Punjab University Journal of Mathematics (2024), 56(11), 753-769
https://doi.org/10.52280/pujm.2024.56(11)04

A novel deep learning technique for solving dynamical model of infertile women with Polycystic Ovarian Syndrome Using the combination of radial basis and log-sigmoid functions

Ume Ayesha
Department of Mathematics,
COMSATS University Islamabad, Islamabad Campus, Pakistan
Email: umeayesha007@gmail.com

Shumaila Javeed
Department of Mathematics,
COMSATS University Islamabad, Islamabad Campus, Pakistan
Email: shumaila javeed@comsats.edu.pk

Mansoor Shaukat Khan
Department of Mathematics,
COMSATS University Islamabad, Islamabad Campus, Pakistan
Email: mansoorkhan@comsats.edu.pk

Dumitru Baleanu
Department of Mathematics and Computer Science,
Lebanese American University, Lebanon
Email: dumitru.baleanu@gmail.com

Mohsin Ali
Department of Mathematics,
COMSATS University Islamabad, Islamabad Campus, Pakistan
Email: mohsin203ali@gmail.com

Mustafa Baryam
Department of Mathematics and Computer Science,
Biruni University, Turkey
Email: mustafabayram@biruni.edu.tr

Received: 23 December, 2024 / Accepted: 18 May, 2025 / Published online: 20 June, 2025

Abstract. The focus of this study is to introduce the novel process of deep neural network (DNN) for the numerical solution of dynamical model based on infertile women with polycystic ovarian syndrome (PCOS). The model consists of five classes such as, Susceptible Women (S), Infertile Women (I), Women under Treatment 1 (Women with Treatment 2 (and

Recovered Women (R). The dual hidden layer neural network procedure is applied using the combination of radial basis function and sigmoid function for the numerical approximations of the nonlinear model. The twenty neurons for radial basis function and forty neurons for sigmoid function are used. The acquired numerical results are validated through the comparison of the solutions. The optimization is implemented by employing a competent scale conjugate gradient procedure. The verification of the proposed numerical method is observed through the reference results, and the reduced absolute error around is obtained. The reference database is achieved using the Runge-Kutta method of order 4. The method is proved reliable when trained, tested and validated using the reference dataset that consists of proportions 72%, 14% and 14%. After using the DNN approach, results are validated and displayed through MSE, regression, fitness, error and comparison plots.

AMS (MOS) Subject Classification Codes: 92C50; 68T07; 35Q92
Key Words: Polycystic ovarian syndrome; Neural network; Hidden layer; Radial basis function; Sigmoid function.

1. Introduction

PCOS is among the most common causes of ovulatory problems. The diagnostic criteria for PCOS developed in Rotterdam in 2003 required the presence of either two .or all three of the following factors: Absent ovulation or irregular periods, enlarged ovaries and increased androgen levels [3]. Some patients have been prescribed ovarian stimulation due to their reduced fertility caused by either improper ovulation or abnormal menstrual cycles. The drug clomiphene citrate is often used to prompt ovulation and help women with PCOS conceive. Approximately one out of every five patients will become pregnant while taking this medication. During their reproductive years, many women develop PCOS which can lead to missed periods, problems with getting pregnant, miscaariages and extra facial hairs. PCOS mainly results from infertility, obesity, problem with lipid metabolism and insulin resistance [7]. Most gynecologists recommend that women taking clomiphene citrate increase ovulation with gonadotropin treatment for six months at most. Infertile couples will no longer benefit from further ovulation stimulation therapy if they have undergone 9-12 cycles of Clomiphene Citrate and Gonadotropin, but have still not conceived. Under such conditions, performing Assisted Reproductive Techniques (ART) becomes necessary. Women just starting therapy, those on Clomiphene Citrate plus Gonadotropin, women receiving ART and women undergoing enhanced therapy. PCOS is a condition that affects reproduction, metabolism and a woman's mind. Since the symptoms are a common to many disorders and there are various causes, this diseases is frequently hard to understand and treat [10]. The most common illness in women is PCOS. Each system is complex, metabolic, endocrine, multifaced and highly. Women in their reproductive years may have heterogeneous disorder, no clear cause or number of cases is yet known and estimates are around 5-10% [12].

Biologists depend a lot on the use of mathematics, mathematics is important in the modeling of biological sciences [21]. Neural networks (NN) derive from the architecture of biological neural networks. The NN design is essential to carry out calculations. A neural network generates an output by combining several responses. Neural networks enhance the ability to produce an accurate plot. DNNs consist of a number of very simple individual process elements called neurons wired together into a network. What sets them apart is that these processing centers are not fixed by design, but instead operate as different layers of abstraction. A single layer neural network consists of both an input layer and an output layer. As layers are added, the models are generally called deep neural networks [14]. Care for PCOS is sometimes based on dietary and lifestyle changes. Losing weight and starting an exercise routine should be part of your lifestyle. The method of birth controlling might help improve the patterns in menstruating, experience acne and excessive hair growth growing. Other usual treatments for acne and method for hair removal can be used as well [16]. Only about half of women with signs and blood tests that show PCOS is shown as abnormalities by ultrasound, as stated in Greece and Spain, about 4-8% women are infected with polycystic ovary. There are more than 250, 000 women living with cancer in US. Apply ultrasound testing in the diagnosis of the disease between 4% and 7% of people have ovarian cysts that are greater than 30mm in size diameter [8]. PCOS occurs in 5-10% of women who are able to have children [2]. A lot of women who are trying to have children experience, polycystic ovary syndrome which is a hard to treat disorder of the hormones [6].

ANN (Artificial Neural Network) models allow us to predict a function by analyzing the given data. They have a major role when handling unorganized and big data. ANNs have ability to do different duties like approximating functions, forecasting, time series, handling data, controlling numbers and using computers [11]. As a result, a deep neural network can use the structure of a function to approximate numerous natural functions using fewer parts [13]. Stochastic numerical analyses dependent on neural networks evaluate the accuracy of solutions to differential equations in various differential equations. First, artificial deep neural networks were inspired by parts of the brain, helping computers to handle tasks similar to those humans do best. Lacking details about these cognitive processes, cognitive scientists now rely on DNNs to learn about the brain and neurology, sparking broad discussion [1]. Numerous studies have resolved demanding challenges by applying feed-forward backpropagation neural networks. In the current work, we develop an efficient DNN algorithm based on the ANN-LMB method to resolve the SIT_1T_2R model dynamical model for infertile women experiencing PCOS. This novel method approaches for the first time in a study to implement SIT_1T_2R model for the investigation of PCOS women. The novelty of this research is defined below.

A dynamic model SIT_1T_2R, was developed to simulate the ability of infertile women with PCOS to conceive. The model analysis considers variations in three distinct sets of parameters. Numerical results are extracted using the ANN-LMB (Artificial Neural Network- Levenberg Marquardt Backpropagation) algorithm. Accuracy, correctness and repeatability are ensured by cross-checking the results with those obtained using the Runge-Kutta numerical algorithm. Graphical analysis is used to evaluate the convergence of the proposed adaptation of the ANN-LMB algorithm.

Ideas and equipment from mathematics play an important role in epidemiology, engineering, research in social science systems and medicine. We use mathematical techniques in medicine, engineering, science and also in learning languages. A mathematical system can be used to explore and predict the actions of various devices. A mathematical model is often evaluated by test-running game theoretic models, statistical networks and dynamical networks. People use mathematical categories to specify logical problems and solutions [4].There have been many developments in artificial intelligence research recently, including speech recognition and recognizing images [5]. There has been great success recently in using deep neural networks for pattern recognitions, classifying items and controlling system like humans do in complex real- world situations [9].

The proposed article is divided into different sections. In section 2, the formulation of dynamical model and parameter selection is described. In section 3, the implementation of the DNN technique is included. Convergence and validation of DNN technique is explained in section 4. In section 5, numerical results are presented. In section 6, the future research and directions are concluded.

2. Model formulation

The considered PCOS women model has five classes: $S(t)$ stands for the women not experiencing infertility and at risk; $I(t)$ for PCOS women who are infertile; $T_1(t)$ stand for women treated with Clomiphene Citrate and Gonadotropin; $T_2(t)$ for women having In Vitro Fertilization (IVF) treatment; $R(t)$ stand for women who have improved infertility issues [17]. Moreover, $N(t)$ is the overall sum of the infertile women, $N = S + I + T_1 + T_2 + R$. The SIT_1T_2R model for infertile due to PCOS is as follows:

$$\begin{cases} S^{'} = \gamma N - \beta(\zeta T_2 + \eta T_1)S - \alpha S, \\ I^{'} = \beta(\zeta T_2 + \eta T_1)S - \lambda I - \alpha I, \\ T_1^{'} = -\alpha T_1 - \mu_1 T_1 + \lambda b I, \\ T_2^{'} = -\alpha T_2 - \mu_2 T_2 + \lambda(-b+1)I + \mu_1(1-u_1)T_1, \\ R^{'} = \mu_2 u_2 T_2 - \alpha R + \mu_1 u_1 T_1, \\ N^{'} = -\alpha N - \mu_2(-u_2+1)T_2 + \gamma N. \end{cases} \tag{2. 1}$$

Initial conditions are

$$S(0) = k_1, I(0) = k_2, T_1(0) = k_{,} T_2(0) = k_4, R(0) = k_5, N(0) = k_6.$$

The SIT_1T_2R model consists of a set of parameters that controls the flow between distinct groups of individuals. The parameter α describes the rate at which patients leave the system either because of death, loss to follow-up, or abandoning their treatment. The parameter γ represents the rate of entrants coming into the treatment facility. β represents the probability at which vulnerable women lose fertility or revisit the treatment class because of miscarriage or treatment unresponsiveness. The parameters η and ζ depict the success rates for two treatment options: Clomiphene Citrate and Gonadotropin and In Vitro Fertilization (IVF). The model also includes λ this parameter governs the movement of women into the treatment phase due to factors such as medical decisions and accessibility to treatments. Moreover, μ_1 and μ_2 represent the rates at which women recover from their infertility after being treated with either Clomiphene Citrate and Gonadotropin or IVF, respectively. Furthermore, u_1 and u_2 show how many patients experience positive outcomes after receiving each of the two treatments. The coefficient b, determines which treatment a patient receives based on medical decision-making or established guidelines. They play an essential role in accurately portraying the treatment process and were chosen to reflect key aspects of the three problems being investigated.

3. Methodology: Deep Neural Network

Although there are several ways of using artificial intelligence in simulating nonlinear models such as Genetic Algorithms, Recurrent Neural Networks, and Physics-Informed Neural Networks, the DNN technique was highly preferred in this work as it has a high computational efficiency, fast convergence, and an ability to make accurate predictions of nonlinear differential systems. Deep neural networks are efficient ways to train a model get outstanding accuracy in picking out patterns a variety of scientific areas [15]. Interaction of deep neural network structure and Levenberg–Marquardt optimization enables the method to approximate the dynamics of the SIT_1T_2R model efficiently outperforming classical and a few modern AI methods on the speed and error reduction of trainings for small to middle-size datasets [19].

DNN is described as a multi-layered procedure that can be classified into hidden output and input categories. The process of mathematical DNN with the activation sigmoid and radial basis function is performed whereby twenty to forty numbers of neurons are used to calculate nonlinear dynamics for SIT_1T_2R model. By using RBNN and SNN, the activation functions are represented by b, w_1, w_2 and weight vectors for layer 1,2 also output layer respectively. The 1^{st} hidden layer is depended on the RB, on the other hand, the SF is applied in the second hidden layer which is written as:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{20} \end{bmatrix} = \delta_1 \left(\begin{bmatrix} w_{1,1} \\ w_{1,2} \\ w_{1,3} \\ \vdots \\ w_{1,20} \end{bmatrix} [x] + \begin{bmatrix} b_{1,1} \\ b_{1,2} \\ b_{1,3} \\ \vdots \\ b_{1,20} \end{bmatrix} \right) \tag{3. 2}$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_{40} \end{bmatrix} = \delta_2 \left(\begin{bmatrix} \psi_{1,1} & \psi_{2,1} & \psi_{3,1} & \psi_{4,1} & \cdots & \psi_{20,1} \\ \psi_{1,2} & \psi_{2,2} & \psi_{3,2} & \psi_{4,1} & \cdots & \psi_{20,2} \\ \psi_{1,3} & \psi_{2,3} & \psi_{3,3} & \psi_{4,1} & \cdots & \psi_{20,3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \psi_{1,40} & \psi_{2,40} & \psi_{3,40} & \psi_{4,40} & \cdots & \psi_{20,40} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{40} \end{bmatrix} + \begin{bmatrix} b_{2,1} \\ b_{2,2} \\ b_{2,3} \\ \vdots \\ b_{2,40} \end{bmatrix} \right) \tag{3. 3}$$

$$\begin{bmatrix} S(t) \\ I(t) \\ T_1(t) \\ T_2(t) \\ R(t) \end{bmatrix} = \left(\begin{bmatrix} \omega_{1,1} & \omega_{2,1} & \omega_{3,1} & \cdots & \omega_{40,1} \\ \omega_{1,2} & \omega_{2,2} & \omega_{3,2} & \cdots & \omega_{40,2} \\ \omega_{1,3} & \omega_{2,3} & \omega_{3,3} & \cdots & \omega_{40,3} \\ \omega_{1,4} & \omega_{2,4} & \omega_{3,4} & \cdots & \omega_{40,4} \\ \omega_{1,5} & \omega_{2,5} & \omega_{3,5} & \cdots & \omega_{40,5} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_{40} \end{bmatrix} + \begin{bmatrix} b_{3,1} \\ b_{3,2} \\ b_{3,3} \\ b_{3,4} \\ b_{3,5} \end{bmatrix} \right) \tag{3. 4}$$

where δ_1 and δ_2 use radial and sigmoid neural networks will denote the activation functions whereas b is the bias to ψ, and w_i for layer 1 (radial basis layer), 2 (log sigmoid layer) and output weight vectors. where y and z are the 1^{st} and 2^{nd} layer and $S(t)$, $I(t)$, $T_1(t)$, $T_2(t)$, $R(t)$ are the outcomes.

4. Radial basis (RB) and Sigmoid functions (SF)

In the past, artificial neural networks commonly used popular functions such as log sigmoid and radial basis function. A Radial Basis Function only provides values that are based on how far points are from the center [18]. Nowadays, however, they do not see much use in practice because the gap in performance between them and more accessible theories is clear in recognition and classification activities [20].

In the procedure, the first hidden layer utilizes the RB neural network and second hidden layer applies the SF. First, hidden layer's RB is often considered an activation function. The activation function determines the configurability of chain which leads to viewing complex patterns in the technical parts. Therefore, most often the choice is made in favor of the RB along a constant form of parameter 1 because it brings model also undermines the uncertainty of overemphasized. The RB offers an estimate of a function X in an everywhere dense subset of F, which demonstrates the possibility of the empirical approximation of the continuous function to the random accuracy. This function is considered to be maintained and running, being computational sufficient to perform and being smooth that is useful to bring a complex relation into a model. They used the logarithmic sigmoid function for the inputs 0 and 1 to arrive at the sigmoid function. It means that the following are the conditions that ensure this function is also differentiable, and so it causes the training duration by using the maximization of gradient illustration. The evaluations from performing NN are fed back into the overall computations in a bid to offer estimated of basically any type of continuous function with respect to raw random accuracy. A Recurrent Radial Basis Function network (RRBFN) that is well-suited for both monitoring and prognosis. Similar to the standard frame of Radial Basis Function networks, the RRBFN have input neurons that feature sigmoid activation functions [22]. Mathematically these RB and SF are shown as follows:

$$\delta_1 = exp(-m^2) \text{ and } \delta_2 = \frac{1}{1+exp(-m)}, \tag{4. 5}$$

where $m = \sum_{j=1}^{s}(w_j z_j) + b$,

where n represents the numbers of neuron. In the suggested ANN-LMB model δ_1 and δ_2 are activation functions utilized for the first and the second hidden layers, respectively. Namely, δ_1 denotes the Radial Basis Function (RBF) utilized on the first hidden layer, whereas δ_2 is the designation of the Log-Sigmoid Function (SF) for the second hidden layer. These functions bring non-linearity to the network so that it can learn the complex interactions between population groups in the SIT_1T_2R model. The RBF works well with localized data sensitivity while the sigmoid function has a smooth transition between states. Taken together, they increase the approximation ability of the model when it comes to the nonlinear differential dynamics.

The DNN process is described in Fig 1. Initially, the single-layer input is crossed through the radial basis taking 20 neurons, while 40 neurons are taking in log sigmoid layer with log sigmoid function to generate output layer.

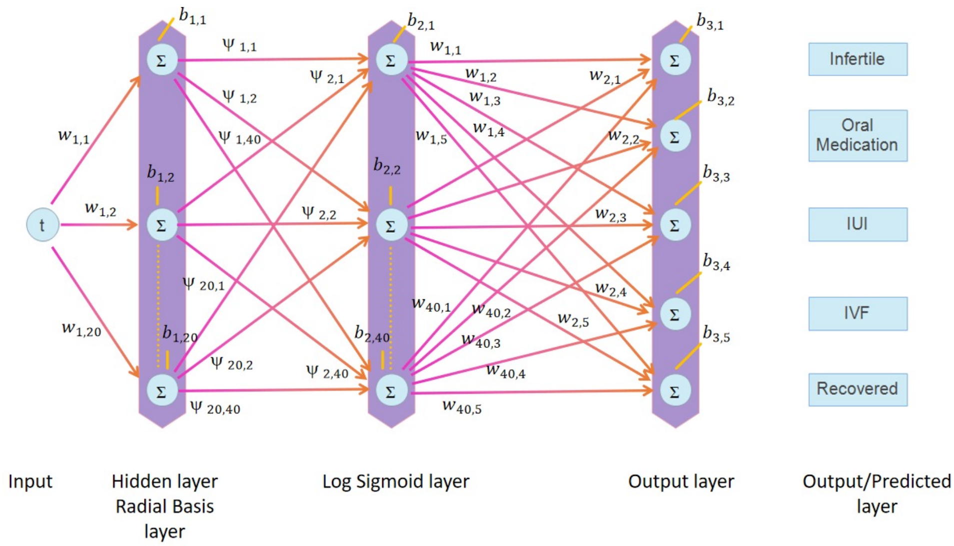


FIGURE 1. A numerical method is used to calculate the nonlinear model using a two-layer DNN.

The application of Artificial Neural Networks (ANNs) to solve dynamical systems provides powerful and flexible framework characteristics, particularly for such highly nonlinear and time dependent models as the SIT_1T_2R formulation for PCOS. ANNs are able to approximate arbitrarily complex nonlinear functions with no use of exact analytical forms. In this research, the ANN-LMB model provides an efficient way of describing the complex dynamics of the population classes within the treatment regimens by learning from reference data calculated by Runge-Kutta. The adaptability of the neural network to various test cases, as well as its possibility to generalise for a variety of initial conditions, define the importance of the neural network in modeling medical and biological systems. Additionally, ANN approach helps to save computation costs, in the case of the subsequent simulations of the processes, which makes it a convenient tool for the forecasting and analysis of the processes in clinical settings. To obtain the numerical solution of the proposed model, we introduce the new DNN approach using the radial basis as well as log sigmoid activation functions in number 20 to 40 neurons located at midpoint layers. Number of choices of the epochs for case 1 to 3 are fixed at 1200. The enhanced level of performance capable of improving neural network training concerning the optimization the training is portrayed below with the use of the structure of SCG. The SCG is an iterative process which is implemented for optimization of the variant of the weights of the network is regulated by cost function. It presents a modified traditional conjugate gradient solver: This new solver includes a scaling factor to make sure that the step made in the course of the optimization process has a size. When compared with other techniques for storing data in the Hessian matrix and the structure in which memory is used, it is more effective. It can therefore be concluded that the convergence requirements for this technique are met through by those intimated no. of cycles and calling for little changes in the number of cost functions. The last process of the Fig 2 supposes that the RBF is applied at the first hidden layer and the SF is applied at the next (second) hidden layer of the network. Over the last few years, the 5CG technique has been used in numerous submissions such as in simulation of accelerating charged polymers also image restitution. Parameters in Table 1 are employed for DNN with hide layers (two) to remove nonlinearity of SIT_1T_2R model.

ANN-LMB Construction: The RK-4 method in MATLAB allows using the reference dataset as a dataset that we start with. Dataset Selection: For each specific dataset the data is assembled in either matrix or vector format. Initialization: Determine the network of neurons also break the data into groups for training, testing and validating:

- No. of **Hidden neurons** are 10
- Allow **Training data** up to 72%
- Allow **Testing data** up to 14%
- Allow **Validation data** up to 14%

Algorithm Construction: Variables are multiplied by different weights and adjusted using biases, to generate a transfer function for each input.

The given dataset is split into the train subset of 72%, the validation subset of 14%, and the test subset of 14%. If we increase the data for training i.e., 100% of the available data for training can lead to overfitting, a lack of evaluation and validation, and an increased risk of data leakage. In overfitting, a model copies every detail of the data it sees and does badly with unseen data. Evaluating the model without a distinct test set becomes very difficult.

If the amount of data used for training is modest, it may not fairly represent the data set, increase the risk of overfitting, result in uncertain performance estimation and make the model less able to be applied in new scenarios. If the unnecessary data is not included, the model does not perform well and provides unconfident forecasts. A good model can be trained and evaluated well if its dataset is large enough.

Stopping Criteria: This approach runs until these conditions are satisfied:

(1) Value of Mu is maximum
(2) Number of iterations is maximum
(3) Performance value is minimum
(4) Results are only saved if the gradient is lower than the minimum gradient; otherwise, the network is retrained.

Retraining: Changes are made to the network's parameters and their resulting outputs are saved.

Results Storage: The results obtained from ANN-LMB are saved as graphs at the end of the conclusion.

TABLE 1. Pseudocode of ANN-LMB

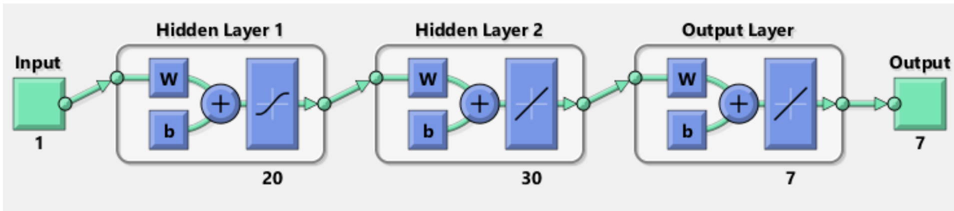


FIGURE 2. Proposed structure for the neurons.

5. RESULTS AND DISCUSSION

The focus of this sectionis to present the mathematical solution of the considered model. The DNN technique is utilized and results are presented for three different cases.

In regression, R is a statistical measure used to evaluate the goodness of fitting the predicted data with the actual data. For our model, the value of R is equal to one which shows the fitting is reliable and trustworthy. The mean square error is also a statistical operator used to evaluate the accuracy and performance of regression models by quantifying the average squared difference between predicted and actual values.

We have used Error histograms to analyse the visual representation of the distribution of errors in a dataset between predicted and actual values. They can help in understanding the performance and characteristics of a predictive model. Each bin in an error histogram represents a range or interval of error values, and the height of each bin corresponds to the frequency or count of data points falling within that error range.

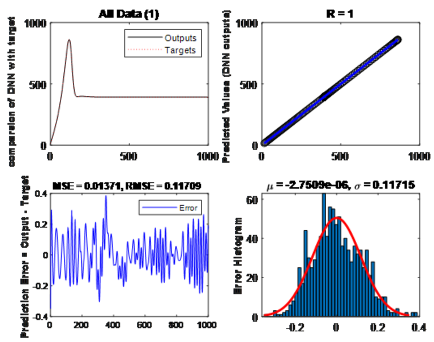
Mean Squared Error (MSE) is a way that calculate the average squared difference between estimated $\hat{y}_i$ and orginally y_i. It gives a value that tells us the approximate accuracy of the network.

$$MSE = \frac{1}{n}\sum(y_i - \hat{y}_i)^2$$

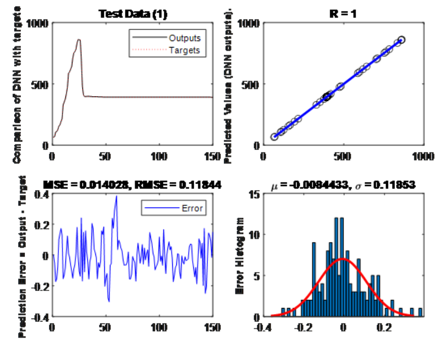
The letter n represents the number of data points in the dataset. The horizontal line in the graph indicates a model that always performs the same. If the horizontal line has the smallest MSE, it proves that the model predicts accurately for every example in the entire dataset. Conversely, when you look at an MSE plot, a vertical line connects the lines for various models being evaluated. The model with the highest number of lowest MSE scores should be seen as the vertical line that outperforms the others.

The functionality of proposed DNN technique is confirmed using the standard statistical measures. RMSE, coming in as the square root measurement of MSE, provides the prediction error in the scale of predicted values thus giving an intuitive insight into the prediction deviation. These validation measures were calculated for all the five compartments in the SIT_1T_2R model on each of the training, testing, and validation dataset sets. The numerical simulations of each case is discussed below:

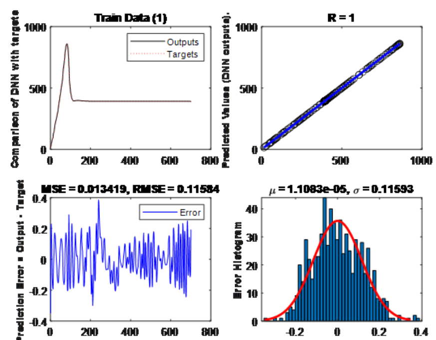
Test Problem 1: The dynamical model given in Eq. (2. 1) is considered. The parameters values are taken from literature. The choosen values are α=0.3, η=0.47, λ=0.47, γ=0.5, β=0.34, ζ=0.16, b=0.34, μ_1=0.9, u_1=0.04 and u_2=0.29, μ_2=0.92. The initial conditions are $S(0)$ = 50, $I(0)$ = 30, $T_1(0)$ = 10, $T_2(0)$ = 5, $R(0)$ = 0, $N(0)$ = 95.



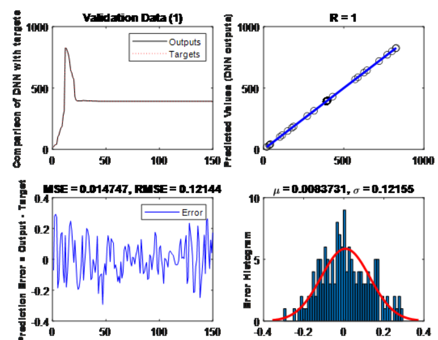
(A) (a)All data for PCOS mathematical model



(B) Test data for PCOS mathematical model



(C) Train data for PCOS mathematical model

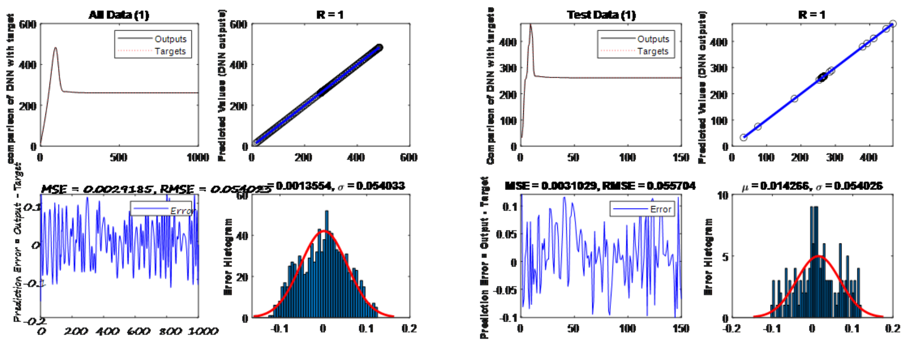


(D) Validation data for PCOS mathematical model

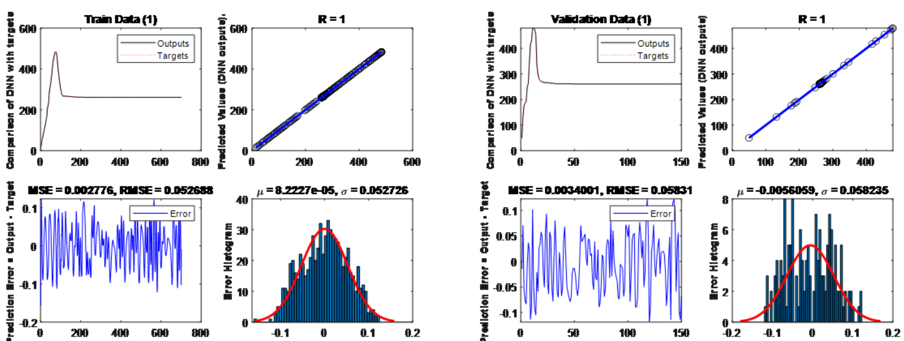
FIGURE 3. DNN was used to conduct MSE performances.

The same has been depicted for all the set of data for S group also the corresponding results is shown below in Fig 3a using ANN. For this class of analysis, the coefficient of regression approximately equal to 1, mean square 0.01371 also the root mean square error is 0.11709. In the histogram presented the mean and variance error and error which are – 0.00027509 and 0.11715 respectively and thereby the efficiency of the presented technique. The test data for S class is as shown below in Fig 3b, and the output can also be as explained as above. In Fig 3b regression coefficient is nearly 1, MSE = 0.0140282 and RMSE = 0.11844 which have been derived for this class. The last coefficient histograms represent mean errors are -0.9984433 also variance error 0.11853 respectively, that underlines effectiveness of the developed approach. Fig 3c case show train data for S group by using the same method, results can be obtained. The regression coefficient was 1, MSE is 0.013419, and RMSE is 0.11585 for this group. Histogram graph shown mean error are 1.1083e-05 also variance errors 0.11593 gained for this group. Fig 3d depicts validation for S class by using the same method the output is achieved. The observed regression coefficient is1, mean square 0.014747 and RMSE is 0.12144 are attained for this class. Histogram shown mean errors are 0.0083731 and variance error 0.12155 for this class.

Test Problem 2: The dynamical model given in Eq. (2. 1) is considered. The parameters values are taken from literature. The updated values of α and γ are taken into accounts i.e α=0.16 and γ=0.23 and remaining are same as above mentioned in test problem 1. The initial conditions are $S(0) = 60$, $I(0) = 25$, $T_1(0) = 8$, $T_2(0) = 6$, $R(0) = 2$, $N(0) = 100$.



(A) (a)All data for PCOS mathematical model (B) Test data for PCOS mathematical model

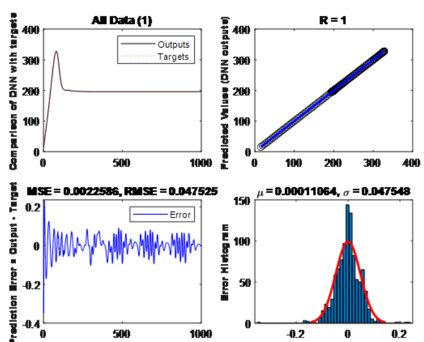


(C) Train data for PCOS mathematical model (D) Validation data for PCOS mathematical model

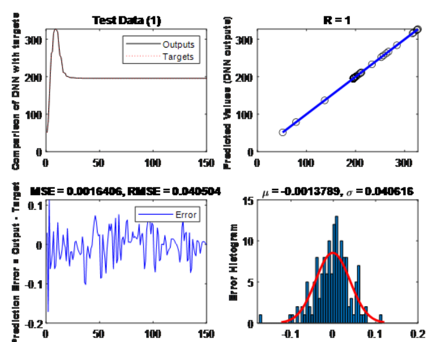
FIGURE 4. DNN was used to conduct MSE performances.

The data presented in the I class for Fig 4a are as follows: ANN generates all the output in the following projects. For this class the regression coefficient is near to 1, mean square of 0.0029185 and root mean square errors of 0.054023 were obtained from this figure. It showed by using histogram the mean error is 0.0013554 and variance error is 0.054033 get explained that how efficient the proposed techniques are. Fig 4b depict the test data for I group also the obtained output is calculated in the same procedure. The regression coefficient (RC) is around 1, MSE is 0.0031029 also RMSE is 0.055704 obtained for this group in the above figure. The histogram graphs described mean error is 0.014266 and variance is 0.0540266 and explained the efficiency of the considered techniques. Train data for I group is as shown in Fig 4c and the following is the output is nearly one regression coefficient is obtained with MSE of 0.002776, RMSE of 0.052688 for this group. The outliers produced by histogram graph shown mean is 8.2227e-05 and variance error is 0.052726, these results hold the efficiency of the considered technique. Likewise for R group, the validation result is presented in Fig 4d and the output is determined using the mentioned above method, regression coefficient is nearly equal to 1, mean square = 0.0034001 RMSE is equal to 0.05831 for it. Histogram represents the mean and variance desired errors whereas the contained error includes -0.0056059 and 0.058235 furnishes the proposed technique efficiency.

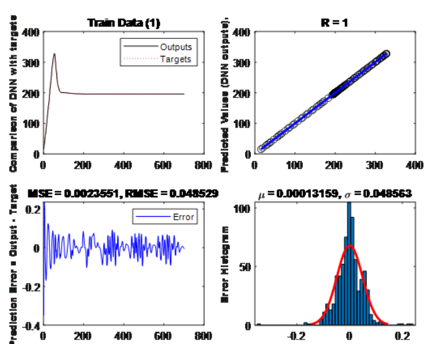
Test Problem 3: The dynamical model given in Eq. (2. 1) is considered. The parameters values are taken from literature. The updated values of α, γ, ζ and u_2 are taken into accounts i.e α=0.16, γ=0.23, ζ=0.08 and u_2=0.19 and remaining are same as above mentioned in test problem 1. The initial conditions are $S(0)$ = 55, $I(0)$ = 28, $T_1(0)$ = 9, $T_2(0)$ = 4, $R(0)$ = 2, $N(0)$ = 98.



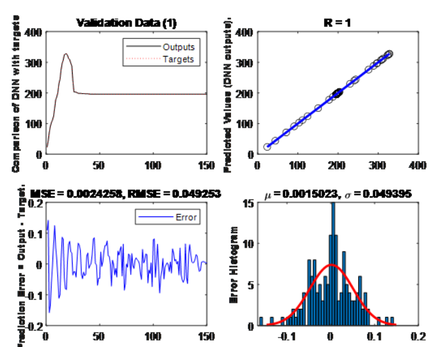
(A) (a)All data for PCOS mathematical model



(B) Test data for PCOS mathematical model



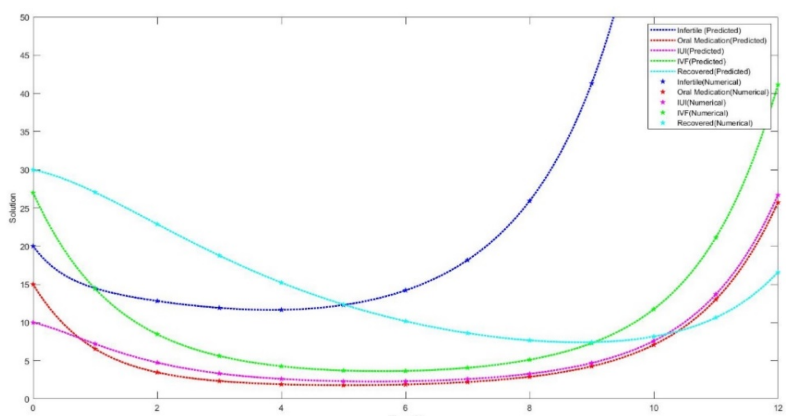
(C) Train data for PCOS mathematical model



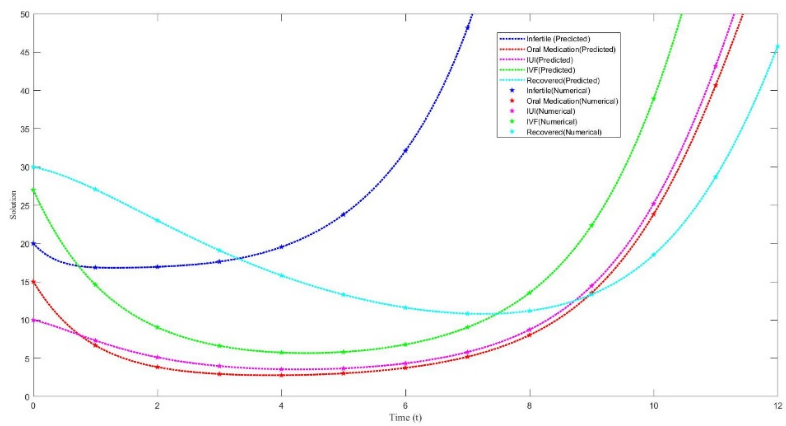
(D) Validation data for PCOS mathematical model

FIGURE 5. DNN was used to conduct MSE performances.

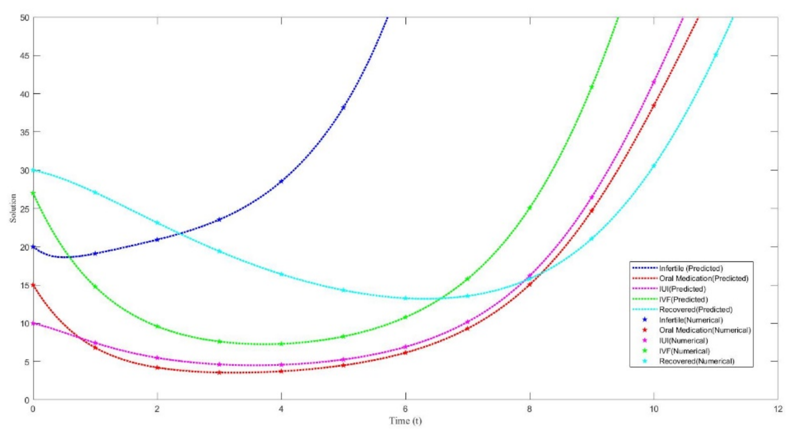
Consequently, all statistics generated for R group are shown in Fig 5a and results has been taken from ANN. In this group the RC = 1, the MSE is 0.0022586 and root mean square errors that I have got for this class is 0.047525. Histogram the mean and variance of the error are represented from that, it is understood that the two errors are 0.00011064 and 0.047548 respectively, are showing the best exactness of the suggested technique. The test data of R class is shown in fig 5b and the outcome is calculated by using the same method as described above. From our data in the figure above we get the regression coefficient in the region of 1, the mean square equal to 0.0016406 and the root man square errors equal to 0.040504 of this group. Histogram chart shown mean error is – 0.0013789 also variance error is 0.040616, these values proved that the suggested technique efficient enough. Figure 5c describes the train data set created in order to R group and the output is generated by a process similar to the one outlined above, regression coefficient about 1, mean squares 0.0023551 and RMSE, 0.048529 for this class. Histogram depicted average and standard deviation variance of the errors and the errors are calculated as 0.00013159 and 0.048563 to depict the efficiency of the proposed approach. In the same manner, the validation data set for R group is provided in next Fig 5d, the results is calculated by the same formula, the RC is of the order of 1; the mean square is 0.0024258 and the root mean square err are 0.049253 for this group. Histogram graphs represented that mean error is 0.0015023 and variance error is 0.049395, it clarifies that the errors are zero it is showing that the suggested technique high accuracy rate.



(A) Case I: Comparison



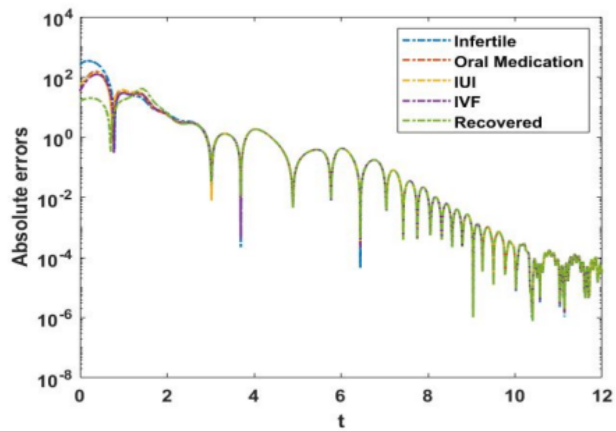
(B) Case II: Comparison



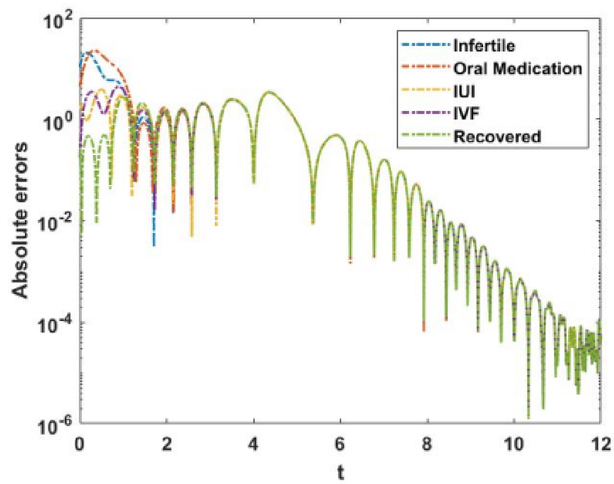
(C) Case III: Comparison

FIGURE 6. Comparisons of solutions of all cases

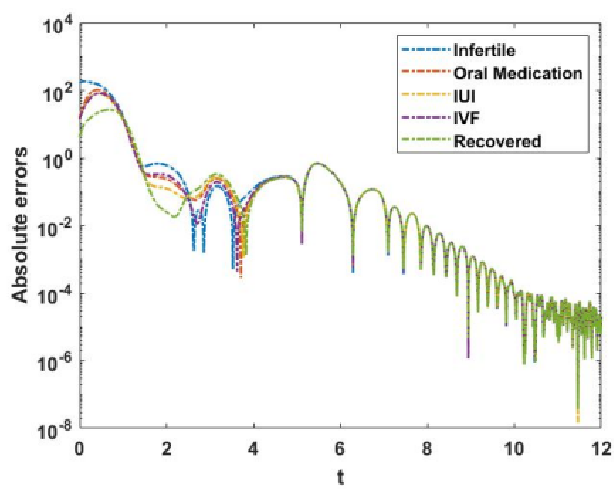
We depicted the overall dynamics of first-class comparison with regard to the ANN in Fig 6a. The same is true for the second model's dynamics of first-class comparison with ANN, as illustrated by Fig 6b. This Fig 6c is an analysis of first-class comparison with ANN.



(A) Absolute Error



(B) Absolute Error



(C) Absolute Error

FIGURE 7. Absolute errors of solutions of all cases

6. Conclusion

A novel computational design based on DNN technique was intoduced to solve the dynamical SIT_1T_2R model related to PCOS women. The proposed SIT_1T_2R model consist of five classes: women who is remain infertile when given gonadotropin and clomiphene citrate treatment, women affected by PCOS, infertile women who have made progress and infertile women undergoing IVF. The neural network procedure considering two hidden layers based on the combination of RB and SF with twenty and forty numbers of neurons have been established for the solutions of the considered model equations. An optimization technique on the competent scale conjugate gradient scheme has been employed to obtain the numerical solutions of the dynamical model. Three cases have been discussed to apply the method in terms of training (72%), testing (14%), and validation (14%) level with 10 hidden neurons. The results that were approximated by the DNN method have been compared to the results of numerical solutions, and both solutions have agreed reasonably well. Plots of regression, correlation, error histograms and state transitions were displayed to demonstrate the efficiency of the suggested DNN approach. In future, the proposed double layer design is intended to solve various nonlinear models of mathematical biology and fluid dynamics.

Conflicts of Interest

The authors declare no conflict of interest.

Funding

No funding has been received for this study.

References

[1] R. M. Cichy and D. Kaiser, *Deep neural networks as scientific models*, Trends in Cognitive Sciences, **23**(4), (2019), 305–317.

[2] M. Dee and A. Serrano, *Effects of metformin in patients with polycystic ovarian syndrome*, Philippine Journal of Reproductive Endocrinology and Infertility, **3**(2), (2006).

[3] R. Eshre, *Revised 2003 consensus diagnostic criteria and long-term health risks related to polycystic ovary syndrome (PCOS), ASRM-sponsored PCOS consensus workshop group*, Human Reproduction, **19**(1), (2004), 41–47.

[4] Z. Faiz, S. Javeed, I. Ahmed, D. Baleanu, M. B. Riaz, and Z. Sabir, *Numerical solutions of the Wolbachia invasive model using Levenberg–Marquardt backpropagation neural network technique*, Results in Physics, **50**, (2023), 106602.

[5] Z. Faiz, S. Javeed, I. Ahmed, and D. Baleanu, *Numerical investigation of a fractional order Wolbachia invasive model using stochastic Bayesian neural network*, Alexandria Engineering Journal, **93**, (2024), 303–327.

[6] C. G. Galindo, I. Hernández, and A. R. Ayala, *Clinical effects of metformin in patients with polycystic ovarian syndrome*, Ginecologia y Obstetricia de Mexico, **75**(4), (2007), 181–186.

[7] A. A. Garcés, L. L. Román, and Y. G. Ríos, *Response to treatment with metformin in patients with polycystic ovarian syndrome*, Pharm Pharmacol Int J, **7**(6), (2019), 302–305.

[8] A. A. Garcés, L. L. Román, and Y. G. Ríos, *Response to treatment with metformin in patients with polycystic ovarian syndrome: Territorial center of care for the infertile couple*, Pharm Pharmacol Int J, **7**(6), (2019), 302–305.

[9] S. G. Hafezi, M. A. Zand, M. Molaei, and M. Eftekhar, *Dynamic model with factors of polycystic ovarian syndrome in infertile*, International Journal of Reproductive BioMedicine, **17**(4), (2019), 231.

[10] L. Huang, Y. Huang, and Q. Pang, *Molecular mechanism of Bushen Huatan Prescription in the treatment of polycystic ovary syndrome based on network pharmacology and bioinformatics*, Pharmacological Research - Modern Chinese Medicine, **5**, (2022), 100174.

[11] S. Javeed, K. S. Alimgeer, W. Javed, M. Atif, and M. A. Uddin, *A modified artificial neural network-based prediction technique for tropospheric radio refractivity*, Plos One, **13**(3), (2018), e0192069.

[12] S. Kandasamy, R. I. Sivagamasundari, A. Bupathy, and S. Sethupathy, *The plasma nitric oxide and homocysteine levels and their association with insulin resistance in South Indian women with polycystic ovary syndrome*, Int J Res Med Sci, **4**(11), (2016), 4829.

[13] N. Kriegeskorte and T. Golan, *Neural network models and deep learning*, Current Biology, **29**(7), (2019), R231–R236.

[14] Y. LeCun, Y. Bengio, and G. Hinton, *Deep learning*, Nature, **521**(7553), (2015), 436–444.

[15] S. M. Moosavi-Dezfooli, A. Fawzi, and P. Frossard, *Deepfool: A simple and accurate method to fool deep neural networks*, In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, (2016), 2574–2582.

[16] R. Obaid, *A prevalence of menstrual dysfunction and hirsutism in women with polycystic ovary syndrome (PCOS)*, Biochemical & Cellular Archives, **21**, (2021).

[17] O. Ogunmolu, X. Gu, S. Jiang, and N. Gans, *Nonlinear systems identification using deep dynamic neural networks*, arXiv preprint arXiv:1610.01439, (2016).

[18] S. Z. Rezaei Lalami, J. Levesley, and M. F. Sajjad, *Radial basis function solution for the LIBOR market model PDE*, Punjab Univ. J. Math, **50**(4), (2020).

[19] Z. Sabir, S. Dirani, S. Bou Saleh, M. K. Mabsout, and A. Arbi, *A novel radial basis and sigmoid neural network combination to solve the human immunodeficiency virus system in cancer patients*, Mathematics, **12**(16), (2024), 2490.

[20] A. N. Samatin Njikam and H. Zhao, *A novel activation function for multilayer feed-forward neural networks*, Applied Intelligence, **45**, (2016), 75–82.

[21] M. F. Tabassum, M. Farman, A. Akgul, and S. Akram, *Mathematical treatment of nonlinear pine wilt disease model: An evolutionary approach*, Punjab Univ. J. Math., **54**(9), (2023).

[22] R. Zemouri, D. Racoceanu, and N. Zerhouni, *Recurrent radial basis function network for time-series prediction*, Engineering Applications of Artificial Intelligence, **16**(5–6), (2003), 453–463.