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Intuitionistic Fuzzy Credibility Implicative Ideals in BCK-algebra for Efficiency Evaluation, Supply Chain and Risk Analysis

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Abstract. This study proposes a robust multi-criteria decision-making (MCDM) framework for efficiency evaluation, supply chain, and risk analysis based on intuitionistic fuzzy credibility sets (IFCSs) and intuitionistic fuzzy credibility implicative ideals (IFCIIs) in BCK-algebras. An integrated LOPCOW–SWARA–ERUNS framework is developed to handle decision-making under uncertainty. The proposed approach is applied to evaluate leading companies, including Amazon, Walmart, and Alibaba, by considering key factors such as cybersecurity, geopolitical instability, environmental impacts, economic conditions, technological aspects, social issues, and logistics-related risks. Conventional MCDM methods often struggle to address uncertainty and subjective bias, resulting in unreliable outcomes for complex decision problems. To overcome these challenges, SWARA is employed to determine subjective weights, while LOPCOW is used to compute objective weights. The ERUNS method is then applied to rank the alternatives, ensuring a comprehensive and reliable evaluation. Sensitivity and comparative analysis are conducted to assess the robustness of the proposed framework. The results confirm that the proposed method significantly enhances MCDM accuracy by effectively managing uncertainty.

AMS (MOS) Subject Classification Codes: 03B52; 03E72 ; 90B50
Key Words: Intuitionistic fuzzy credibility sets; LOPCOW; SWARA; ERUNS; Efficiency Evaluation; Supply Chain.

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1. INTRODUCTION

Currently, the concept of security has broadened considerably beyond the conventional scope of conflict between nations and state-centric issues. Unconventional security threats significantly affect various domains, including economic, political, and social aspects, with particular implications for supply chain finance (SCF) in E-commerce companies such as Walmart, Alibaba, and Amazon. Complex occurrences such as cyberattacks, pandemics, and climate change generate non-traditional threats. These issues impact individuals worldwide and complicate the differentiation between international and domestic security.

The global economy is undergoing a transformation as companies shift towards on information operations facilitated by online technologies. In response to the global economic crisis, businesses were compelled to reduce expenditures amid diminished economic activity. Expanding the scope of online business operations is a prevalent strategy for reducing costs. We are currently experiencing a major transformation: the transition to an Internet-based society. The global network facilitating information exchange among businesses, corporations, clients, and various divisions is significantly increasing the volume of accessible business data.

Information technology (IT) has transformed the operational practices of businesses. The IT system has been deployed and integrated across all businesses that have invested significantly in IT infrastructure to facilitate their growth. It has emerged as a primary focus for numerous businesses. The increasing prevalence and swift advancement of the Internet and network technology have rendered the electronic industry essential for contemporary enterprises. Currently, major corporations are executing their operations through online platforms. Individuals engage in the online business and services, with numerous facilities proving challenging in the absence of IoT.

E-commerce, defined as electronic commerce, involves the direct sale of goods or services to customers via an online vendor's website. E-commerce facilitators include the internet, payment gateways, analytics, social media, and autonomous vehicles operated by artificial intelligence [20]. The types of E-commerce business include business-to-business, business-to-consumer, business-to-administration (B2A), and consumer-to-administration (C2A).

Supply chain management (SCM), serves as key function in ensuring that products move from their origin to the final consumer. It involves a series of interconnected business processes, including sourcing raw materials, manufacturing, warehouse management, shipping, and distribution. It involves coordinating and working together with different supply chain partners, such as suppliers, middlemen, third-party service providers, and customers [11]. The emerging intelligent technologies that facilitate SCM, like the internet of things (IoT), blockchain (BC) [5], physical internet, and artificial intelligence (AI), can be applied in the industry, including business and logistics, marketing, procurement, and sustainable SCM. E-commerce relies on supply chain essential to E-commerce in order to fulfill on its promises to clients. When an order is placed on an E-commerce platform, the supply chain ensures the product is sourced, stored, and delivered to the customer efficiently. Certainly, novel opportunities for supply chain management have been made possible by integration of AI tools.

The research work aims to achieve the following main objectives.

- Investigate IFCSs as a framework for managing uncertainties in the process of making decisions.
- Explore IFCSs in BCK-algebras.
- Introduce the concept of IFC implicative ideals in BCK-algebras and examine associated properties.
- Examine the non-traditional threats to E-commerce and supply chain companies.
- Propose a decentralized model based on IFCSs, with eight multinational supply chain and E-commerce companies (as alternatives): Amazon, Walmart, Costco, Alibaba, Best Buy, Carrefour Group, IKEA and Tesco and Nontraditional threats (as criteria).
- Examine the non-traditional threats and challenges to these multinational supply chain and E-commerce companies.
- Systematically evaluate and prioritize these multinational supply chain and E-commerce companies using a robust MCDM model.

1.1. **Literature review.** BCK-algebras are a fundamental algebraic structure with applications in logic and mathematics, attracting significant attention due to their rich algebraic properties. BCK/BCI-algebras were first introduced in the mathematical literature in 1966 [16]. The concepts are developed through two distinct methods: the characteristics of set-theoretic difference and the propositional calculus of Meredith [17]. BCK-algebras are regarded as the algebraic counterpart of Meredith's BCK-implicational calculus [26]. BCK/BCI-algebras serve as algebraic representations of the BCK/BCI-system within the framework of combinatory logic. The term BCK/BCI-algebras is based on combinators B, C, K, and I in combinatory logic. Numerous studies investigate the properties of BCK/BCI-algebras [28]. Iseki proposed the concept of ideals in BCK-algebras [17], which was subsequently elaborated upon in [18]. Implicative ideals in BCK-algebras were introduced by Meng [30].

In the dynamic realm of decision-making, decision-makers (DMs) face significant challenges in selecting the optimal option from the available options as systems become increasingly complex. Measuring the difficulty of doing a task is a challenging but manageable endeavor. Businesses usually have to balance a lot of conflicting interests when making choices since employee motivation, goal formulation, and organizational perspective are complicated processes. DMs are focusing more on creating reliable and useful methods to successfully navigate and manage the issues that arise in real-world situations.

When given complicated situations that call for the evaluation of many competing criteria, DMs may depend on the rigorous analytical framework known as MCDM. In MCDM, DMs attempt to determine the optimal choice from a range of options by concurrently assessing each option's performance across many factors. The challenges of decision-making have led to the development of several models, techniques, and processes within MCDM. These technologies aim to assist DMs in making more deliberate and knowledgeable judgments by obtaining pertinent data, evaluating viable solutions according to a number of conflicting criteria, and ultimately finding a compromise option. Karakoc *et al*. [22] developed a bibliometric review of integrated MCDM methods for sustainable supplier selection.

Zadeh proposed the concept of fuzzy sets (FSs) in 1965 to tackle the uncertainties commonly encountered in daily life [55]. Fuzzy set theory serves as an effective framework for characterising situations involving imprecise or unidentified d ata. To d eal w ith such circumstances, FSs assign a degree to which an item is a member of a set. However, in real life, a person could preassume, to a certain extent, that an item x belongs to a set A, but he might not be quite convinced. To put it another way, there can be hesitancy or doubt about x's level of membership (MS) in set A. Numerous problems in the fields of applied mathematics, information science, and decision-making have been greatly aided by fuzzy sets (FSs).

Atanassov's [4] groundbreaking work established the novel idea of "intuitionistic FSs (IFSs)" as an extension of conventional FSs. In contrast to conventional FSs that only take membership degrees into account, IFSs also take into account a parameter known as the "non-membership" (N-MS). This improvement makes it possible to depict ambiguity and uncertainty inside sets in a more sophisticated manner. Fuzzy set theory was greatly enhanced by Atanassov's seminal contribution, which offered a more complex framework for dealing with ambiguous and uncertain data. Since its invention, IFSs have been used extensively in a variety of fields where uncertainty calls for a more sophisticated approach, such as pattern recognition, decision support systems, medical diagnostics, and control systems. The foundation for future investigation and advancement in IFSs was established by Atanassov's groundbreaking work, which encouraged continuous study in uncertainty modeling and management. Al-Quran et al. [2] suggested the integration tropical artificial forests are efficient tools in data-driven modeling for real-life problems.

Human judgments in complex decision-making contexts are often unpredictable and ambiguous. Current fuzzy decision-making methods primarily present fuzzy evaluation values without offering degrees of credibility for these values in attribute selection. The credibility degree of the fuzzy evaluation value in fuzzy decision-making indicates its necessity and significance. F uzzy a ssessment v alues m ust b e c losely a ligned w ith t heir corresponding credibility measures to enhance credibility. This will enhance the accuracy and validity of the assessment data. Ye *et al.* [54] introduced the concept of a fuzzy credibility number (FCN) which is a robust extension of the fuzzy numbers, with a pair of membership grade and its credibility degree.

The intuitionistic fuzzy evaluation value must closely match its credibility measure in ambiguous and uncertain situations where human understanding is involved in complicated decision-making scenarios, including uncertain information and judgments. However, the absence of a reliable metric for IFSs suggests that they are inadequate for accurately assessing uncertain information. Jun *et al.* [21] presented the idea of an IFCS to improve the credibility measure of an IFS. Both a MS and a credibility degree, as well as a N-MS degree and a credibility degree, may be represented with the help of IFCS. They also pro-vide generalized similarity and distance metrics for IFCSs. Additionally, it incorporates trigonometric function-based similarity measurements, such as cosine, sine, tangent, and cotangent similarity measures, into the weighted generalized distance measure of IFCSs. Riaz *et al.* [38] introduced bipolar fuzzy credibility numbers (BFCNs) as an extension of bipolar fuzzy numbers for data-driven sustainable SCM. They also developed BFC Einstein weighted averaging and weighted geometric aggregation operators to aggregate experts' fuzzy preferences. In addition, numerous other extensions of FCSs have been proposed by various researchers.

In 1991, Xi utilized the concept of FSs in the context of BCK-algebras [51]. Jun and Kim

applied Atanassov's concept to develop the intuitionistic fuzzification of subalgebras and ideals within BCK-algebras [21]. Satyanarayan and Prasad [41] presented findings regarding intuitionistic fuzzy implicative ideals (IFIIs), intuitionistic fuzzy positive implicative ideals, and intuitionistic fuzzy commutative ideals. Ceylan et al. [7] proposed FPF soft matrices-based FMEA. Ilgin et al. [15] introduced the idea of soft union tri-bi-ideals of semigroups. Oner et al. [34] introduced the notion of subalgebras of Sheffer Stroke BCK-algebras and intuitionistic fuzzy implicative WSBG-ideals. Senturk et al. [42] developed algorithmic approach for Sheffer stroke BCK-algebras. Meng et al. [27] proposed bipolar-valued fuzzy ideals of BCK/BCI-algebras. Akram and Zhan [1] suggested the sensible fuzzy ideals of BCK-algebras by integrating t-conorm.
Figure 1 provides a comprehensive view of the interconnected components ensuring efficient operations and optimization in supply chain.

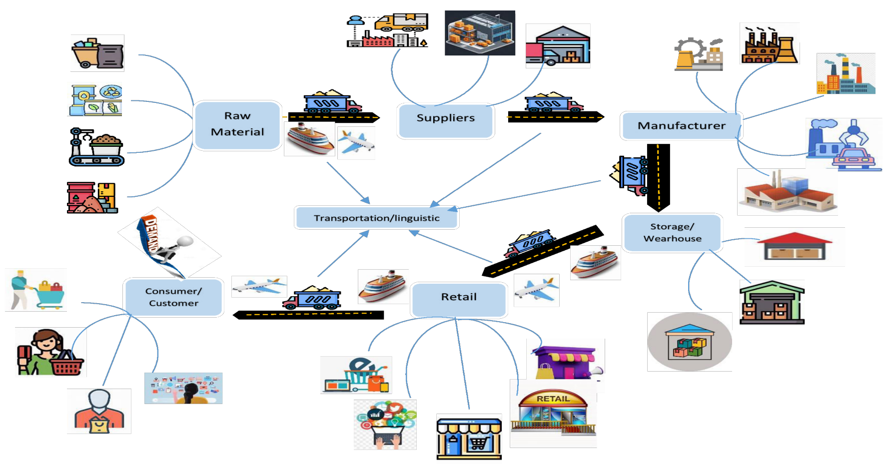

FIGURE 1. Representation of the proposed method

1.2. **Research Gap.** Despite the extensive use of MCDM methods in various fields, several research gaps remain, particularly when it comes to addressing uncertainty in decision-making processes. Traditional MCDM methods, such as AHP and Technique for Order of TOPSIS, often struggle to accurately model uncertainty and imprecision inherent in decision-making, especially when multiple conflicting criteria are involved. These methods rely on precise input data and may not effectively handle subjective judgments, which are crucial when evaluating alternatives in complex environments.
Additionally, conventional decision-making approaches fail to integrate both subjective and objective weights in a systematic manner. While some methods focus on subjective judgments, they lack robust mechanisms for incorporating objective data, or vice versa. Moreover, existing frameworks often neglect the interdependencies between criteria, which can

lead to biased or unreliable results. The gap in the literature lies in the lack of a comprehensive framework that effectively integrates IFCS to manage uncertainty within a BCK-algebraic structure, and the absence of a combined subjective-objective weighting mechanism for handling both types of influences in decision-making. Furthermore, there is a need for an advanced method that can account for both uncertainty and complexity while evaluating real-world cases with diverse and conflicting criteria, such as assessing companies across various operational dimensions.

This paper addresses these gaps by proposing a novel approach that combines IFCS within BCK-algebras, IFCII, SWARA [24] for subjective weights, LOPCOW [10] for objective weights, and ERUNS [6] for ranking alternatives. This integrated framework improves decision-making accuracy in uncertain, complex environments, offering a more holistic and reliable evaluation process.

1.3. **Motivation.** In the modern business landscape, organizations are faced with complex and uncertain environments when making critical decisions. The presence of multiple conflicting criteria—ranging from cybersecurity and geopolitical instability to economic, environmental, and technological factors—requires a more sophisticated decision-making approach that can effectively handle uncertainty and manage subjective judgment. Traditional decision-making methods often fall short when dealing with such uncertainty, leading to inconsistent results. Additionally, existing frameworks may struggle to capture the nuanced interplay between various factors and the inherent imprecision of expert opinions. Therefore, there is a strong need for an innovative, robust framework that incorporates both subjective and objective weights, while managing uncertainty in a systematic manner.

This paper is motivated by the need to enhance decision-making processes by addressing these challenges. The introduction of IFCS and IFCII within the framework of BCK-algebras aims to provide a novel and effective approach to decision-making under uncertainty. By combining advanced fuzzy logic techniques with proven MCDM methods, the framework enables more accurate, reliable, and transparent decision evaluations in dynamic, uncertain environments.

1.4. **Main contribution.** This paper contributes to the field of MCDM by proposing a comprehensive framework that integrates IFCS within BCK-algebras and introduces IFCII for advanced decision-making. The key contributions of this paper include:

- The paper demonstrates the use of IFCS and IFCII in a BCK-algebraic context, providing a new way to handle uncertainty and imprecision in decision-making. This integration enhances the ability to make more informed and consistent decisions in environments characterized by uncertainty and subjectivity.
- The paper introduces the combined use of SWARA and LOPCOW, ensuring that both subjective and objective factors are accurately incorporated into the decision-making process.
- The proposed framework is applied to evaluate leading companies such as Amazon, Walmart, and Alibaba across diverse criteria, such as cybersecurity, geopolitical instability, environmental impacts, and others. This real-world application demonstrates the practical utility of the framework.

- The paper includes a sensitivity analysis to test the robustness of the framework and a comparative analysis with traditional methods, providing evidence of its superior performance in handling complex decision-making scenarios.

Overall, this work offers a powerful decision-making tool for evaluating organizations across multiple, often conflicting, criteria while managing uncertainty effectively. The framework's application to real-world scenarios underscores its practical relevance and the potential for wider use in business and management decision-making.

2. Preliminaries

The term BCK/BCI-algebras is based on combinators B, C, K, and I in combinatory logic [17]. A number of BCK/BCI-algebra characteristics are investigated in [18, **?**, 28].

Definition 1. [16, 17] A BCK-algebra $\mathcal{K}$ is a general algebra $(\mathcal{K}, *, 0)$ of type $(2, 0)$ satisfying the following conditions:

- ($\mathcal{K}$-1) $d * d = 0$
- ($\mathcal{K}$-2) $0 * d = d$
- ($\mathcal{K}$-3) $(d * (d * e)) * e = 0$
- ($\mathcal{K}$-4) $((d * e) * (d * f)) * (f * e) = 0$
- ($\mathcal{K}$-5) $d * e = 0$ and $e * d = 0$ imply $d = e$
  for all $d, e, f \in \mathcal{K}$.

We can define a partial order "$\leq$" on $\mathcal{K}$ by $d \leq e$ if and only if $d * e = 0$.

Example 1. Let $\mathcal{K}$ = {0, i, j, k} be a BCK-algebra with the following Cayley Table 1 [21].

Table 1. BCK-algebra

*	0	i	j	k
0	0	0	0	0
i	i	0	0	i
j	j	i	0	j
k	k	k	k	0

Definition 2. [21] A non-empty subset $\mathcal{S}$ of a BCK-algebra $\mathcal{K}$ is referred to as a subalgebra if for all elements $d, e \in \mathcal{S}$, the result of the operation $d*e$ also lies in $\mathcal{S}$. Similarly, a subset $I \subseteq \mathcal{K}$ is considered an ideal of $\mathcal{K}$ if it contains the zero element (i.e., $0 \in I$), and for any $d, e \in \mathcal{K}$, whenever $d * e \in I$ and $e \in I$, it follows that $d \in I$. Furthermore, a fuzzy subset $\mathcal{F}$ of a universe $\mathcal{K}$ is a mapping that assigns to each element in $\mathcal{K}$ a MS value within the closed interval $[0, 1]$, reflecting the degree to which each element belongs to the FS.

$$\mathcal{F} : \mathcal{K} \to [0, 1]$$

Definition 3. [51]. *A fuzzy subset $\mathcal{F}$ in $\mathcal{K}$ is said to be a fuzzy sub-algebra of $\mathcal{K}$ if it satisfies*

$$\mathcal{F}(d * e) \geq \mathcal{F}(d) \wedge \mathcal{F}(e), \quad \textit{for all } d, e \in \mathcal{K}$$

Definition 4. [21]. *A fuzzy subset $\mathcal{F}$ of $\mathcal{K}$ is called a fuzzy ideal (FI) of $\mathcal{K}$ if it satisfies the following conditions:*

$$\text{(D1)} \quad \mathcal{F}(0) \geq \mathcal{F}(d), \quad \text{for all } d \in \mathcal{K}.$$

$$\text{(E1)} \quad \mathcal{F}(d) \geq \mathcal{F}(d * e) \wedge \mathcal{F}(e), \quad \text{for all } d, e \in \mathcal{K}.$$

Definition 5. *An object of the following kind is an IFS $\mathcal{D}$ of a non-empty set $\mathcal{K}$:*

$$\mathcal{D} = \{\langle d, \gamma^\mu(d), \xi^\mu(d)\rangle \mid d \in \mathcal{K}\}$$

where $\gamma^\mu : \mathcal{K} \to [0,1]$ and $\xi^\mu : \mathcal{K} \to [0,1]$ with the condition, $\forall d \in \mathcal{K}$:

$$0 \leq \gamma^\mu(d) + \xi^\mu(d) \leq 1$$

The degrees of MS and N-MS of the element d in the set $\mathcal{D}$[21] are indicated by the values $\gamma^\mu(d)$ and $\xi^\mu(d)$, respectively. For the sake of simplicity,

$$\mathcal{D} = (\gamma^\mu, \xi^\mu)$$

for the IFS.

$$\mathcal{D} = \{\langle d, \gamma^\mu(d), \xi^\mu(d)\rangle \mid d \in \mathcal{K}\}$$

Definition 6. [21] An IFS $\mathcal{D}$ of $\mathcal{K}$ is called an intuitionistic fuzzy ideal (IFI) of $\mathcal{K}$ if it satisfies the conditions (IFI-1) - (IFI-4), where $\forall d, e \in \mathcal{K}$:

- (IFI-1) $\gamma^\mu(0) \geq \gamma^\mu(d)$
- (IFI-2) $\xi^\mu(0) \leq \xi^\mu(d)$
- (IFI-3) $\gamma^\mu(d) \geq \gamma^\mu(d * e) \wedge \gamma^\mu(e)$
- (IFI-4) $\xi^\mu(d) \leq \xi^\mu(d * e) \vee \xi^\mu(e)$

Example 2. Let $\mathcal{K} = \{0, i, j, k\}$ be a BCK-algebra with the Cayley Table 1. Define $\mathcal{D} = (\gamma^\mu, \xi^\mu) : \mathcal{K} \to [0,1]$ by:

$$\gamma^\mu(0) = 0.65, \quad \xi^\mu(0) = 0.34$$

$$\gamma^\mu(i) = \gamma^\mu(j) = \gamma^\mu(k) = 0.42, \quad \xi^\mu(i) = \xi^\mu(j) = \xi^\mu(k) = 0.58$$

Then $\mathcal{D}$ is an IFI of $\mathcal{K}$.

Definition 7. [41] IFS $\mathcal{D}$ of $\mathcal{K}$ is called an IFII of $\mathcal{K}$ if it satisfies the conditions (IFII-1) - (IFII-4), where $\forall d, e \in \mathcal{K}$:

- (IFII-1) $\gamma^\mu(0) \geq \gamma^\mu(d)$
- (IFII-2) $\xi^\mu(0) \leq \xi^\mu(d)$
- (IFII-3) $\gamma^\mu(d) \geq \gamma^\mu(d * (e * d)) * f) \wedge \gamma^\mu(f)$
- (IFII-4) $\xi^\mu(d) \leq \xi^\mu((d * (e * d)) * f) \vee \xi^\mu(f)$

Theorem 1. Let $\{\mathcal{D}_i \mid i \in I\}$ be a family of IFIIs of a BCK-algebra $\mathcal{K}$. Then $\mathcal{D} = \bigcap_{i \in I} \mathcal{D}_i$ is an IFII of $\mathcal{K}$.

Proof. Let $\{\mathcal{D}_i \mid i \in I\}$ be a family of IFIIs of $\mathcal{K}$, where $\mathcal{D} = \{\langle d, \gamma^\mu(d), \xi^\mu(d)\rangle \mid d \in \mathcal{K}\}$. Since $\gamma^\mu_{\mathcal{D}_i}(0) \geq \gamma^\mu_{\mathcal{D}_i}(d)$ and $\xi^\mu_{\mathcal{D}_i}(0) \leq \xi^\mu_{\mathcal{D}_i}(d)$ for all $i \in I$, we have:

$$(\bigwedge_{i \in I} \gamma^\mu_{\mathcal{D}_i})(0) = \bigwedge_{i \in I}(\gamma^\mu_{\mathcal{D}_i}(0)) \geq \bigwedge_{i \in I}(\gamma^\mu_{\mathcal{D}_i}(d)) = (\bigwedge_{i \in I} \gamma^\mu_{\mathcal{D}_i})(d)$$

$$(\bigvee \xi^{\mu}_{\mathcal{D}_i})(0) = \bigvee_{i\in I} \xi^{\mu}_{\mathcal{D}_i}(0)) \leq \bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}(d)) = (\bigvee \xi^{\mu}_{\mathcal{D}_i})(d)$$

Thus, $\mathcal{D}$ satisfies conditions (IFII-1) and (IFII-2). For conditions (IFII-3) and (IFII-4), since each $\mathcal{D}_i$ is an IFII:

$$\gamma^{\mu}_{\mathcal{D}_i}(d) \geq \gamma^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \wedge \gamma^{\mu}_{\mathcal{D}_i}(f), \quad \xi^{\mu}_{\mathcal{D}_i}(d) \leq \xi^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \vee \xi^{\mu}_{\mathcal{D}_i}(f)$$

for all $i \in I$. Therefore,

$$(\bigwedge_{i\in I} \gamma^{\mu}_{\mathcal{D}_i})(d) = \bigwedge_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}(d)) \geq \bigwedge_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \wedge \gamma^{\mu}_{\mathcal{D}_i}(f)) = (\bigwedge_{i\in I} \gamma^{\mu}((d*(e*d))*f)) \wedge (\bigwedge_{i\in I} \gamma^{\mu}_{\mathcal{D}_i}(f))$$

$$(\bigvee_{i\in I} \xi^{\mu}_{\mathcal{D}_i})(d) = \bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}(d)) \leq \bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \vee \xi^{\mu}_{\mathcal{D}_i}(f)) = (\bigvee_{i\in I} \xi^{\mu}((d*(e*d))*f)) \vee (\bigvee_{i\in I} \xi^{\mu}_{\mathcal{D}_i}(f))$$

Hence, $\mathcal{D}$ satisfies conditions (IFII-3) and (IFII-4), proving $\mathcal{D}$ is an IFII of $\mathcal{K}$. □

Theorem 2. Let $\{\mathcal{D}_i \mid i \in I\}$ be a family of IFIIs of a BCK-algebra $\mathcal{K}$. Then $\mathcal{D} = \bigcup_{i\in I} \mathcal{D}_i$ is an IFII of $\mathcal{K}$.

Proof. Let $\{\mathcal{D}_i \mid i \in I\}$ be a family of IFIIs of $\mathcal{K}$, where $\mathcal{D} = \{\langle d, \gamma^{\mu}(d), \xi^{\mu}(d)\rangle \mid d \in \mathcal{K}\}$. Since $\gamma^{\mu}_{\mathcal{D}_i}(0) \geq \gamma^{\mu}_{\mathcal{D}_i}(d)$ and $\xi^{\mu}_{\mathcal{D}_i}(0) \leq \xi^{\mu}_{\mathcal{D}_i}(d)$ for all $i \in I$, we have:

$$(\bigvee_{i\in I} \gamma^{\mu}_{\mathcal{D}_i})(0) = \bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}(0)) \geq \bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}(d)) = (\bigvee_{i\in I} \gamma^{\mu}_{\mathcal{D}_i})(d)$$

$$(\bigwedge \xi^{\mu}_{\mathcal{D}_i})(0) = \bigwedge_{i\in I} \xi^{\mu}_{\mathcal{D}_i}(0)) \leq \bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}(d)) = (\bigwedge \xi^{\mu}_{\mathcal{D}_i})(d)$$

Thus, $\mathcal{D}$ satisfies conditions (IFII-1) and (IFII-2). For conditions (IFII-3) and (IFII-4), since each $\mathcal{D}_i$ is an IFII:

$$\gamma^{\mu}_{\mathcal{D}_i}(d) \geq \gamma^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \wedge \gamma^{\mu}_{\mathcal{D}_i}(f), \quad \xi^{\mu}_{\mathcal{D}_i}(d) \leq \xi^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \vee \xi^{\mu}_{\mathcal{D}_i}(f)$$

for all $i \in I$. Therefore,

$$(\bigvee_{i\in I} \gamma^{\mu}_{\mathcal{D}_i})(d) = \bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}(d)) \geq \bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \wedge \gamma^{\mu}_{\mathcal{D}_i}(f)) = (\bigvee_{i\in I} \gamma^{\mu}((d*(e*d))*f)) \wedge (\bigvee_{i\in I} \gamma^{\mu}_{\mathcal{D}_i}(f))$$

$$(\bigwedge_{i\in I} \xi^{\mu}_{\mathcal{D}_i})(d) = \bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}(d)) \leq \bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}((d*(e*d))*f) \vee \xi^{\mu}_{\mathcal{D}_i}(f)) = (\bigwedge_{i\in I} \xi^{\mu}((d*(e*d))*f)) \vee (\bigwedge_{i\in I} \xi^{\mu}_{\mathcal{D}_i}(f))$$

Hence, $\mathcal{D}$ satisfies conditions (IFII-3) and (IFII-4), proving $\mathcal{D}$ is an IFII of $\mathcal{K}$. □

Example 3. Let $\mathcal{K}$ = {0, i, j, k} be a BCK-algebra with the Cayley Table 1. Define two IFIIs $\mathcal{D}_1$ and $\mathcal{D}_2$:

$$\mathcal{D}_1 = (\gamma^{\mu}_{\mathcal{D}_\infty}, \xi^{\mu}_{\mathcal{D}_1}) : \mathcal{K} \to [0,1] \text{ by } (\gamma^{\mu}_{\mathcal{D}_1}, \xi^{\mu}_{\mathcal{D}_1})(0) = (0.75, 0.25)$$
$$(\gamma^{\mu}_{\mathcal{D}_1}, \xi^{\mu}_{\mathcal{D}_1})(i) = (\gamma^{\mu}_{\mathcal{D}_1}, \xi^{\mu}_{\mathcal{D}_1})(j) = (\gamma^{\mu}_{\mathcal{D}_1}, \xi^{\mu}_{\mathcal{D}_1})(k) = (0.8, 0.19)$$

$$\mathcal{D}_2 = (\gamma^{\mu}_{\mathcal{D}_\in}, \xi^{\mu}_{\mathcal{D}_2}) : \mathcal{K} \to [0,1] \text{ by } (\gamma^{\mu}_{\mathcal{D}_2}, \xi^{\mu}_{\mathcal{D}_2})(0) = (0.83, 0.16)$$
$$(\gamma^{\mu}_{\mathcal{D}_2}, \xi^{\mu}_{\mathcal{D}_2})(i) = (\gamma^{\mu}_{\mathcal{D}_2}, \xi^{\mu}_{\mathcal{D}_2})(j) = (\gamma^{\mu}_{\mathcal{D}_2}, \xi^{\mu}_{\mathcal{D}_2})(k) = (0.62, 0.36)$$

, Then, $\mathcal{D} = \mathcal{D}_1 \cap \mathcal{D}_2$ will be:

$$\mathcal{D}(0) = (\mathcal{D}_1 \cap \mathcal{D}_2)(0) = (0.75, 0.25)$$
$$\mathcal{D}(i) = \mathcal{D}(j) = \mathcal{D}(k) = (0.62, 0.36)$$

and $\mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2$ will be:

$$\mathcal{D}(0) = (\mathcal{D}_1 \cup \mathcal{D}_2)(0) = (0.83, 0.16)$$

$$\mathcal{D}(i) = \mathcal{D}(j) = \mathcal{D}(k) = (0.8, 0.19)$$

By calculations, $\mathcal{D}$ is an IFII of $\mathcal{K}$.

Definition 8. [53] Let X be a non-empty universe. An IFCS in X can be expressed as

$$R = \left\{\left\langle d, ({\gamma^{\mu}}_d, {\pi^{\gamma}}_d), ({\xi^{\mu}}_d, {\nu^{\xi}}_d)\right\rangle \mid d \in X\right\},$$

where for each element $d \in X$, the ordered pair $({\gamma^{\mu}}_d, {\pi^{\gamma}}_d)$ gives the degree MS and its credibility, while the pair $({\xi^{\mu}}_d, {\nu^{\xi}}_d)$ gives to the degree of N-MS and its credibility. All values ${\gamma^{\mu}}_d, {\pi^{\gamma}}_d, {\xi^{\mu}}_d, {\nu^{\xi}}_d$ lie in the interval $[0,1]$, such that $0 \leq {\gamma^{\mu}}_d + {\xi^{\mu}}_d \leq 1$. For the convenient representation, the component $\langle d, ({\gamma^{\mu}}_d, {\pi^{\gamma}}_d), ({\xi^{\mu}}_d, {\nu^{\xi}}_d)\rangle$ in R is simply denoted as $r = \langle ({\gamma^{\mu}}_d, {\pi^{\gamma}}_d), ({\xi^{\mu}}_d, {\nu^{\xi}}_d)\rangle$, which is named IFCN. In IFS, MS and N-MS represent the subjective evaluations made by DMs when assessing various alternatives across one or more criteria.

An Intuitionistic fuzzy credibility number (IFCN) Φ is defined as

$$\Phi = \left\langle (\gamma^{\mu}, \pi^{\gamma}), (\xi^{\mu}, \nu^{\xi})\right\rangle$$

Definition 9. [53] Let $\Phi_1 = \left\langle (\gamma_1^{\mu}, \pi_1^{\gamma}), \left(\xi_1^{\mu}, \nu_1^{\xi}\right)\right\rangle$ and $\Phi_2 = \left\langle (\gamma_2^{\mu}, \pi_2^{\gamma}), \left(\xi_2^{\mu}, \nu_2^{\xi}\right)\right\rangle$ as two IFCNs, and let $\rho > 0$. The operational relations of IFCNs are defined as follows.

- $\Phi_1 \supseteq \Phi_2 \Leftrightarrow \gamma_1^{\mu} \geq \gamma_2^{\mu}, \pi_a^{\gamma}1 \geq \pi_2^{\gamma}, \xi_1^{\mu} \leq \xi_2^{\mu}$ and $\nu_1^{\xi} \leq \nu_2^{\xi}$
- $\Phi_1 = \Phi_2 \Leftrightarrow \Phi_1 \supseteq \Phi_2$ and $\Phi_2 \supseteq \Phi_1$
- $\Phi_1 \cup \Phi_2 = \left\langle (\gamma_1^{\mu} \vee \gamma_2^{\mu}, \pi_1^{\gamma} \vee \pi_2^{\gamma}), \left(\xi_1^{\mu} \wedge \xi_2^{\mu}, \nu_1^{\xi} \wedge \nu_2^{\xi}\right)\right\rangle$
- $\Phi_1 \cap \Phi_2 = \left\langle (\gamma_1^{\mu} \wedge \gamma_2^{\mu}, \pi_1^{\gamma} \wedge \pi_2^{\gamma}), \left(\xi_1^{\mu} \vee \xi_2^{\mu}, \nu_1^{\xi} \vee \nu_2^{\xi}\right)\right\rangle$
- $(\Phi_1)^c = \left\langle (\xi_1^{\mu}, {\nu^{\theta}}_1), (\gamma_1^{\mu}, \pi_1^{\gamma})\right\rangle$ (the complement of Φ_1)
- $\Phi_1 \oplus \Phi_2 = \left\langle (\gamma_1^{\mu} + \gamma_2^{\mu} - \gamma_1^{\mu}\gamma_2^{\mu}, \pi_1^{\gamma} + \pi_2^{\gamma} - \pi_1^{\gamma}\pi_2^{\gamma}), \left(\xi_1^{\mu}\xi_2^{\mu}, \nu_1^{\xi}\nu_2^{\xi}\right)\right\rangle$
- $\Phi_1 \otimes \Phi_2 = \left\langle (\gamma_1^{\mu}\gamma_2^{\mu}, \pi_1^{\gamma}\pi_2^{\gamma}), (\xi_1^{\mu} + \xi_2^{\mu} - \xi_1^{\mu}\xi_2^{\mu}, {\nu^{\xi}}_1 + {\nu^{\xi}}_2 - {\nu^{\xi}}_1{\nu^{\xi}}_2)\right\rangle$
- $\rho\Phi_1 = \left\langle \left(1 - (1 - \gamma_1^{\mu})^{\rho}, 1 - (1 - \pi_1^{\gamma})^{\rho}\right), \left((\xi_1^{\mu})^{\rho}, \left(\nu_1^{\xi}\right)^{\rho}\right)\right\rangle$
- $(\Phi_1)^{\rho} = \left\langle \left((\gamma_1^{\mu})^{\rho}, (\pi_1^{\gamma})^{\rho}\right), \left(1 - (1 - \xi_1^{\mu})^{\rho}, 1 - \left(1 - \nu_1^{\xi}\right)^{\rho}\right)\right\rangle$

3. Intuitionistic fuzzy credibility sets (IFCS) in BCK-algebras

Definition 10. An IFCS $\mathcal{D}$ in BCK-algebra $\mathcal{K}$ is an object of the form:

$$\mathcal{D} = \{\langle d, [(\gamma^{\mu}(d), \pi^{\gamma}(d)), (\xi^{\mu}(d), \nu^{\xi}(d))]\rangle \mid d \in \mathcal{K}\}$$

where $(\gamma^{\mu}, \pi^{\gamma}) : \mathcal{K} \to [0,1]^2$ and $(\xi^{\mu}, \nu^{\xi}) : \mathcal{K} \to [0,1]^2$ with the condition $\forall d \in \mathcal{K}$, $0 \leq \gamma^{\mu}(d) + \xi^{\mu}(d) \leq 1$. The numbers $\gamma^{\mu}(d)$ and $\xi^{\mu}(d)$ denote, respectively, the degree of MS and the degree of N-MS of the element $d \in \mathcal{D}$, and the numbers $\pi^{\gamma}(d)$ and $\nu^{\xi}(d)$ denote, respectively, the credibility of the degree of MS and the degree of N-MS of the element $d \in \mathcal{D}$.

Definition 11. In BCK-algebra, an intuitionistic fuzzy credibility subset $\mathcal{D}$ of $\mathcal{K}$ and $\tau, \omega, \sigma, \upsilon \in (0,1]$, the crisp set

$$\mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]} = \{d \in \mathcal{K} \mid \mathcal{D}(d) \geq [(\tau,\omega),(\sigma,\upsilon)]\}$$

is called the *level subset* of $\mathcal{D}$.

Definition 12. In BCK-algebra, an IFCS $\mathcal{D}$ of $\mathcal{K}$ of the form:

$$\mathcal{D}(e) = [(\gamma^{\mu}, \pi^{\gamma})(e), (\xi^{\mu}, \nu^{\xi})(e)]$$

where

$$(\gamma^{\mu}, \pi^{\gamma})(e) = \begin{cases} (\tau, \omega) & \text{if } e = d \\ (0, \rho) & \text{if } e \neq d \end{cases}$$

and

$$(\xi^{\mu}, \nu^{\xi})(e) = \begin{cases} (\sigma, \upsilon) & \text{if } e = d \\ (0, \theta) & \text{if } e \neq d \end{cases}$$

is said to be an *intuitionistic fuzzy credibility point* with support d and value $\tau, \omega, \sigma, \upsilon$ (where $\tau, \omega, \rho, \sigma, \upsilon, \theta \in (0, 1]$) and is denoted by $d_{[(\tau,\omega),(\sigma,\upsilon)]}$. For an intuitionistic fuzzy credibility point $d_{[(\tau,\omega),(\sigma,\upsilon)]}$ and an IFCS $\mathcal{D}$ in a set $\mathcal{K}$, consider the symbol $d_{[(\tau,\omega),(\sigma,\upsilon)]} \in \mathcal{D}$ (resp. $d_{[(\tau,\omega),(\sigma,\upsilon)]} q\mathcal{D}$) if

$$(\gamma^{\mu}, \pi^{\gamma})(d) \geq (\tau, \omega) \quad \text{(resp. } (\gamma^{\mu}, \pi^{\gamma})(d) + (\tau, \omega) > 1)$$

and

$$(\xi^{\mu}, \nu^{\xi})(d) \leq (\sigma, \upsilon) \quad \text{(resp. } (\xi^{\mu}, \nu^{\xi})(d) + (\sigma, \upsilon) < 1).$$

In this paper, the following notations will be used:

(i) If $d_{[(\tau,\omega),(\sigma,\upsilon)]} \in \mathcal{D}$ or $d_{[(\tau,\omega),(\sigma,\upsilon)]} q\mathcal{D}$, then we write $d_{[(\tau,\omega),(\sigma,\upsilon)]} \in q\mathcal{D}$.

(ii) If $(\gamma^{\mu}, \pi^{\gamma})(d) < (\tau, \omega)$ (resp. $(\gamma^{\mu}, \pi^{\gamma})(d) + (\tau, \omega) \leq 1$) and $(\xi^{\mu}, \nu^{\xi})(d) \geq (\sigma, \upsilon)$ (resp. $(\xi^{\mu}, \nu^{\xi})(d) + (\sigma, \upsilon) > 1$), then we say that $d_{[(\tau,\omega),(\sigma,\upsilon)]} \in \overline{\mathcal{D}}$ (resp. $d_{[(\tau,\omega),(\sigma,\upsilon)]} q\overline{\mathcal{D}}$). The symbol $\overline{\mathcal{D}}$ indicates that q does not hold.

3.1. Intuitionistic fuzzy credibility implicative ideals (IFCII) in BCK-algebras.

Definition 13. An intuitionistic fuzzy credibility subset $\mathcal{D}$ of a BCK-algebra $\mathcal{K}$ is called an *intuitionistic fuzzy credibility implicative ideal* of $\mathcal{K}$ if it satisfies the conditions (IFCII-1) - (IFCII-4), where $\mathcal{D} = ((\gamma^{\mu}, \pi^{\gamma}), (\xi^{\mu}, \nu^{\xi}))$:

(IFCII-1) $(\gamma^{\mu}, \pi^{\gamma})(0) \geq (\gamma^{\mu}, \pi^{\gamma})(d)$,
(IFCII-2) $(\xi^{\mu}, \nu^{\xi})(0) \leq (\xi^{\mu}, \nu^{\xi})(d)$,
(IFCII-3) $(\gamma^{\mu}, \pi^{\gamma})(d) \geq (\gamma^{\mu}, \pi^{\gamma})((d * (e * d)) * f) \wedge (\gamma^{\mu}, \pi^{\gamma})(f)$,
(IFCII-4) $(\xi^{\mu}, \nu^{\xi})(d) \leq (\xi^{\mu}, \nu^{\xi})((d * (e * d)) * f) \vee (\xi^{\mu}, \nu^{\xi})(f)$,

for all $d, e, f \in \mathcal{K}$.

Example 4. Let $\mathcal{K} = \{0, i, j, k\}$ be a BCK-algebra with the following Cayley Table 2 [21] Consider an IFCS $\mathcal{D} = [(\gamma^{\mu}, \pi^{\gamma}), (\xi^{\mu}, \nu^{\xi})]$, defined as $[(\gamma^{\mu}, \pi^{\gamma}), (\xi^{\mu}, \nu^{\xi})] : \mathcal{K} \rightarrow$

TABLE 2. BCK-algebra

*	0	i	j	k
0	0	0	0	0
i	i	0	0	i
j	j	i	0	j
k	k	k	k	0

$([0,1]^2 \times [0,1]^2)$ by
$[(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](0) = [(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](i) = [(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](j)$
$= [(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](k) = [(\kappa_0,\zeta_0),(\kappa_1,\zeta_1)]$, where $0 \le \kappa_0+\kappa_1 \le 1$ and $\kappa_0,\zeta_0,\kappa_1,\zeta_1 \in [0,1]$. By Definition 13, $\mathcal{D} = [(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)]$ is an IFCII of $\mathcal{K}$.

Example 5. Let $\mathcal{K}$ = {0, i, j, k} be a BCK-algebra with the following Cayley Table 2. We define $\mathcal{D} = [(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)]$ as $[(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)] : \mathcal{K} \to ([0,1]^2 \times [0,1]^2)$ by

$$[(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](i) = [(\kappa_0,\zeta_0),(v_0,h_0)],$$
$$[(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](i) = [(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](k) = [(\kappa_1,\zeta_1),(v_1,h_1)],$$
$$[(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)](j) = [(\kappa_2,\zeta_2),(v_2,h_2)],$$

where $\kappa_0,\zeta_0,\kappa_1,\zeta_1,\kappa_2,\zeta_2,v_0,h_0,v_1,h_1,v_2,h_2 \in [0,1]$ and satisfy $(\kappa_0,\zeta_0) \le (\kappa_1,\zeta_1) \le (\xi_2,\zeta_2)$ and $(v_2,h_2) \le (v_1,h_1) \le (v_0,h_0)$.
By Definition 13, $\mathcal{D}$ is not an IFCII of BCK-algebra.

Theorem 3. An intuitionistic fuzzy credibility subset $\mathcal{D}$ of $\mathcal{K}$ is an IFCII of $\mathcal{K}$ if and only if, for every $[(\tau,\omega),(\sigma,\upsilon)] \in (0,1]$, $\mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$ is either empty or an implicative ideal of $\mathcal{K}$.

Proof. Suppose that $\mathcal{D} = [(\gamma^\mu,\pi^\gamma),(\xi^\mu,\nu^\xi)]$ is an IFCII of $\mathcal{K}$ and $\mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]} \neq \emptyset$ for any $d,\omega,\sigma,\upsilon \in (0,1]$. Then there exists $d \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$ and so

$$\mathcal{D}(d) \ge (\tau,\omega).$$

It follows from Definition [(IFCII-1) & (IFCII-2)] that

$$(\gamma^\mu,\pi^\gamma)(0) \ge (\gamma^\mu,\pi^\gamma)(d) \ge (\tau,\omega),$$
$$(\xi^\mu,\nu^\xi)(0) \le (\xi^\mu,\nu^\xi)(d) \le (\sigma,\upsilon),$$

so that

$$0 \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}.$$

Let $d,e,f \in \mathcal{K}$ and $\tau,\omega,\sigma,\upsilon \in (0,1]$ be such that $((d*(e*d))*f) \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$ and $f \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$. Then

$$(\gamma^\mu,\pi^\gamma)((d*(e*d))*f) \ge (\tau,\omega) \quad \text{and} \quad (\gamma^\mu,\pi^\gamma)(f) \ge (\tau,\omega),$$
$$(\xi^\mu,\nu^\xi)((d*(e*d))*f) \le (\sigma,\upsilon) \quad \text{and} \quad (\xi^\mu,\nu^\xi)(f) \le (\sigma,\upsilon).$$

Thus, by Definition [(IFCII-3) & (IFCII-4)], we have

$$(\gamma^\mu,\pi^\gamma)(d) \ge (\gamma^\mu,\pi^\gamma)((d*(e*d))*f) \wedge (\gamma^\mu,\pi^\gamma)(f) \ge (\tau,\omega) \wedge (\tau,\omega) = (\tau,\omega),$$

and

$$(\xi^\mu,\nu^\xi)(d) \le (\xi^\mu,\nu^\xi)((d*(e*d))*f) \vee (\xi^\mu,\nu^\xi)(f) \le (\sigma,\upsilon) \vee (\sigma,\upsilon) = (\sigma,\upsilon),$$

and so

$$d \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$$

Hence, $\mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$ is an implicative ideal of $\mathcal{K}$.

Conversely, suppose that $\mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$ is an ideal of $\mathcal{K}$ for every $\tau,\omega,\sigma,\upsilon \in (0,1]$. For $d \in \mathcal{K}$, let

$$((\gamma^\mu,\pi^\gamma)(d) = (\tau,\omega)$$

and

$$(\xi^{\mu}, \nu^{\xi})(d) = (\sigma, \upsilon)$$

Then $d \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$. Since $0 \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$, it follows that

$$(\gamma^{\mu}, \pi^{\gamma})(0) \geq (\tau, \omega) = (\gamma^{\mu}, \pi^{\gamma})(d),$$
$$(\xi^{\mu}, \nu^{\xi})(0) \leq (\sigma, \upsilon) = (\xi^{\mu}, \nu^{\xi})(d),$$

so

$$(\gamma^{\mu}, \pi^{\gamma})(0) \geq (\gamma^{\mu}, \pi^{\gamma})(d),$$
$$(\xi^{\mu}, \nu^{\xi})(0) \leq (\xi^{\mu}, \nu^{\xi})(d),$$

for all $d \in \mathcal{K}$. Now we prove that $\mathcal{D} = [(\gamma^{\mu}, \pi^{\gamma}), (\xi^{\mu}, \nu^{\xi})]$ satisfies conditions (IFCII-3) & (IFCII-4). On the contrary, suppose that there exist $d, e, f \in \mathcal{K}$ such that

$$(\gamma^{\mu}, \pi^{\gamma})(d) < (\gamma^{\mu}, \pi^{\gamma})((d * (e * d)) * f) \wedge (\gamma^{\mu}, \pi^{\gamma})(f),$$
$$(\xi^{\mu}, \nu^{\xi})(d) > (\xi^{\mu}, \nu^{\xi})((d * (e * d)) * f) \vee (\xi^{\mu}, \nu^{\xi})(f).$$

Select $\tau, \omega, \sigma, \upsilon \in (0, 1]$ such that

$$(\gamma^{\mu}, \pi^{\gamma})(d) < (\tau, \omega) \leq (\gamma^{\mu}, \pi^{\gamma})((d * (e * d)) * f) \wedge (\gamma^{\mu}, \pi^{\gamma})(f),$$
$$(\xi^{\mu}, \nu^{\xi})(d) > (\sigma, \upsilon) \geq (\xi^{\mu}, \nu^{\xi})((d * (e * d)) * f) \vee (\xi^{\mu}, \nu^{\xi})(f).$$

This implies that

$$(((d * (e * d)) * f) \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$$

and

$$f \in \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}$$

but

$$f \notin \mathcal{D}_{[(\tau,\omega),(\sigma,\upsilon)]}.$$

This is a contradiction. Therefore,

$$(\gamma^{\mu}, \pi^{\gamma})(d) \geq (\gamma^{\mu}, \pi^{\gamma})((d * (e * d)) * f) \wedge (\gamma^{\mu}, \pi^{\gamma})(f)$$
$$(\xi^{\mu}, \nu^{\xi})(d) \leq (\xi^{\mu}, \nu^{\xi})((d * (e * d)) * f) \vee (\xi^{\mu}, \nu^{\xi})(f)$$

This shows that $\mathcal{D}$ is an IFCII of $\mathcal{K}$. □

Theorem 4. Let $\{\mathcal{D}_i \mid i \in I\}$ be a family of IFCIIs of a BCK-algebra $\mathcal{K}$. Then $\mathcal{D} = \bigcap \mathcal{D}_i = [(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}), (\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})]$ is an IFCII of $\mathcal{K}$.

Proof. Suppose $\{\mathcal{D}_i \mid i \in I\}$ is a family of IFCII of $\mathcal{K}$ and $d \in \mathcal{K}$. Since

$$(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(0) \geq (\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(d)$$

and

$$(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(0) \leq (\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(d)$$

for all $i \in I$, we have

$$(\bigwedge_{i \in I}(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(0) = \bigwedge_{i \in I}((\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(0)) \geq \bigwedge_{i \in I}((\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(d)) = (\bigwedge_{i \in I}(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(d),$$

and

$$(\bigvee_{i \in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(0)) = \bigvee_{i \in I}((\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(0)) \leq \bigvee_{i \in I}((\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(d)) = (\bigvee_{i \in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i}))(d).$$

Thus,

$$\bigwedge_{i\in I}(\gamma^{\mu},\pi^{\gamma})(0)\geq\bigwedge_{i\in I}(\gamma^{\mu},\pi^{\gamma})(d),\quad \bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})(0)\leq\bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})(d).$$

Let $d,e,f\in\mathcal{K}$. Since each $\mathcal{D}_i=(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i}),(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})$ is an IFCII of $\mathcal{K}$, we have

$$(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i})(d)\geq(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i})((d*(e*d))*f)\wedge(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i})(f),$$

and

$$(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})\leq(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})((d*(e*d))*f)\vee(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})(f).$$

Therefore,
$(\bigwedge_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i}))(d)=\bigwedge_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i})(d))\geq\bigwedge_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i})((d*(e*d))*f))\wedge\bigwedge_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i})(f))=(\bigwedge_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i})((d*(e*d))*f)\wedge(\bigwedge_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i}))(f)$,
and
$(\bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i}))(d)=\bigvee_{i\in I}((\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})(d))\leq\bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})((d*(e*d))*f)\vee\bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})(f)$
$=(\bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i}))((d*(e*d))*f)\vee(\bigvee_{i\in I}(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i}))(f)$.
Hence, $\mathcal{D}=\bigcap\mathcal{D}_i=(\gamma^{\mu}_{\mathcal{D}_i},\pi^{\gamma}_{\mathcal{D}_i}),(\xi^{\mu}_{\mathcal{D}_i},\nu^{\xi}_{\mathcal{D}_i})$ is an IFCII of $\mathcal{K}$. □

Example 6. Let B = {0, i, j, k} be a BCK-algebra with the following Cayley Table 3 Consider two intuitionistic fuzzy credibility ideals $\mathcal{D}_1$ and $\mathcal{D}_2$:

TABLE 3. BCK-algebra

*	0	i	j	k
0	0	0	0	0
i	i	0	0	i
j	j	i	0	j
k	k	k	k	0

- $\mathcal{D}_1=[(\gamma^{\mu}_{\mathcal{D}_1},\pi^{\gamma}_{\mathcal{D}_1}),(\xi^{\mu}_{\mathcal{D}_1},\nu^{\xi}_{\mathcal{D}_1})]$, defined as $(\gamma^{\mu}_{\mathcal{D}_1},\pi^{\gamma}_{\mathcal{D}_1}),(\xi^{\mu}_{\mathcal{D}_1},\nu^{\xi}_{\mathcal{D}_1}):\mathcal{K}\to([0,1]^2\times[0,1]^2)$ by

$=[(0.76,0.5),(0.22,0.4)]$,

$[(\gamma^{\mu}_{\mathcal{D}_1},\pi^{\gamma}_{\mathcal{D}_1}),(\xi^{\mu}_{\mathcal{D}_1},\nu^{\xi}_{\mathcal{D}_1})](i)=[(\gamma^{\mu}_{\mathcal{D}_1},\pi^{\gamma}_{\mathcal{D}_1}),(\xi^{\mu}_{\mathcal{D}_1},\nu^{\xi}_{\mathcal{D}_1})](j)=[(\gamma^{\mu}_{\mathcal{D}_1},\pi^{\gamma}_{\mathcal{D}_1}),(\xi^{\mu}_{\mathcal{D}_1},\nu^{\xi}_{\mathcal{D}_1})](k)$
$=[(0.67,0.55),(0.33,0.45)]$.

- $\mathcal{D}_2=[(\gamma^{\mu}_{\mathcal{D}_2},\pi^{\gamma}_{\mathcal{D}_2}),(\xi^{\mu}_{\mathcal{D}_2},\nu^{\xi}_{\mathcal{D}_2})]$, defined as $[(\gamma^{\mu}_{\mathcal{D}_2},\pi^{\gamma}_{\mathcal{D}_2}),(\xi^{\mu}_{\mathcal{D}_2},\nu^{\xi}_{\mathcal{D}_2})]:\mathcal{K}\to([0,1]^2\times[0,1]^2)$ by

$=[(0.8,0.6),(0.2,0.4)]$,

$[[(\gamma^{\mu}_{\mathcal{D}_2},\pi^{\gamma}_{\mathcal{D}_2}),(\xi^{\mu}_{\mathcal{D}_2},\nu^{\xi}_{\mathcal{D}_2})]](i)=[[(\gamma^{\mu}_{\mathcal{D}_2},\pi^{\gamma}_{\mathcal{D}_2}),(\xi^{\mu}_{\mathcal{D}_2},\nu^{\xi}_{\mathcal{D}_2})]](j)=[[(\gamma^{\mu}_{\mathcal{D}_2},\pi^{\gamma}_{\mathcal{D}_2}),(\xi^{\mu}_{\mathcal{D}_2},\nu^{\xi}_{\mathcal{D}_2})]](k)$
$=[(0.65,0.55),(0.34,0.45)]$.

Then $\mathcal{D}=\mathcal{D}_1\wedge\mathcal{D}_2$ will be:

$$\mathcal{D}(0)=[(0.76,0.5),(0.22,0.4)],$$
$$\mathcal{D}(i)=\mathcal{D}(j)=\mathcal{D}(k)=[(0.65,0.55),(0.34,0.45)].$$

By calculations, $\mathcal{D}$ is an IFCII of $\mathcal{K}$.

Theorem 5. Let $\{\mathcal{D}_i \mid i \in I\}$ be a family of IFCIIs of a BCK-algebra $\mathcal{K}$. Then

$$\mathcal{D} = \bigcup_{i\in I} \mathcal{D}_i = (\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}), (\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})$$

is a IFCII of $\mathcal{K}$.

Proof. Suppose $\{\mathcal{D}_i \mid i \in I\}$ is a family of IFCII of $\mathcal{K}$ and $d \in \mathcal{K}$. Since

$$(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(0) \geq (\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(d)$$

and

$$(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(0) \leq (\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(d)$$

for all $i \in I$, we have

$$(\bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(0) = \bigvee_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(0)) \geq \bigvee_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(d)) = (\bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(d),$$

and

$$(\bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(0)) = \bigwedge_{i\in I}((\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(0)) \leq \bigwedge_{i\in I}((\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(d)) = (\bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i}))(d).$$

Thus,

$$\bigvee_{i\in I}(\gamma^{\mu}, \pi^{\gamma})(0) \geq \bigvee_{i\in I}(\gamma^{\mu}, \pi^{\gamma})(d), \quad \bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(0) \leq \bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(d).$$

Let $d, e, f \in \mathcal{K}$. Since each $\mathcal{D}_i = (\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}), (\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})$ is an IFCII of $\mathcal{K}$, we have

$$(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(d) \geq (\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})((d * (e * d)) * f) \wedge (\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(f),$$

and

$$(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i}) \leq (\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})((d * (e * d)) * f) \vee (\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(f).$$

Therefore,
$(\bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(d) = \bigvee_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(d)) \geq \bigvee_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})((d * (e * d)) * f)) \wedge \bigvee_{i\in I}((\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})(f)) = (\bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i})((d * (e * d)) * f) \wedge (\bigvee_{i\in I}(\gamma^{\mu}_{\mathcal{D}_i}, \pi^{\gamma}_{\mathcal{D}_i}))(f),$
and
$(\bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i}))(d) = \bigwedge_{i\in I}((\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(d)) \leq \bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})((d*(e*d))*f) \vee \bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i})(f)$
$= (\bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i}))((d * (e * d)) * f) \vee (\bigwedge_{i\in I}(\xi^{\mu}_{\mathcal{D}_i}, \nu^{\xi}_{\mathcal{D}_i}))(f).$
Therefore, we conclude that

$$\mathcal{D} = \bigcup_{i\in I} \mathcal{D}_i$$

is an IFCII of $\mathcal{K}$. □

Example 7. Consider two IFCII $\mathcal{D}_1$ and $\mathcal{D}_2$ defined as in Example 6. Then, we define $\mathcal{D} = \mathcal{D}_1 \vee \mathcal{D}_2$ as follows:

$$\mathcal{D}(0) = [(0.8, 0.6), (0.2, 0.4)], \mathcal{D}(i) = \mathcal{D}(j) = \mathcal{D}(k) = [(0.67, 0.55), (0.33, 0.45)].$$

Hence $\mathcal{D}$ is an intuitionistic fuzzy credibility ideal of $\mathcal{K}$.

4. LOPCOW-SWARA-ERUNS Method for MCDM

The proposed model is systematically summarised in this section. The IFC LOPCOW approach is applied to establish the objective weights that are given to each criterion. Then, following the highlighted limitations, solutions are evaluated using the IFC SWARA-ERUNS approach. By contributing their expertise based on the language factors listed in Table 4, DMs aid in the assessment process.

TABLE 4. Linguistic terms for global supply chain systems

Linguistic Terms	Abbreviations	CIFNs
Highly Resilient Supply Chain	$HRSC$	$\langle(0.92, 0.88), (0.03, 0.12)\rangle$
Moderately Resilient Supply Chain	$MRSC$	$\langle(0.85, 0.80), (0.08, 0.15)\rangle$
Low Resilience in Supply Chain	$LRSC$	$\langle(0.78, 0.75), (0.12, 0.20)\rangle$
High Cybersecurity Effectiveness	HCE	$\langle(0.75, 0.72), (0.15, 0.18)\rangle$
Moderate Cybersecurity Effectiveness	MCE	$\langle(0.68, 0.65), (0.20, 0.25)\rangle$
Low Cybersecurity Effectiveness	LCE	$\langle(0.60, 0.58), (0.25, 0.30)\rangle$
High Environmental Adaptability	HEA	$\langle(0.55, 0.50), (0.30, 0.35)\rangle$
Moderate Environmental Adaptability	MEA	$\langle(0.48, 0.45), (0.35, 0.40)\rangle$
Low Environmental Adaptability	LEA	$\langle(0.42, 0.40), (0.40, 0.45)\rangle$
High Geopolitical Stability	HGS	$\langle(0.38, 0.35), (0.45, 0.50)\rangle$
Moderate Geopolitical Stability	MGS	$\langle(0.30, 0.28), (0.50, 0.55)\rangle$
Low Geopolitical Stability	LGS	$\langle(0.25, 0.22), (0.55, 0.60)\rangle$

Algorithm: LOPCOW-SWARA-ERUNS method

Step 1: In a decision-making scenario using DMs, we are given a set of criteria indicated by $\mathrm{C} = \{\mathrm{U}^{\zeta}{}_1, \mathrm{U}^{\zeta}{}_2, ..., \mathrm{U}^{\zeta}{}_n\}$ and a set of alternatives denoted by $\mathrm{A} = \{\Phi^{\Theta}{}_1, \Phi^{\Theta}{}_2, ..., \Phi^{\Theta}{}_m\}$. Each criteria $\mathrm{U}^{\zeta}{}_j$ for each alternative $\Phi^{\Theta}{}_i$ is evaluated by DMs using IFCNs. The IFC-DM matrix $T = (\upsilon_{ij}^{*(k)})_{m\times n}$ is used by DMs, identified as $\mathfrak{E}_1$, $\mathfrak{E}_2$,..., $\mathfrak{E}_k$, to express their evaluations.

Step 2: Analyze the DMs' weights Δ_k by:

$$\begin{bmatrix} \mathrm{U}^{\zeta}{}_1 & \mathrm{U}^{\zeta}{}_2 & & \mathrm{U}^{\zeta}{}_n \\ \left(\langle\gamma^{\mu}{}_{11}, \beta^{\nu}{}_{11}\rangle, \langle\xi^{\mu}{}_{11}, \nu^{\xi}{}_{11}\rangle\right) & \left(\langle\gamma^{\mu}{}_{12}, \beta^{\nu}{}_{12}\rangle, \langle\xi^{\mu}{}_{12}, \nu^{\xi}{}_{12}\rangle\right) & \cdots & \left(\langle\gamma^{\mu}{}_{1n}, \beta^{\nu}{}_{1n}\rangle, \langle\xi^{\mu}{}_{1n}, \nu^{\xi}{}_{1n}\rangle\right) \\ \left(\langle\gamma^{\mu}{}_{21}, \beta^{\nu}{}_{21}\rangle, \langle\xi^{\mu}{}_{21}, \nu^{\xi}{}_{21}\rangle\right) & \left(\langle\gamma^{\mu}{}_{22}, \beta^{\nu}{}_{22}\rangle, \langle\xi^{\mu}{}_{22}, \nu^{\xi}{}_{22}\rangle\right) & \cdots & \left(\langle\gamma^{\mu}{}_{2n}, \beta^{\nu}{}_{2n}\rangle, \langle\xi^{\mu}{}_{2n}, \nu^{\xi}{}_{2n}\rangle\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\langle\gamma^{\mu}{}_{m1}, \beta^{\nu}{}_{m1}\rangle, \langle\xi^{\mu}{}_{m1}, \nu^{\xi}{}_{m1}\rangle\right) & \left(\langle\gamma^{\mu}{}_{m2}, \beta^{\nu}{}_{m2}\rangle, \langle\xi^{\mu}{}_{m2}, \nu^{\xi}{}_{m2}\rangle\right) & \cdots & \left(\langle\gamma^{\mu}{}_{mn}, \beta^{\nu}{}_{mn}\rangle, \langle\xi^{\mu}{}_{mn}, \nu^{\xi}{}_{mn}\rangle\right) \end{bmatrix}$$

$$\Delta_k = \frac{\gamma^{\mu}{}_{ij} + \beta^{\nu}{}_{ij} - \xi^{\mu}{}_{ij} - \nu^{\xi}{}_{ij}}{\sum_{k=1}^{t} \left[\gamma^{\mu}{}_{ij} + \beta^{\nu}{}_{ij} - \xi^{\mu}{}_{ij} - \nu^{\xi}{}_{ij}\right]} \tag{4.1}$$

where $w_k \geq 0$ and $\sum_{k=1}^{t} \Delta_k = 1$.

Step 3: Obtain the combined IFCNs table. According to the DMs' views, all decision tables provided must be gathered into a group in order to create the aggregated decision table. Apply Intuitionistic fuzzy credibility Dombi weighted averaging (IFCDWA) operator which is explained in Equation 4.2.

$$\left\{ \begin{array}{l} \left(1 - \frac{1}{1 + \left(\sum_{i=1}^{n} w_k \theta \left(\frac{\gamma^{\mu}{}_i}{1 - \gamma^{\mu}{}_i} \right)^{\Omega} \right)^{\frac{1}{\Omega}}}, 1 - \frac{1}{1 + \left(\sum_{i=1}^{n} w_k \theta \left(\frac{\beta^{\nu}{}_i}{1 - \beta^{\nu}{}_i} \right)^{\Omega} \right)^{\frac{1}{\Omega}}} \right), \\ \left(\frac{1}{1 + \left(\sum_{i=1}^{n} w_k \theta \left(\frac{1 - \xi^{\mu}{}_i}{\xi^{\mu}{}_i} \right)^{\Omega} \right)^{\frac{1}{\Omega}}}, \frac{1}{1 + \left(\sum_{i=1}^{n} w_k \theta \left(\frac{1 - \nu^{\xi}{}_i}{\nu^{\xi}{}_i} \right)^{\Omega} \right)^{\frac{1}{\Omega}}} \right) \end{array} \right\} \tag{4.2}$$

Step 4: Establish the score matrix $S^* = (\pi(R_{ij}))_{m \times n}$, $\forall$ of each IFCNs $\daleth_{ij}$ by using:

$$\pi_{(R_{ij})} = \frac{\gamma^{\mu}{}_{ij} + \beta^{\nu}{}_{ij} - \xi^{\mu}{}_{ij} - \nu^{\xi}{}_{ij}}{2}. \tag{4.3}$$

Step 5: The mean and standard deviation of matrix $\pi_{(R_{ij})}$ are computed using Equations 4.5 and 4.6, respectively, followed by applying the Gaussian membership function to each element using Equation 4.4.

$$\mu_{\pi_{(R_{ij})}} = \exp\left(-\frac{(\pi_{(R_{ij})} - \mu_{\pi_{(R_{ij})}})^2}{2\sigma^2_{\pi_{(R_{ij})}}} \right) \tag{4.4}$$

where

$$\nu_{\pi_{(R_{ij})}} = \frac{1}{m \cdot n} \sum_{i=1}^{m} \sum_{j=1}^{n} \pi_{(R_{ij})} \tag{4.5}$$

$$\sigma_{\pi_{(R_{ij})}} = \sqrt{\frac{1}{m \cdot n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\pi_{(R_{ij})} - \mu_{\pi_{(R_{ij})}} \right)^2} \tag{4.6}$$

LOPCOW method. Step 6: The normalized matrix $T^{\varrho} = (\wp_{ij})_{x \times o}$ constructed for standardises various criteria with differing units and scales to a uniform scale. It guarantees consistency across criteria.

$$\wp^{G}_{ij} = \begin{cases} \frac{\mu_{\pi_{(R)}}{}^{ij} - \mu_{\pi_{(R)}}{}^{j^-}}{\mu_{\pi_{(R)}}{}^{j^+} - \mu_{\pi_{(R)}}{}^{j^-}}, & \text{if } j \in \omega^{\nu}{}_b, \\ \frac{\mu_{\pi_{(R)}}{}_{j^+} - \mu_{\pi_{(R)}}{}_{ij}}{\mu_{\pi_{(R)}}{}_{j^+} - \mu_{\pi_{(R)}}{}_{j^-}}, & \text{if } j \in \omega^{\nu}{}_c, \end{cases} \quad i = 1, 2, \ldots, o; \; j = 1, 2, \ldots, x; \tag{4.7}$$

where $J^+ = \max_i \mu_{\pi_{(R)}}{}_{ij}$ and $\mu_{\pi_{(R)}}{}_{j^-} = \min_i \mu_{\pi_{(R)}}{}_{ij}$, $\omega^{\nu}{}_b$ and $\omega^{\nu}{}_c$ represent the cost-type and benefit-type criteria, respectively.

Step 7: Equation 4.8 yields the percentage values (P) for every criteria.

$$T^{Y}{}_{j} = \left| \ln \left(\frac{\sqrt{\frac{\sum_{i=1}^{m} \wp_{ij}^{\varrho 2}}{m}}}{\aleph\wp_{j}} \right) .100 \right| \quad j = 1, 2, \ldots, o; j = 1, 2, \ldots, x; \tag{4.8}$$

where m is the number of choices and $¶^{\daleth}{}_{j}$ is the SD, which is computed as follows.

$$¶^{\daleth}{}_{j} = \sqrt{\frac{\sum_{i=1}^{m} \left(\wp_{ij} - \bar{\varrho}_{j}\right)^{2}}{m}}, \text{where} \bar{\varrho}_{j} = \sum_{i=1}^{m} \wp_{ij}/m \quad j = 1, 2, \ldots, x;$$

Step 8: Calculated objective weights by using Equation 4.9.

$$\mathrm{C_{w}}_{j} = \frac{T^{Y}{}_{j}}{\sum_{i=1}^{m} T^{Y}{}_{j}} \quad j = 1, 2, \ldots, x; \tag{4.9}$$

SWARA method. Step 9: To begin, decision-makers prioritize attributes according to their perceived relative importance. The calculation of the attribute coefficient (Ξ) for each decision-maker is determined using Equation 4.10.

$$\Xi_{j} = \begin{cases} 1 & \text{if } j = 1 \\ \mathrm{C_{w}} + 1 & \text{if } j > 1 \end{cases} \quad j = 1, \ldots, n \tag{4.10}$$

Step 10: The preliminary importance value of every criterion matching the available options is computed by Equation 4.11.

$$\Omega^{\Im}{}_{j} = \begin{cases} 1 & \text{if } j = 1 \\ \frac{\Omega^{\Im}{}_{j}}{\Xi_{j}} & \text{if } j > 1 \end{cases} \quad j = 1, \ldots, q \tag{4.11}$$

Step 11 Applying Equation 4.12, one decides the relative weight of a criterion for every alternative.

$$W_{j} = \frac{\Omega^{\Im}{}_{j}}{\sum_{j=1}^{n} \Omega^{\Im}{}_{j}} \tag{4.12}$$

ERUNS method. Step 12: Apply Equation 4.13 to scale the matrix $\mathfrak{o} = \left[\mu_{\pi_{(R)}ij}\right]_{r \times o}$ within the specified range $[B, Z]$.

$$\nabla^{\ell}{}_{ij} = \left(\frac{Z_{B_{(R)}j}^{\min}}{Z_{B_{(R)}ij}} \right)^{3} Z + \frac{B}{Z_{B_{(R)}j}^{\min}} \quad i = 1, 2, \ldots, o;\ j = 1, 2, \ldots, x \tag{4.13}$$

In this context, $Z_{B_{(R)}j}^{\min} = \min_{1 \le i \le r} (Z_{B_{(R)}ij})$. The bounds B and Z of the interval are selected based on the judgment of the decision-maker and the particularities of the evaluation scenario.

Step 13: The matrix $\mathfrak{o}^N = [\Lambda_{ij}]$ contains the values that fulfill the condition for the maximum-type criterion. To update these values, apply Equation 4.14.

$$\natural_{ij} = \begin{cases} -\nabla^{\ell}{}_{ij} + \max\limits_{1\leq i\leq m}\left(\nabla^{\ell}{}_{ij}\right) + \min\limits_{1\leq i\leq m}\left(\nabla^{\ell}{}_{ij}\right), & j \in \omega^{\nu}{}_b \\ \nabla^{\ell}{}_{ij}, & j \in \omega^{\nu}{}_c \end{cases} \tag{4.14}$$

Step 14: Obtain the weighted standardised decision matrix using Equation 4.15.

$$\aleph_{\surd ij} = \frac{\exp\left(J\left(\natural_{ij}\right)/\delta^D\right) W_j}{\sum_{j=1}^{n}\exp\left(J\left(\natural_{ij}\right)/\delta^D\right) W_j} \quad i = 1,2,\ldots,o; j = 1,2,\ldots,x \tag{4.15}$$

Where $J\left(\natural_{ij}\right) = \frac{\natural_{ij}}{\sum_{j=1}^{n}\natural_{ij}}, j \in \{1,2,3\ldots n\}$.

Step 15: Find the utility values of the perfect and anti-ideal solutions. The utility degrees of the i^{th} choice for the ideal and anti-ideal solutions are shown below:

$$Q_{a_i}^{+} = \frac{\prod\limits_{j=1}^{n}\left(\natural_{ij}\right)^{\aleph_{\surd ij}}}{\sum\limits_{j=1}^{n}\aleph_{\surd j}^{+}}, \quad i = 1,2,\ldots,o$$

$$Q_{a_i}^{-} = -\frac{\sum_{j=1}^{n}\aleph_{\surd j}^{-}}{\prod_{j=1}^{n}\left(\natural_{ij}\right)^{\aleph_{\surd ij}}} + \max_{1\leq i\leq m}\left(\frac{\sum_{j=1}^{n}\aleph_{\surd j}^{-}}{\prod_{j=1}^{n}\left(\natural_{ij}\right)^{\aleph_{\surd ij}}}\right) + \min_{1\leq i\leq m}\left(\frac{\sum_{j=1}^{n}\aleph_{\surd j}^{-}}{\prod_{j=1}^{n}\left(\natural_{ij}\right)^{\aleph_{\surd ij}}}\right),$$

$$where$$

$$\aleph_{\surd j}^{+} = \max_{1\leq i\leq m}\left(\natural_{ij}\cdot W_j\right),$$

$$\aleph_{\surd j}^{-} = \min_{1\leq i\leq m}\left(\natural_{ij}\cdot W_j\right),$$

Step 16: Based on the general degrees of utility, Equations 4.16 and 4.17 are used to calculate the values of the utility function.

$$J\left(Q_{a_i}^{+}\right) = \frac{Q_{a_i}^{+}}{Q_{a_i}^{+} + Q_{a_i}^{-}} \quad i = 1,2,\ldots,o \tag{4.16}$$

$$J\left(Q_{a_i}^{-}\right) = \frac{Q_{a_i}^{-}}{Q_{a_i}^{+} + Q_{a_i}^{-}} \quad i = 1,2,\ldots,o \tag{4.17}$$

Step 17: The evaluation scores of the alternatives are set using Equation 4.18 and the parameter $Z \in [0,1]$.

$$F_i = \left(Q_{a_i}^{+} + Q_{a_i}^{-}\right)\frac{\left(1+J\left(Q_{a_i}^{+}\right)\right)^Z\left(1+J\left(Q_{a_i}^{-}\right)\right)^{1-Z} - \left(1-J\left(Q_{a_i}^{+}\right)\right)^Z\left(1-J\left(Q_{a_i}^{-}\right)\right)^{1-Z}}{\left(1+J\left(Q_{a_i}^{+}\right)\right)^Z\left(1+J\left(Q_{a_i}^{-}\right)\right)^{1-Z} + \left(1-J\left(Q_{a_i}^{+}\right)\right)^Z\left(1-J\left(Q_{a_i}^{-}\right)\right)^{1-Z}} \quad i = 1,2,\ldots,o \tag{4.18}$$

Figure 2 presents the complete algorithm of the proposed model.

5. Case study

Non-traditional security has become a major problem that affects supply chain corporations as well as countries around the world. Many facets of life, including society, security, defence, and the economy, are being directly affected by the growingly clear risks, challenges, and hazards to security. The COVID-19 epidemic, wars, geopolitical tensions, economic sanctions, the growing relevance of ESG initiatives, and natural disasters related to climate change have all contributed to modern supply chain networks' heightened volatility.

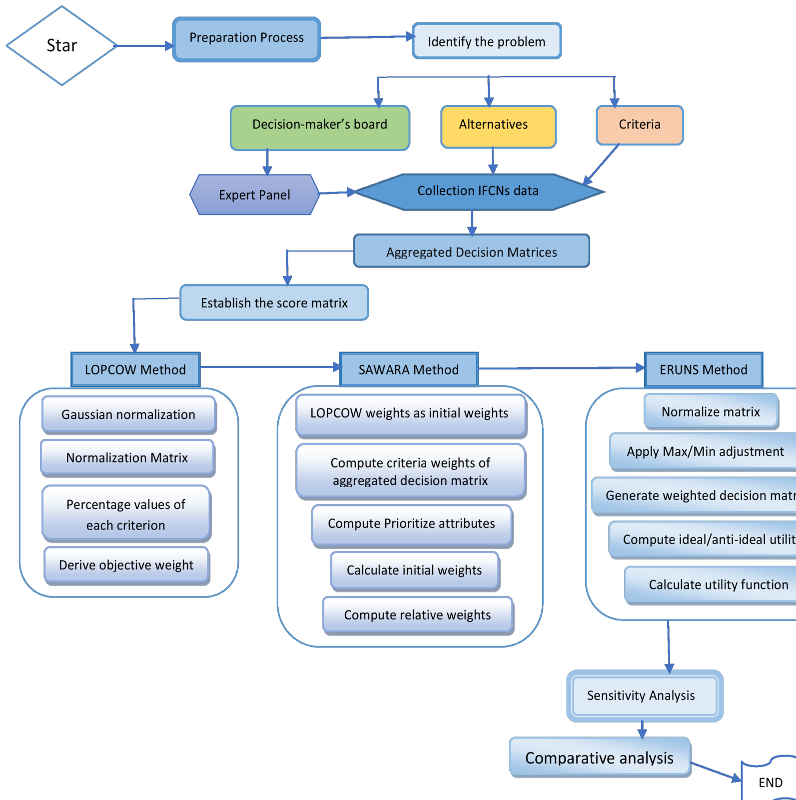

FIGURE 2. Flow chart of LOPCOW-SWARA-ERUNS Method

The supply chain is the economic and operational system moving a product from the warehouse to the manufacturing centre and finally to the end consumers. Therefore, effective execution of policies including production, storage, transportation, retailing, and customer service defines SCM. It can therefore be thought of as one of the essential pillars of international trade. It is clear that top-notch factories and commercial facilities are threatened as the reach of global trade keeps growing. However, this expansion has also made the market more competitive. Navigating this very competitive climate is a big challenge for businesses who want to provide clients affordable goods or services without compromising quality. E-commerce businesses have become important participants in the SCM field in the present era of online e-transactions. While serving as a link between suppliers and customers, E-commerce is not without its limitations and challenges. Therefore, it will be crucial to understand how supply chain networks rank and how challenges emerge one another in order to determine their overall effectiveness.

The factors have a complex effect on manufacturing, corporate operations, human resources, and the general consumption of goods, which causes problems for the smooth. When analyzing and prioritizing the identified criteria, consideration is given to how they affect supply chain efficiency, customer satisfaction, cost management, and adaptability. The success of identifying the most significant and relevant issues is guaranteed by this methodical approach.

In the context of supply chain companies, DMs in different cities the globe are tasked with prioritizing various companies(as alternatives). It is the responsibility of these DMs to rate various solutions according to their possible advantages. Five possibilities and four criteria with sub-criteria make up the hierarchical structure used to do this. This framework seeks to direct the assessment procedure and make it easier to rank the alternatives.

5.1. **Definition of Alternatives.**

(1) **Amazon**
Jeff Bezos, established Amazon in 1994, making it one of the biggest multinational tech companies for supply chain and logistics operations. The largest online retailer in the world, Amazon.com, ships goods to individual customers from its own warehouses and partner inventory. It gives independent sellers the chance to sell on the Amazon platform. On the same platform, it offers its own products as well. Amazon's supply chain mechanism uses a large number of fulfillment and distribution centers, powerful data analytics, and sophisticated robot and drone applications. This company offers a variety of logistical services, such as Amazon Prime Air, Amazon Air, Amazon Flex, and Amazon Logistics. The production line of Amazon can be described as having effective and adaptable inventory control, prompt delivery fulfillment, fruitful partner relationships, astute acquisitions of related firms and technology, and a high level of customer service [19].

(2) **Walmart**
The Sam brothers established Walmart, another worldwide corporation for supply chain and logistics operations with headquarters in the United States, in 1962. It runs a chain of supermarkets, inexpensive department stores, and hypermarkets in the United States and 23 other countries.Suppliers use the vendor-managed inventory model that Walmart implemented to obtain information from the business's system. Walmart also developed an application that reloads inventory in the storage area and continuously delivers information about sales data [19].

(3) **Costco**
Costco Wholesale Corporation, headquartered in the United States, operates a network of membership-only big-box warehouse clubs globally. Costco operates as a membership-based warehouse club, focusing on delivering significant discounts on both quantity and quality of products. Along with warehouse sales, Costco generates revenue by providing premium goods in tens of thousands of product categories at affordable prices. Costco made 235.442 dollars in revenue for the 12 months that ended on May 31, 2023. Costco ranks as the second largest retailer globally, attributed to its significant buying power and extensive membership base, alongside its ongoing commitment to efficiency, resulting in optimal pricing for its members [48].

(4) **Alibaba**
Alibaba, established by Jack Ma in 1999, is a globally recognized online shopping site that attracts customers from all over the world. Alibaba distinguishes itself through the integration of its diverse platforms and services, exemplifying a collaborative and ecosystem-centric approach to SCM. The company now operates in

over 190 countries and regions. Alibaba focuses on a mix of B2B, B2C, and C2C models. In particular, the Cainiao Network is essential since it serves as a central location for the consolidation of logistics data. It uses artificial intelligence and analytics to improve package tracking accuracy, speed up shipments, and optimize delivery routes. In addition to increasing the effectiveness of Alibaba's supply chain operations, this data-driven approach promotes more openness and dependability, which increases customer satisfaction and trust [36].

(5) **Best Buy**
Best Buy was founded by James Wheeler and Richard M. Schulze in 1966. Best Buy Co., Inc., the American international retailer of consumer electronics, is headquartered in the Minnesota town of Richfield. In 2022, Best Buy generated revenues of 51.761 billion dollars. This indicates a 9.52 percent increase in year-over-year growth for the company, which generated 47.262 billion dollars in 2021. The consumer electronics retailer operates 1,043 locations across the United States, with more than 50% utilizing its Ship-from-Store service. Best Buy has been able to greatly strengthen its pickup and delivery operations because of a smart technique, which has increased domestic internet sales by 242 percent [13].

(6) **Carrefour Group**
Carrefour Group, established by Denis Defforey, Jacques Defforey, Marcel Fournier in 1959. The headquarters of the international French retail and wholesale company Carrefour Group, S.A. are located in Massy, France. It runs a chain of supermarkets, convenience stores, and hypermarkets. The company operated 14,000 outlets in 40 countries by 2024. In terms of revenue, it ranks as the seventh-largest retailer globally. Carrefour has positioned its business as a retailer offering premium goods at affordable prices. Customers have many options when they shop at Carrefour stores because they also provide a wide variety of products from numerous brands [39].

(7) **IKEA**
IKEA, established by Ingvar Kamprad(17 years old) in 1943, is a Swedish multinational company. IKEA is a furniture manufacturer that often sells kitchen appliances, home accents, self-assembled furniture, and a wide range of other goods. The company handles restaurants, residences, and apartments in addition to its main business of designing, manufacturing, and selling furniture. IKEA is the most valuable furniture retail brand, with a 2021 valuation of over 21 billion dollars, making it the seventh most valuable retailer globally. In addition, the corporation operates 458 physical shops worldwide and 50 E-commerce marketplaces [14]

(8) **Tesco**
Tesco was founded by Jack Cohen in Hackney, London, in 1919. The company is a global entity specialising in grocery and everything else, headquartered in Welwyn Garden City, in the United Kingdom. Tesco, the largest grocery store in the UK, specialises in food items and offers a limited range of non-food services, including apparel, home appliances, and banking. Tesco's operations management has demonstrated success, evidenced by the company's significant revenue and net profit growth over the past five years [29].

5.2. **Definition of Criteria.**

(1) **Cybersecurity**
- **Ransomware attacks (Effect):** By compromising important suppliers and components, ransomware supply chain attacks take advantage of the network of trust that exists inside the supply chain. Because digital supply chains are linked together, these attacks are more dangerous [25].
- **Data breaches(Cause):**A data breach is related to the unauthorised acquisition or access to electronic data, documents, including files, and media that contain sensitive personally identifiable information. Numerous companies, such as Alibaba and Amazon, are increasingly experiencing data breaches [44].
- **IoT vulnerabilities (Cause):** In SCM, the IoT enhances the integration of smart connected products. Intelligent systems and services can identify, gather, distribute, and analyze data due to the Internet of Things, a network of diverse components [3].

(2) **Geopolitical Instability**
- **Trade wars and tariffs (Effect):** Trade disputes affect pricing strategies, industry trends, and international trade models, all of which have an impact on global supply networks. Without the integrated and cross-border supply chains, the modern global economy might not operate efficiently. However, trade disputes and other international conflicts create a major threat to their sustainability in the future [12].
- **Political Sanctions (Effect):** In Political sanctions, the target country's access to global markets is restricted by financial and business constraints [8].
- **Political unrest (Effect):** Political instability is defined as the propensity for social violence, political upheaval, or government change, as well as the ambiguity and volatility of government policies such as taxation, regulations, or human rights legislation. Political unrest poses a significant danger to the success of companies engaged in international supply networks. Companies that require foreign sourcing must create risk-reduction strategies and be mindful of the potential damage that political upheaval may bring to quality, relationships, and productivity [32].

(3) **Environment**
- **Pandemics and health crises (Cause):** Several of the effects that the coronavirus has had on logistics and supply chains have proven adverse. Given the significance of supply chains and logistics worldwide, any interruption or poor management results in several issues for both the impacted nations and the entire world [9].
- **Climate change and natural disasters (Cause):** Climate change is marked by unknown uncertainty, particularly in the near and medium future, since weather findings provide a noisy signal of potential changes in the underlying distribution. Determining the indirect climate change exposure that suppliers and customers are responsible for may be challenging. In this context, the effects of progressively changing exposure to climate hazards on businesses'

decisions to terminate existing supply chain agreements and form new ones are uncertain [35].
- **Natural Resources scarcity and Energy crises (Cause) :**Energy consumption from fossil fuels, the depletion of natural resources, and overall greenhouse gas emissions are serious problems that need to be addressed right away. The world's limited resources are under a lot of strain due to rapid population increase and growing industrialization, which is causing shortages in many places [49].

(4) **Economic**
- **Economic sanction (Cause):** Economic sanction is the threat or actual use of measures to limit economic interchange unless a targeted party authorizes some sort of non-economic policy concession.
- **Currency rate fluctuations (Effect):** Currency fluctuations can have a significant impact on global supply chains, affecting each step from final sales to procurement. Additionally, it may result in price volatility, necessitating the use of hedging techniques or long-term agreements by businesses to control expenses. However, as goods become more expensive for overseas buyers, a substantial increase in the value of the currency in a company's primary market may result in a decline in export competitiveness. Reduced demand may arise from this, requiring changes to inventory control and production levels [46].

(5) **Technology**
- **Blockchain failure:**Blockchain projects strengthen current relationships of power, making sustainability governance harder to locate and remove from the communities and individuals that are directly influenced by the functioning of global supply chains [5].
- **Legacy system incompatibilities:** The difficulties that develop when older technology systems do not function properly with newer technologies are referred to as legacy system incompatibility. To properly utilize new technology, organizations must address compatibility issues with legacy systems [33].

(6) **Social**
- **Social Instability:** As 2025 approaches, social instability is a major worry for organizations. Global supply chains impacted by social instability, with differing degrees of harm to businesses and the economy [47].
- **Terrorism:** One of the biggest risks nowadays is terrorism. ”The threatened or actual use of illegal force and violence by a nonstate actor” is its definition. Businesses' connections with their primary clientele are impacted by terrorist attacks, which are unanticipated negative shocks to the environment in which they operate, for some reasons [45].

(7) **Logistics-Driven Threats**
- **Port inefficiencies:** Port congestion is becoming an increasingly significant problem in the world of logistics, impacting supply chains' dependability and efficiency globally. Logistics businesses, freight forwarders, and shippers

must comprehend the causes, effects, and potential solutions of port congestion as global trade grows. Ports are important hubs in maritime supply lines and are essential to international maritime commerce [50].

- **Transportation Costs:** One of the most important problems with communication in supply chain phenomena is transportation. The logistics and supply chain might be negatively impacted by transportation interruptions. Additionally, transportation costs have an influence on the supply chain [31].

5.3. **Numerical approach.** **Step 1:** The DMs used the terminology from Table 4 to express their ideas, and the responses are recorded in the Table 5, Table 6, and Table 7.

Step 2: A crucial step in establishing the weights of DMs is analysing the importance

TABLE 5. Decision-maker 1 evaluation

U^{ζ}_1	U^{ζ}_2	U^{ζ}_3	U^{ζ}_4	U^{ζ}_5	U^{ζ}_6	U^{ζ}_7	U^{ζ}_8
HRSC	LCE	MGS	HCE	LEA	HRSC	HEA	LGS
MRSC	HEA	LGS	MCE	HGS	MRSC	MEA	HRSC
LRSC	MEA	HRSC	LCE	MGS	LRSC	LEA	MRSC
HCE	LEA	MRSC	HEA	LGS	HCE	HGS	LRSC
MCE	HGS	LRSC	MEA	HRSC	MCE	MGS	HCE
LCE	MGS	HCE	LEA	MRSC	LCE	LGS	MCE
HEA	LGS	MCE	HGS	LRSC	HEA	HRSC	LCE
MEA	HRSC	LCE	MGS	HCE	MEA	MRSC	HEA
LEA	MRSC	HEA	LGS	MCE	LEA	LRSC	MEA
HGS	LRSC	MEA	HRSC	LCE	HGS	HCE	LEA
MGS	HCE	LEA	MRSC	HEA	MGS	MCE	HGS
LGS	MCE	HGS	LRSC	MEA	LGS	LCE	MGS
HRSC	LCE	MGS	HCE	LEA	HRSC	HEA	LGS
MRSC	HEA	LGS	MCE	HGS	MRSC	MEA	HRSC
LRSC	MEA	HRSC	LCE	MGS	LRSC	LEA	MRSC
HCE	LEA	MRSC	HEA	LGS	HCE	HGS	LRSC

assigned to each DM's area of expertise. In the current case study, three DMs took part. Equation 4.1 was used to apply the IFCS linguistic values to the different DMs shown in Table 8.

Step 3: Using a methodical manner, individual tables are collected and classified based on the perspectives of the corresponding DMs to create the aggregated IFCNs. Equation 4.2 was used to combine them and displaying the outcome in Table 9.

TABLE 6. Decision-maker 2 evaluation

$U^{\zeta}{}_1$	$U^{\zeta}{}_2$	$U^{\zeta}{}_3$	$U^{\zeta}{}_4$	$U^{\zeta}{}_5$	$U^{\zeta}{}_6$	$U^{\zeta}{}_7$	$U^{\zeta}{}_8$
HCE	HEA	MRSC	HGS	LGS	LGS	HGS	HCE
MRSC	HGS	HEA	LGS	LEA	LEA	MRSC	MCE
LGS	LGS	HRSC	MEA	HGS	HGS	LGS	LEA
HEA	HRSC	LCE	MRSC	LCE	LCE	LCE	MRSC
LRSC	LCE	LEA	LEA	MRSC	MRSC	MCE	LCE
MEA	MEA	HGS	LCE	HEA	HEA	MEA	MGS
HRSC	LRSC	LGS	HRSC	MGS	MGS	HCE	LEA
MCE	MCE	MGS	MCE	HRSC	MEA	MGS	MGS
LCE	HCE	MEA	LRSC	MEA	MEA	LGS	HEA
HGS	MRSC	LRSC	HCE	HRSC	HRSC	LRSC	MEA
MGS	LEA	MRSC	MGS	MEA	MEA	HCE	HCE
LEA	HGS	MCE	MEA	HCE	HCE	LEA	MCE
LCE	MGS	HCE	HEA	MCE	MCE	MGS	MRSC
MRSC	HRSC	MEA	HGS	HRSC	HRSC	LEA	LCE
HCE	MCE	HRSC	LGS	MEA	MEA	MGS	MGS
MEA	LGS	MGS	LEA	HCE	HCE	LEA	MEA
LGS	MEA	HCE	MGS	MCE	MCE	HGS	HRSC

TABLE 7. Decision-maker 3 evaluation

$U^{\zeta}{}_1$	$U^{\zeta}{}_2$	$U^{\zeta}{}_3$	$U^{\zeta}{}_4$	$U^{\zeta}{}_5$	$U^{\zeta}{}_6$	$U^{\zeta}{}_7$	$U^{\zeta}{}_8$
LEA	HGS	MCE	MEA	LGS	HEA	LRSC	MGS
LGS	HRSC	HGS	MGS	HGS	LCE	HCE	LEA
MRSC	LCE	LRSC	HRSC	HEA	MCE	MRSC	HGS
HCE	MRSC	LGS	HGS	MCE	MRSC	HEA	MEA
HRSC	LEA	LEA	LCE	HCE	MEA	LGS	HRSC
HEA	LGS	HRSC	MCE	HRSC	HGS	LCE	MCE
MCE	LGS	MRSC	LEA	MGS	LGS	HGS	HCE
MGS	HCE	MEA	LRSC	LRSC	HRSC	LEA	HGS
LCE	HCE	LCE	LGS	LCE	LGS	MCE	MRSC
LRSC	HCE	HCE	MGS	LEA	HRSC	MCE	HRSC
MEA	MEA	HEA	MEA	MCE	MGS	MGS	MGS
HGS	LGS	MGS	MCE	MEA	HCE	HCE	LRSC
LEA	HGS	HCE	HGS	HRSC	HGS	HGS	LEA
HCE	HRSC	HRSC	LCE	LEA	HCE	LGS	MGS
HRSC	MEA	LCE	MGS	MRSC	HRSC	MRSC	HEA
MCE	MGS	LCE	LEA	HEA	HEA	MEA	MRSC
HGS	HGS	MCE	MRSC	MCE	MEA	HCE	HEA

Step 4: Equation 4.3 was used to calculate the score matrix $\pi_{(R_{ij})}$ as follows.

$$\pi_{(R_{ij})} = \begin{bmatrix} 0.6879 & 0.1764 & 0.5018 & 0.3186 & -0.1991 & 0.5484 & 0.3531 & 0.2265 \\ 0.6157 & 0.6191 & -0.0385 & 0.1179 & -0.0727 & 0.6995 & 0.5617 & 0.6675 \\ 0.5534 & 0.0883 & 0.7897 & 0.6395 & -0.0205 & 0.3955 & 0.4133 & 0.4580 \\ 0.4923 & 0.7062 & 0.4869 & 0.4688 & 0.2383 & 0.5080 & 0.1649 & 0.5819 \\ 0.7023 & 0.1046 & 0.3520 & 0.1520 & 0.7489 & 0.4140 & 0.1028 & 0.6896 \\ 0.2151 & -0.1290 & 0.6699 & 0.2859 & 0.7147 & 0.0528 & 0.0823 & 0.3165 \\ 0.6655 & 0.4581 & 0.4951 & 0.6155 & 0.2971 & -0.0126 & 0.6847 & 0.2808 \\ 0.1909 & 0.6711 & 0.1156 & 0.4060 & 0.7233 & 0.5963 & 0.4485 & 0.4762 \\ 0.2289 & 0.5826 & 0.2105 & 0.2564 & 0.3129 & 0.2888 & 0.4150 & 0.4921 \\ 0.2976 & 0.5652 & 0.4921 & 0.7256 & 0.0716 & 0.3362 & 0.7195 & 0.6160 \\ -0.1094 & 0.2795 & 0.4748 & 0.6128 & 0.4372 & 0.1009 & 0.6509 & -0.1737 \\ -0.1393 & 0.4233 & 0.1429 & 0.3965 & 0.3670 & 0.7489 & 0.4065 & 0.2538 \\ 0.6548 & 0.2583 & 0.2279 & 0.3295 & 0.6478 & 0.4322 & 0.1453 & -0.0443 \\ 0.6781 & 0.7618 & 0.4228 & 0.3995 & -0.2169 & 0.3659 & 0.2464 & 0.8016 \\ 0.7197 & 0.2464 & 0.7656 & 0.5949 & 0.4302 & 0.2461 & 0.4078 & 0.7105 \\ 0.4284 & 0.2519 & 0.4484 & 0.4420 & 0.6192 & 0.1777 & 0.4299 & 0.2865 \\ 0.1399 & -0.0662 & 0.6683 & 0.0608 & 0.7020 & 0.1387 & -0.1925 & 0.5641 \end{bmatrix}$$

TABLE 8. Linguistic terms about DMs

DMs	Qualification	Expertise	Aptitude	Experience	Weights
Δ_1	HRSC	HGS	Of	MRSC	0.3521
Δ_2	HRSC	MGS	MCE	LEA	0.3315
Δ_3	HEA	MEA	MEA	MCE	0.3164

TABLE 9. Aggregated IFCNs

$U^{\zeta}{}_1$	$U^{\zeta}{}_2$	$U^{\zeta}{}_3$	$U^{\zeta}{}_4$	$U^{\zeta}{}_5$	$U^{\zeta}{}_6$	$U^{\zeta}{}_7$	$U^{\zeta}{}_8$
⟨(0.841, 0.786)(0.068, 0.183)⟩	⟨(0.530, 0.497)(0.311, 0.363)⟩	⟨(0.530, 0.497)(0.311, 0.363)⟩	⟨(0.608, 0.573)(0.251, 0.293)⟩	⟨(0.320, 0.304)(0.486, 0.537)⟩	⟨(0.755, 0.701)(0.141, 0.218)⟩	⟨(0.637, 0.597)(0.220, 0.308)⟩	⟨(0.555, 0.518)(0.287, 0.333)⟩
⟨(0.799, 0.738)(0.110, 0.197)⟩	⟨(0.810, 0.740)(0.080, 0.232)⟩	⟨(0.417, 0.375)(0.408, 0.462)⟩	⟨(0.499, 0.465)(0.334, 0.394)⟩	⟨(0.394, 0.375)(0.432, 0.482)⟩	⟨(0.847, 0.797)(0.066, 0.179)⟩	⟨(0.759, 0.708)(0.138, 0.206)⟩	⟨(0.833, 0.774)(0.070, 0.201)⟩
⟨(0.759, 0.707)(0.476, 0.450)⟩	⟨(0.900, 0.856)(0.817, 0.755)⟩	⟨(0.426, 0.387)(0.652, 0.619)⟩	⟨(0.683, 0.617)(0.708, 0.645)⟩	⟨(0.133, 0.226)(0.348, 0.402)⟩	⟨(0.039, 0.137)(0.078, 0.216)⟩	⟨(0.401, 0.453)(0.218, 0.261)⟩	⟨(0.184, 0.290)(0.169, 0.269)⟩
⟨(0.707, 0.672)(0.180, 0.215)⟩	⟨(0.854, 0.798)(0.063, 0.177)⟩	⟨(0.722, 0.662)(0.159, 0.251)⟩	⟨(0.715, 0.649)(0.166, 0.260)⟩	⟨(0.563, 0.534)(0.282, 0.338)⟩	⟨(0.733, 0.677)(0.155, 0.239)⟩	⟨(0.524, 0.491)(0.316, 0.369)⟩	⟨(0.774, 0.725)(0.125, 0.210)⟩
⟨(0.848, 0.799)(0.066, 0.175)⟩	⟨(0.485, 0.467)(0.345, 0.398)⟩	⟨(0.632, 0.604)(0.220, 0.312)⟩	⟨(0.510, 0.491)(0.323, 0.374)⟩	⟨(0.873, 0.825)(0.056, 0.145)⟩	⟨(0.666, 0.635)(0.197, 0.276)⟩	⟨(0.490, 0.457)(0.340, 0.401)⟩	⟨(0.839, 0.786)(0.070, 0.176)⟩
⟨(0.848, 0.799)(0.066, 0.175)⟩	⟨(0.485, 0.467)(0.345, 0.398)⟩	⟨(0.632, 0.604)(0.220, 0.312)⟩	⟨(0.510, 0.491)(0.323, 0.374)⟩	⟨(0.873, 0.825)(0.056, 0.145)⟩	⟨(0.666, 0.635)(0.197, 0.276)⟩	⟨(0.490, 0.457)(0.340, 0.401)⟩	⟨(0.839, 0.786)(0.070, 0.176)⟩
⟨(0.550, 0.518)(0.293, 0.344)⟩	⟨(0.360, 0.332)(0.449, 0.501)⟩	⟨(0.830, 0.773)(0.073, 0.190)⟩	⟨(0.588, 0.565)(0.264, 0.317)⟩	⟨(0.858, 0.802)(0.062, 0.169)⟩	⟨(0.464, 0.433)(0.370, 0.421)⟩	⟨(0.473, 0.447)(0.351, 0.405)⟩	⟨(0.610, 0.578)(0.250, 0.305)⟩
⟨(0.538, 0.507)(0.303, 0.360)⟩	⟨(0.835, 0.776)(0.070, 0.198)⟩	⟨(0.490, 0.466)(0.336, 0.390)⟩	⟨(0.664, 0.630)(0.200, 0.282)⟩	⟨(0.857, 0.811)(0.062, 0.159)⟩	⟨(0.800, 0.727)(0.083, 0.252)⟩	⟨(0.703, 0.639)(0.171, 0.274)⟩	⟨(0.717, 0.654)(0.165, 0.254)⟩
⟨(0.551, 0.535)(0.288, 0.340)⟩	⟨(0.772, 0.720)(0.130, 0.196)⟩	⟨(0.548, 0.515)(0.295, 0.346)⟩	⟨(0.583, 0.542)(0.251, 0.361)⟩	⟨(0.605, 0.577)(0.252, 0.304)⟩	⟨(0.590, 0.560)(0.264, 0.308)⟩	⟨(0.670, 0.635)(0.195, 0.279)⟩	⟨(0.710, 0.677)(0.170, 0.233)⟩
⟨(0.606, 0.568)(0.241, 0.339)⟩	⟨(0.765, 0.715)(0.130, 0.220)⟩	⟨(0.710, 0.677)(0.170, 0.233)⟩	⟨(0.857, 0.809)(0.062, 0.153)⟩	⟨(0.465, 0.447)(0.357, 0.412)⟩	⟨(0.627, 0.586)(0.225, 0.316)⟩	⟨(0.854, 0.808)(0.064, 0.160)⟩	⟨(0.810, 0.741)(0.079, 0.239)⟩
⟨(0.369, 0.344)(0.440, 0.492)⟩	⟨(0.584, 0.553)(0.267, 0.311)⟩	⟨(0.716, 0.655)(0.165, 0.257)⟩	⟨(0.797, 0.737)(0.111, 0.198)⟩	⟨(0.677, 0.642)(0.201, 0.243)⟩	⟨(0.489, 0.456)(0.341, 0.402)⟩	⟨(0.824, 0.762)(0.074, 0.211)⟩	⟨(0.344, 0.314)(0.477, 0.528)⟩
⟨(0.355, 0.338)(0.460, 0.511)⟩	⟨(0.675, 0.640)(0.195, 0.274)⟩	⟨(0.514, 0.481)(0.325, 0.384)⟩	⟨(0.660, 0.622)(0.203, 0.286)⟩	⟨(0.642, 0.607)(0.214, 0.300)⟩	⟨(0.873, 0.825)(0.056, 0.145)⟩	⟨(0.662, 0.631)(0.200, 0.279)⟩	⟨(0.580, 0.541)(0.254, 0.360)⟩
⟨(0.827, 0.766)(0.072, 0.211)⟩	⟨(0.573, 0.546)(0.274, 0.329)⟩	⟨(0.556, 0.519)(0.287, 0.332)⟩	⟨(0.615, 0.576)(0.245, 0.287)⟩	⟨(0.821, 0.761)(0.076, 0.211)⟩	⟨(0.697, 0.626)(0.175, 0.283)⟩	⟨(0.516, 0.477)(0.326, 0.376)⟩	⟨(0.414, 0.373)(0.411, 0.464)⟩
⟨(0.828, 0.780)(0.094, 0.158)⟩	⟨(0.887, 0.836)(0.044, 0.156)⟩	⟨(0.689, 0.621)(0.180, 0.284)⟩	⟨(0.655, 0.622)(0.217, 0.261)⟩	⟨(0.317, 0.290)(0.495, 0.545)⟩	⟨(0.641, 0.609)(0.214, 0.304)⟩	⟨(0.569, 0.538)(0.280, 0.334)⟩	⟨(0.906, 0.863)(0.037, 0.128)⟩
⟨(0.855, 0.809)(0.064, 0.160)⟩	⟨(0.569, 0.538)(0.280, 0.334)⟩	⟨(0.890, 0.840)(0.042, 0.156)⟩	⟨(0.779, 0.734)(0.122, 0.201)⟩	⟨(0.692, 0.626)(0.178, 0.279)⟩	⟨(0.566, 0.529)(0.279, 0.324)⟩	⟨(0.658, 0.629)(0.214, 0.257)⟩	⟨(0.856, 0.800)(0.063, 0.172)⟩
⟨(0.670, 0.638)(0.205, 0.247)⟩	⟨(0.568, 0.537)(0.278, 0.323)⟩	⟨(0.705, 0.637)(0.171, 0.274)⟩	⟨(0.700, 0.631)(0.174, 0.273)⟩	⟨(0.812, 0.740)(0.078, 0.235)⟩	⟨(0.529, 0.504)(0.310, 0.368)⟩	⟨(0.692, 0.629)(0.180, 0.282)⟩	⟨(0.599, 0.560)(0.240, 0.346)⟩
⟨(0.513, 0.479)(0.326, 0.386)⟩	⟨(0.397, 0.369)(0.423, 0.474)⟩	⟨(0.833, 0.776)(0.071, 0.202)⟩	⟨(0.462, 0.440)(0.365, 0.415)⟩	⟨(0.847, 0.795)(0.066, 0.172)⟩	⟨(0.506, 0.480)(0.328, 0.381)⟩	⟨(0.327, 0.304)(0.483, 0.533)⟩	⟨(0.760, 0.711)(0.137, 0.205)⟩

Step 5: The mean of matrix $\mu_{\pi_{(R_{ij})}}$ is calculated using Equation 4.5, and its standard deviation is determined by using Equation 4.6. Each element of $\mu_{\pi_{(R_{ij})}}$ is then processed through the Gaussian function using Equation 4.4.

$$\mu_{\pi_{(R_{ij})}} = \begin{bmatrix} 0.4884 & 0.7179 & 0.8974 & 0.9680 & 0.0725 & 0.8102 & 0.9929 & 0.8266 \\ 0.6591 & 0.6510 & 0.2523 & 0.5797 & 0.2000 & 0.4619 & 0.7821 & 0.5359 \\ 0.7999 & 0.5099 & 0.2790 & 0.6023 & 0.2831 & 0.9989 & 0.9931 & 0.9579 \\ 0.9124 & 0.4468 & 0.9206 & 0.9453 & 0.8496 & 0.8870 & 0.6912 & 0.7375 \\ 0.4556 & 0.5482 & 0.9924 & 0.6608 & 0.3560 & 0.9928 & 0.5438 & 0.4845 \\ 0.8032 & 0.1313 & 0.5302 & 0.9290 & 0.4280 & 0.4294 & 0.4958 & 0.9660 \\ 0.5405 & 0.9578 & 0.9080 & 0.6594 & 0.9440 & 0.2974 & 0.4957 & 0.9218 \\ 0.7508 & 0.5273 & 0.5742 & 0.9961 & 0.4093 & 0.7044 & 0.9678 & 0.9357 \\ 0.8314 & 0.7359 & 0.7934 & 0.8826 & 0.9622 & 0.9331 & 0.9923 & 0.9128 \\ 0.9446 & 0.7745 & 0.9128 & 0.4045 & 0.4715 & 0.9829 & 0.4176 & 0.6581 \\ 0.1529 & 0.9199 & 0.9375 & 0.6657 & 0.9779 & 0.5395 & 0.5753 & 0.0907 \\ 0.1209 & 0.9878 & 0.6392 & 0.9987 & 0.9979 & 0.3560 & 0.9959 & 0.8781 \\ 0.5659 & 0.8860 & 0.8293 & 0.9777 & 0.5826 & 0.9818 & 0.6449 & 0.2430 \\ 0.5110 & 0.3306 & 0.9881 & 0.9980 & 0.0616 & 0.9976 & 0.8649 & 0.2587 \\ 0.4171 & 0.8649 & 0.3233 & 0.7077 & 0.9832 & 0.8642 & 0.9954 & 0.4374 \\ 0.9845 & 0.8747 & 0.9679 & 0.9738 & 0.6507 & 0.7208 & 0.9834 & 0.9299 \\ 0.6320 & 0.2094 & 0.5341 & 0.4470 & 0.4562 & 0.6292 & 0.0769 & 0.7770 \end{bmatrix}$$

Step 6: Equation 4.7 was used to produce the normalized matrix $\wp_{ij}^{G}$ from the score matrix $\mu_{\pi_{(R_{ij})}}$.

$$\begin{pmatrix}
0.4518 & 0.7012 & 0.8962 & 0.9729 & 0.0000 & 0.8015 & 1.0000 & 0.8193\\
0.7885 & 0.7747 & 0.0898 & 0.6523 & 0.0000 & 0.4498 & 1.0000 & 0.5769\\
0.7236 & 0.3207 & 0.0000 & 0.4492 & 0.0056 & 1.0000 & 0.9920 & 0.9431\\
0.0659 & 1.0000 & 0.0495 & 0.0000 & 0.1920 & 0.1169 & 0.5097 & 0.4168\\
0.1563 & 0.3017 & 1.0000 & 0.4786 & 0.0000 & 1.0000 & 0.2948 & 0.2017\\
0.8050 & 0.0000 & 0.4780 & 0.9557 & 0.3555 & 0.3572 & 0.4368 & 1.0000\\
0.6319 & 0.0000 & 0.0753 & 0.4517 & 0.0209 & 1.0000 & 0.6996 & 0.0545\\
0.4181 & 0.7989 & 0.7190 & 0.0000 & 1.0000 & 0.4971 & 0.0481 & 0.1029\\
0.6278 & 1.0000 & 0.7758 & 0.4279 & 0.1175 & 0.2310 & 0.0000 & 0.3103\\
0.0663 & 0.3603 & 0.1213 & 1.0000 & 0.8843 & 0.4941 & 0.4538 & 0.5615\\
0.9299 & 0.0654 & 0.0455 & 0.3519 & 0.0000 & 0.7321 & 0.0032 & 1.0000\\
1.0000 & 0.0124 & 0.4096 & 0.0000 & 0.0009 & 0.0004 & 0.4559 & 0.1374\\
0.5629 & 0.1296 & 0.2064 & 0.0055 & 0.5404 & 0.7321 & 0.1422 & 1.0000\\
0.5200 & 0.7127 & 0.0106 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.7896\\
0.8604 & 0.1942 & 1.0000 & 0.4281 & 0.0182 & 0.1953 & 0.0032 & 0.8303\\
0.0000 & 0.3290 & 0.0498 & 0.0321 & 1.0000 & 0.7899 & 0.4559 & 0.1635
\end{pmatrix}$$

Step 7: Equation 4.8 calculates P for each condition.

Step 8: Equation 4.9 calculates the objective weights of the criteria, which are then shown

TABLE 10. Standard deviation

SD	U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
¶	0.3336	0.3474	0.4216	0.3381	0.3776	0.3438	0.3828	0.3750	0.3422	0.4193	0.3873	0.3899	0.3432	0.4074	0.4026	0.3895	0.3298
T^{Y}	46.1337	22.0768	10.1600	13.2444	0.9606	23.7919	8.4996	4.0105	8.1360	3.4506	2.1037	20.2745	3.2473	7.6268	1.2190	19.0290	10.8654

in Table 11.

Step 9: The calculation of Ξ for each criterion is determined using Equation 4.10 and the

TABLE 11. Objective weights by LOPCOW

Weight	U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
C_{w_j}	0.2252	0.1078	0.0496	0.0647	0.0047	0.1162	0.0415	0.0196	0.0397	0.0167	0.0103	0.0990	0.0159	0.0372	0.0060	0.0929	0.0530

results are displayed in Table 12.

Step 10: The weight of criteria is determined using Equation 4.11, and the results are

TABLE 12. Relative importance

U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
1	1.1078	1.0496	1.0647	1.0047	1.1162	1.0415	1.0196	1.0397	1.0168	1.0103	1.0990	1.0159	1.0372	1.0060	1.0929	1.0530

given in Table 13.

Step 11 The determination of the relative weight of each criterion is carried out by applying Equation 4.12 and given in Table 14.

TABLE 13. Initial weight of criteria

U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
1	0.9027	0.8600	0.8078	0.8040	0.7203	0.6916	0.6783	0.6524	0.6416	0.6351	0.5779	0.5688	0.5484	0.5452	0.4988	0.4737

TABLE 14. Relative weight

U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
0.0862	0.0778	0.0741	0.0696	0.0693	0.0621	0.0596	0.0584	0.0562	0.0553	0.0547	0.0498	0.0490	0.0473	0.0470	0.0430	0.0408

TABLE 15. Provided Matrix Data

U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
0.0033	0.0280	0.0424	0.1174	0.4773	0.0044	0.1665	0.1620	0.6936	0.0785	0.2085	1.0000	0.0791	0.0018	0.4657	0.2887	0.0018
0.0010	0.0290	0.1639	1.0000	0.2740	1.0000	0.0299	0.4678	1.0000	0.1425	0.0010	0.0018	0.0206	0.0065	0.0522	0.4117	0.0495
0.0005	0.4983	1.0000	0.1143	0.0462	0.0152	0.0351	0.3623	0.7980	0.0871	0.0009	0.0068	0.0252	0.0002	1.0000	0.3039	0.0030
0.0004	0.0411	0.0994	0.1056	0.1564	0.0028	0.0917	0.0694	0.5797	1.0000	0.0025	0.0018	0.0153	0.0002	0.0954	0.2984	0.0051
1.0000	1.0000	0.9575	0.1455	1.0000	0.0288	0.0313	1.0000	0.4474	0.6318	0.0008	0.0018	0.0726	1.0000	0.0356	1.0000	0.0048
0.0007	0.0812	0.0218	0.1278	0.0461	0.0286	1.0000	0.1962	0.4906	0.0697	0.0047	0.0391	0.0152	0.0002	0.0524	0.7356	0.0018
0.0004	0.0167	0.0222	0.2701	0.2807	0.0186	0.2158	0.0756	0.4079	0.9092	0.0039	0.0018	0.0535	0.0004	0.0343	0.2897	1.0000
0.0007	0.0520	0.0247	0.2223	0.3969	0.0025	0.0336	0.0837	0.5241	0.2322	1.0000	0.0026	1.0000	0.0135	0.4040	0.3426	0.0010

5.4. **Proposed Method: ERUNS.** **Step 12:** Equation 4.13 is used to normalize the matrix $\mathfrak{o} = \left[Z_{B_{(R)}{}_{ij}} \right]$ to the interval $[B, Z]$ and given in Table 16.
Step 13: To bring uniformity across the dataset, Equation 4.14 is applied to transform the

TABLE 16. Normalized matrix

U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
0.0033	0.0280	0.0424	0.1174	0.4773	0.0044	0.1665	0.1620	0.6936	0.0785	0.2085	1.0000	0.0791	0.0018	0.4657	0.2887	0.0018
0.0010	0.0290	0.1639	1.0000	0.2740	1.0000	0.0299	0.4678	1.0000	0.1425	0.0010	0.0018	0.0206	0.0065	0.0522	0.4117	0.0495
0.0005	0.4983	1.0000	0.1143	0.0462	0.0152	0.0351	0.3623	0.7980	0.0871	0.0009	0.0068	0.0252	0.0002	1.0000	0.3039	0.0030
0.0004	0.0411	0.0994	0.1056	0.1564	0.0028	0.0917	0.0694	0.5797	1.0000	0.0025	0.0018	0.0153	0.0002	0.0954	0.2984	0.0051
1.0000	1.0000	0.9575	0.1455	1.0000	0.0288	0.0313	1.0000	0.4474	0.6318	0.0008	0.0018	0.0726	1.0000	0.0356	1.0000	0.0048
0.0007	0.0812	0.0218	0.1278	0.0461	0.0286	1.0000	0.1962	0.4906	0.0697	0.0047	0.0391	0.0152	0.0002	0.0524	0.7356	0.0018
0.0004	0.0167	0.0222	0.2701	0.2807	0.0186	0.2158	0.0756	0.4079	0.9092	0.0039	0.0018	0.0535	0.0004	0.0343	0.2897	1.0000
0.0007	0.0520	0.0247	0.2223	0.3969	0.0025	0.0336	0.0837	0.5241	0.2322	1.0000	0.0026	1.0000	0.0135	0.4040	0.3426	0.0010

matrix $\mathfrak{o}^N = [\Lambda_{ij}]_{m \times n}$, where all entries are adjusted to lie within the range $[0, 1]$. This normalization is consistent with the principles of a benefit-oriented evaluation framework. The revised matrix is presented below.

U^{Θ}_{1}	U^{Θ}_{2}	U^{Θ}_{3}	U^{Θ}_{4}	U^{Θ}_{5}	U^{Θ}_{6}	U^{Θ}_{7}	U^{Θ}_{8}	U^{Θ}_{9}	U^{Θ}_{10}	U^{Θ}_{11}	U^{Θ}_{12}	U^{Θ}_{13}	U^{Θ}_{14}	U^{Θ}_{15}	U^{Θ}_{16}	U^{Θ}_{17}
0.00327	0.98877	0.97935	0.11741	0.56877	0.00437	0.16654	0.16204	0.69364	0.07855	0.20846	1	0.07914	0.00175	0.46574	0.28872	0.00180
0.00103	0.98772	0.85792	1	0.77215	1	0.02993	0.46775	1	0.14247	0.00096	0.00183	0.02062	0.00647	0.05225	0.41171	0.04953
0.00053	0.51839	0.02180	0.11431	0.99994	0.01517	0.03512	0.36225	0.79797	0.08705	0.00090	0.00676	0.02515	0.00024	1	0.30386	0.00299
0.00042	0.97565	0.92240	0.10559	0.88972	0.00282	0.09170	0.06939	0.57969	1	0.00253	0.00177	0.01535	0.00024	0.09538	0.29835	0.00510
1	0.01673	0.06427	0.14545	0.04612	0.02885	0.03126	1	0.44741	0.63175	0.00080	0.00178	0.07256	1	0.03556	1	0.00479
0.00072	0.93549	1	0.12780	1	0.02857	1	0.19621	0.49058	0.06971	0.00474	0.03914	0.01516	0.00024	0.05238	0.73562	0.00183
0.00039	1	0.99962	0.27009	0.76542	0.01856	0.21582	0.07564	0.40788	0.90921	0.00391	0.00179	0.05347	0.00036	0.03427	0.28967	1
0.00067	0.96472	0.99708	0.22233	0.64927	0.00251	0.03357	0.08371	0.52411	0.23223	1	0.00261	1	0.01351	0.40404	0.34259	0.00097

Step 14: Equation 4.15 is used to display the weighted standardised decision matrix V once it has been obtained.

$U^{\Theta}{}_1$	$U^{\Theta}{}_2$	$U^{\Theta}{}_3$	$U^{\Theta}{}_4$	$U^{\Theta}{}_5$	$U^{\Theta}{}_6$	$U^{\Theta}{}_7$	$U^{\Theta}{}_8$	$U^{\Theta}{}_9$	$U^{\Theta}{}_{10}$	$U^{\Theta}{}_{11}$	$U^{\Theta}{}_{12}$	$U^{\Theta}{}_{13}$	$U^{\Theta}{}_{14}$	$U^{\Theta}{}_{15}$	$U^{\Theta}{}_{16}$	$U^{\Theta}{}_{17}$
0.0002	0.1452	0.1261	0.1470	0.1135	0.1470	0.0044	0.0688	0.1470	0.0209	0.0001	0.0003	0.0030	0.0010	0.0077	0.0605	0.0073
0.0001	0.1208	0.0051	0.0266	0.2330	0.0035	0.0082	0.0844	0.1859	0.0203	0.0002	0.0016	0.0059	0.0001	0.2330	0.0708	0.0007
0.0001	0.1930	0.1824	0.0209	0.1760	0.0006	0.0181	0.0137	0.1147	0.1978	0.0005	0.0004	0.0030	0.0000	0.0189	0.0590	0.0010
0.1809	0.0030	0.0116	0.0263	0.0083	0.0052	0.0057	0.1809	0.0809	0.1143	0.0001	0.0003	0.0131	0.1809	0.0064	0.1809	0.0009
0.0001	0.1642	0.1755	0.0224	0.1755	0.0050	0.1755	0.0344	0.0861	0.0122	0.0008	0.0069	0.0027	0.0000	0.0092	0.1291	0.0003
0.0001	0.1654	0.1653	0.0447	0.1266	0.0031	0.0357	0.0125	0.0675	0.1504	0.0006	0.0003	0.0088	0.0001	0.0057	0.0479	0.1654
0.0001	0.1490	0.1540	0.0343	0.1003	0.0004	0.0052	0.0129	0.0810	0.0359	0.1545	0.0004	0.1545	0.0021	0.0624	0.0529	0.0001

$U^{\Theta}{}_1$	$U^{\Theta}{}_2$	$U^{\Theta}{}_3$	$U^{\Theta}{}_4$	$U^{\Theta}{}_5$	$U^{\Theta}{}_6$	$U^{\Theta}{}_7$	$U^{\Theta}{}_8$	$U^{\Theta}{}_9$	$U^{\Theta}{}_{10}$	$U^{\Theta}{}_{11}$	$U^{\Theta}{}_{12}$	$U^{\Theta}{}_{13}$	$U^{\Theta}{}_{14}$	$U^{\Theta}{}_{15}$	$U^{\Theta}{}_{16}$	$U^{\Theta}{}_{17}$
0.0861	0.0779	0.0743	0.0695	0.0693	0.0620	0.0595	0.0584	0.0563	0.0552	0.0547	0.0499	0.0490	0.0472	0.0470	0.0430	0.0408
0.0860	0.0779	0.0742	0.0697	0.0693	0.0622	0.0595	0.0584	0.0563	0.0552	0.0546	0.0497	0.0489	0.0472	0.0469	0.0430	0.0408
0.0861	0.0779	0.0740	0.0696	0.0695	0.0620	0.0595	0.0585	0.0564	0.0552	0.0547	0.0497	0.0490	0.0472	0.0471	0.0430	0.0408
0.0860	0.0780	0.0743	0.0695	0.0694	0.0620	0.0595	0.0584	0.0563	0.0554	0.0546	0.0497	0.0489	0.0472	0.0469	0.0430	0.0408
0.0864	0.0777	0.0740	0.0696	0.0692	0.0620	0.0595	0.0586	0.0562	0.0553	0.0547	0.0497	0.0490	0.0474	0.0469	0.0431	0.0408
0.0860	0.0779	0.0743	0.0695	0.0694	0.0620	0.0597	0.0584	0.0562	0.0552	0.0546	0.0497	0.0489	0.0472	0.0469	0.0430	0.0408
0.0861	0.0779	0.0743	0.0696	0.0694	0.0620	0.0596	0.0584	0.0562	0.0554	0.0547	0.0497	0.0489	0.0472	0.0469	0.0430	0.0409
0.0861	0.0779	0.0742	0.0696	0.0693	0.0620	0.0595	0.0584	0.0562	0.0553	0.0548	0.0497	0.0491	0.0472	0.0470	0.0430	0.0408

Step 15, 16 and 17: Table 17 presents the usefulness levels for the ideal, anti-ideal, and utility functions. The alternative appraisal scores are calculated utilizing the parameters $Z = 5$ and 4.18.

TABLE 17. Normalized decision matrix

Alternative	Q_a-	Q_a+	$J(Q_a+)$	$J(Q_a-)$	AS	Ranking
$\Phi^{\Theta}1$	1.6490	0.1054	0.0601	0.9400	1.2556	1
$\Phi^{\Theta}2$	1.5529	0.0904	0.0550	0.9450	1.1920	2
$\Phi^{\Theta}3$	0.5809	0.0371	0.0601	0.9400	0.4423	8
$\Phi^{\Theta}4$	0.7986	0.0428	0.0508	0.9492	0.6175	7
$\Phi^{\Theta}5$	1.4283	0.0764	0.0507	0.9493	1.1045	4
$\Phi^{\Theta}6$	1.2862	0.0649	0.0480	0.9520	0.9997	6
$\Phi^{\Theta}7$	1.3401	0.0688	0.0488	0.9512	1.0400	5
$\Phi^{\Theta}8$	1.5053	0.0845	0.0531	0.9469	1.1592	3

5.5. **Sensitive Analysis.** The intricate interaction between the parameters δ^D and Z in our decision-making framework is comprehensively analyzed in this study, as summarized in Table 18. The data reveals that increasing δ^D significantly reduces individual scores while maintaining the overall ranking of alternatives across a wide range of values, from 5 to 1000. This behavior emphasizes the model's focus on specific attributes, ensuring robust decision support. Notably, the computational efficiency of the method is evident—variations in δ^D and Z do not affect the ranking results, making the framework highly suitable for large datasets and dynamic scenarios.

Table 18 demonstrates that Φ_1^{Θ} consistently ranks highest, followed by Φ_2^{Θ}, with other alternatives retaining their positions as δ^D increases. For instance, at $\delta^D = 5$ and $Z = 0.1$, the ranking order is $\Phi_1^{\Theta} \succ \Phi_2^{\Theta} \succ \Phi_8^{\Theta} \succ \Phi_5^{\Theta} \succ \Phi_7^{\Theta} \succ \Phi_6^{\Theta} \succ \Phi_4^{\Theta} \succ \Phi_3^{\Theta}$. This pattern remains stable even as δ^D reaches 1000, showcasing the method's reliability and adaptability. Figure 3 and Figure 4 show the graphical representation of the results.

TABLE 18. The effect of parameters δ^D and Z on the outcome of the alternatives

Z	δ^D	$(\Phi^{\Theta}1)$	$(\Phi^{\Theta}2)$	$(\Phi^{\Theta}3)$	$(\Phi^{\Theta}4)$	$(\Phi^{\Theta}5)$	$(\Phi^{\Theta}6)$	$(\Phi^{\Theta}7)$	$(\Phi^{\Theta}8)$	Ranking
	$\delta^D = 5$	1.2556	1.1920	0.4423	0.6175	1.1045	0.9997	1.0400	1.1592	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 10$	1.2502	1.1963	0.4398	0.6193	1.1023	1.0024	1.0385	1.1641	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
$Z = 0.1$	$\delta^D = 100$	1.2614	1.1874	0.4456	0.6201	1.1067	1.0005	1.0432	1.1576	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 500$	1.2543	1.1902	0.4404	0.6154	1.1090	0.9978	1.0397	1.1610	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 1000$	1.2589	1.1941	0.4441	0.6180	1.1073	0.9990	1.0414	1.1602	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 5$	1.2517	1.1937	0.4412	0.6163	1.1041	1.0006	1.0420	1.1559	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 10$	1.2602	1.1886	0.4400	0.6198	1.1059	0.9982	1.0379	1.1587	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
$Z = 0.3$	$\delta^D = 100$	1.2593	1.1924	0.4436	0.6170	1.1100	1.0034	1.0441	1.1615	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 500$	1.2530	1.1913	0.4420	0.6147	1.1064	0.9989	1.0408	1.1565	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 1000$	1.2574	1.1892	0.4407	0.6183	1.1082	1.0021	1.0424	1.1596	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 5$	1.2526	1.1950	0.4449	0.6159	1.1070	1.0013	1.0393	1.1631	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 10$	1.2610	1.1889	0.4416	0.6202	1.1055	0.9967	1.0433	1.1582	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
$Z = 0.6$	$\delta^D = 100$	1.2550	1.1919	0.4393	0.6178	1.1043	1.0045	1.0401	1.1598	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 500$	1.2606	1.1932	0.4442	0.6191	1.1094	0.9995	1.0416	1.1603	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 1000$	1.2571	1.1897	0.4427	0.6165	1.1069	1.0003	1.0399	1.1555	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 5$	1.2549	1.1944	0.4409	0.6186	1.1104	1.0017	1.0422	1.1612	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 10$	1.2615	1.1907	0.4419	0.6152	1.1057	0.9973	1.0378	1.1572	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
$Z = 0.9$	$\delta^D = 100$	1.2584	1.1923	0.4453	0.6161	1.1095	0.9998	1.0434	1.1608	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 500$	1.2588	1.1923	0.4393	0.6170	1.1038	1.0044	1.0412	1.1612	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$
	$\delta^D = 1000$	1.2597	1.1883	0.4433	0.6173	1.1084	0.9984	1.0410	1.1584	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$

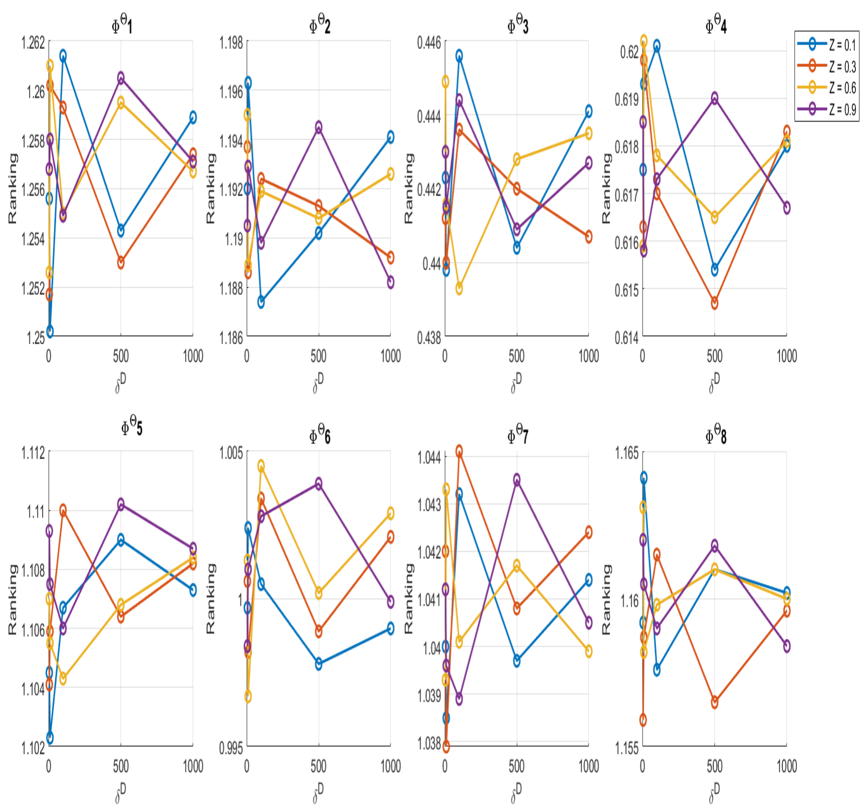

FIGURE 3. Sensitivity analysis shown visually

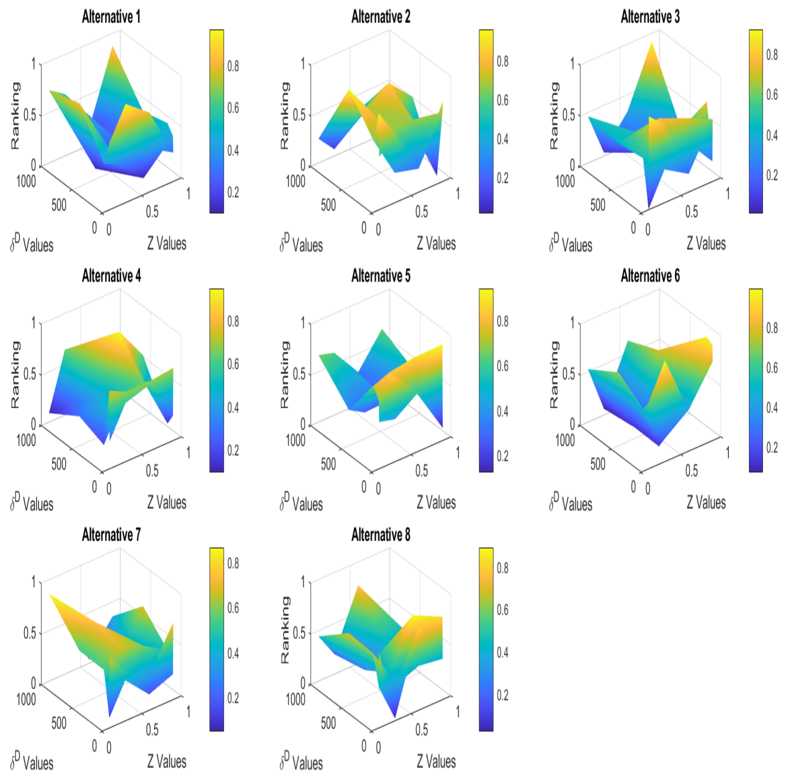

FIGURE 4. 3D representation of each alternative by surface

TABLE 19. Comparison of the newly proposed method with existing methods.

Authors	Methodology	Ranking of alternatives	Optimal alternative
Kumari and Mishra [23]	COPRAS method	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3 \succ \Phi^{\Theta}7$	$\Phi^{\Theta}1$
Ecer and Pamucar [10]	LOPCOW-DOBI	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}3 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}4$	$\Phi^{\Theta}1$
Stanujkic and Karabasevic [43]	WASPAS method	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}3$	$\Phi^{\Theta}1$
Riaz et al. [37]	RFE approach	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}3 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}4$	$\Phi^{\Theta}1$
Saqlain et al. [40]	Fuzzy MCDM	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}3 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6$	$\Phi^{\Theta}1$
Yasin et al. [52]	ECI-LOPCOW	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}3 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6$	$\Phi^{\Theta}1$
Proposed	LOPCOW-ERUNS	$\Phi^{\Theta}1 \succ \Phi^{\Theta}2 \succ \Phi^{\Theta}8 \succ \Phi^{\Theta}5 \succ \Phi^{\Theta}7 \succ \Phi^{\Theta}6 \succ \Phi^{\Theta}4 \succ \Phi^{\Theta}3$	$\Phi^{\Theta}1$

5.6. **Discussion.** In the modern era of business, companies such as Amazon, Walmart, and Alibaba operate in highly competitive and uncertain environments. Making reliable decisions across various, often conflicting, criteria is a challenge for decision-makers. Traditional decision-making methods often fail to account for the complexity and uncertainty inherent in such multi-dimensional decision-making processes. Furthermore, classical methods tend to ignore the subjective nature of human judgment and do not sufficiently manage the fuzziness of decision data. This study aims to address these challenges by providing a robust framework that integrates advanced mathematical models to capture both subjective and objective influences on the decision-making process. The solution proposed in this paper involves the use of IFCS and IFCII within BCK-algebras to effectively handle uncertainty in the decision-making process. The methodology ensures that decision-makers can systematically evaluate and rank alternatives across multiple criteria while managing both subjective and objective factors. By incorporating these advanced fuzzy logic concepts, this paper presents a comprehensive framework that supports more accurate and reliable decision-making in uncertain environments. The model developed in this study uses a combination of subjective and objective weighting methods to evaluate alternatives. The subjective weights are calculated using the SWARA method, which captures the expert judgment of decision-makers. Objective weights, on the other hand, are calculated using the LOPCOW method, which helps to reflect the inherent importance of each criterion based on numerical data and objective metrics. The final ranking of alternatives is done using the ERUNS method, which combines relative utility and nonlinear standardization to ensure a robust evaluation.

The LOPCOW method is used to calculate the objective weights of the criteria. It works by evaluating the relative change in importance between each criterion based on the available objective data. This method is particularly useful in capturing the underlying importance of each criterion when objective data is available, ensuring that the final decision-making process incorporates both subjective and objective perspectives. The ERUNS method is employed to rank the alternatives based on the combined subjective and objective weights. This method uses relative utility values and applies nonlinear standardization to ensure that the ranking process accounts for the differing scales and units of measurement among the criteria. By integrating both subjective and objective aspects, ERUNS provides a balanced and comprehensive evaluation of the alternatives.

Upon applying the proposed algorithm to evaluate leading companies such as Amazon, Walmart, and Alibaba, the results showed that Amazon emerged as the optimal alternative. This outcome can be attributed to Amazon's superior performance across several key criteria, such as cybersecurity, economic factors, technological advancements, and logistics-driven threats. Amazon's advanced use of technology in supply chain and logistics operations, coupled with its robust data analytics, inventory control, and customer service, positions it as the most favorable option among the evaluated alternatives. The application of the IFCS and IFCII models effectively handled the inherent uncertainty in evaluating these complex criteria. By employing the SWARA, LOPCOW, and ERUNS methods, the proposed decision-making framework provided a comprehensive, systematic, and reliable approach to ranking the companies. As a result, Amazon stands out as the optimal alternative, showcasing the effectiveness and robustness of the proposed methodology in real-world decision-making scenarios.

6. Conclusion

This paper introduces a novel MCDM framework that integrates IFCSs in BCK-algebras, extending the concept to IFCII. The framework effectively manages uncertainty and provides a robust approach to evaluating alternatives across multiple conflicting criteria. By incorporating both subjective and objective weighting techniques, SWARA and LOPCOW, the model ensures a comprehensive evaluation of complex decision-making problems. Through the ERUNS method, the framework ranks alternatives in a systematic and objective manner. The case study conducted on leading companies like Amazon, Walmart, and Alibaba demonstrates the effectiveness of the model in real-world decision-making contexts. Among the alternatives, Amazon emerges as the optimal choice due to its superior performance in supply chain and logistics operations, driven by advanced technologies and efficient inventory control systems.

Despite its effectiveness, the model has certain limitations, such as its dependence on expert judgments, data availability, and the assumption of independence between criteria. Future research could explore the integration of temporal dynamics and correlations between criteria to further enhance the robustness of the framework. Overall, the proposed decision-making model offers valuable insights for evaluating companies and can be adapted for other complex decision-making scenarios in various domains.

6.1. **Limitations.** The limitations of proposed methodology are listed as follows.

- The integrated LOPCOW–SWARA–ERUNS framework involves increased computational complexity, which may limit its practical applicability for large-scale problems.
- The obtained results are sensitive to normalization techniques and ERUNS-related parameters, potentially affecting ranking stability.
- The framework requires reliable, complete, and consistent data; data incompleteness or imprecision may degrade decision quality.
- The SWARA method heavily depends on expert judgments, making the results vulnerable to subjectivity, bias, and inconsistency.
- Limited diversity or a small number of experts may further reduce the robustness of subjective weighting.
- The approach assumes independence among evaluation criteria, ignoring possible interdependencies and interactions.
- Uncertainty and inconsistencies may propagate across the sequential stages of weighting and ranking.
- The performance of the framework is sensitive to the parameters chosen for the SWARA, LOPCOW, and ERUNS methods. In particular, the choice of thresholds and the weighting factors can significantly influence the results, making the model potentially sensitive to minor variations in input values.

These limitations suggest that while the model provides a useful and advanced decision-making tool, its practical application should be carefully considered, especially in environments with high uncertainty, interdependent criteria, or complex implementation requirements.

6.2. **Future directions.** While the proposed framework provides a robust decision-making approach using IFCS and IFCII, there are several avenues for future research and enhancement:

- Future research could focus on refining the subjective weight assessment process by incorporating expert consensus mechanisms. This would reduce the subjectivity associated with expert judgments and improve the reliability of the weights assigned to criteria.
- The current model assumes independence between criteria, which may not always hold true in real-world decision-making scenarios. Future work could explore methods to account for inter-criteria correlations, thereby providing more accurate evaluations.
- Many decision-making problems evolve over time, particularly in dynamic environments. Integrating temporal factors into the model would enhance its applicability to real-time decision-making and long-term planning.
- Extending the model to accommodate multi-stage decision-making processes could be beneficial. By incorporating feedback loops and adjusting weights dynamically based on interim results, the model could better handle evolving and uncertain environments.
- The proposed framework can be extended beyond corporate evaluations to other domains, such as SCM, risk assessment, and sustainability. Further research could explore its potential in sectors such as healthcare, finance, and environmental management, where complex, multi-criteria decision-making is crucial.

By addressing these areas, future work can significantly enhance the proposed model's robustness and broaden its applicability to a wide range of real-world decision-making problems.

CREDIT AUTHORSHIP CONTRIBUTION'S STATEMENT

Laraib Hayat: Draft Writing, Investigation, Methodology. **Asim Naseem**: Conceptualization, Data Curation, Review, and Editing **Muhammad Riaz**: Conceptualization, Data Curation, Review and Editing, Supervision, Formal Analysis, Software. The final version of the manuscript has been read and approved by all authors.

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