, \ldots \xi$ then qROFZNWPG$_D$ is as follows:*

$$qROFZNWPG_D(\gimel_{Q_1}, \gimel_{Q_2}, \gimel_{Q_3}, \gimel_{Q_4} \ldots \gimel_{Q_\xi}) = \prod_{i=1}^{\xi} \gimel_{Q_i}^{\widetilde{\Psi}_i}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{1-\varrho_{C_i}^q}{\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{1-\varrho_{D_i}^q}{\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{1-\frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{\varpi_{C_i}^q}{1-\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{i=1}^{\xi}\left(\widetilde{\Psi_i}\right)\left(\frac{\varpi_{D_i}^q}{1-\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\} \tag{3. 22}$$

Proof. This proof is similar as theorem 3.5. ☐

Property 3.21. ***Idempotency:*** *Let $\gimel_Q = \gimel_{Q_i} = \{(\varrho_L, \varrho_M), (\varpi_L, \varpi_M)\}$ and $i = 1, 2, 3, \ldots \xi$ are identical then*

$$qROFZWPG_D(\gimel_{Q_1}, \gimel_{Q_2}, \gimel_{Q_3}, \gimel_{Q_4} \ldots \gimel_{Q_\xi}) = \gimel_Q \tag{3. 23}$$

Property 3.22. ***Monotonicity:*** *Let $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i})(\varpi_{C_i}, \varpi_{D_i})\}$ and $\gimel_{Q_{i'}} = \left\{(\varrho_{C_{i'}}, \varrho_{D_{i'}})(\varpi_{C_{i'}}, \varpi_{D_{i'}})\right\}$ be two sets of qROFZN such that $\gimel_{Q_i} \subseteq \gimel_{Q_{i'}}$ then*

$$qROFZNWPG_D(\gimel_{Q_1}, \gimel_{Q_2}\gimel_{Q_3} \ldots \gimel_{Q_\xi}) \leq qROFZNWPG_D(\gimel_{Q_{1'}}, \gimel_{Q_{2'}}\gimel_{Q_{3'}} \ldots \gimel_{Q_{\xi'}}) \tag{3. 24}$$

Property 3.23. ***Boundedness:*** *For any $\gimel_{Q_i} = \{(\varrho_{C_i}, \varrho_{D_i}), (\varpi_{C_i}, \varpi_{D_i})\}$, $i = 1, 2, 3, \ldots, \xi$ be any set of qROFZN such that $\gimel_{Q_{DX}} = max\gimel_{Q_i}$ and $\gimel_{Q_{DN}} = min\gimel_{Q_i}$ then*

$$\gimel_{Q_{DN}} \leq qROFZNWPG_D(\gimel_{Q_1}, \gimel_{Q_2}\gimel_{Q_3} \ldots \gimel_{Q_\xi}) \leq \gimel_{Q_{DX}} \tag{3. 25}$$

4. A Hybrid qROFZN$_D$ Aggregation Framework for Multi-Attribute Decision Analysis

This section outlines the developed algorithmic approach to solve uncertain complex MADM problems. The proposed method seeks to extend the capabilities of traditional MADM methods by skillfully combining the use of q-rung orthopair fuzzy Z-numbers (qROFZNs) with Dombi parametric aggregation operators. The use of qROFZNs allows for the simultaneous representation of membership, non-membership and reliability values, which effectively captures both the uncertainty and reliability of decision-making experts' opinions. Moreover, by integrating Dombi T-norm and T-conorm operators, the approach provides a dynamic aggregation framework, offering an adjustable compensation degree between conflicting criteria, a crucial feature in environmental decision-making. The developed approach also leverages both averaging and geometric aggregation operators to achieve stability and robustness in information aggregation.

For the numerical illustration of the MADM procedure, let the set of alternatives be denoted as $\widetilde{Q} = \{Q_1, Q_2, Q_3, ..., Q_\imath\}$, $\widetilde{\ell} = \{\ell_1, \ell_2, \ell_3, ..., \ell_\jmath\}$ as attributes and weights of attributes as $\widetilde{\omega} = \{\widetilde{\omega}_1, \widetilde{\omega}_2, \widetilde{\omega}_3\}$, where sum of the weights equal to 1 and weight is between 0 and 1. These components form the basis of the decision matrix, which is subsequently processed using the proposed qROFZN$_D$ -based aggregation framework to obtain a reliable ranking of alternatives under uncertainty.

Algorithm A Novel qROFZN$_D$ -Based MADM Procedure with Dombi Prioritized Aggregation

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- Collect the initial decision information in the form of a q-rung orthopair fuzzy Z-number (qROFZN) based decision matrix, where expert evaluations of $\imath$ alternatives $\widetilde{Q} = \{Q_1, Q_2, Q_3, ..., Q_\hbar\}$, where $(\imath = 1, 2, ..., \hbar)$ with respect to attributes $\widetilde{\ell} = \{\ell_1, \ell_2, \ell_3, ..., \ell_{\Bbbk}\}$, where $(\jmath = 1, 2, ..., \Bbbk)$ are expressed using linguistic terms and then converted into qROFZN values to properly represent both uncertainty and reliability in the assessment data.
- Usually the MADM compare two type of criteria: favorable and un-favorable, if we use the complement to unfavorable criteria then it will converted in favorable.
- To carry out the MADM process, the collection of attributes of every alternative integrated by using proposed arithmetic and geometric aggregation in accordance of Dombi T-norm and T-conorm.
- By using definition 2.6, calculate the score function for every alternatives.
- Ranked the alternative after calculating score function and select the optimum option.

The following figure 1 showing flowchart of above proposed algorithm:

5. Case Study: Selection of Optimum Strategy among Climate Change Adaptation Strategies

In 21st, the world concerns to climate change anxiously, which has a predominantly negative impact on cities and urban areas. Powerful heat waves, long time droughts, more

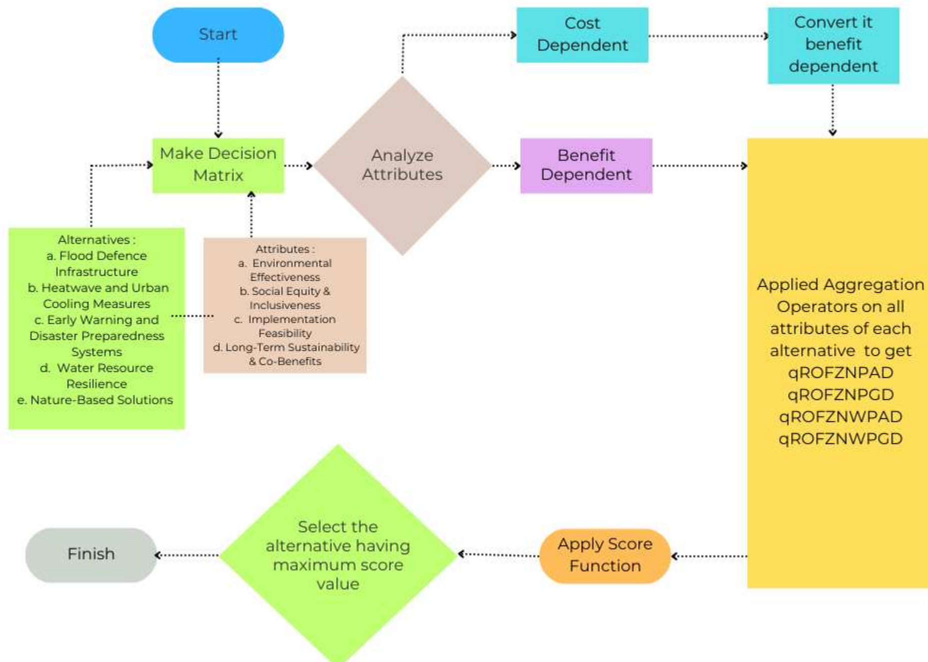

FIGURE 1. Decision-Making Flowchart

frequent floods, increasing sea levels, and irregular weather patterns are intimidating urban infrastructure, public health, and socioeconomic stability. Due to the concentration of human, resources, and serious services, cities are far more vulnerable to climate-related threats than rural areas. In order to safeguard urban resilience and protection populations from future unexpected occurrences, developing effective adaptation plans has become a main priority. In metropolitan and regional settings, adaptation strategies might be operational (like flood walls, resilient housing, and cultured drainage systems) or ecosystem-based (like urban greening, wetlands renovation, and coastal protection). Social and economic initiatives like disaster preparedness plans, good governance, and community-based timely warning systems are correspondingly crucial. A number of variables, including population densities, governance organisms, economic capabilities, and regional disparities, affect how effective these strategies are. In order to choose effective results, it is vital to carefully consider long-term sustainability and effective goals, available resources, and local susceptibilities.

Establishing primacies for strategies is a difficult task, even with the increasing worldwide attention and significant financial funds in adaptation. The need to settle short-term viability with long-term flexibility, competing demands on metropolitan budgets, and uncertain climate forecasts sometimes confuse decision-making. Additionally, adaptation approaches must be tailored to precise geographical conditions because procedures that are effective in one atmosphere may not be as appropriate in another. Urban' limited capacity to overcome all susceptibilities simultaneously and the urgency of climate significances underscore the necessity for a systematic review and prioritization of adaptation alternatives. This assurances that funds are used for the most appropriate and efficient projects.

There are some adaptation strategies (alternatives) for changing climate:

- **Flood Defense Infrastructure (Q_1)-** Levees, sea walls, storm outpouring barriers, and unconventional drainage systems are illustrations of engineering solutions that make up Flood Defense Infrastructure, which guards urban areas from increasing sea levels and heavy rains [27]. These structures are dynamic for guarding flooding, preserving vital infrastructure, and saving human lives and resources. Despite their great success, they require ongoing renovation, maintenance and may have ecological trade-offs if not balanced with natural resources.
- **Heatwave and Urban Cooling Measures (Q_2)-** Provide infrastructure like cool roofs, reflecting pavements, shaded green passages, and increased urban ventilation are examples of "Heatwave and Urban Cooling Measures" that help minimize extreme heat in metropolitan areas [26]. By using these steps, residents' thermal ease increases, minimize health risks and use less energy for cooling. By undertaking the growing effects of climate-induced heatwaves, this aids to build long-term metropolitan resilience.
- **Early Warning and Disaster Preparedness Systems (Q_3)-** Early Warning and Disaster Preparedness Systems employ real-time observing, monitoring, weather forecasting, and community-based alarm systems to predict extreme climate events. They enhance cities' ability to quick respond, which lowers the number of causalities, property damage, and dis-connected to essential services. By enhancing awareness and readiness, these solutions help communities in becoming stronger to the increasingly recurrent climate-related disasters.
- **Water Resource Resilience (Q_4)-** Water Resource Resilience pursues to ensure ecological water supplies by putting practices like rainwater collection, groundwater recharge, well-organized irrigation, and drought-resistant structure into practice. These policies help metropolises adapt to longer droughts and increased rainfall unpredictability brought on by climate change [12]. Improving water elasticity ensures long-term supply, reduces susceptibility, and supports ecosystem health and human needs.
- **Nature-Based Solutions (Urban Greening & Wetlands Restoration) (Q_5)-** The "Nature-Based Solutions (Urban Greening & Wetlands Restoration)" refers to the use of natural ecologies, such as metropolitan plantations, wetlands, green top roofs and stagnated corridors, to increase climate flexibility. They absorb extra rainfall, clean and improve air quality, minimize urban heat islands, and save wildlife. These policies make cities healthier and more habitable while also endorsing social well-being and the atmosphere [14].

The following evaluation criteria (attributes) which are discussed for each alternatives:

(1) **Environmental Effectiveness** (ℓ_1)- Environmental Effectiveness refers to an adaptation policy's ability to minimize climate-related risks such as floods, heat waves, or water scarcity. It validates how much the actions protects ecologies and increases overall environmental flexibility.
(2) **Social Equity & Inclusiveness** (ℓ_2)- Social Equity & Inclusiveness emphasizes how an adaptation policy fairly welfare a variety of social groups, especially disadvantaged and marginalized communities. It ensures that policies that support weather resilience improve social well-being, equity, and equality.
(3) **Implementation Feasibility** (ℓ_3)- Implementation Feasibility gauges how operational it is to put an adaptation policy into action in terms of governance, institutional readiness, and technological volume. It reflects how easily a measure can be put into effect given the limitations of the existing resources and legal agenda.
(4) **Long-Term Sustainability & Co-Benefits** (ℓ_4)- The Long-Term Sustainability & Co-Benefits of an adaptation policy measures whether it provides long-term resilience composed with additional benefits like improved public health, biodiversity protection, or metropolitan livability. It showoff activities that have frequent beneficial benefits in addition to their core function in environmental adaptation.

In order to handle the complication of measuring different and unpredictable adaption keys, MADM provides a mathematical framework that strikes a balance among social, environmental, and economic factors. Because it takes into consideration both the accuracy of data in climate adaptation planning and the unpredictability of expert sentiments, the q-rung orthopair fuzzy Z-number (qROFZ) approach is mainly well-suited for this subject under context of Dombi aggregations.

5.1. **Numerical Example.** We will now look at a numerical example to demonstrate the application and value of the proposed methodology in resolving the case study.

The decision-making dataset in this study is built using a case study approach rather than large-scale empirical data. The dataset is made up of a limited number of climate change adaptation options that have been assessed based on several sustainability and environmental efficacy factors. Domain experts provide the evaluation data, which uses linguistic factors to convey qualitative assessments like efficacy, resilience, and dependability. These language evaluations are then transformed using preset linguistic scales into qROFZN representations, which enable the simultaneous modeling of expert opinion confidence and uncertainty. The gathered data go through a preprocessing step before analysis, which involves creating the qROFZN-based decision matrix, normalizing the criteria values, and confirming the consistency of expert inputs. In the end, this systematic preprocessing guarantees that the data are appropriate for aggregate using the suggested Dombi prioritized operators, allowing for a solid and trustworthy assessment of the adaptation strategies under consideration.

: Step 1. Five options are examined, and each is evaluated based on specific attributes $\{\ell_1, \ell_2, \ell_3, ..., \ell_\jmath\}$. Three expert decision matrices are taken into account, as indicated in Table 1 with q taken as 3.
: Step 2: There is no need to normalized because the attributes are favorable attributes.

TABLE 1. Decision matrix by experts

	$\boldsymbol{\ell_1}$	$\boldsymbol{\ell_2}$	$\boldsymbol{\ell_3}$	$\boldsymbol{\ell_4}$
Q_1	((0.50, 0.40), (0.30, 0.20))	((0.60, 0.30), (0.40, 0.70))	((0.70, 0.40), (0.50, 0.50))	((0.70, 0.60), (0.40, 0.30))
Q_2	((0.80, 0.70), (0.40, 0.20))	((0.60, 0.40), (0.50, 0.40))	((0.50, 0.60), (0.40, 0.30))	((0.40, 0.50), (0.80, 0.50))
Q_3	((0.50, 0.50), (0.80, 0.10))	((0.60, 0.90), (0.30, 0.20))	((0.40, 0.70), (0.20, 0.40))	((0.60, 0.70), (0.30, 0.40))
Q_4	((0.70, 0.30), (0.30, 0.20))	((0.40, 0.40), (0.20, 0.60))	((0.50, 0.50), (0.60, 0.10))	((0.30, 0.60), (0.20, 0.20))
Q_5	((0.70, 0.90), (0.60, 0.20))	((0.60, 0.90), (0.70, 0.30))	((0.60, 0.90), (0.50, 0.20))	((0.80, 0.90), (0.30, 0.30))

: Step 3: Table 2 presents the prioritized values and the corresponding qROPFZNPA$_D$ and qROFZNPG$_D$ results obtained by applying the prioritized values and aggregation operators. The weights used here are $\widetilde{\omega} = \{0.3, 0.4, 0.2, 0.1\}$.

TABLE 2. Prioritized Values along with corresponding qROFZNPA$_D$ & qROFZNPG$_D$

Alternatives	Prioritized Values				qROFZNPA$_D(\widetilde{\Psi_i})$	qROFZNPG$_D(\widetilde{\Psi_i})$
Q_1	0.242	0.138	0.247	0.372	((0.669 , 0.530) , (0.357 , 0.247))	((0.584 , 0.381) , (0.431 , 0.555))
Q_2	0.206	0.152	0.259	0.381	((0.692 , 0.608) , (0.444 , 0.254))	((0.452 , 0.490) , (0.733 , 0.440))
Q_3	0.206	0.120	0.274	0.398	((0.559 , 0.797) , (0.241 , 0.129))	((0.468 , 0.603) , (0.686 , 0.375))
Q_4	0.229	0.131	0.251	0.387	((0.592 , 0.540) , (0.221 , 0.125))	((0.345 , 0.371) , (0.495 , 0.449))
Q_5	0.188	0.142	0.267	0.402	((0.749 , 0.900) , (0.345 , 0.224))	((0.656 , 0.900) , (0.579 , 0.274))

Similarly, Table 3 presents the weighted prioritized values along with the corresponding qROPFZNWPA$_D$ and qROFZNWPG$_D$ results obtained by applying the prioritized values and aggregation operators.

TABLE 3. Weighted Prioritized Values along with corresponding qROFZNWPA$_D$ & qROFZNWPG$_D$

Alternatives	Weighted Prioritized Values				qROFZNWPA$_D(\widetilde{\Psi_i})$	qROFZNWPG$_D(\widetilde{\Psi_i})$
Q_1	0.338	0.257	0.230	0.173	((0.592 , 0.401) , (0.979 , 0.910))	((0.561 , 0.354) , (0.425 , 0.599))
Q_2	0.290	0.286	0.243	0.179	((0.746 , 0.641) , (0.936 , 0.983))	((0.492 , 0.470) , (0.678 , 0.406))
Q_3	0.302	0.236	0.267	0.194	((0.551 , 0.852) , (0.835 , 0.990))	((0.467 , 0.582) , (0.715 , 0.354))
Q_4	0.326	0.250	0.238	0.184	((0.647 , 0.419) , (0.972 , 0.949))	((0.378 , 0.350) , (0.492 , 0.494))
Q_5	0.272	0.274	0.258	0.194	((0.678 , 0.900) , (0.898 , 0.993))	((0.637 , 0.900) , (0.622 , 0.269))

: Step 4-5. By applying Definition 2.6, the score values of each alternative under all the above operators are computed, and based on these scores, the alternatives are ranked in ascending order, as summarized in Table 4 :

TABLE 4. Score Values according to qROFZN$_D$ aggregation operators

Sr.No.	Operator	Score Values					Ranking
		$F(\daleth_{Q_1})$	$F(\daleth_{Q_2})$	$F(\daleth_{Q_3})$	$F(\daleth_{Q_4})$	$F(\daleth_{Q_5})$	
1	qROFZNPA$_D$	0.633	0.653	0.707	0.646	0.798	$F(\daleth_{Q_1} < F(\daleth_{Q_4}) < F(\daleth_{Q_2}) < F(\daleth_{Q_3}) < F(\daleth_{Q_5})$
2	qROFZNPG$_D$	0.491	0.449	0.512	0.452	0.715	$F(\daleth_{Q_2}) < F(\daleth_{Q_4}) < F(\daleth_{Q_1}) < F(\daleth_{Q_3}) < F(\daleth_{Q_5})$
3	qROFZNWPA$_D$	0.206	0.292	0.305	0.195	0.368	$F(\daleth_{Q_4}) < F(\daleth_{Q_1}) < F(\daleth_{Q_2}) < F(\daleth_{Q_3}) < F(\daleth_{Q_5})$
4	qROFZNWPG$_D$	0.472	0.477	0.509	0.444	0.703	$F(\daleth_{Q_4}) < F(\daleth_{Q_1}) < F(\daleth_{Q_2}) < F(\daleth_{Q_3}) < F(\daleth_{Q_5})$

The ranking results are shown in figure 2

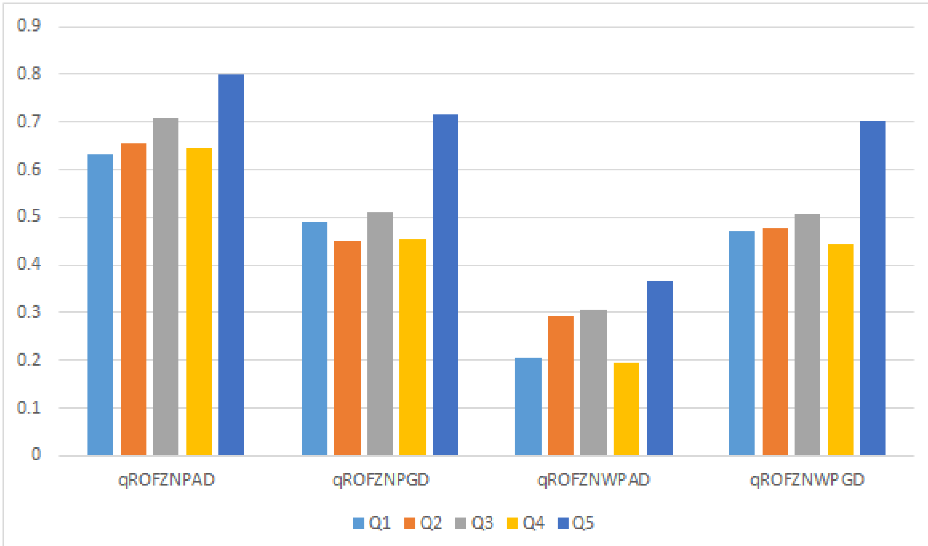

FIGURE 2. Graphical Representation of qROFZN$_D$ Ranking

6. IMPROVED DECISION ANALYSIS BY USING EDAS METHOD

In this section we use EDAS method over qROFZN and a numerical example.

The EDAS (Evaluation Based on Distance from Average Solution) method is a MADM strategy that rates alternatives by evaluating how faraway each one deviates from the average performance value. It facilitate a fair and balanced evaluation by selecting options with larger positive distances and smaller negative ones. Ghorabaee et al. [6] suggested this strategy for better choice analysis in complicated environment.

6.1. EDAS method over qROFZN's Algorithm Flow.

Here we use EDAS method for selection of optimum adaptation climate change strategy to check the authenticity of our proposed qROFZNPA$_D$ method. The selection of optimum strategy has following algorithm:

Algorithm EDAS-Based Multi-Criteria Decision-Making Approach Using qROFZN Information

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Step-1:: Collect the initial decision information in the form of a q-rung orthopair fuzzy Z-number (qROFZN) based decision matrix, where expert evaluations of $\imath$ alternatives $\widetilde{Q} = \{Q_1, Q_2, Q_3, ..., Q_\hbar\}$, where $(\imath = 1, 2, ..., \hbar)$ with respect to attributes $\widetilde{\ell} = \{\ell_1, \ell_2, \ell_3, ..., \ell_{\Bbbk}\}$, where $(\jmath = 1, 2, ..., \Bbbk)$ are expressed using linguistic terms and then converted into qROFZN values to properly represent both uncertainty and reliability in the assessment data.

Step-2:: Usually the MADM compare two type of criteria: favorable and un-favorable, if we use the complement to unfavorable criteria then it will converted in favorable.

Step-3:: Use qROFZNWPG$_D$ to evaluate performance of alternatives.

$$qROFZNWPG_D(\daleth_{Q_1}, \daleth_{Q_2}, \daleth_{Q_3}, \daleth_{Q_4} \dot{\daleth}_{Q_\xi}) = \prod_{i=1}^{\xi} \daleth_{Q_i}^{\widetilde{\Psi}_i}$$

$$= \left\{ \begin{array}{c} \left(\sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi_i})\left(\frac{1-\varrho_{C_i}^q}{\varrho_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{\frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi_i})\left(\frac{1-\varrho_{D_i}^q}{\varrho_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right), \\ \left(\sqrt[q]{1-\frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi_i})\left(\frac{\varpi_{C_i}^q}{1-\varpi_{C_i}^q}\right)^p\right\}^{\frac{1}{p}}}}, \sqrt[q]{1-\frac{1}{1+\left\{\sum_{i=1}^{\xi}(\widetilde{\Psi_i})\left(\frac{\varpi_{D_i}^q}{1-\varpi_{D_i}^q}\right)^p\right\}^{\frac{1}{p}}}} \right) \end{array} \right\}$$

Step-4:: Calculate average solution (A_vS) of all alternatives of each attribute attained by
qROFZNWPG$_D$ i.e.,

$$A_vS_j = \frac{\sum_{i=1}^{n} \daleth_{Q_{ij}}}{n} \; where \; j = 1, 2, 3, ... \tag{6. 26}$$

Step-5:: Measure the positive distance from the average solution (PDA_vS) and negative distance from the average solution (NDA_vS) i.e.,

$$PDA_vS = [PDA_vS_{ij}]_{n\times m}$$

$$NDA_vS = [NDA_vS_{ij}]_{n\times m}$$

where

$$PDA_vS_{ij} = \frac{\Delta(0, (\daleth_{Q_{ij}} - A_VS_j))}{A_VS_j}$$

and

$$NDA_vS_{ij} = \frac{\Delta(0, (A_VS_j - \daleth_{Q_{ij}}))}{A_VS_j}$$

Step-6:: Calculate weighted sum of PDA_vS and NDA_vS :

$$SP_i = \sum_{j=1}^{m} \omega_j PDA_vS_j \tag{6. 27}$$

and

$$SN_i = \sum_{j=1}^{m} \omega_j NDA_vS_j, \tag{6. 28}$$

where $\omega_j \in [0, 1]$ and $\sum_{j=1}^{m} \omega_j = 1$, $i = 1, 2, 3, ...n$.

Step-7:: Now find the normalized values of $SPDA_vS$ and $SNDA_vS$ in the following ways:

$$NSP_i = \frac{SPDA_vS_i}{\Delta(SPDA_vS_i)} \tag{6. 29}$$

and

$$NSN_i = \frac{SNDA_vS_i}{\Delta(SNDA_vS_i)}, \tag{6. 30}$$

where $i = 1, 2, 3, ...n$

Step-8:: Calculate the appraisal scores (AS) of all alternatives in following way:

$$AS = \frac{NSP_i + 1 - NSN_i}{2}, \tag{6. 31}$$

where $0 \leq AS \leq 1$

Step-9:: Ranked the result of AS in ascending order, the largest value AS is the optimum alternative.

6.2. **Numerical Example.** Here,the previous numerical MADM problem is gain computed by EDAS method to verify the validity, effectiveness and efficiency.

: **Step-1,2:** Same procedure follows as previously proposed operating methodology adopt.

: **Step-3:** For evaluation of information utilize the qROFZNWPG$_D$ is given in table 5:

TABLE 5. The qROFZNWPG$_D$ Operator.

$\beth_{Q_i}$	$\widetilde{\Psi}_i$
Q_1	((0.5619 , 0.3546) , (0.4255 , 0.5990))
Q_2	((0.4925 , 0.4701) , (0.6781 , 0.4066))
Q_3	((0.4670 , 0.5821) , (0.7153 , 0.3544))
Q_4	((0.3782 , 0.3507) , (0.4924 , 0.4949))
Q_5	((0.6373 , 0.9000) , (0.6224 , 0.2691))

: **Step-4:** The average of all alternatives for each attribute is obtained by equation 6. 26 is as follows:

$$A_vS_j = \{\ 0.507, 0.531, 0.586, 0.424\ \}$$

: **Step-5:** The table 8 includes the positive and negative distance from A_vS_j:

TABLE 6. Positive and Negative distance from average solution

TABLE 7. *

Positive distance from average solution

PDA_vS			
0.1074	0	0	0.4100
0	0	0.1557	0
0	0.0952	0.2190	0
0	0	0	0.1650
0.2559	0.6931	0.0606	0

TABLE 8. *

Negative distance from average solution

NDA_vS			
0	0.3327	0.2747	0
0.0293	0.1154	0	0.0428
0.0795	0	0	0.1657
0.2546	0.3402	0.1608	0
0	0	0	0.3664

: Step-6 to 9: Calculate weighted positive and negative distance from average of all alternative with weights $= \{0.25, 0.25, 0.25, 0.25\}$, normalized values and appraisal score with their descending ranking index are shown in consolidated table 9.

TABLE 9. Weighted positive and negative distance from average solution

Alternative	SP_i	SN_i	NSP_i	NSN_i	AS	RI
Q_1	0.1293	0.1518	0.5124	0.8038	0.3542	4
Q_2	0.0389	0.0469	0.1542	0.2483	0.4529	3
Q_3	0.0785	0.0613	0.3112	0.3245	0.4933	2
Q_4	0.0412	0.1889	0.1633	1	0.0816	5
Q_5	0.2524	0.0916	1	0.4849	0.7575	1

7. DISCRETE EVOLUTION SEMIGROUP

Using these premises of children's learning as her guideline, the teacher initiated the mutual construction of a different set of norms for mathematics lessons as she acted to help the students reconceptualize their role during mathematics instruction. Her intention was for the children to figure things out for themselves and to express their ideas in the public arena of whole-class discussions. Additionally, during small-group work she expected them to cooperate and work together to solve problems and to agree on an answer. Her expectation that the children would express their thoughts placed the students under the obligation of having to recall their solutions and explain them to others during the whole-class discussion.

8. RESULT & ANALYSIS

In this section we will provide the results and make a through discussion on the validity, robustness, effectiveness and efficiency of proposed method. To complete evaluation process we compared the proposed method with traditionally renowned EDAS method. Additionally performed a sensitivity analysis to verified the results' robustness for different parameter configurations, presenting more details regarding how the decision-making process would be stable. At the end, overall practical implications of the results are examined in terms of how applicable the proposed methodology is to real-world applications.

8.1. **Comparative Analysis.** It is crucial to compare the suggested framework's consistency and dependability to a recognized benchmark approach before presenting the findings of the comparison analysis. Such a comparison offers insights into the relative performance of the created qROFZNDombi-based MADM strategy under the identical decision environment in addition to helping to validate its efficacy. Thus, using the identical numerical case study, a thorough comparison with the EDAS method is carried out in the subsequent subsection. The validity, stability, and discriminative power of the suggested method are evaluated by analyzing the outcomes in terms of score values and ranking consistency.

To demonstrate the versatility and validity of the proposed approach, the proposed algorithms are applied in the framework of q-rung orthopair fuzzy Z-numbers (qROFZNs) and compared with the EDAS method. The comparative study is conducted based on the efficiency, effectiveness, stability and validity. The proposed method and the EDAS method are applied to the numerical case study and the overall scores and their subsequent rankings are presented in Table 10.

TABLE 10. Comparative Analysis.

Operator	$F(\gimel_{Q_1})$	$F(\gimel_{Q_2})$	$F(\gimel_{Q_3})$	$F(\gimel_{Q_4})$	$F(\gimel_{Q_5})$	Ranking
qROFZNPA$_D$	0.633	0.653	0.707	0.646	0.798	$F(\gimel_{Q_1} < F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
qROFZNPG$_D$	0.491	0.449	0.512	0.452	0.715	$F(\gimel_{Q_2}) < F(\gimel_{Q_4}) < F(\gimel_{Q_1}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
qROFZNWPA$_D$	0.206	0.292	0.305	0.195	0.368	$F(\gimel_{Q_4}) < F(\gimel_{Q_1}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
qROFZNWPG$_D$	0.472	0.477	0.509	0.444	0.703	$F(\gimel_{Q_4}) < F(\gimel_{Q_1}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
$qROPFZN_{EDAS}$	0.354	0.452	0.493	0.081	0.757	$F(\gimel_{Q_4}) < F(\gimel_{Q_1}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$

While there are slight differences in the scores, the same alternative is consistently ranked as the best alternative by both approaches. The comparison result of qROFZN$_D$ and EDAS method, in both cases shows the optimum alternative is Q_5 in Table 10. The results with two different methods confirms that the final ranking appropriately reflects the underlying criteria and expert opinions rather than the specific mathematical formulation used. This consistency is because both approaches are essentially based on multi-criteria assessment of the same normalized decision matrix; but the proposed framework uses Dombi's flexible aggregation operator and reliability-sensitive qROFZN model, which provides more flexibility in expressing uncertain information while preserving the relative preference orderings of highly competitive alternatives. On the other hand, EDAS is based on proximity-based evaluation (from average solutions), which may lead to some different distributions of intermediate scores, but preserves the final ranking consistency in most cases when alternatives are well-distributed.

Thus, the final ranking consistency justifies the reliability and accuracy of the proposed method, while the capacity to express more uncertainty information in complex decision-making demonstrates its superiority over traditional EDAS. The figure 3 combine the result attain by qROFZN$_D$ and EDAS method.

FIGURE 3. Graphical Representation of qROFZN$_D$ and EDAS method Ranking

The result's alignment with EDAS method shows the validity and reliability of proposed qROFZN$_D$ method independently.

8.2. **Sensitivity Analysis.** Analyzing the impact of parameter modification on the final decision results is crucial because the suggested methodology uses a parametric aggregation mechanism based on the Dombi operator. In order to assess the stability and robustness of the suggested method, a thorough sensitivity analysis is carried out.

A comprehensive sensitivity analysis is carried out by varying the aggregation parameter p of Dombi aggregation operators to assess the stability and robustness of the proposed decision-making model under varying degrees of interaction and compensatory behavior. Because parameter p regulates the interaction and compensatory effect between criteria, varying p enables us to observe the sensitivity of the global ranking of alternatives to the aggregation environment. The sensitivity analysis is conducted for several discrete values of p, and the results of the rankings is presented in Table 11 with the comparative information of the alternatives provided in the corresponding figure.

TABLE 11. Ranking of alternatives by D's parameter p at $3, 4, 5$

P	D Operators	Score Values					Ranking
		$F(\gimel_{Q_1})$	$F(\gimel_{Q_2})$	$F(\gimel_{Q_3})$	$F(\gimel_{Q_4})$	$F(\gimel_{Q_5})$	
$p=3$	qROFZNPA$_D$	0.646	0.677	0.722	0.661	0.806	$F(\gimel_{Q_1}) < F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNPG$_D$	0.469	0.430	0.490	0.426	0.703	$F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_1}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNWPA$_D$	0.228	0.322	0.328	0.218	0.381	$F(\gimel_{Q_4}) < F(\gimel_{Q_1}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNWPG$_D$	0.454	0.451	0.488	0.420	0.694	$F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_1}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
$p=4$	qROFZNPA$_D$	0.654	0.691	0.731	0.669	0.812	$F(\gimel_{Q_1}) < F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNPG$_D$	0.454	0.419	0.478	0.398	0.691	$F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_1}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNWPA$_D$	0.242	0.340	0.341	0.232	0.390	$F(\gimel_{Q_4}) < F(\gimel_{Q_1}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNWPG$_D$	0.471	0.457	0.493	0.425	0.681	$F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_1}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
$p=5$	qROFZNPA$_D$	0.659	0.700	0.737	0.675	0.815	$F(\gimel_{Q_1}) < F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNPG$_D$	0.444	0.411	0.470	0.402	0.689	$F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_1}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNWPA$_D$	0.251	0.351	0.349	0.241	0.397	$F(\gimel_{Q_4}) < F(\gimel_{Q_1}) < F(\gimel_{Q_2}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$
	qROFZNWPG$_D$	0.435	0.425	0.469	0.399	0.684	$F(\gimel_{Q_4}) < F(\gimel_{Q_2}) < F(\gimel_{Q_1}) < F(\gimel_{Q_3}) < F(\gimel_{Q_5})$

Based on the results, it is noted that while minor fluctuations in the numerical values are caused by different values of p (such as variations in the degree of interaction and compensatory behavior), the ranking order of the alternatives is extremely stable over all considered values of p. This suggests that the qROFZN-Dombi-based model proposed in this paper is not highly sensitive to the specific value of the parameter p, and the decision results are quite consistent even in the presence of varying degrees of compensatory effects. The minimal ranking variations also demonstrate that the model exhibits good robustness and stability, implying that it can be used for complex MADM problems with uncertain parameters.

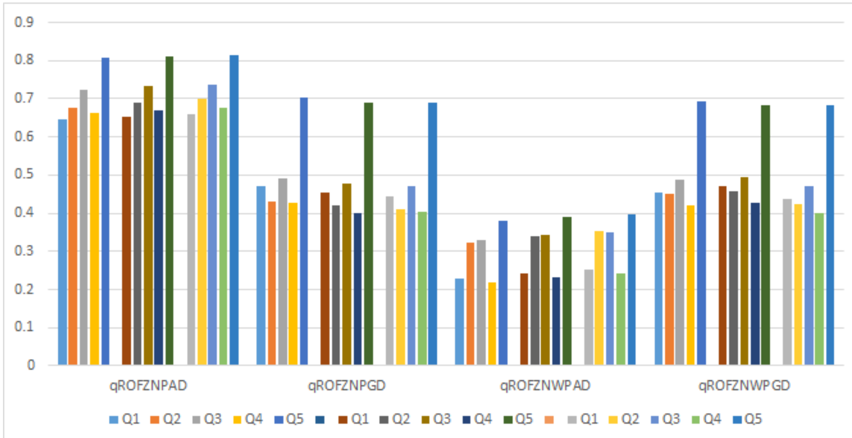

FIGURE 4. Graphical Representation of Sensitivity Analysis

The table 11 and figure 4 show the finding in which the alternative Q_5 got the highest score with every changing value of parameter p that is the optimum alternative. The absolute score values change due changing in parameter p but the overall ranking of alternative remain same. The consistent of ranking shows the validity, credibility and robustness of proposed method that not extensively sensitive with changing values of parameter p.

8.3. **Practical Implication.** The findings of this research have significant practical implications for prioritizing climate change adaptation strategies in an uncertain and complex decision-making environment. Decision-makers must weigh several trade-offs between Social Equity and Inclusivity, Environmental Effectiveness, Implementation Feasibility, and Long-Term Sustainability with Co-Benefits in real-world planning scenarios. Such evaluations are often imprecise, subjective, and fuzzy in nature, and thus decision-making is complex. Existing MADM approaches generally cannot account for these trade-offs as they are based on crisp or limited fuzzy numbers that limit their ability to represent higher-order uncertainty and reliability in expert assessments. On the other hand, the proposed qROFZNs offer a more realistic and adaptable representation of uncertain and reliability-sensitive information.

Through the incorporation of Dombis parametric aggregation operators and prioritized aggregation techniques, the proposed approach supports flexible integration of multi-criteria information while ensuring the prioritized nature of decision attributes. This allows key sustainability factors to have the desired impact on decision-making. Pragmatically, the approach can be readily applied by environmental managers, policymakers and urban authorities to assess and rank climate change adaptation strategies for urban and regional areas. Its capacity to capture uncertainty, imprecision, and reliability offers a sound and user-friendly approach to address real-world environmental management problems, leading to more reliable, transparent and sustainable decision-making processes.

9. Conclusion

In the climate change adaptation strategies an organized and comprehensive MADM process is required to manage the conflicting and diverse strategic option in imprecision and uncertain environment. Due to rigid and restricted oriented assessment the conventional method are not much liable to handle inherent uncertainty, imprecisions and reliability related to aspects. The qROFS has the ability to cover the inherent uncertainty, higher level of ambiguity and hesitation than conventional method. In addition the Z-number is enrich to provide reliable expert evaluation, truthiness of data and credible decision outcomes in MADM process. Moreover the Dombi's aggregation provide highly flexible, adaptable and parameterized framework to integrate the MADM process and expert suggestion to balance synthesize the heterogeneous perspective. The integration of all these advance concepts can make a powerful, adaptable and flexible MADM framework that enhance transparency, credibility, robustness and interpretability of climate change adaptation strategies.

In this research article, firstly we use operations of qROFZN, then combine the qROFZN with Dombi's aggregation operators and define novel prioritize aggregation operators. In this research we provide essential definition, theorems and properties for the authentication of proposed prioritize aggregation operators. Then we define the qROFZN$_D$'s algorithm for MADM process. We applied these aggregation operators on numerical case study of adaptation climate change strategies for the effectiveness, reliability and validity of proposed method. Ranking the alternative by applying score function to the result of aggregation operators and select the alternative having highest score value. For validity and authentication of proposed method we compare with EDAS method, for robustness we use sensitivity analysis by using different values of Dombi's parameter p. The result attaining by comparison and sensitivity analysis assure the flexibility, adaptability, novelty and robustness of proposed method over conventional methods. The practical implementation assures by applying proposed method on relevant factors such as environmental effectiveness, social equity & Inclusiveness, Implementation feasibility and long term sustainability & co-benefits etc. The proposed method provide reliable tools not only theoretical discussion but also helps the professional to make knowledgable decision in uncertain environment.

The results of this research show that the proposed qROFZN-based MADM method integrated with Dombi prioritized aggregation operators is a powerful and versatile technique for the assessment of climate change adaptation strategies under uncertainty. The case study findings suggest that the method is able to successfully differentiate among alternative strategies while taking into account uncertainty and confidence of the experts' opinions. The results of the comparative study using the EDAS method demonstrate the consistency and validity of the proposed method, and the sensitivity analysis shows that the method is robust to changes in the input parameters. In conclusion, the proposed method is more flexible, robust, and reliable than traditional methods, and can serve as an effective decision support tool for dealing with complex environmental and sustainability issues.

9.1. Limitations.

Every method has limitations, the qROFZN-framework has some limitations too. The authentications of proposed method is checked by only one numerical case study which is a restriction of method and it may effect the generalization of proposed method. The high level of computational complexity may occurs if we use the proposed method to a large scale of MADM process. Here assumed that the decision inputs are static but these are variables in real-world. Actually these limitations are opportunities for future research. In future the proposed method could evaluate the different field like operations management related to health care, alternates energy sectors and financial assets etc., it could be use for large scale MADM process by combining it with others effective and efficient method like machine learning and AI etc. or other dynamic decision making process. These benefits would be attain in future by using the proposed method.

9.2. **Future Research.** In future research can be focused to minimize the limitations by applying the proposed method on different real-life MADM problems where data is keeping on change. There is possibility to investigate other case studies like smart cities, traffic flow management, operations management, environmental engineering for developing purpose the effectiveness and efficiency of model will be studied.

CREDIT AUTHORSHIP CONTRIBUTION'S STATEMENT

Abid Ali: Conceptualization, Methodology, Formal Analysis, Writing – Original Draft, Investigation. Hisam Uddin Shaikh: Supervision, Validation, Resources, Writing – Review & Editing. Shahzaib Ashraf: Conceptualization, Supervision, Methodology, Validation, Project administration, Resources, Writing – review editing, Correspondence.

DECLARATIONS

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APPENDIX

The lists of the symbols used in the paper are given in Table 1.

TABLE 1. Symbols list.

Sr.	Full Name	Symbol	Sr.	Full Name	Symbol
1	Universal Set	$\mathfrak{U}$	14	Scalar Value	λ
2	Set	$\beth$	15	Dombi Parameter	p
3	Element of Set	$\mathfrak{u}$	16	Weights	ω
4	Membership	ϱ	17	Prioritized Weights	Ψ
5	Non-Membership	ϖ	18	Average Solution	A_vS_j
6	Distance	Δ	19	Positive distance	PDA_vS
7	Reliability	D	20	Negative distance	NDA_vS
8	Scoring Function	$\digamma$	21	Weighted Sum from PDA_vS	SP_i
9	Accuracy Function	$\Re$	22	Weighted Sum from NDA_vS	NP_i
10	Appraisal Scores	AS	23	Normalized values of $SPDA_vS$	NSP_i
11	qROFS Parameter	q	24	Normalized values of $SNDA_vS$	NSN_i
12	Alternatives	Q	26	Criteria	ℓ

The lists of the abbreviations used in the paper are given in Table 2.

TABLE 2. Abbreviation list.

Sr.	Full Name	Abbreviation
1	Fuzzy Set	FS
2	Decision Support	DS
3	Intuitionistic Fuzzy Set	IFS
4	Circular Intuitionistic Fuzzy Set	C-IFS
5	Pythagorean FSs	PFSs
6	q-Rung Orthopair Fuzzy Set	qROFSs
7	q-Rung Orthopair Fuzzy Numbers	qROFZNs
8	qROFZN Dombi Prioritized Averaging Aggregation Operator	qROFZNPA$_D$
9	qROFZN Dombi Weighted Prioritized Averaging Aggregation Operator	qROFZNWPA$_D$
10	qROFZN Dombi Prioritized Geometric Aggregation Operator	qROFZNPG$_D$
11	qROFZN Dombi Weighted Prioritized Geometric Aggregation Operator	qROFZNWPG$_D$
12	Positive Distance from Average	PDAS
13	Negative Distance from Average	NDAS
14	Multi Attribute Decision Making	MADM
15	Evaluation Based on Distance from Average Solution	EDAS