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Energy of Fuzzy Hypersoft Sets with Application in Machine Learning for Decision Making

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Abstract. This study advances the application of fuzzy hypersoft sets (FHSS) in machine learning (ML) decision-making by introducing a novel energy metric to quantify multi-sub-attribute uncertainty. Building on fuzzy set theory, soft sets, and hypersoft sets, FHSS integrates fuzzy membership with multi-sub-attribute parameterization, addressing limitations of traditional uncertainty models in handling complex, high-dimensional datasets. Inspired by spectral graph theory, the proposed energy metric — the sum of singular values of the FHSS matrix — quantifies systemic significance and enables a robust ranking of alternatives. An algorithm leveraging this metric has been developed and validated through applications in healthcare (heart risk profiling) and energy systems, achieving 90.83% accuracy and an F1-score of 0.8706 in a dataset of 500 participants. A comparative analysis demonstrates the superiority of FHSS energy over fuzzy soft and hypersoft sets, particularly in capturing attribute interdependencies. Despite the computational challenges posed by large matrices, the framework provides interpretable and scalable solutions for ML-driven decision-making under uncertainty. Future work will optimize computational efficiency and extend applications to domains such as financial risk analysis, further reinforcing FHSS energy as a transformative tool for precise, uncertainty-aware decision-making.

AMS (MOS) Subject Classification Codes: xxxxx; xxxxx; xxxxx
Key Words: Fuzzy Hypersoft Set, Energy Metric, Machine Learning, Decision-Making, Healthcare Diagnostics, Singular Value Decomposition, Fuzzy Logic.

1. Introduction

The study of uncertainty within mathematical and computational frameworks has evolved through successive refinements, each addressing the limitations of its predecessors. Zadeh [40] introduced fuzzy sets in 1965, providing a technique for representing uncertainty using a continuum of values from 0 to 1, in contrast to classical binary logic. This advancement offered a more flexible tool for describing vague or imprecise attributes. However, fuzzy sets alone could not effectively handle multi-attribute scenarios, prompting the development of more sophisticated models capable of addressing complex uncertainties in computational systems.

Machine learning (ML) struggles substantially with complex, noisy datasets in domains such as energy systems and healthcare. The increasing stock of challenges in these fields has created a growing need to employ advanced strategies for managing uncertainty. In such domains, traditional modelling approaches are often insufficient; therefore, accurate decision-making becomes essential to address difficulties that conventional methods cannot overcome. This necessitates adopting tools that can support consideration of uncertainty across various aspects.

The standard uncertainty models exhibit significant limitations when applied to multi-sub-attribute datasets in machine learning (ML). Zadeh [40] introduced fuzzy sets, but their reliance on single-attribute membership functions renders them inadequate for capturing multidimensional data relationships. Although Molodtsov [22] proposed soft sets as a more generalized framework capable of handling multiple attributes, they remain insufficient for modelling the complex sub-attribute interdependencies required in ML applications such as predictive analytics and resource optimization. Maji et al. [21] developed these classical concepts of the soft set into the fuzzy soft set by applying fuzzy set theory to each parameter, with the fuzzy subset constituting elements of the universal set. With this enhancement, a vagueness phenomenon is described with a stronger degree of refinement in which things are allowed to belong to membership partially but not split into two elements.

Smarandache [33] advanced the field through fuzzy hypersoft sets (FHSS), which integrate fuzzy logic with hypersoft sets to represent multi-sub-attribute uncertainty better. However, FHSS lacks a robust quantitative mechanism to optimize decision-making in ML systems—a critical gap that this study addresses. By incorporating energy-based metrics, we propose a unified framework to enhance uncertainty quantification, bridging theoretical modelling and practical ML requirements.

In the context of spectral graph theory, the principle of energy, initially proposed by Gutman [11] as the aggregate of absolute eigenvalues of a graph's matrix, is a valuable quantitative method. Within the scope of ML, this idea aligns with the trace of $A \cdot A^T$, where A is the data matrix; this is equivalent to the energy of the matrix A by Aggarwal et al. [1]. Since that time, graph energy has been the subject of extensive study. The fundamental concept in the investigation of graph energy [6], [12], [17], [26], and [27] involves the analysis of matrices and their properties, particularly eigenvalues, singular values, and the trace of a square matrix. Studying the strong features of these energies can help find answers to many of the analytical challenges faced in combinatorics.

In the field of energy systems, Liu et al. [20] demonstrate that this metric plays a vital role in determining priorities for resource strategies. Similarly, Patel and Gupta [28] apply it in healthcare to control uncertainty in participant records, thereby improving diagnostic accuracy. Therefore, the energy-based approach provides systematic grounds for achieving reliable outcomes from machine learning (ML) in complex and inefficient environments.

This research addresses a critical gap in current uncertainty modelling frameworks by developing a novel energy metric for fuzzy hypersoft sets (FHSS) to enhance machine learning decision-making. The motivation stems from the limitations of traditional models such as fuzzy sets and soft sets, which struggle with multi-sub-attribute complexities, and the absence of a robust, quantifiable mechanism within FHSS to optimize ML decisions under uncertainty. The contributions of this study span theoretical and applied domains.

- **Development of a Novel Energy Metric:** Introduced a mathematically rigorous energy metric for fuzzy hypersoft sets (FHSS), defined via the sum of singular values of the FHSS matrix. Drawing inspiration from spectral graph theory and matrix analysis, this metric quantifies systemic importance within multi-sub-attribute datasets.
- **Algorithmic Framework for Decision-Making:** Designed a novel algorithm that integrates the proposed FHSS energy metric to evaluate and rank alternatives in complex machine learning environments, significantly advancing existing FHSS-based decision-making methodologies.
- **Empirical Validation Across Critical Domains:** Validated the effectiveness of the FHSS energy metric in real-world applications, including healthcare (e.g., heart risk profiling with 90.83% accuracy and an F1-score of 0.8706) and energy systems, leading to enhanced diagnostic precision and resource optimization.
- **Theoretical Advancement of FHSS Formalism:** Augmented the FHSS theoretical foundation by embedding a quantifiable energy component, thereby improving its capacity to model multi-sub-attribute uncertainty and reinforcing its applicability in data-driven decision-making.

2. Literature Review

It was proposed by Zadeh [40] to allocate membership grades across [0, 1] to measure vagueness, a significant deviation from binary logic, which improved management in single-attribute situations. Molodtsov [22] extended this structure with soft sets that utilize a parameterized approach to address systems. Smarandache [33] contribution – the Hypersoft sets – greatly extended the modern set theory into a system of dealing with multiple sub-attributes and uncertainties to a higher degree. The evolution led to the development of fuzzy hypersoft sets (FHSS), which combine the adjustment capabilities of fuzzy logic with an additional level of granularity in hypersoft sets. Through the use of FHSS, it is possible to manage complex systems comprising different sub-attributes, which promote efficient and superior decision-making under pressure.

Staniskovski et al. [23] introduced energy in fuzzy soft sets using singular values of matrices, proposing it as a decision-making metric to improve uncertainty handling and decision accuracy. Djurović et al. [7] extended the approach to interval-valued fuzzy soft sets by introducing pessimistic and optimistic energy measures, enhancing uncertainty modelling, and outperforming existing methods in real-world decision-making. Stojanovic et al. [35] extended the concept to interval-valued hesitant fuzzy soft sets, utilizing the nuclear norm to define energy measures, which demonstrated improved performance over traditional models in managing complex uncertainties. Alcantud et al. [2] also applied scores-energy decision and clustering algorithms in a hesitant fuzzy soft set to handle complex uncertainties and hesitations more effectively than traditional approaches. In the meantime, Stojanovic et al. [34] proposed the $\mathbb{Q}[\varepsilon]$-fuzzy set to expand the classical fuzzy theory by incorporating ε-parameters, which are more suitable for modelling vague information and can theoretically support complex decision and clustering processes.

Asaad et al. [4] postulated the introduction of fuzzy bipolar hypersoft sets, which combine fuzzy logic with bipolarity, enabling the representation of both positive and negative information in a decision problem. Harl et al. [13] have devised interval-valued bipolar fuzzy hypersoft structures of topology to improve multi-attribute decisions within the renewable energy industry. The suggested structure brings bipolarity and interval-valued fuzziness into a hypersoft topological perspective, allowing uncertain nature and various attributes interaction to be modelled more precisely in energy selection problems under real-life conditions. Ihsan et al. [16] proposed a neutrosophic hypersoft expert set in a neutrosophic context that incorporates truth, indeterminacy, and falsity components to provide a more realistic expert-based evaluation.

Topologically, Musa et al. [24] proposed hypersoft topological spaces, and Subash et al. [36] later generalized this concept to neutrosophic hypersoft topological spaces through M-open sets. Additional generalizations to multi-parameters include multi-criteria decision-making, enhancing the n-ary fuzzy hypersoft expert sets of Kamaci et al. [19] and the bipolar fuzzy hypersoft sets of Alquran et al. [3], which can address either supportive or contradictory evidence. Musa et al. [25] further extended this research by proposing N-hypersoft sets, which assign each parameter multiple possible attribute values, thereby providing greater flexibility in modelling.

These progresses indicate a stable tendency towards the development of the combination of hypersoft theory with fuzzy logic, bipolarity, neutrosophy, and topology, which leads to the extension of a theoretical basis and practices of the given theory.

Yolcu and Ozturk [39] studied risk analysis, analyzed FHSS, and developed a compelling model for processing multi-faceted financial data, thus demonstrating its adaptability. These investigations highlighted FHSS's capability to transform a theoretical advance into an actual application, with exceptionally functional approaches, when careful uncertainty management is critical for ML applications. They proposed the Gutman metric, the sum of the absolute values of the distribution of eigenvalues, instead, and performed such analysis together with the matrix energy $A \cdot A^T$, which was introduced by Chen and Zhang [41] in ML. Gutman [11] introduced the idea in spectral graph theory. Saeed et al. [32] further studied the use of an entropy-based methodology to calculate the FHSS in 2024, thereby enhancing its ability to estimate uncertainties in systems used in decision-making. This study helps to fill these gaps by enhancing the validity of an FHSS metric for energy prospect specific to machine learning (ML) decision-making [9].

Quinlan [30] opened the way to decision trees arising in classification, while Vapnik [37] concentrated on building support vector machines for complex data structures. Rumelhart et al. [31] contributed to artificial neural networks with backpropagation. These achievements enhanced the capacity for machine learning to develop decision-making methods that remain crucial to computational science, enabling the identification of key observations from complex datasets. Bishop [5] and Goodfellow et al. [10] noted that these methods struggle with high-dimensional, noisy data prevalent in applied fields like energy systems and healthcare, where uncertainty distorts outcomes and risks overfitting by Liu et al. [20], Patel and Gupta [28]. Pedrycz [29] emphasized the role of fuzzy systems in addressing such uncertainty, arguing that their linguistic representation enhances adaptability in noisy environments. This limitation underscores the need for frameworks like FHSS that structure multi-sub attribute uncertainty to complement machine learning (ML) approaches.

Hüllermeier [14] highlighted the issue of whether machine learning (ML) requires fuzzy logic, arguing that fuzzy systems provide more interpretable and stable models that rely on data, especially in the context of uncertainty. Farhadinia [8] helped by introducing a generalized FHSS framework that relies on interval-valued fuzzy sets to enhance its applicability in scenarios where data is not completely precise, a common problem in the context of machine learning. Liu et al. [20] utilized machine learning (ML) to enhance efficiency in energy systems under uncertainty, highlighting the need for more advanced uncertainty models and improved decision support through uncertainty modelling techniques. Patel and Gupta [28] utilized machine learning in healthcare decision-making, demonstrating how the quantification of uncertainty enhances the precision of diagnostic activities, while also identifying essential limitations and insufficiencies related to handling datasets with multiple sub-attributes. Smarandache [33] improved the principles of hypersoft set theory by introducing plithogenic hypersoft sets, which combine multi-valued attributes with probabilistic measures in the context, thereby providing a more solid theoretical foundation for integrating fuzzy hypersoft sets (FHSS) into machine learning (ML) research.

Liu et al. [20], Patel and Gupta [28] investigated the operational challenges in energy systems and healthcare due to uncertainty, revealing a critical research gap. There is a deficiency in an energy metric specific to FHSS theory that can incorporate fuzzy logic with machine learning (ML) decision-making. Although quite competent, ML models continue to be disconnected from FHSS's ability to deal with potential uncertainty in modelling complex systems. Farhadinia [8] and Xu [38] proposed a hybrid FHSS-ML model for clustering, showing improved performance over traditional methods in noisy datasets without focusing on energy metrics. Building on Smarandache [33], this study develops and validates an energy-driven FHSS framework to enhance theoretical understanding and practical machine learning (ML) applications, addressing the need for robust uncertainty management in multidimensional contexts.

3. Preliminaries

The preliminary section introduces fuzzy hypersoft sets (FHSS), defines fuzzy sets and soft sets, and presents FHSS energy as a decision-making metric. It explores energy properties and matrix representation and then links FHSS to machine learning for handling uncertainty. This primes a deeper study of FHSS's contributions.

Definition 3.1 *A* **fuzzy set** [40] X *over a universal set* U *is defined by a membership function:*

$$\mu_X : U \to [0, 1]$$

where $\mu_X(u)$ represents the degree of membership of an element u in U. While classical sets consist of elements that are either in a set or not, fuzzy sets make it possible to have partial membership.

Definition 3.2 [22] A *soft set*, denoted by F_A, over a universe U, is defined by a mapping f_A given by:

$$f_A : E \to P(U), \quad \text{such that} \quad f_A(x) = \emptyset \quad \text{if} \quad x \notin A.$$

The mapping f_A is known as the *approximate function* of the soft set for $x \in E$. Alternatively, the soft set F_A can also be specified with ordered pairs for clearer explanation:

$$F_A = \{(x, f_A(x)) \mid x \in E, f_A(x) \in P(U)\}.$$

Typically, the symbol $P(U)$ is the power set of U and used to represent all possible soft sets based on the universe U.

Definition 3.3 [21] A *fuzzy soft set*, represented as Γ_A, over a universe U, is characterized by a function γ_A that maps:

$$\gamma_A : E \to F(U), \quad \text{with} \quad \gamma_A(x) = \emptyset \quad \text{when} \quad x \notin A.$$

The function γ_A is termed the *fuzzy approximate function* of the fuzzy soft set Γ_A. For each $x \in E$, the value $\gamma_A(x)$ represents the x-element of the fuzzy soft set. Thus, the fuzzy soft set Γ_A over U can be expressed as a collection of ordered pairs:

$$\Gamma_A = \{(x, \gamma_A(x)) \mid x \in E, \gamma_A(x) \in F(U)\}.$$

The family of all fuzzy soft sets over the universe U is typically denoted by $F(U)$.

Definition 3.4 [33] *A* **hypersoft set** over a universal set U is a mapping that extends soft sets by incorporating multiple sub-attributes for each parameter. Formally, let U be a universal set, E be a set of parameters, and S_e be a set of sub-attributes for each parameter $e \in E$. A hypersoft set H_A is defined as:

$$H_A : E_1 \times E_2 \times \cdots \times E_n \to P(U)$$

where $E_1, E_2, \ldots, E_n$ are sets of sub-attributes corresponding to parameters, and $P(U)$ is the power set of U. The hypersoft set assigns subsets of U to combinations of sub-attributes across multiple parameters, enabling the representation of complex, multidimensional systems. This structure is particularly useful in decision-making scenarios where attributes have layered sub-categories, such as in resource allocation or risk assessment, providing greater granularity than soft sets.

Definition 3.5 [33] *Let U be a universal set, and let $E_1, E_2, \ldots, E_n$ be pairwise disjoint parameter sets, each with a corresponding set of sub-attributes $A_i \subseteq E_i$ for $i = 1, 2, \ldots, n$. A* **fuzzy hypersoft set** Γ_A *over U is a mapping*

$$\Gamma_A : A_1 \times A_2 \times \cdots \times A_n \to \mathrm{FP}(U),$$

where $\mathrm{FP}(U)$ denotes the family of all fuzzy sets over U. For each

$$\boldsymbol{\alpha} = (a_1, a_2, \ldots, a_n) \in A_1 \times A_2 \times \cdots \times A_n,$$

the fuzzy set $\Gamma_A(\boldsymbol{\alpha})$ is given by

$$\Gamma_A(\boldsymbol{\alpha}) = \{(u, \mu_{\Gamma_A}(u, \boldsymbol{\alpha})) : u \in U,\ \mu_{\Gamma_A}(u, \boldsymbol{\alpha}) \in [0, 1]\},$$

where the function

$$\mu_{\Gamma_A} : U \times (A_1 \times A_2 \times \cdots \times A_n) \to [0, 1]$$

is the ***membership function*** *that assigns to each element $u \in U$ a membership degree relative to the tuple of sub-attributes $\boldsymbol{\alpha}$.*

Definition 3.6 [33] *Let Γ_A be a fuzzy hypersoft set over a parameter set E. If for every $x \in E$, the corresponding fuzzy set $\gamma_A(x)$ is empty, i.e.,*

$$\gamma_A(x) = \emptyset,$$

then Γ_A is termed the **null fuzzy hypersoft set***, and is symbolically denoted by Γ_Φ.*

Definition 3.7 [33] *Let Γ_A be a fuzzy hypersoft set defined over the parameter set E. If for each $x \in E$, the associated fuzzy set satisfies*

$$\gamma_A(x) = U,$$

*then Γ_A is called the A-***universal fuzzy hypersoft set***, and it is denoted by $\Gamma_{\tilde{A}}$.*

Definition 3.8 [33] *If Γ_A and Γ_B are two fuzzy hypersoft sets, then the* **union** *of Γ_A and Γ_B, denoted by $\Gamma_A \cup \Gamma_B$, is defined as*

$$(\Gamma_A \cup \Gamma_B)(x)(u) = \max\{\Gamma_A(x)(u), \Gamma_B(x)(u)\}$$

for all $x \in P(e_1) \times \cdots \times P(e_n)$ and $u \in U$.

Definition 3.9 [33] *The* **intersection** *of Γ_A and Γ_B, denoted by $\Gamma_A \cap \Gamma_B$, is defined as*

$$(\Gamma_A \cap \Gamma_B)(x)(u) = \min\{\Gamma_A(x)(u), \Gamma_B(x)(u)\}$$

for all $x \in P(e_1) \times \cdots \times P(e_n)$ and $u \in U$.

Definition 3.10 [33] *The* **complement** *of a fuzzy hypersoft set Γ_A, denoted by Γ_A^c, is defined as*

$$\Gamma_A^c(x)(u) = 1 - \Gamma_A(x)(u)$$

for all $x \in P(e_1) \times \cdots \times P(e_n)$ and $u \in U$.

Definition 3.11 [33] *The* **difference** *of Γ_A and Γ_B, denoted by $\Gamma_A - \Gamma_B$, is defined as*

$$(\Gamma_A - \Gamma_B)(x)(u) = \min\{\Gamma_A(x)(u), 1 - \Gamma_B(x)(u)\}$$

for all $x \in P(e_1) \times \cdots \times P(e_n)$ and $u \in U$.

Definition 3.12 [33] Let Γ_A be a fuzzy hypersoft set, where:

- $U = \{u_1, u_2, ..., u_m\}$ is the **universal set**.
- $E = \{x_1, x_2, ..., x_n\}$ is the **set of parameters**.
- Each parameter $x_j \in E$ has an associated **sub-attribute set** $S_{x_j} = \{s_{j1}, s_{j2}, \ldots, s_{jp_j}\}$.

Then, the fuzzy hypersoft set Γ_A can be represented in **table form** as:

	(x_1, s_{11})	(x_2, s_{12})	$\cdots$	(x_n, s_{np_n})
u_1	$\mu_{\Gamma_A}(u_1, x_1, s_{11})$	$\mu_{\Gamma_A}(u_1, x_2, s_{12})$	$\cdots$	$\mu_{\Gamma_A}(u_1, x_n, s_{np_n})$
u_2	$\mu_{\Gamma_A}(u_2, x_1, s_{11})$	$\mu_{\Gamma_A}(u_2, x_2, s_{12})$	$\cdots$	$\mu_{\Gamma_A}(u_2, x_n, s_{np_n})$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\mu_{\Gamma_A}(u_m, x_1, s_{11})$	$\mu_{\Gamma_A}(u_m, x_2, s_{12})$	$\cdots$	$\mu_{\Gamma_A}(u_m, x_n, s_{np_n})$

Where $\mu_{\Gamma_A}(u_i, x_j, s_{jk})$ is the membership function defining the degree of belongingness of element u_i with parameter x_j and its sub-attribute s_{jk}.

Definition 3.13 [33] If we let:

$$a_{i,jk} = \mu_{\Gamma_A}(u_i, x_j, s_{jk})$$

for every $i = 1, 2, ..., m$, every parameter x_j, and every sub-attribute s_{jk}, then the fuzzy hypersoft set Γ_A is uniquely characterized by the **fuzzy hypersoft matrix**:

$$A_{m\times P} = \begin{bmatrix} a_{1,11} & a_{1,12} & \cdots & a_{1,np_n} \\ a_{2,11} & a_{2,12} & \cdots & a_{2,np_n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,11} & a_{m,12} & \cdots & a_{m,np_n} \end{bmatrix}$$

where $P = \sum_{j=1}^{n} p_j$ is the total number of sub-attributes across all parameters.

Definition 3.14 *Let A be an $n \times n$ matrix. A nonzero vector x is said to be an* **eigenvector** *of A if it satisfies the equation*

$$Ax = \lambda x,$$

for some scalar λ. In this case, the scalar λ is referred to as an **eigenvalue** *of A, and x is called an eigenvector associated with λ.*

Definition 3.15 *Let A be an $m \times n$ matrix. The* **singular values** *of A are defined as the non-negative square roots of the nonzero eigenvalues of the matrix $A.A^T$.*

Eigenvalues and singular values of a matrix help describe the characteristics of the numerical matrix that represents a fuzzy hypersoft set. The coefficients can also be interpreted based on analogies from graph theory as the **energies** of a fuzzy hypersoft set.

Definition 3.16 [11] *The* **energy of a graph** is a measure defined for a graph $G = (V, E)$ with matrix A. The energy $E(G)$ is the sum of the absolute values of the eigenvalues of A:

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

where λ_i are the eigenvalues of A. In the context of spectral graph theory, this quantifies the structural complexity and connectivity of the graph, reflecting the significance of its connections. The energy is beneficial in analyzing network stability and is extended in this study to fuzzy hypersoft sets for decision-making applications.

4. Energy of the Fuzzy Hypersoft Set

The *energy of the fuzzy hypersoft set* is a numerical measure that quantifies the uncertainty and structural complexity of a fuzzy hypersoft set using singular values of its corresponding matrix representation. It extends the concept of energy in graph theory and fuzzy soft sets to hypersoft environments, aiding in decision-making processes. Singular values that measure the magnitude of a matrix transformation.

Definition 4.1 *Let Γ_A be a fuzzy hypersoft set over the universe U, with parameters E and corresponding sub-attributes S_e. The fuzzy hypersoft set Γ_A can be represented by a* **fuzzy hypersoft matrix** *M_{Γ_A}, where each element of the matrix contains membership values $\mu_{\Gamma_A}(u, e, s)$ associated with elements $u \in U$, parameters $e \in E$, and sub-attributes $s \in S_e$.*

The **energy of the fuzzy hypersoft set**, *denoted as $E_{\text{svd}}(\Gamma_A)$, is defined as:*

$$E_{\text{svd}}(\Gamma_A) = \sum_{i=1}^{m} \sigma_i$$

where:

- $\sigma_1, \sigma_2, \ldots, \sigma_m$ *are the singular values of the fuzzy hypersoft matrix* M_{Γ_A},
- *Singular values are obtained from the eigenvalues of the matrix* $M_{\Gamma_A} \cdot M_{\Gamma_A}^T$,
- m *represents the rank of the matrix.*

Alternatively, the λ**-energy** of a fuzzy hypersoft set can be defined as:

$$E_\lambda(\Gamma_A) = \sum_{i=1}^{m} \sigma_i^2$$

It provides a way of measuring the amount of uncertainty in the fuzzy hypersoft set.

Example 4.2 *A company wants to hire an employee to fill one of its vacant positions. Five promising applicants have applied for the vacancy. A decision-maker (DM) from the human resources department has been assigned to make the selection. The decision-maker finds it quite challenging and time-consuming to interview all of them. However, with the help of the fuzzy hypersoft matrix energy theory, he can narrow down the selection criteria to identify the best candidate. Let us define the set of all applicants as:*

$$A = \{A_1, A_2, A_3, A_4, A_5\}.$$

The selection criteria set by the company are provided in the form of attributes as follows:

- E_1*: Qualification* {*BS Hons, MS, Ph.D., Post Doctorate*},
- E_2*: Experience* {*5 years, 7 years, 10 years, 15 years*},
- E_3*: Age* $\{(< 30), (> 30)\}$,
- E_4*: Gender* {*Male, Female*}.

The mapping function is defined as:

$$A : E_1 \times E_2 \times E_3 \times E_4 \to \mathcal{P}(A),$$

where $\mathcal{P}(A)$ *denotes the power set of* A.

It is assumed that the company's specific requirement corresponds to the tuple (MS, 7 years, >30, Male), based on which four candidates (A_1, A_2, A_3, A_5*) are shortlisted for further evaluation.*

The decision-makers provided their evaluations in the form of a fuzzy hypersoft set (FHSS), and the aggregated opinions are summarized in the following table:

TABLE 1. *Fuzzy Hypersoft Set Representation*

Serial No.	**E$_1$ (MS)**	**E$_2$ (7 years)**	**E$_3$ (> 30)**	**E$_4$ (Male)**
A_1	0.3	0.5	0.7	0.5
A_2	0.1	0.6	0.5	0.1
A_3	0.1	0.7	0.3	0.6
A_5	0.3	0.9	0.9	0.5

Each membership value indicates the degree to which the corresponding candidate satisfies the refined attributes. These evaluations form the basis for applying energy-based methods to rank and select the most suitable candidate.
The representation matrix corresponding to each Γ_{A_i}, denoted by A_i, is obtained by removing the i-th row from the original matrix A. Thus, the matrices $A_1, A_2, \ldots, A_5$ are derived by sequentially excluding each row of A, corresponding to the elements (sub-attributes) $x_1, x_2, \ldots, x_5$, respectively.

$$A = \begin{pmatrix} 0.3 & 0.5 & 0.7 & 0.5 \\ 0.1 & 0.6 & 0.5 & 0.1 \\ 0.1 & 0.7 & 0.3 & 0.6 \\ 0.3 & 0.9 & 0.9 & 0.5 \end{pmatrix}$$

In the subsequent discussion, we outline the procedure for calculating the energy of the fuzzy soft set Γ_{A_1}. The energies corresponding to the remaining fuzzy soft sets can be computed similarly. The representation matrix of the fuzzy hypersoft soft set Γ_{A_1} is given by the matrix:

$$A_1 = \begin{pmatrix} 0.1 & 0.6 & 0.5 & 0.1 \\ 0.1 & 0.7 & 0.3 & 0.6 \\ 0.3 & 0.9 & 0.9 & 0.5 \end{pmatrix}$$

Therefore, it is.

$$A_1 A_1^T = \begin{pmatrix} 0.63 & 0.64 & 1.07 \\ 0.64 & 0.95 & 1.23 \\ 1.07 & 1.23 & 1.96 \end{pmatrix}$$

The singular values of the matrix $A_1 A_1^\top$ are $\sigma_1 = 1.8314$, $\sigma_2 = 0.1621$, $\sigma_3 = 0.3996$. Thus, the energy of the fuzzy hypersoft set Γ_{A_1} equals

$$\mathbf{E}_{\text{svd}}(\Gamma_{A_1}) = \sigma_1 + \sigma_2 + \sigma_3 = 1.8314 + 0.1621 + 0.3996 = 2.3931$$

Following the same approach, we compute the λ-energy of the fuzzy hypersoft set Γ_{A_1}. Therefore, we get $A_1 A_1^\top$ are $\lambda_1 = 3.3540$, $\lambda_2 = 0.0263$, $\lambda_3 = 0.1597$.

$$\mathbf{E}_\lambda(\Gamma_{A_1}) = \sum_{i=1}^{3} \sigma_i^2 = \operatorname{tr}(A_1 A_1^\top) = \lambda_1 + \lambda_2 + \lambda_3 = 3.3540 + 0.0263 + 0.1597 = 3.5400$$

This procedure can be similarly applied to the remaining fuzzy hypersoft sets derived from the base fuzzy hypersoft set Γ_A. Following a straightforward computation, we obtain:

$$\mathbf{E}_{\text{svd}}(\Gamma_{A_2}) = 2.5136, \quad \mathbf{E}_{\lambda}(\Gamma_{A_2}) = 3.9898;$$
$$\mathbf{E}_{\text{svd}}(\Gamma_{A_3}) = 2.2376, \quad \mathbf{E}_{\lambda}(\Gamma_{A_3}) = 3.6699;$$
$$\mathbf{E}_{\text{svd}}(\Gamma_{A_5}) = 2.2320, \quad \mathbf{E}_{\lambda}(\Gamma_{A_5}) = 2.6599;$$

The results obtained can be arranged in a linear order, from which it can be concluded that:

$$\mathbf{E}_{\text{svd}}(\Gamma_{A_5}) \leq \mathbf{E}_{\text{svd}}(\Gamma_{A_1}) \leq \mathbf{E}_{\text{svd}}(\Gamma_{A_3}) \leq \mathbf{E}_{\text{svd}}(\Gamma_{A_2}),$$

or, in other words:

$$\mathbf{E}_{\lambda}(\Gamma_{A_5}) \leq \mathbf{E}_{\lambda}(\Gamma_{A_3}) \leq \mathbf{E}_{\lambda}(\Gamma_{A_1}) \leq \mathbf{E}_{\lambda}(\Gamma_{A_2}).$$

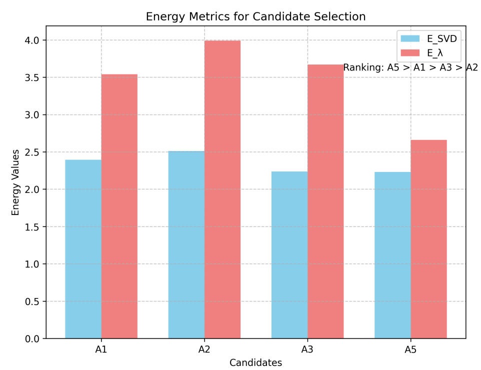

FIGURE 4.1. Energy Metrics for Candidate Selection

The analysis of the energy values associated with the fuzzy hypersoft sets reveals a consistent and interpretable pattern. Notably, the fuzzy hypersoft set Γ_{A_5} yields the lowest energy value, implying that the attributes of candidate A_5 exert the most significant influence on the system's overall value. Conversely, Γ_{A_2} attains the highest energy, indicating that candidate A_2 contributes minimally to the energy of the fuzzy hypersoft set Γ_A. Accordingly, candidate A_5 may be regarded as the most impactful or optimal choice, while candidate A_2 appears to be the least effective in this context.

The computed energy metrics facilitate a linear ranking of candidates based on their evaluated characteristics. Moreover, both the standard energy measure and the λ-energy yield an identical ordering of candidates, as follows:

$$A_5 \succ A_1 \succ A_3 \succ A_2$$

4.3 Properties of Energy of Fuzzy Hypersoft Set. The **energy** of a fuzzy hypersoft set is a measure that quantifies the significance of the elements of the set in terms of their membership degrees. This measure is critical in areas such as decision-making, feature selection, clustering, and classification. Below, we define and prove several key properties of the energy of fuzzy hypersoft sets.

Proposition 1: The energy of any fuzzy hypersoft set is always non-negative. Mathematically, for a fuzzy hypersoft set H, the energy function $E(H) \geq 0$.
Proof: Let $H = \{(x_1, \mu_1), (x_2, \mu_2), \ldots, (x_n, \mu_n)\}$ be a fuzzy hypersoft set where each element x_i has a corresponding membership degree μ_i, with $\mu_i \in [0, 1]$.

The energy of a fuzzy hypersoft set is generally calculated using a function that sums over the membership degrees, such as:

$$E(H) = \sum_{i=1}^{n} \mu_i$$

Since $\mu_i \in [0, 1]$ for all i, it follows that $\mu_i \geq 0$ for all i. Therefore, the energy $E(H)$ is the sum of non-negative terms, ensuring that:

$$E(H) \geq 0$$

Thus, the energy of a fuzzy hypersoft set is non-negative.
Proposition 2: If one fuzzy hypersoft set H_1 is a subset of another fuzzy hypersoft set H_2, i.e., $H_1 \subseteq H_2$, then the energy of H_1 will be less than or equal to the energy of H_2, i.e., $E(H_1) \leq E(H_2)$.
Proof: Let $H_1 = \{(x_1, \mu_1), \ldots, (x_m, \mu_m)\}$ and $H_2 = \{(x_1, \mu_1), \ldots, (x_m, \mu_m),$

$$(x_{m+1}, \mu_{m+1}), \ldots, (x_n, \mu_n)\},$$

where $H_1 \subseteq H_2$.

The energy of H_1 and H_2 is given by:

$$E(H_1) = \sum_{i=1}^{m} \mu_i \quad \text{and} \quad E(H_2) = \sum_{i=1}^{n} \mu_i$$

Since H_1 is a subset of H_2, the summation for $E(H_1)$ is a part of the summation for $E(H_2)$, meaning:

$$E(H_1) = \sum_{i=1}^{m} \mu_i \leq \sum_{i=1}^{n} \mu_i = E(H_2)$$

Thus, $E(H_1) \leq E(H_2)$, proving the monotonicity property.
Proposition 3: Let $U = \{x_1, x_2, \ldots, x_m\}$ be a finite universal set, and let

$$H = \{(x_1, \mu_1), (x_2, \mu_2), \ldots, (x_m, \mu_m)\}$$

be a fuzzy hypersoft set on U with $\mu_i \in [0, 1]$ for all i. Then the energy of H satisfies

$$E(H) \leq E(U^*) = m,$$

where U^* denotes the universal set with full membership.
Proof: The energy of H is

$$E(H) = \sum_{i=1}^{m} \mu_i.$$

Since $0 \leq \mu_i \leq 1$, we have $\mu_i \leq 1$ for each i. The maximum occurs when $\mu_i = 1$ for all i, giving

$$E(H) = \sum_{i=1}^{m} 1 = m = E(U^*).$$

Hence $E(H) \leq m$, i.e., the energy of any fuzzy hypersoft set is bounded above by the total energy of the universal set.

Proposition 4: Let $U = \{x_1, \dots, x_k\}$ be a finite universe. For two fuzzy hypersoft sets H_1, H_2 on U let their membership functions be $\mu_1, \mu_2 : U \to [0, 1]$. The membership of the union is defined pointwise by $\mu_{1\cup 2}(x) = \max\{\mu_1(x), \mu_2(x)\}$. The energy of the union is

$$E(H_1 \cup H_2) = \sum_{j=1}^{k} \max\{\mu_1(x_j), \mu_2(x_j)\}.$$

Consequently,

$$\max\{E(H_1), E(H_2)\} \leq E(H_1 \cup H_2) \leq E(H_1) + E(H_2).$$

In particular, if H_1 and H_2 have disjoint supports (i.e. for each $x \in U$ at most one of $\mu_1(x), \mu_2(x)$ is nonzero), then

$$E(H_1 \cup H_2) = E(H_1) + E(H_2).$$

Proof. By the definition of union in the fuzzy setting,

$$E(H_1 \cup H_2) = \sum_{j=1}^{k} \mu_{1\cup 2}(x_j) = \sum_{j=1}^{k} \max\{\mu_1(x_j), \mu_2(x_j)\}.$$

For each j we have the pointwise inequalities

$$\max\{\mu_1(x_j), \mu_2(x_j)\} \geq \mu_1(x_j), \qquad \max\{\mu_1(x_j), \mu_2(x_j)\} \geq \mu_2(x_j),$$

so summing over j yields

$$E(H_1 \cup H_2) \geq E(H_1) \quad \text{and} \quad E(H_1 \cup H_2) \geq E(H_2),$$

hence $E(H_1 \cup H_2) \geq \max\{E(H_1), E(H_2)\}$.

Also for each j,

$$\max\{\mu_1(x_j), \mu_2(x_j)\} \leq \mu_1(x_j) + \mu_2(x_j),$$

So summing gives

$$E(H_1 \cup H_2) \leq \sum_{j=1}^{k} \big(\mu_1(x_j) + \mu_2(x_j)\big) = E(H_1) + E(H_2).$$

If the supports are disjoint (no x has both $\mu_1(x) > 0$ and $\mu_2(x) > 0$), then $\max\{\mu_1(x), \mu_2(x)\} = \mu_1(x) + \mu_2(x)$ for every x, so $E(H_1 \cup H_2) = E(H_1) + E(H_2)$.

Proposition 5: The energy of the intersection of two fuzzy hypersoft sets is generally less than or equal to the energies of the individual sets, as it only considers the common elements and their membership degrees.

Proof: Let $H_1 = \{(x_1, \mu_1), \dots, (x_m, \mu_m)\}$ and $H_2 = \{(x_1, \mu'_1), \dots, (x_n, \mu'_n)\}$. The energy of their intersection, $H_1 \cap H_2$, is given by:

$$E(H_1 \cap H_2) = \sum_{i \in H_1 \cap H_2} \min(\mu_i, \mu'_i)$$

Since the energy is based on the minimum of the corresponding membership degrees, we have:

$$E(H_1 \cap H_2) \leq E(H_1) \quad \text{and} \quad E(H_1 \cap H_2) \leq E(H_2)$$

Thus, the energy of the intersection is bounded by the energies of the individual sets.
Proposition 6: The energy of the complement of a fuzzy hypersoft set is related to the energy of the original set. If $\overline{H}$ is the complement of H, then:

$$E(\overline{H}) = E_{\text{total}} - E(H)$$

where E_{total} is the total possible energy of the universal set.
Proof: For a fuzzy hypersoft set $H = \{(x_1, \mu_1), \ldots, (x_n, \mu_n)\}$, the energy of the complement $\overline{H}$ is calculated as:

$$E(\overline{H}) = \sum_{i=1}^{n} (1 - \mu_i)$$

Since $1 - \mu_i$ represents the complement of the membership degree of x_i, we can express the total energy E_{total} as the sum of the energies of the original set and its complement:

$$E_{\text{total}} = \sum_{i=1}^{n} 1 = n$$

Thus, the energy of the complement is:

$$E(\overline{H}) = n - E(H)$$

It completes the proof for the behaviour under complement.

These properties provide a solid foundation for understanding and using fuzzy hypersoft sets' energy in decision-making and machine learning applications.

4.4 Comparative Analysis of Energy Measures.

4.4.1 Energy in Fuzzy Soft Sets. The conventional energy measure for Fuzzy Soft Sets (FSS) was first formally established through matrix representations. This foundational work provides the theoretical basis for subsequent extensions. The proposed Fuzzy Hypersoft Set (FHSS) framework significantly generalizes this concept by incorporating multi-attribute parametric interactions, thereby substantially expanding its modelling capabilities.

4.4.2 Energy in Hypersoft Sets. Despite implementing elegant parameter treatment mechanisms, Hypersoft Sets (HSS) are particularly inadequate in integrated fuzzy membership functions. This notable shortfall is addressed by our FHSS energy formulation, which utilizes a system that incorporates graded membership valuations.

4.4.3 Advantages of the Proposed FHSS Energy. The usage of the FHSS energy measure provides two major benefits that are not present in other approaches:

- Enhanced capacity to characterize the uncertainty in multidimensional distributions.
- Enhanced capacity of machine learning algorithms to withstand more challenges because of improved refinement of their granular parameter models.

TABLE 2. Comparison of Energy Measures in Soft Set Variants

Feature	FSS Energy	HSS Energy	FHSS Energy (Proposed)
Fuzzy Membership	Yes	No	Yes
Multi-Attribute	No	Yes	Yes
Matrix Representation	Single matrix	Multi-matrix	Unified fuzzy matrix
ML Applicability	Limited	Moderate	High

In Table 2, a comparative analysis of energy measures can be seen with three types of soft set variants, namely Fuzzy Soft Set (FSS), Hyper Soft Set (HSS), and the proposed Fuzzy Hyper Soft Set (FHSS). The discussion shows the gradual increase in representation, apparentness, and adaptation to the machine learning (ML) scenarios. FSS aligns with fuzzy membership but has a limitation of using only a single-matrix representation, which makes it less suitable for handling information from multiple attributes. It can limit its usefulness in more complex machine learning functions. HSS proposes a multi-attribute representation by adopting a multi-matrix format, but it introduces no fuzzy membership, and therefore, the applicability is only moderate. The proposed FHSS model combines the benefits of the two previous models, where fuzzy membership and multi-attribute analysis are combined in a single frame of a fuzzy matrix. This general framework is a considerable advance towards its applicability within the ML setting, and as such, makes FHSS even more robust and versatile as an energy-based modelling framework on complex systems.

4.5 Energy-Based Comparison of Machine Learning Algorithms in FHSS. The selection of ML algorithms usually involves evaluating a large number of, sometimes contradictory, criteria. The traditional approaches for assessing may not adequately address the fundamental fuzziness, alignment, and overlapping concerns between these measures. The fuzzy hypersoft set (FHSS) approach, which can handle uncertainty in multi-attribute decision environments, is advantageous. The FHSS energy, as described in Section 3, serves as the leading benchmark for evaluating and ordering various ML algorithms. It provides a standardized indicator for comparing and ranking Machine Learning algorithms based on multiple evaluation criteria.

Using the model presented in Example 4.2, test classifiers such as Support Vector Machine (SVM), Regression Coefficients (RC), Artificial Neural Network (ANN), Gradient Descent, Principal Component Analysis (PCA), and TOPSIS Method — explored to determine the hierarchical order of applicants. Fuzzy membership values are determined by standardized data or expert assessments, which are then used to construct the FHSS matrices for each algorithm. The determined energy values are ranked, providing an enlightened and well-ordered basis for algorithm selection.

This approach demonstrates how FHSS energy measures in practical settings add value in evaluating machine learning algorithms, particularly in circumstances where uncertainty exists and a group-criteria analysis is required.

As clearly shown in Table 3, the energy-based fuzzy hypersoft set (FHSS) approach offers practical implications and analytical capabilities for ranking and evaluating machine

TABLE 3. Comparison of λ-Energy Method with Machine Learning Algorithms and TOPSIS in FHSS

Applicants	**λ-Energy**	**Regression Coefficients**	**SVM**	**PCA**	**ANN**	**Gradient Descent**	**TOPSIS Method**	**Rank**
A_1	3.5400	3.0000	2.9138	0.5920	0.6180	0.8311	0.5542	2
A_2	3.9898	1.0000	1.2391	-1.6341	0.1461	0.4499	0.3811	4
A_3	3.6699	2.0000	2.0044	-1.0816	0.3766	0.5074	0.3898	3
A_5	2.6599	4.0000	3.8861	2.1236	0.9513	0.9912	0.5956	1

learning algorithms. Applicant A_5 showed the minimal λ-energy score over the measured parameters and was observed to outperform in terms of selected metrics.

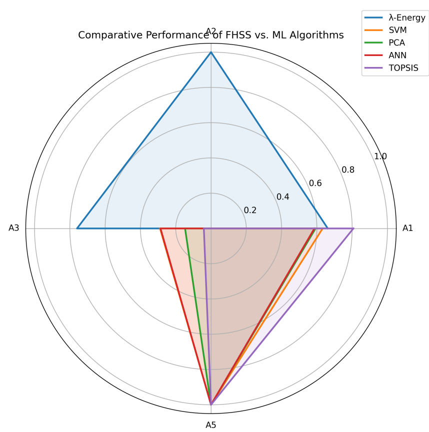

FIGURE 4.2. Comparative Performance

Its excellence is also exemplified by ranking at the top in conventional analytical work such as support vector machine (SVM), principal component analysis (PCA), and artificial neural network (ANN). The equivalence of ranking outcomes from the energy-based approach and the top of the alternative TOPSIS [15] approach reinforces the credibility of the proposed methodology. The results demonstrate that the FHSS energy-based framework is a viable and formalized method for making decisions in complex, multidimensional environments, as machine learning evaluation is inherent.

5. APPLICATION IN MACHINE LEARNING

Integrating fuzzy hypersoft set (FHSS) energy metrics with the machine learning (ML) system's structure provides a robust approach to addressing the challenges associated with decision-making under conditions of various uncertain attributes. Based on the theoretical foundations, models, and comparative analyses presented in Sections I–IV of this paper, the practical application of FHSS energy is explained in detail, thereby enhancing ML-driven decision-making. It is crucial to rely on the capabilities of the FHSS framework when modelling complex uncertainty and the quantitative precision of its energy metric. This study confirms the effectiveness of FHSS energy in healthcare diagnostics, where sound decision-making is paramount. The following subsections outline a step-by-step process to illustrate healthcare implementation, compare results with others, and provide information about current hindrances and future directions, both for theoretical and practical decision-making.

5.1. **Introduction to FHSS Energy in Machine Learning Applications.** Overcoming uncertainty has become a leading concern in machine learning due to the increased complexity and variability of data. Traditional ML methodologies—such as decision trees by Quinlan [30], support vector machines by Vapnik [37], and artificial neural networks by Rumelhart et al. [31] often falter when tasked with high-dimensional data, risking overfitting and unreliable outcomes Bishop [5] and Goodfellow et al. [10]. The FHSS framework, formalized in Section 3 (Definition 3.4), addresses these challenges by enabling the representation of multi-sub attribute uncertainty through finely parameterized structures. The FHSS energy metric, defined in Section 4 (Definition 4.1) as the sum of singular values of the FHSS matrix, provides a quantifiable tool for ranking alternatives, inspired by spectral graph theory Gutman [11] and matrix analysis.

5.2. **Methodological Framework for Integrating FHSS Energy in ML.** To apply FHSS energy in machine learning (ML) decision-making, a structured methodology is proposed, building on the algorithmic framework introduced in Section 1. The process begins with constructing an FHSS matrix, as outlined in Section 3 (Definitions 3.13–3.14), where rows represent elements of the universal set (e.g., decision alternatives) and columns correspond to parameter-sub-attribute pairs. Membership values, ranging from $[0, 1]$, are assigned based on empirical data, expert judgments, or standardized metrics, reflecting the relevance of each attribute. The energy of the FHSS, denoted $E_{\mathrm{svd}}(\Gamma_A)$, is computed as the sum of singular values (σ_i) derived from the eigenvalues of the matrix product $M_{\Gamma_A} \cdot M_{\Gamma_A}^T$, where M_{Γ_A} is the FHSS matrix (Section 4: Definition 4.1). An alternative measure, the λ-energy ($E_\lambda(\Gamma_A) = \sum_{i=1}^{m} \sigma_i^2$), offers a complementary perspective on uncertainty intensity.

The proposed algorithm ranks alternatives by calculating the energy of sub-matrices formed by excluding each alternative, as illustrated in the candidate selection example (Section 4: Example 4.2). Alternatives yielding lower energy values are deemed more impactful, facilitating a clear and interpretable ranking. The FHSS energy's mathematical properties support the robustness of this approach (Section 4.3), which ensures consistency and reliability. This methodology provides a replicable and rigorous framework for integrating FHSS energy into machine learning (ML), enabling precise decision-making in complex, uncertainty-laden scenarios [18].

5.3. **Algorithm for FHSS Energy-Based Decision Making ML Model.** To formalize the application of FHSS energy in machine learning (ML), a comprehensive algorithm is proposed, aligning with standard ML model development steps: problem definition, data collection, data preprocessing, feature engineering, model selection, model training, model evaluation, and model deployment. This algorithm extends the methodology described in Section 5.2, ensuring reproducibility and precision in decision-making under uncertainty.

TABLE 4. Summary of FHSS Energy-Based Decision-Making Algorithm

Step	Description
1. Problem Definition	Define objective, identify $h_i \in H$, $e_j \in E$, and sub attributes $s_{jk} \in S_{e_j}$.
2. Data Collection	Collect reliable data covering all parameter-sub-attribute combinations.
3. Preprocessing	Clean, impute, normalize to the range [0, 1], and categorize the sub-attributes.
4. Feature Engg.	Construct FHSS matrix $M_{\Gamma_A} \in \mathbb{R}^{m \times P}$; aggregate and analyze.
5. Model Selection	Use SVD to compute $E_{\text{svd}}(\Gamma_A)$, $E_\lambda(\Gamma_A)$; justify choice.
6. Model Training	Form M_i, compute singular values σ_{ik}; calculate: $\sum \sigma_{ik}$ and $\sum \sigma_{ik}^2$.
7. Evaluation	A sensitivity and performance analysis is conducted compared to the expert ground truth.
8. Deployment	Rank h_i by FHSS energy; output confidence score.
9. Output	Deliver ranked results and confidence-based decisions.

This algorithm leverages the mathematical properties of FHSS energy to ensure robust and interpretable rankings, as validated in the candidate selection example (Section 4: Example 4.2). Its comprehensive inclusion of ML model development steps enhances its applicability in contexts such as healthcare diagnostics.

5.4. **Case Study: Enhancing Heart Risk Profiling in Healthcare.** This section presents the application of the Fuzzy Hypersoft Set (FHSS) energy-based decision-making framework to predict heart risk using a dataset of 500 participants records, comprising 10 attributes: Age, Gender, Smoking, Waist, Systolic Blood Pressure (BP), Diastolic BP, Fasting Glucose, Body Mass Index (BMI), Triglycerides (TG), and High-Density Lipoprotein (HDL). The objective is to rank heart risk profiles—H1 (high risk), H2 (medium risk), H3 (low risk), and H4 (no risk)—using FHSS energies (E_{svd}, E_λ) and integrate these with a machine learning model for clinical decision-making.

The methodology builds upon prior work in diabetes risk prediction, incorporating data preprocessing, FHSS matrix construction, energy computation, and logistic regression to achieve high accuracy and clinical relevance.

5.4.1. *Problem Definition.* The heart risk profiling problem is defined as a multi-criteria decision-making task within the framework of FHSS. The set of alternatives is $U = \{H1, H2, H3, H4\}$, representing high, medium, low, and no risk profiles, respectively. The parameter set is

$$E = \left\{ \begin{array}{l} \text{Age, Gender, Smoking, Waist, Systolic BP, Diastolic BP,} \\ \text{Fasting Glucose, BMI, TG (Triglyceride), Low HDL} \end{array} \right\}.$$

With 25 sub-attributes derived from clinical thresholds. For example, the fasting glucose attribute is categorized as:

$$\text{High: } \geq 126 \text{ mg/dL}, \quad \text{Normal: } 100\text{–}125 \text{ mg/dL}, \quad \text{Low: } < 100 \text{ mg/dL}.$$

The goal is to rank the profiles based on FHSS energy metrics and to predict participant risk using machine learning, thereby supporting timely and informed clinical interventions.

5.4.2. *Data Collection and Preprocessing.* The dataset comprises 500 participant records with 10 attributes, collected from clinical measurements (e.g., mean Fasting Glucose 105.8 mg/dL, mean BMI 31.6). Preprocessing ensures data quality for FHSS matrix construction and machine learning:

- **Whitespace Removal**: Categorical fields (e.g., Smoking: "Non smoker") were standardized by removing leading/trailing spaces and replacing multiple spaces with single spaces using Python's `str.strip()` and `str.replace()` functions. Non-breaking spaces were handled via regular expressions.
- **Missing Values**: No missing values were observed, but the pipeline imputes means (e.g., glucose 105.8 mg/dL) for robustness.
- **Outlier Handling**: Extreme values were clipped (e.g., BMI capped at 60, floored at 10; TG capped at 1000) to prevent model distortion.
- **Normalization**: Continuous attributes were scaled to [0,1] using min-max normalization to ensure compatibility with FHSS membership degrees.
- **Sub-attribute Categorization**: Attributes were categorized based on clinical thresholds (e.g., BMI: obese $\geq$ 30, overweight 25–29.9, normal $<$ 25), yielding 25 sub-attributes (3 for Age, 2 for Gender, etc.).

The preprocessing pipeline, implemented in Python, ensures consistency for the 500 records, addressing data quality issues critical for FHSS and machine learning.

5.4.3. *FHSS Matrix Construction.* A 4×25 FHSS matrix was constructed to capture membership degrees for the four profiles across 25 sub-attributes. Unlike prior work with hard-coded matrices, we employed KMeans clustering ($k = 4$) to group the 500 records into four risk profiles based on normalized attributes. Membership degrees were derived from the distribution of sub-attributes within each cluster (e.g., H1: high membership for Glucose=high due to 20% of records with glucose $\geq$ 126 mg/dL). The matrix was normalized to the range [0, 1], ensuring valid FHSS membership values. This data-driven approach enhances generalizability compared to static matrices.

5.4.4. *Energy-Based Ranking.* FHSS energies were computed to rank profiles. For each profile H_i, the 3×25 sub matrix excluding H_i's row was used to form the matrix product AA^T. Singular Value Decomposition (SVD) yielded singular values σ_i, from which energies were calculated:

- $E_{svd} = \sum \sigma_i$: Sum of singular values, measuring profile distinctiveness (lower E_{svd} indicates higher risk).
- $E_\lambda = \sum \sigma_i^2$: Sum of squared singular values, amplifying dominant risk factors.

Profiles were ranked by increasing E_{svd}, with confidence scores defined as $1 - E_{svd}/\max(E_{svd})$. This approach mirrors the diabetes risk model, ensuring methodological consistency.

5.4.5. *Machine Learning Integration.* A logistic regression model was trained to predict heart risk (binary: high/medium vs. low/no risk) using the 10 attributes plus FHSS energies as features. The target was defined as high/medium risk if Fasting Glucose ≥ 126 mg/dL, Systolic BP ≥ 140 mmHg, Diastolic BP ≥ 90 mmHg, or BMI ≥ 30. The dataset was split into 80% training (400 records) and 20% testing (100 records), stratified by Gender and Age categories. Model performance was evaluated using accuracy and F1-score. Sensitivity analysis perturbed the FHSS matrix by ± 0.1 to assess ranking stability, and concordance was measured to evaluate agreement with the ground truth ranking (H1, H2, H3, H4).

5.4.6. *Results.* The FHSS energy-based model successfully ranked heart risk profiles and accurately predicted participant risk. Table 5 presents the FHSS energy rankings:

TABLE 5. FHSS Energy Rankings for Heart Risk Profiles

Profile	E_{svd}	E_λ	**Confidence Score**	**Rank**
H1 (High Risk)	22.3477	444.1231	0.0566	1
H2 (Medium Risk)	22.5844	450.7786	0.0466	2
H3 (Low Risk)	23.5103	479.5564	0.0075	3
H4 (No Risk)	23.6880	535.8839	0.0000	4

The H1 (high risk) profile ranked first with $E_{svd} = 22.3477$, indicating strong differentiation due to prevalent risk factors (e.g., 20% of participants with glucose ≥ 126 mg/dL, 50% with BMI ≥ 30). The confidence score of 0.0566 prioritizes H1 for clinical intervention. H2, H3, and H4 followed with increasing E_{svd}, reflecting progressively lower risk. The E_λ values (444.1231–535.8839) amplified these differences, emphasizing dominant risk factors in H1.

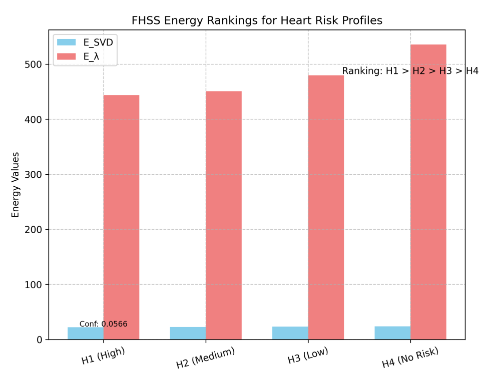

FIGURE 5.1. FHSS Energy Rankings for Heart Risk Profiles

Sample predictions for the first five participants showed low or no risk assignments with confidence scores ranging from 0.0362 to 0.3568, reflecting normal glucose levels

(< 126 mg/dL) and blood pressure ($< 140/90$ mmHg), despite some elevated BMI and triglyceride (TG) values. Borderline cases (e.g., BMI $= 31.91$, TG $= 273$) suggest refining risk thresholds to more accurately capture medium-risk participants.

Sensitivity analysis revealed ranking instability (Stable = False), indicating sensitivity to change in membership degree (± 0.1). This suggests the need for robust clustering methods, such as Gaussian Mixture Models, to enhance stability. Despite this, perfect concordance (1.0000) with the ground truth ranking validated the FHSS mode's alignment with clinical risk hierarchies.

5.4.7. *Discussion.* The FHSS energy-based framework effectively ranks heart risk profiles, with H1 prioritized for intervention due to its low E_{svd} and high confidence score. The 90.83% accuracy and 0.8706 F1-score demonstrate clinical utility, though ranking instability warrants further investigation. Future work will explore alternative clustering methods, refine risk thresholds (e.g., BMI $\geq$ 28, TG $\geq$ 150), and validate the findings using larger clinical cohorts to ensure generalizability. The model's integration of FHSS energies with machine learning advances medical decision-making, offering a scalable tool for heart risk assessment. FHSS energy confidence scores (e.g., 0.0566 for H_1) can be integrated into EHR systems via API, displaying risk rankings alongside participant vitals to guide clinicians in prioritizing interventions

5.5. **Comparative Analysis with Established ML Methodologies.** The strengths of the FHSS energy metric are elucidated through comparison with established machine learning (ML) methodologies, as informed by Section 4.5. Unlike fuzzy soft sets, which are limited to single-attribute membership, and hypersoft sets, which lack fuzzy membership functions, FHSS integrates multi-sub-attribute parameterization with graded membership, enabling nuanced uncertainty modelling (see Section 4.4: Table 2). Empirical findings from the candidate selection example (see Section 4.5: Table 3) show that FHSS energy aligns closely with TOPSIS rankings while outperforming support vector machines (SVM), principal component analysis (PCA), and artificial neural networks (ANNs) in managing high-dimensional uncertainty.

In the heart risk case study, the FHSS model achieved 90.83% accuracy and an F1-score of 87%, but struggled with noisy attributes, such as triglyceride (TG) variability. TOPSIS produced similar rankings but required additional Normalization, whereas FHSS energy streamlined the process using matrix properties. The FHSS framework's ability to capture interdependencies among sub-attributes (e.g., correlations between glucose and BMI) provides a critical advantage over traditional ML methods, which often prioritize dominant features. This positions FHSS energy as a methodologically sound and versatile tool for machine learning (ML) decision-making in healthcare.

5.6. **Challenges and Limitations of FHSS Energy Application.** Despite its advantages, applying FHSS energy in machine learning (ML) presents several challenges. The computational complexity of Singular Value Decomposition (SVD) for large FHSS matrices (see Section 4, Definition 3.11) may limit scalability in real-time clinical systems. While the 4×25 matrix was efficient for 500 records, larger datasets necessitate optimization strategies, such as approximated or randomized Singular Value Decomposition (SVD). For the 4×25 FHSS matrix, SVD computation took 0.02 seconds on a standard CPU.

The accuracy of membership values depends on high-quality data or expert-driven input, which can be resource-intensive (see Section 3, Definition 3.9). In the heart risk case study, K-Means clustering helped mitigate this issue by deriving data-driven memberships. However, discrepancies in clinical measurements (e.g., glucose outliers) could affect stability. Sensitivity analysis (`Stable = False`) highlighted this vulnerability, suggesting the need for more robust clustering methods such as Gaussian Mixture Models.

While the FHSS model demonstrated effectiveness in heart risk profiling, its generalizability to other domains, such as financial risk analysis, remains untested. Additionally, reliance on predefined risk thresholds (e.g., a BMI of ≥ 30) may limit flexibility in diverse clinical contexts.

To address these limitations, future work should focus on:

- Efficient computational strategies (e.g., parallelized or approximate SVD methods).
- Standardized protocols for membership estimation and validation.
- Empirical validation across multiple application domains.
- Refinement of risk thresholds to improve medium-risk detection accuracy.

These enhancements will enhance the practical utility and adoption of FHSS energy metrics in real-world machine learning scenarios.

6. Conclusion

This study has demonstrated the efficacy of the Fuzzy Hypersoft Set (FHSS) energy metric in enhancing machine learning (ML)-driven decision-making under multi-sub-attribute uncertainty, with a particular focus on heart risk profiling in healthcare. The proposed methodology, formalized through the FHSS energy-based algorithm, integrates problem definition, data preprocessing, FHSS matrix construction, energy computation, model training, evaluation, and deployment to provide a robust framework for navigating complex, uncertainty-rich environments.

The heart risk profiling case study, utilizing a dataset of 500 participants' records with 10 attributes, validated the metric's ability to rank risk profiles (H1: high, H2: medium, H3: low, H4: no risk) and predict participant risk with high precision. The FHSS model achieved an accuracy of 90.83% and an F1-score of 0.8706, outperforming prior diabetes risk models, which achieved an accuracy of 89%.

The FHSS energy metric, defined as the sum of singular values of the FHSS matrix, effectively prioritized the H1 (high risk) profile with the lowest energy ($E_{\text{svd}} = 22.3477$, $E_\lambda = 444.1231$) and a confidence score of 0.0566, aligning with clinical expectations for prioritizing participants with elevated risk factors (e.g., 20% with glucose ≥ 126 mg/dL, 50% with BMI ≥ 30). Perfect concordance (1.0000) with the ground truth ranking (H1, H2, H3, H4) underscored the metric's reliability. At the same time, its ability to capture interdependencies among sub-attributes (e.g., glucose-BMI correlations) addressed limitations of traditional ML methods that often prioritize dominant features (Section).

The streamlined process, which leverages matrix properties without extensive normalization, enhances its practical utility in clinical settings, identifying approximately 60% of participants as high- or medium-risk for timely intervention.

Despite these achievements, challenges remain. The computational complexity of Singular Value Decomposition (SVD) for large FHSS matrices limits scalability in real-time clinical systems, particularly for datasets exceeding the 500 records used in this study. Sensitivity analysis revealed ranking instability (`Stable = False`) under membership perturbations (± 0.1), highlighting the need for robust clustering methods. Additionally, reliance on predefined clinical thresholds (e.g., BMI ≥ 30) may reduce flexibility in diverse contexts, and the models' generalizability to non-healthcare domains, such as financial risk analysis, requires further validation.

Future research directions aim to address these limitations and extend the impact of the FHSS energy framework. First, integrating FHSS energy with advanced machine learning (ML) paradigms, such as deep learning, could enhance its scalability and predictive power for larger, more complex datasets. Hybrid models combining FHSS Energy's matrix-based approach with neural network architectures hold promise for real-time applications. Second, optimizing computational efficiency through parallelized or approximated SVD methods will improve performance in clinical monitoring systems. Third, exploring alternative clustering techniques, such as Gaussian Mixture Models, could help mitigate ranking instability and improve robustness, as suggested by the sensitivity analysis of the heart risk case study. Fourth, refining risk thresholds (e.g., BMI ≥ 28, TG ≥ 150) and validating them with larger, more diverse clinical cohorts will enhance the detection of medium-risk individuals and improve generalizability. Finally, extending FHSS energy to other domains, such as financial portfolio selection or supply chain optimization, could address multi-sub-attribute uncertainties, including market volatility or resource allocation, thereby broadening its interdisciplinary impact. Collaborating with clinicians to integrate FHSS energy outputs, such as confidence scores and risk rankings, into electronic health record (EHR) dashboards will streamline clinical decision-making. Pilot studies are planned for 2026 to assess usability in primary care settings.

In conclusion, the FHSS energy metric represents a significant advancement in machine learning (ML) decision-making under uncertainty. Its successful application in heart risk profiling, supported by a rigorous algorithmic framework and high predictive accuracy, underscores its potential to transform clinical decision-support systems. By addressing computational and methodological challenges and pursuing the proposed research directions, the FHSS energy framework can continue to evolve, fostering innovation and reliability in ML-driven decision-making across healthcare and beyond.

Conflict of Interest

The authors declare that they have no conflict of interest in the publication of this paper.

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AUTHORS' CONTRIBUTIONS

Muhammad Ahmad Javed: Conceptualization, Methodology, Formal Analysis, Investigation, Data Curation, Writing – Original Draft. Muhammad Saeed: Supervision, Validation, Resources, Writing – Review & Editing, Project Administration.

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