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Enhanced multi-criteria decision-making through fuzzy soft set parameter reduction and score optimization

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Abstract.: Parameter reduction is a crucial task in multi-criteria decision-making (MCDM), particularly when dealing with high-dimensional and uncertain data. Fuzzy soft set (FSS) theory, which integrates the parameterization capability of soft sets with the uncertainty-handling strength of fuzzy sets, provides an effective framework for such problems. In this paper, we focus on parameter reduction in FSS and its impact on score-based decision-making. Existing approaches, including S-normal and I-S-normal parameter reduction methods, often lead to identical or indistinguishable decision scores and involve considerable computational complexity. To overcome these limitations, a new and efficient decision-making algorithm is proposed within the fuzzy soft set framework, yielding unique and more discriminative scores after parameter reduction. The proposed algorithm is applied to illustrative examples and real-life decision-making problems, demonstrating improved accuracy and reduced computational effort. A comparative analysis with the TOPSIS method further confirms the effectiveness and reliability of the proposed approach for MCDM under uncertainty.

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1. Introduction

The problems of conventional techniques have been removed by the SS and It has an excellent implementation capability to solve the uncertainties. The SS theory was for the first time in 1999 introduced by Molodtsov [20]. Finally, through the development of the SS, many other extended theories were developed [10]. Currently, decision-makers observe undefined, inaccurate fuzzy information, including various problems in daily routine work in the environmental, medical, computer, and engineering fields. Furthermore, the theory of the classical set could be more effective due to having many restrictions. Fuzzy and soft set research has seen significant progress in recent decades, both in theoretical developments and practical applications [28, 25, 35]. Consequently, the mathematical models such as probability model [33], intuitionistic fuzzy set model [8], interval mathematics model, and rough set model [22] to manage these decisions in a better way. Although, Molodtsov [20] pointed out the intrinsic restrictions of the models mentioned above. Therefore, he developed an innovative, informal, and direct mathematical theory of SS to overcome and resolve these restrictions. Thus, attribution is used in the theory of SS due to the primary implementation in its expansion and use. The identified attribution of facts and figures in a SS is applicable as a standard to make the decision quickly. Various set-theoretical procedures were developed in the theory of SS by Maji et al. [18] and Ali et al. [3]. Moreover, soft sets have been applied in practical areas such as medical analysis [19], handling incomplete or uncertain information [24], and supporting decision-making [1, 4]. Moreover, Alcantud [2] developed an algorithm for decision-making based on FSS, which have input parameters set by multiple observers.

The reduction of parameters is one of the most active topics in the SS theory. This segmentation has been initiated by Maji et al. [18] in the year 2002 where they introduced a concept known as reduce SS. With the help of these, researchers started to focus on attribute reduction in FSS [32]. Chen et al. [9] described the reduction of attributes in SS, indicated that the reduction in rough set theory was Maji and Roy's reduct-soft set method, and compared them. Zhan and Alcantud [34] also made a novel contribution to multicriteria group decision-making. Kong et al. [14] defined some new reductions regarding normal parameters for parameter addition problems in SS. The article of Ma et al. [15] also included a large number of computation issues that they think are not possible for normal parameter reduction in real-life problems of FSS. She also developed distance-based parameter reduction of a FSS, which is more suitable for higher levels. In decision-making, Peng et al. [23] presented a few algorithms based on regret and prospect theory for interval-valued fuzzy sets.

Therefore, the FSS are the most effective comprehensive models of SS. Maji et al. [18] explained the theory of FSS based on the collaboration between SS and FS, elaborated in a general form. Besides, the FSS are primarily used in collective estimating [21], assembled decisions [13], and medical sciences [11]. Maji and Roy [26] put forward a technique of object identification through indefinite multiobserver data to decide with the help of FSS that are enhanced in this paper. The FSS are primarily used to calculate various valuable parameters in the method of accurate decision-making. The reduction of attributes is the minimum subgroup of attributes. The minimum attributes set maintains the identical description capability or decision-making as the original attribute set due to minimized valuable attributes. Consequently, reducing attributes is highly significant in the FSS to make the decision. Many researchers [29, 6, 12, 7] have made very notable contributions in the field of decision-making. Recently, the limitations in SS were overcome by the hypersoft set theory, an extension of SS introduced by Smarandache [30]. Saeed et al. [27] studied some fundamentals and matrix representations of hypersoft sets. In [5], Amman et al. employed Pythagorean fuzzy hypersoft sets in a decision-making framework.

In [16], Ma et al. further reduced the computation of parameters, but the resulting scores obtained after applying the method differ. Instead, two or more scores are the same, and this is an issue because one cannot choose the best or worst among them if the end scores are similar. Lately, Ma et al. [17] presented another method that improved the parameter reduction process, but the technique could not provide more accurate results after the score decision criteria. There are typically uncertainties and unpredictabilities associated with high-dimensional data. The proposed method (AA-algorithm) is explained in detail in this paper, along with further reductions. The proposed algorithm improves the method of Ma et al. by enhancing the score-based decision outcome, addressing limitations observed in their earlier approach. SS theory offers a method for handling ambiguous and imprecise information through parameter reduction. It is vital in real-world situations where data might need to be more precise and complete. Therefore, the proposed AA-algorithm provides more reliable and discriminative decision scores by eliminating score ties observed in existing methods. In addition, it improves both parameter reduction and score-based decision-making within the fuzzy soft set framework. With its systematic technique to rank options according to their distance from the negative ideal solution and their proximity to the ideal answer, TOPSIS excels in managing complicated choice scenarios [21, 24]. Using TOPSIS to validate the novel FSS parameter reduction and score decision criteria enhancement suggested in this study, we aim to improve the decision-making accuracy and robustness and offer a thorough and cutting-edge solution for MCDM problems.

MOTIVATION

One useful framework that has evolved is the concept of soft sets, which provides a potential path for handling uncertainty. Even though SS and parameter reduction have advanced significantly in current research, there is still a vital research gap.

- This work is motivated by the need to solve problems involving high-dimensional data when traditional methods are frequently insufficient.
- The main objective of this research is to apply sophisticated parameter reduction approaches to improve decision-making based on score decision criteria.
- Prior algorithms, including the ones put out by Ma et al. [17] have had trouble generating unique scores following the parameter reduction procedure. Making the best or worst selection amongst similar scores is hampered by the inability to distinguish between them.
- By putting forth a better approach that not only streamlines the parameter reduction procedure but also guarantees more precise and distinct outcomes in the score decision criteria, this research aims to close this gap and advance decision-making models that deal with high-dimensional data and uncertainties.

CONTRIBUTION

The main contributions of this research are as follows:

- The research advances practical applications by introducing a unique mathematical tool that blends SS and fuzzy sets.
- Aiming to address the computational complexity associated with existing methods, the newly offered decision-making algorithm builds upon the basis provided by the proposed algorithms, S parameter, and I-S parameter reduction.
- The study offers decision-makers a useful framework for handling uncertainty in real-world situations by proposing a unique score decision technique and applying parameter reduction to dispensable sets.
- This hybrid technique is beneficial in sectors like environmental, medical, computer science, and engineering because it tackles the difficulties presented by unknown information.

The rest of the paper is organized as follows: basic definitions related to this study are introduced in Section 2. The algorithms generated from parameter reduction techniques and score decision criteria are explained in section 3. The described proposed method and the detailed example of its application are presented in Section 4. In Sect. 5, two real-world real-world applications are solved using the algorithms given in this paper. In Section 6, we discuss how the proposed method is related to some of the existing solution methods. As the next step to assess the results of the proposed method, the ranking comparison with TOPSIS is given in Section 7 while Section 8 gives the conclusion of the research.

2. PRELIMINARIES

This section includes some basic definitions of the related research.

Definition 2.1. *(see* [20]*). Let X be a universe set and Y be a parameter set associated with X. Let $P(X)$ denote the power set of X and $M \subseteq Y$. A pair (F, M) is called a* soft set (SS) *over X, where F is a mapping $F : M \to P(X)$. Formally, (F, M) is defined as*

$$(F, M) = \{F(m) \in P(X) : m \in M,\ F(m) = \emptyset \text{ if } m \notin M\}.$$

If two soft sets (F, M) and (G, N) over X satisfy $M \subseteq N$ and $F(m) \subseteq G(m)$ for all $m \in M$, then (F, M) is called a soft subset *of (G, N).*

A soft set (F, M) represents objects in X with respect to parameters in M, where $F(m)$ indicates which objects satisfy parameter m. It provides a structured way to represent knowledge about the objects and their attributes [31]. In decision-making, the membership values can be used to construct a comparison table and define score criteria, enabling object ranking and identification of dispensable parameters whose removal does not change the decision outcomes. This representation makes it easier to analyze, compare, and select the most suitable alternatives based on the available information.

Definition 2.2. *(see* [26]*). Let $X = \{\text{Ą}_1, \text{Ą}_2, \ldots, \text{Ą}_n\}$ be the universe of objects and $A = \{\text{Ç}r_1, \text{Ç}r_2, \ldots, \text{Ç}r_m\}$ be a set of parameters. A* fuzzy soft set *(G, A) over X is a mapping $G : A \to \tilde{P}(X)$, where $\tilde{P}(X)$ denotes the set of fuzzy subsets of X. That is, for each parameter $\text{Ç}r_j \in A$, $G(\text{Ç}r_j)$ is a fuzzy set on X that assigns to each object Ą_i a membership value in* $[0, 1]$.

Definition 2.3. *(see* [14]*). Let* (G, A) *be an FSS with attribute set* A *and object set* X*. A subset* $S \subseteq A$ *is called a* dispensable set *if the FSS* (G, A) *and* $(G, A - S)$ *yield the same decision ranks according to a given score-based decision criterion.*

Definition 2.4. *(see* [16]*). For an FSS* (G, A)*, let* S *be the largest dispensable set of* A*. The* parameter reduction *of* A *is then defined as* $A - S$*.*

By leveraging S-normal parameter reduction based on score criteria, the proposed method allows decision-making without altering the ranks of objects, even if dispensable parameters are removed.

2.5. **Case of dispensable sets.** Dispensable sets determined by score decision in a FSS. There are a few ways in which one can get dispensable sets.

(1) The first case is when the choice value in a column is in increasing order while the other is in decreasing order. Therefore, same number without repetition by adding the choice values is obtained.
(2) In the second case, if a table is given where one column is arranged in ascending order and another in descending order, the same sum can be achieved when adding these columns.
(3) If the attribute set S is ignored, and the prime concern of sets does not change, a fair conclusion can be drawn that the parameter set S dispensable in the FSS, (G,A) based on the score criterion.

3. Related algorithms

This section discusses a few algorithms developed to reduce some parameters that could not be more worth calculating scores in decision-making problems. Kong et al. [14] proposed an algorithm for parameter reduction using a dispensable set, the S-normal parameter reduction method. Kong et al. discussed the dispensable sets in detail so one can easily understand the concept of dispensable sets. The algorithm and examples are as follows.

3.1. **S-normal Reduction Method.** For calculating the score to find the final selection or to choose the best value, kong et al. use Maji and Roy's algorithm, shown below as Algorithm 2.

3.2. **Algorithm 2.**

3.3. **I-S-normal parameter reduction method.** The reduction technique of I-S normal parameters in the decision criteria of FSS with the help of dispensable sets for getting the score table that Ma and Qin introduced [15] is discussed. The related algorithm is as follows.

The details for finding the comparison table are

1. Calculate the upper and lower triangles of the comparison table using as described in 2.5. Additionally, mark the number of membership values that are equal between two objects. $B_{ij} = count\{\xi_{S_{\text{Çr}_i}}(\xi_j) > \xi_{S_{\text{Çr}_j}}(\xi_i)\}$
$B' = count\{\xi_{S_{\text{Çr}_i}}(\xi_j) = \xi_{S_{\text{Çr}_j}}(\xi_i)\}$.

Algorithm 1 S-normal Reduction Method

Input: Fuzzy Soft Set (FSS) (G, A) with parameter set $A = \{\text{Çr}_1, \text{Çr}_2, \ldots, \text{Çr}_m\}$ and universal set $X = \{\text{A̧}_1, \text{A̧}_2, \ldots, \text{A̧}_n\}$
Output: S-normal parameter reduction $A - S$
1. For any subset $S \subset A$:
  Compute the comparison matrix K_A
  Compute the matrix K_{A-S}
  Compute the difference $K_A - K_{A-S}$
2. Check if the matrix $K_A - K_{A-S}$ is symmetric
if the matrix is symmetric **then**
    S is identified as the dispensable set D
end if
3. Determine if S is the maximum dispensable set among all subsets of A
if S is the maximum dispensable set **then**
    The final s-normal parameter reduction is $A - S$
end if

Algorithm 2 FSS Attribute Comparison

Input: Fuzzy Soft Set (FSS) and objects $\{w_1, w_2, \ldots, w_k\}$
Output: Object p_n with the maximum score
1. Initialize a table T with dimensions $k \times k$
for each pair of objects (w_i, w_j) **do**
  Compute FSS result using the minimum (AND) operator
  Store the result in cell $T[i, j]$
end for
2. Create an $n \times n$ comparison matrix W
for each pair of attributes (p_i, p_j) **do**
  Set $W[i, j]$ to the count of attributes where the membership value of p_i is $\geq$ the membership value of p_j
  This is equivalent to counting positive values in $\{(T[i, m] - T[j, m])$ for $m = 1$ to $k\}$
end for
3. For each row i from 1 to n
for each row i **do**
  Compute r_i as the sum of elements in row i of matrix W
  Compute t_i as the sum of elements in column i of matrix W
  Compute score S_i as $S_i = z_i - t_i$
end for
4. Select the object with the maximum score
Find the object p_n such that $S_n = \max(S_i)$ for $i = 1, \ldots, n$
5. Return: Object p_n with the highest score

2. Now calculate the upper and lower triangle matrix by joining them according to dispensable sets,

$$B_{ij} = N - B_{ij} - B' = N - (B_{ij} + B') \tag{3. 1}$$

Algorithm 3 I-S-normal Parameter Reduction Method

Input: Fuzzy Soft Set (FSS) (G, A) with parameter set $A = \{\text{Çr}_1, \text{Çr}_2, \ldots, \text{Çr}_m\}$ and object set $X = \{\text{A̧}_1, \text{A̧}_2, \ldots, \text{A̧}_n\}$
Output: I-S normal parameter reduction $A - S'$
1. For any subset $S' \subset A$:
  Compute the comparison matrix $K_{S'}$
if $K_{S'}$ is symmetric **then**
    S' is identified as the dispensable set
end if
2. Compute the I-S normal parameter reduction:
  The final reduction is $A - S'$

3. Make a whole comparison table, then find the comparison matrix from this.

4. Description of the proposed method

This section includes the proposed algorithm to improve the score in decision-making problems, which was noticed in the S-normal reduction method and I-S algorithm. The proposed algorithm increases the accuracy of the given algorithms while requiring fewer steps in the parameter reduction process as shown in Figure 1.

In the proposed AA-algorithm, all criteria are considered equally important; explicit criteria weights are not used. Decision-makers provide the parameter set, object set, and the corresponding fuzzy membership values for each object, which serve as the input to the algorithm. The AA-algorithm then computes net dominance scores and ranks the alternatives, assisting decision-makers in making informed choices. In the AA-algorithm, the score of each object considers both its total advantage over other objects and its total disadvantage, providing a net dominance measure. This ensures that the AA-score is more discriminative than simple aggregation methods by simultaneously accounting for strengths and weaknesses.

As shown in Table 1, the proposed AA-algorithm is more computationally efficient than the S-normal and I-S-normal methods, particularly in terms of time complexity. While all three methods require similar space for storing the comparison matrix and position table, the AA-algorithm reduces the number of computational steps by focusing only on dispensable sets rather than checking all subsets of parameters. Furthermore, the AA-scores are more discriminative, providing stable and reliable rankings, which have been validated through comparison with the TOPSIS method. This demonstrates that the AA-algorithm offers a practical improvement in both efficiency and ranking performance over the existing approaches.

Applying AA-algorithm on example 1 to show better accuracy in the score table. The stepwise explanation is given below.

Example 1. Supposing that $X = \{\text{A̧}_1, \text{A̧}_2, \text{A̧}_3, \text{A̧}_4, \text{A̧}_5, \text{A̧}_6\}$ be the set of six objects and $A = \{\text{Çr}_1, \text{Çr}_2, \text{Çr}_3, \ldots, \text{Çr}_{10}\}$ are a few parameters which are to be considered while deciding that which object has a better score value. (G, A) is the FSS whose tabular representation is shown in Table 2 with comparison values in Table 3. To calculate the comparison table of S-normal and I-S normal reduction methods, find dispensable sets whose few

Algorithm 4 AA-algorithm

Input: Fuzzy Soft Sets (FSS) (G, A) with parameter set $A = \{\text{Çr}_1, \text{Çr}_2, \text{Çr}_3, \ldots, \text{Çr}_n\}$ and object set $U = \{d_1, d_2, d_3, \ldots, d_n\}$

Step 1: Generate Position Table
- For each column, find the positions of values
- Create a position table for the given values

Step 2: Identify Dispensable Sets
- For each subset $T \subseteq A$:
  - Add T to dispensable set R

Step 3: Determine Common Dispensable Sets
- For each subset $T = \{t_1, t_2, \ldots, t_m\}$:
  - For each $\text{Çr}_i \in T$:
    - Define $F(\text{Çr}_i) = \{d_1/f_{1i}, d_2/f_{2i}, \ldots, d_n/f_{ni}\}$
    - If all values in $F(\text{Çr}_i)$ are equal:
      - Then T is a common dispensable set

Step 4: Compute Parameter Reduction
- Set $A - R$ as the parameter reduction of R

Step 5: Construct Comparison Matrix
- For each parameter j:
  - Compute $M_j = \max(t_{ij})$ for $i = 1, \ldots, k$
- Create $k \times k$ comparison matrix $A = (\text{Çr}_{ij})_{k\times k}$
- For each pair (i, j):
  - Compute Çr_{ij} as the sum of the non-negative values in:
    - $\frac{t_{i1}-t_{j1}}{M_1}, \frac{t_{i2}-t_{j2}}{M_2}, \ldots, \frac{t_{iq}-t_{jq}}{M_q}$

Step 6: Compute Scores
- For each object $i = 1, \ldots, k$:
  - Compute row sum $R_i = \sum_{j=1}^{k} \text{Çr}_{ij}$
  - Compute column sum $T_i = \sum_{j=1}^{k} \text{Çr}_{ji}$
  - Compute score:

$$S_i = R_i - T_i = \sum_{j=1}^{k} \text{Çr}_{ij} - \sum_{j=1}^{k} \text{Çr}_{ji}$$

Step 7: Identify Best Object
- Find object d_k such that $S_k = \max(S_i)$ for $i = 1, \ldots, k$

cases are discussed above. Find the score values and put them in a table as shown in Table 6. Now the score of each object i.e, $f = \{9, -9, -1, -3, -4, 8\}$,and the priority of these objects are $\text{A̧}_1 > \text{A̧}_6 > \text{A̧}_3 > \text{A̧}_4 > \text{A̧}_5 > \text{A̧}_2$ are available.

A few dispensable sets by applying the S-parameter reduction method to this problem are obtained. Using this method, dispensable sets $\{\text{Çr}_1, \text{Çr}_4, \text{Çr}_5, \text{Çr}_6\}$ are obtained from table 4, which gives the same membership values, which can be neglected while solving for the score decision. After removing the dispensable set, find the comparison table for it, shown in table 5. Now, we have a comparison table and can easily find the scores.

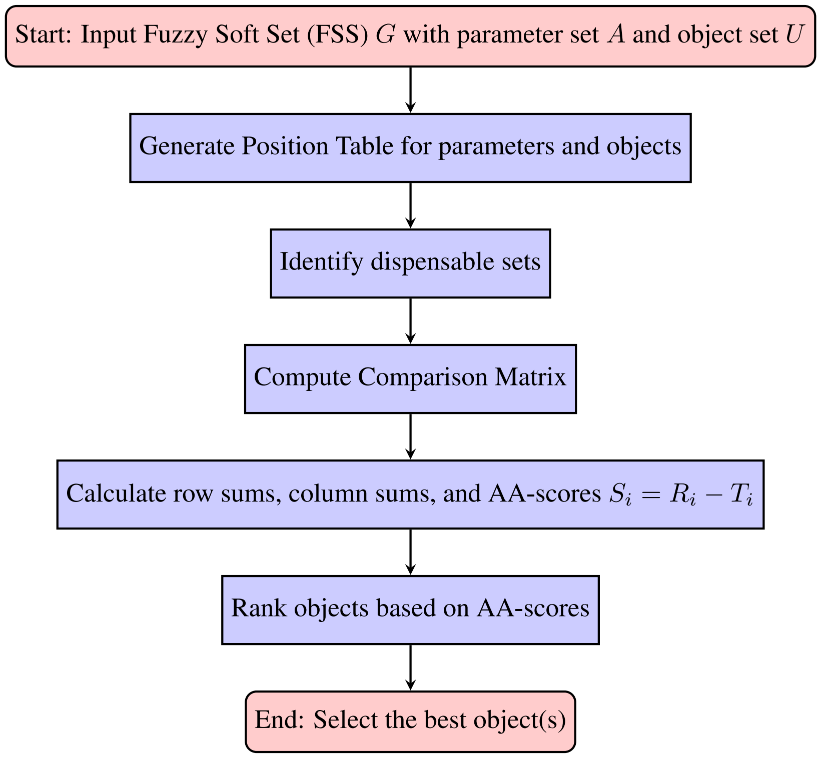

FIGURE 1. Flowchart of the proposed AA-algorithm for parameter reduction and ranking.

TABLE 1. Comparison of computational efficiency and score discrimination between S-normal, I-S-normal, and AA-algorithm

Aspect	**S-normal**	**I-S-normal**	**AA-algorithm (proposed)**
Time Complexity	$O(2^n \times k^2)$	$O(2^n \times k^2)$	$O(k^2 \times n)$
Space Complexity	$O(k^2 + k \times n)$	$O(k^2 + k \times n)$	$O(k^2 + k \times n)$
Number of computational steps	High: all subsets of parameters are checked	Moderate: some subset optimizations	Reduced: only dispensable sets are processed
Score discrimination	May produce ties	May produce ties	More discriminative due to net dominance computation
Ranking stability	Sometimes ambiguous	Sometimes ambiguous	Stable and consistent with TOPSIS results

TABLE 2. The fuzzy soft set (G, A) associated with Example 1.

	$Çr_1$	$Çr_2$	$Çr_3$	$Çr_4$	$Çr_5$	$Çr_6$	$Çr_7$	$Çr_8$	$Çr_9$	$Çr_{10}$
$A̧_1$	0.62	0.6	0.7	0.3	0.3	0.7	0.8	0.7	0.1	0.6
$A̧_2$	0.58	0.57	0.5	0.63	0.6	0.4	0.4	0.3	0.2	0.6
$A̧_3$	0.4	0.4	0.3	0.7	0.68	0.3	0.5	0.8	0.8	0.4
$A̧_4$	0.59	0.9	0.2	0.5	0.5	0.57	0.3	0.6	0.8	0.3
$A̧_5$	0.5	0.66	0.55	0.66	0.8	0.2	0.6	0.4	0.5	0.2
$A̧_6$	0.2	0.8	0.36	0.8	0.6	0.4	0.9	0.5	0.6	0.5

TABLE 3. The comparison table of (G, A) by S-normal parameter reduction method.

	$A̧_1$	$A̧_2$	$A̧_3$	$A̧_4$	$A̧_5$	$A̧_6$
$A̧_1$	10	7	6	6	6	5
$A̧_2$	4	10	5	5	3	5
$A̧_3$	4	5	10	7	5	4
$A̧_4$	4	5	4	10	6	5
$A̧_5$	4	7	5	4	10	3
$A̧_6$	5	7	6	5	7	10

TABLE 4. The reduced fuzzy soft set (G, A-S) associated with Example 1.

	$Çr_2$	$Çr_3$	$Çr_7$	$Çr_8$	$Çr_9$	$Çr_{10}$
$A̧_1$	0.6	0.7	0.8	0.7	0.1	0.6
$A̧_2$	0.57	0.5	0.4	0.3	0.2	0.6
$A̧_3$	0.4	0.3	0.5	0.8	0.8	0.4
$A̧_4$	0.9	0.2	0.3	0.6	0.8	0.3
$A̧_5$	0.66	0.55	0.6	0.4	0.5	0.2
$A̧_6$	0.8	0.36	0.9	0.5	0.6	0.5

For parameter $Çr_1$, the choice values are arranged in columns as $\{6, 4, 2, 5, 3, 1\}$. Now, for attribute $Çr_2$, the choice values are arranged by their position, and have a result that is $\{1, 3, 5, 2, 4, 6\}$. Consider these two arrangements $\{6, 4, 2, 5, 3, 1\}$ and $\{1, 3, 5, 2, 4, 6\}$. The result of the sums of these columns gives 7. Similarly, by adding the choice values of $Çr_5$ and $Çr_6$ parameters, it gives answer 6. Thus, these two matrices also have no impact on the final result, so consider them as dispensable sets and remove them. Let us present the matrices of K_A and K_{A-S} displayed in Table 3 and Table 5, respectively.

TABLE 5. The comparison table of (G, A-S) associated with Example 1.

	$Ą_1$	$Ą_2$	$Ą_3$	$Ą_4$	$Ą_5$	$Ą_6$
$Ą_1$	6	5	4	4	4	3
$Ą_2$	2	6	3	3	1	2
$Ą_3$	2	3	6	5	3	2
$Ą_4$	2	3	2	6	4	3
$Ą_5$	2	5	3	2	6	1
$Ą_6$	3	4	4	3	5	6

$$K_A = \begin{bmatrix} 10 & 7 & 6 & 6 & 6 & 5 \\ 4 & 10 & 5 & 5 & 3 & 5 \\ 4 & 5 & 10 & 7 & 5 & 4 \\ 4 & 5 & 4 & 10 & 6 & 5 \\ 4 & 7 & 5 & 4 & 10 & 3 \\ 5 & 7 & 6 & 5 & 7 & 10 \end{bmatrix}.$$

$$K_{A-S} = \begin{bmatrix} 6 & 5 & 4 & 4 & 4 & 3 \\ 2 & 6 & 3 & 3 & 1 & 2 \\ 2 & 3 & 6 & 5 & 3 & 2 \\ 2 & 3 & 2 & 6 & 4 & 3 \\ 2 & 5 & 3 & 2 & 6 & 1 \\ 3 & 4 & 4 & 3 & 5 & 6 \end{bmatrix}.$$

After computing $K_A - K_{A-S}$ is symmetric, this matrix is the resultant matrix. Therefore, the dispensable sets did not impact the final results. By reducing the dispensable sets it will reduce the complex computations.

$$K_A - K_{A-S} = \begin{bmatrix} 4 & 2 & 2 & 2 & 2 & 2 \\ 2 & 4 & 2 & 2 & 2 & 3 \\ 2 & 2 & 4 & 2 & 2 & 2 \\ 2 & 2 & 2 & 4 & 2 & 2 \\ 2 & 2 & 2 & 2 & 4 & 2 \\ 2 & 3 & 2 & 2 & 2 & 4 \end{bmatrix}.$$

From the above comparison, matrix M''_A of a FSS is presented as

$$M''_A = \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 & 1 \\ 2 & 2 & 0 & 2 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 2 & 0 & 2 \\ 2 & 1 & 2 & 2 & 2 & 0 \end{bmatrix}.$$

The attribute set A'' is dispensable and comaprison values are presented in Table 7. Reduce the following parameters $\{Çr_1, Çr_4, Çr_5, Çr_6\}$ so that proposed algorithm becomes more robust and gives us accurate scores in the end, compared to the S-normal reduction

TABLE 6. The score table of (G, A-S) for S-normal parameter reduction method associated with Example 1.

	$Row - sum(R_i)$	$Col - sum(C_i)$	$Score(S_i)$
A_1	26	17	9
A_2	17	26	-9
A_3	21	22	-1
A_4	20	23	-3
A_5	19	23	-4
A_6	25	17	8

TABLE 7. The comparison table of (G, A'').

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	2	2	2	2	2
A_2	2	0	2	2	2	1
A_3	2	2	0	2	2	2
A_4	2	2	2	0	2	2
A_5	2	2	2	2	0	2
A_6	2	1	2	2	2	0

method and I-S-parameter reduction method. Now, find the comparison table for the above parameters after reducing the dispensable parameters, which do not affect the proposed algorithm. The comparison table is shown in Table 8,

TABLE 8. The comparison table of (G, A) by AA-algorithm of Example 1.

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	1.263	1.46	1.89	1.48	0.902
A_2	0.125	0	0.808	1.039	0.667	0.367
A_3	1	1.486	0	0.782	1.208	0.625
A_4	1.208	1.49	0.555	0	1.058	0.486
A_5	.5667	0.893	0.757	0.833	0	0.271
A_6	0.958	1.561	1.14	1.23	1.24	0

The scores in Table 9 shows almost the same choice values as in the S-normal and I-S-parameter reduction methods. However, the scores of AA-algorithm has accurate values than others, i.e., A_1, A_6 are similar in both algorithms. While in AA-algorithm, all scores are different which shows clearly that (A_6) is the best choice. The proposed algorithm shows the same score order but differentiates all score values, making it easier for the selector to choose the best among all values.

TABLE 9. The score table of (G, T) for AA-algorithm of Example 1.

	$Row-sum(R_i)$	$Col-sum(C_i)$	$Score(S_i)$
A_1	6.995	3.857	3.1373
A_2	3.005	6.693	−3.689
A_3	5.101	4.720	0.3811
A_4	4.797	5.774	−0.977
A_5	3.320	5.643	−2.323
A_6	6.129	2.650	3.4783

Furthermore, this paper verifies AA-algorithm's superiority by discussing two real-life problems.

5. REAL-LIFE APPLICATIONS

5.1. **Recruitment system for universities.** University of Management and Technology, Lahore, a major private sector university of Pakistan, released information about recruitment for one assistant professor in mathematics. By the due date, the university's mathematics department received five CVs from five applicants. Eight parameters were set to evaluate the five applicants while getting them interviewed by the Dean's panel to find the most competitive applicant among the five. An FSS was utilized to model the assessment framework. Let $X = \{A_1, A_2, A_3, A_4, A_5\}$ be the set of five candidates. $A = \{Çr_1, Çr_2, Çr_3, Çr_4, Çr_5, Çr_6, Çr_7, Çr_8\}$ as the primary criteria set, where $Çr_i$ stands for "Ph.D. degree institute", "Ph.D. passing year", "working experience", "area of specialization", "Research papers", "Research students", "management experience", "self-motivation", respectively. A tabular representation in Table 10 illustrates the FSS of the provided application.

TABLE 10. The fuzzy soft set (G, A) associated with recruitment system for the university.

	$Çr_1$	$Çr_2$	$Çr_3$	$Çr_4$	$Çr_5$	$Çr_6$	$Çr_7$	$Çr_8$
A_1	0.2	0.7	0.5	0.3	0.2	0.4	0.9	0.1
A_2	0.6	0.65	0.6	0.2	0.5	0.2	0.3	0.1
A_3	0.7	0.5	0.2	0.8	0.2	0.9	0.9	0.1
A_4	0.75	0.3	0.8	0.5	0.4	0.3	0.8	0.1
A_5	0.9	0.2	0.9	0.4	0.7	0.1	0.1	0.1

Table 10 presents four specific criteria in the FSS. The set $T = \{Çr_1, Çr_2, Çr_5, Çr_7\}$ is classified as the dispensable criteria R. Since the criteria for each object in $Çr_8$ are identical, the S-normal parameter reduction of the FSS is determined to be $E = \{Çr_3, Çr_4, Çr_6\}$, as illustrated in Table 11. The comparison and score values, following the computations, are presented in Table 11 and Table 12, respectively.

TABLE 11. Reduced fuzzy soft set (G, A) associated with recruitment system for the university.

	Çr$_3$	Çr$_4$	Çr$_6$
A̧$_1$	0.5	0.3	0.4
A̧$_2$	0.6	0.2	0.2
A̧$_3$	0.2	0.8	0.9
A̧$_4$	0.8	0.5	0.3
A̧$_5$	0.9	0.4	0.1

TABLE 12. The comparison table of (G, A-S) by S-parameter reduction.

	A̧$_1$	A̧$_2$	A̧$_3$	A̧$_4$	A̧$_5$
A̧$_1$	3	2	1	1	1
A̧$_2$	1	3	1	0	1
A̧$_3$	2	2	3	2	2
A̧$_4$	2	3	1	3	2
A̧$_5$	2	2	1	1	3

TABLE 13. The score table of (G, A-S) by S-parameter reduction.

	$Row-sum(R_i)$	$Col-sum(C_i)$	$Score(S_i)$
A̧$_1$	8	10	-2
A̧$_2$	6	12	-6
A̧$_3$	11	7	4
A̧$_4$	11	7	4
A̧$_5$	9	9	0

$$K_A = \begin{bmatrix} 8 & 5 & 5 & 4 & 4 \\ 4 & 8 & 4 & 3 & 4 \\ 6 & 5 & 8 & 5 & 5 \\ 5 & 6 & 4 & 8 & 5 \\ 5 & 5 & 4 & 4 & 8 \end{bmatrix}.$$

$$K_{A-S} = \begin{bmatrix} 6 & 4 & 4 & 3 & 3 \\ 3 & 6 & 3 & 2 & 4 \\ 5 & 4 & 6 & 5 & 5 \\ 4 & 5 & 4 & 6 & 5 \\ 4 & 5 & 4 & 4 & 6 \end{bmatrix}.$$

The resultant matrix is generated after subtracting K_A, K_{A-S} and shows symmetry. Thus, these parameters are dispensable and do not disturb the ranking order in the score table.

$$K_A - K_{A-S} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

Using the I-S-parameter reduction method show the difference between these two parameters. In the recruitment system for the university, a set of some attributes $T'' =$ {Çr$_1$, Çr$_2$, Çr$_5$, Çr$_7$, Çr$_8$}. Calculate the lower and upper triangles of a FSS using the 2.5 to know whether the attribute set is dispensable. $C_{ii} = 0$ as it has little useful value. The FSS has five attributes, so the value of N is 5 to use for the formula, i.e., $B_{ij} = N - B_{ij} - B' = N - (B_{ij} + B')$. Eventually, a comparison table of the whole triangle matrix is calculated, and the score of every value is given in the complete table.

M''_A shows the comparison matrix of the above FSS

$$M''_A = \begin{bmatrix} 0 & 2 & 1 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 1 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix}.$$

Therefore, the results obtained from the above discussion are shown in Table 14, where before and after reduction were calculated. The corresponding scores of each scheme are similar (Table 14). Hence, the attribute set A'' is a dispensable attribute.

TABLE 14. The comparison table of (G, A'').

	A̧$_1$	A̧$_2$	A̧$_3$	A̧$_4$	A̧$_5$
A̧$_1$	0	2	1	2	2
A̧$_2$	2	0	2	2	2
A̧$_3$	1	2	0	2	2
A̧$_4$	2	2	2	0	2
A̧$_5$	2	2	2	2	0

TABLE 15. The score table of (G, A'').

	$Row - sum(R_i)$	$Col - sum(C_i)$	$Score(S_i)$
A̧$_1$	7	7	0
A̧$_2$	8	8	0
A̧$_3$	7	7	0
A̧$_4$	8	8	0
A̧$_5$	8	8	0

TABLE 16. The score comparison of (G, A'').

	$\underset{\sim}{A}_1$	$\underset{\sim}{A}_2$	$\underset{\sim}{A}_3$	$\underset{\sim}{A}_4$	$\underset{\sim}{A}_5$
S_i	-2	-6	4	4	0
$S_i^{'}$	-2	-6	4	4	0

Both S-reduction and I-S reduction methods have similar score tables. However, both have the same problem: they have less accurate values, as shown in Table 16. Decision-maker cannot decide the best value among $\underset{\sim}{A}_3$ and $\underset{\sim}{A}_4$ as they show 4. Thus, this issue can be resolved by the proposed algorithm. Which is as follows:

TABLE 17. The comparison table of (G, T) of AA-algorithm with recruitment system for the university.

	$\underset{\sim}{A}_1$	$\underset{\sim}{A}_2$	$\underset{\sim}{A}_3$	$\underset{\sim}{A}_4$	$\underset{\sim}{A}_5$
$\underset{\sim}{A}_1$	0	0.347	0.333	0.111	0.333
$\underset{\sim}{A}_2$	0.111	0	0.444	0	0.111
$\underset{\sim}{A}_3$	1.18	1.53	0	1.04	1.39
$\underset{\sim}{A}_4$	0.583	0.708	0.667	0	0.347
$\underset{\sim}{A}_5$	0.569	0.583	0.778	0.111	0

TABLE 18. The score table of (G, T) of AA-algorithm with recruitment system for university.

	$Row - sum(R_i)$	$Col - sum(C_i)$	$Score(S_i)$
$\underset{\sim}{A}_1$	1.124	2.443	-1.319
$\underset{\sim}{A}_2$	0.666	3.168	-2.502
$\underset{\sim}{A}_3$	5.143	2.222	2.921
$\underset{\sim}{A}_4$	2.305	1.262	1.043
$\underset{\sim}{A}_5$	2.041	2.181	-0.140

AA-algorithm does not need to find symmetric matrices to show whether the parameters are dispensable. Instead, sum the dispensable columns and check whether they have the same value throughout the column. If it has the same value, they are dispensable sets and can be deleted from the given data. Table 17 shows the comparison values. Table 18 indicates that $\underset{\sim}{A}_3$ is the best applicant to be recruited. However, the other two methods lack these scoring criteria, which the proposed method makes better and more accurate. Therefore, AA-algorithm has fewer steps while reducing parameters and gives many accurate results.

Now, another real-life problem is discussed to verify the reliability of AA-algorithm.

5.2. **Attractiveness of cars for purchase.** Let universe set $X = \{Ą_1, Ą_2, Ą_3, Ą_4, Ą_5, Ą_6\}$ represent a fuzzy set of cars. Each car in X can be evaluated based on a set of relevant attributes. The set of attributes $A = \{Çr_1, Çr_2, Çr_3, Çr_4, Çr_5, Çr_6, Çr_7, Çr_8, Çr_9, Çr_{10}\}$ includes factors such as $A = \{$fuel efficiency, interior space, design appeal, reliability, safety, price, comfort, resale value, warranty, brand reputation$\}$ which are essential considerations when selecting a car. Let G be the mapping of set A to all the fuzzy subsets of the universal set U. Now consider an FSS (G, A) as illustrated in Table 19 for the given scenario.

TABLE 19. The fuzzy soft set (G, A) associated with attractiveness of cars for purchase.

	$Çr_1$	$Çr_2$	$Çr_3$	$Çr_4$	$Çr_5$	$Çr_6$	$Çr_7$	$Çr_8$	$Çr_9$	$Çr_{10}$
$Ą_1$	0.1	0.9	0.2	0.2	0.1	0.8	0.9	0.5	0.9	0.5
$Ą_2$	0.2	0.6	0.3	0.6	0.3	0.6	0.4	0.6	0.3	0.4
$Ą_3$	0.3	0.5	0.8	0.2	0.3	0.6	0.5	0.1	0.9	0.9
$Ą_4$	0.5	0.4	0.5	0.4	0.5	0.4	0.6	0.2	0.8	0.8
$Ą_5$	0.7	0.2	0.6	0.7	0.8	0.2	0.8	0.7	0.1	0.2
$Ą_6$	0.8	0.1	0.4	0.5	0.9	0.1	0.2	0.4	0.7	0.7

Firstly, after solving this real-life problem using S-parameter reduction method and the resulting scores obtained are shown in Table 20.

TABLE 20. The score table of (G, A) for attractiveness of cars for purchase.

	$Row - sum(R_i)$	$Col - sum(C_i)$	$Score(S_i)$
$Ą_1$	36	36	0
$Ą_2$	32	39	-6
$Ą_3$	39	35	4
$Ą_4$	36	34	2
$Ą_5$	38	32	6
$Ą_6$	32	38	-6

K_A, K_{A-S} shows the values from table 22 respectively.

$$K_A = \begin{bmatrix} 10 & 5 & 6 & 5 & 5 & 5 \\ 5 & 10 & 5 & 4 & 4 & 5 \\ 6 & 7 & 10 & 5 & 5 & 6 \\ 5 & 6 & 5 & 10 & 4 & 6 \\ 5 & 6 & 5 & 6 & 10 & 6 \\ 5 & 5 & 4 & 4 & 4 & 10 \end{bmatrix}.$$

TABLE 21. The parameter reduction table for attractiveness of cars for purchase.

	$\text{\c{C}r}_3$	$\text{\c{C}r}_7$
$\text{\c{A}}_1$	0.2	0.9
$\text{\c{A}}_2$	0.3	0.4
$\text{\c{A}}_3$	0.8	0.5
$\text{\c{A}}_4$	0.5	0.6
$\text{\c{A}}_5$	0.6	0.8
$\text{\c{A}}_6$	0.4	0.2

TABLE 22. The comparison table of (G, A) for attractiveness of cars for purchase.

	$\text{\c{A}}_1$	$\text{\c{A}}_2$	$\text{\c{A}}_3$	$\text{\c{A}}_4$	$\text{\c{A}}_5$	$\text{\c{A}}_6$
$\text{\c{A}}_1$	2	1	1	1	1	1
$\text{\c{A}}_2$	1	2	0	0	0	1
$\text{\c{A}}_3$	1	2	2	1	1	2
$\text{\c{A}}_4$	1	2	1	2	0	2
$\text{\c{A}}_5$	1	2	1	2	2	2
$\text{\c{A}}_6$	1	1	0	0	0	2

$$K_{A-S} = \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & 0 & 0 & 1 \\ 1 & 2 & 2 & 1 & 1 & 2 \\ 1 & 2 & 1 & 2 & 0 & 2 \\ 1 & 2 & 1 & 2 & 2 & 2 \\ 1 & 1 & 0 & 0 & 0 & 2 \end{bmatrix}.$$

To check the symmetry in these matrices, check the difference between them. Hence, by computing $K_A - K_{A-S}$, gives symmetric resultant matrix.

$$K_A - K_{A-S} = \begin{bmatrix} 8 & 4 & 5 & 4 & 4 & 4 \\ 4 & 8 & 5 & 4 & 4 & 4 \\ 5 & 5 & 8 & 4 & 4 & 4 \\ 4 & 4 & 4 & 8 & 4 & 4 \\ 4 & 4 & 4 & 4 & 8 & 4 \\ 4 & 4 & 4 & 4 & 4 & 8 \end{bmatrix}.$$

After seeing the symmetric resultant matrix, compute Table 22 using a reduced parameters given in Table 21. Apply the I-S normal parameter reduction method to solve the same problem. Here, it shows that the scores in Table 20 and Table 23 are the same.

M''_A is shown below which is based on both upper and lower triangle comparison tables.

TABLE 23. The score table of (G, S) attractiveness of cars for purchase.

	$Row - sum(R_i)$	$Col - sum(C_i)$	$Score(S_i)$
A_1	7	7	0
A_2	4	10	−6
A_3	9	5	4
A_4	8	6	2
A_5	10	4	6
A_6	4	10	−6

$$M''_A = \begin{bmatrix} 0 & 2 & 2 & 2 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 & 1 \\ 2 & 2 & 0 & 2 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 2 & 0 & 2 \\ 2 & 1 & 2 & 2 & 2 & 0 \end{bmatrix}.$$

TABLE 24. The comparison table of (G, A'').

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	4	3	4	4	4
A_2	4	0	3	4	4	4
A_3	3	3	0	4	4	4
A_4	4	4	4	0	4	4
A_5	4	4	4	4	0	4
A_6	4	4	4	4	4	0

Now compute these dispensable matrices' score table in Table 25. And calculating the scores of each car, same score are achieved as by S-parameter reduction as shown in Table 26. From these real-life examples, conclude that S-parameter and I-S parameter reduction techniques have the same score tables. In contrast, the I-S parameter technique has used fewer computations to reach the same result.

TABLE 25. The score table of (G, A'').

	$Row - sum(R_i)$	$Col - sum(C_i)$	$Score(S_i)$
A_1	19	19	0
A_2	19	19	0
A_3	18	18	0
A_4	20	20	0
A_5	20	20	0

TABLE 26. The score comparison table of (G, A'').

	$\underset{\varsigma}{A}_1$	$\underset{\varsigma}{A}_2$	$\underset{\varsigma}{A}_3$	$\underset{\varsigma}{A}_4$	$\underset{\varsigma}{A}_5$	$\underset{\varsigma}{A}_6$
S_i	0	−6	4	2	6	−6
S_i'	0	−6	4	2	6	−6

Now, implementing AA-algorithm on the same example and show how it dominates the score comparison for decision-making. The comparison values of FSS are presented in Table 27.

TABLE 27. The comparison table of (G, T) for attractiveness of cars for purchase.

	$\underset{\varsigma}{A}_1$	$\underset{\varsigma}{A}_2$	$\underset{\varsigma}{A}_3$	$\underset{\varsigma}{A}_4$	$\underset{\varsigma}{A}_5$	$\underset{\varsigma}{A}_6$
$\underset{\varsigma}{A}_1$	0	0.55	0.444	0.333	0.111	0.777
$\underset{\varsigma}{A}_2$	0.125	0	0	0	0	0.222
$\underset{\varsigma}{A}_3$	0.75	0.736	0	0.375	0.25	0.833
$\underset{\varsigma}{A}_4$	0.375	0.472	0.111	0	0	0.569
$\underset{\varsigma}{A}_5$	0.55	0.819	0.333	0.347	0	0.917
$\underset{\varsigma}{A}_6$	0.25	0.125	0	0	0	0

TABLE 28. The score table of (G, T) attractiveness of cars for purchase.

	$Row - sum(R_i)$	$Col - sum(C_i)$	$Score(S_i)$
$\underset{\varsigma}{A}_1$	2.220	2.000	0.220
$\underset{\varsigma}{A}_2$	0.347	2.707	−2.37
$\underset{\varsigma}{A}_3$	2.944	0.888	2.036
$\underset{\varsigma}{A}_4$	1.527	1.055	0.427
$\underset{\varsigma}{A}_5$	2.916	0.361	2.555
$\underset{\varsigma}{A}_6$	0.375	3.318	−2.94

Here are the results, which show every car has its unique score to get selected by the customers. While, in other algorithms this problem was present which needs to be improve. Previous algorithms only focus on reducing parameters, while reducing the parameters and improve the score criteria (Table 28). Both these examples exhibit the proposed algorithm dominance over the other two algorithms.

5.3. **Sensitivity Analysis of the AA-Algorithm.** To assess the robustness of the proposed AA-algorithm, we performed a sensitivity analysis based on attractiveness of cars for purchase by varying the fuzzy membership value of a single attribute. Specifically, for the car selection case study, we increased the membership value of $\c{C}r_5$ (safety) for car $\underset{\varsigma}{A}_2$ from

TABLE 29. Sensitivity analysis: Effect of increasing $\text{\c{C}r}_5$ for $\text{\c{A}}_2$ on AA-scores and ranking.

Car	Original Score S_i	Perturbed Score S_i'	Rank Change
$\text{\c{A}}_1$	0.220	0.220	0
$\text{\c{A}}_2$	-2.370	-0.985	+1
$\text{\c{A}}_3$	2.036	2.036	0
$\text{\c{A}}_4$	0.427	0.427	0
$\text{\c{A}}_5$	2.555	2.555	0
$\text{\c{A}}_6$	-2.940	-2.940	0

0.3 to its maximum value 1.0, while keeping all other values unchanged. The AA-scores were recomputed to observe the effect on the ranking of cars.

As seen in Table 29, increasing the safety score of car $\text{\c{A}}_2$ improves its AA-score, causing only a minor change in ranking. All other cars retain their relative positions, demonstrating that the proposed algorithm is robust to moderate changes in fuzzy membership values. Similar analyses can be performed for other attributes or objects, and the results are expected to show comparable stability, highlighting the reliability of the AA-algorithm in real-world decision-making scenarios.

6. COMPARISON OF DISCUSSED METHODS

The comparison presented in this section focuses on both the theoretical distinctions and numerical performance of the S-normal, I-S-normal, and proposed AA-algorithm. The S-normal and I-S-normal approaches primarily aim at reducing parameters through normalization procedures or matrix-based comparisons, followed by direct aggregation of decision scores. In contrast, the proposed AA-algorithm integrates parameter reduction with an optimized score construction mechanism, allowing relative dominance information among alternatives to be preserved. This theoretical distinction explains the improved score discrimination observed in the proposed method.

In this article, a brief discussion of the S-normal and I-S-normal algorithms is presented along with illustrative examples. The proposed AA-algorithm is then applied to the same examples and two real-life decision-making applications to facilitate a fair comparison. The comparison highlights differences in computational complexity, parameter usage, and score discrimination among the methods. In particular, the proposed algorithm produces more clearly differentiated decision scores in cases where the existing methods yield identical or weakly informative results.

Thus, the comparison presented in Table 30 demonstrates that the proposed AA-algorithm improves score-based decision outcomes relative to the S-normal and I-S-normal approaches. While the existing methods focus primarily on parameter reduction, the proposed algorithm enhances the effectiveness of decision-making by refining the score evaluation process after reduction, as illustrated through numerical examples and real-life applications.

1. All three methods achieve parameter reduction by identifying a subset of significant parameters.

2. The S-normal and I-S-normal approaches rely on matrix-based comparisons and normalization processes for parameter reduction, whereas the proposed AA-algorithm employs dispensable parameter sets, thereby reducing computational overhead.
3. For score computation, the S-normal method uses all parameters, while both the I-S-normal method and the proposed AA-algorithm utilize only the reduced parameter set.
4. The S-normal and I-S-normal methods may yield identical or weakly differentiated decision scores after reduction, whereas the proposed AA-algorithm provides more consistent and discriminative scores across the considered examples and real-life applications.

Based on the comparative analysis and the tables presented, it is evident that while the S-normal and I-S-normal algorithms mainly address parameter reduction, the proposed AA-algorithm further improves the quality of score-based decision-making. This enhancement is supported by numerical results and graphical illustrations, as shown in Figure 6.

TABLE 30. Comparison of methods for attractiveness of cars for purchase.

	S-normal	I-S normal	AA-algorithm	Improvement of AA-algorithm
Parameters after reduction	$\c{C}r_3, \c{C}r_7$	$\c{C}r_3, \c{C}r_7$	$\c{C}r_3, \c{C}r_7$	same
Matrices compared	3	1	0	reduced
Parameters in comparison	10	2	2	reduced
Accurate decision scores	$4/6$	$4/6$	$6/6$	improved

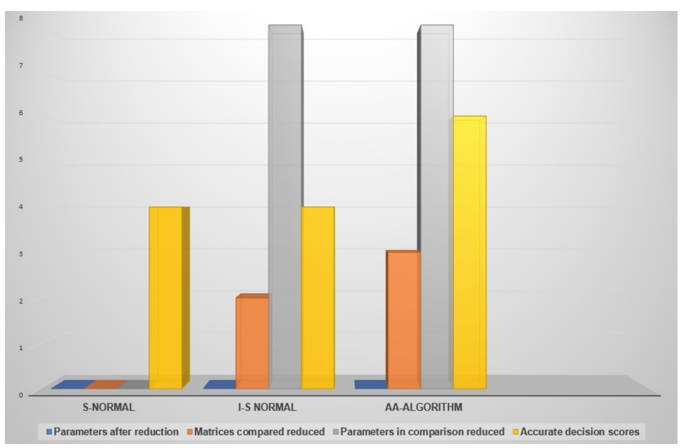

FIGURE 2. Graphical comparison of methods for attractiveness of cars for purchase.

The reduction in computational complexity achieved by the proposed AA-algorithm is primarily reflected in the number of matrix constructions, parameter comparisons, and score aggregation operations required during the decision-making process. The S-normal approach involves multiple matrix-based comparisons across all parameters, leading to higher computational overhead, while the I-S-normal method reduces this burden by limiting the number of matrices but still relies on normalization and aggregation steps. In contrast, the proposed AA-algorithm eliminates matrix comparisons by identifying dispensable parameter sets and directly operating on the reduced parameter subset. This reduction is quantified through the number of matrices compared, the number of parameters involved in score computation, and the resulting decision scores, as summarized in Table 30. The comparative results demonstrate a consistent decrease in computational steps while maintaining reliable and discriminative decision outcomes.

7. Ranking Comparison with TOPSIS

Table 31,32 exhibits a comparative study of the AA-algorithm and TOPSIS in two real-world applications: the university recruitment system and the attractiveness of cars for purchase. The evaluation of several criteria, represented by Ą_i, which individually add to the total ranking of the options, yields the performance metrics. The ranks of the AA-algorithm and TOPSIS for the recruitment system show a reasonably close agreement. Interestingly, both algorithms rate the options equally for criteria ranging from Ą_1 to Ą_5, indicating a consistent evaluation alignment. When evaluating how desirable automobiles are to buy, the outcomes demonstrate that the AA-algorithm and TOPSIS perform similarly. Rankings from Ą_1 to Ą_6 are often consistent, and both approaches converge to the same results. It is clear from both applications that the rankings produced by the AA-algorithm are generally in close agreement with those obtained from TOPSIS. While there are some minor discrepancies, particularly in the attractiveness of cars for purchase, the general trend indicates that the AA-algorithm is reliable in approximating the TOPSIS outputs. The table presents the ranks determined by both algorithms based on several criteria, facilitating an easy comparison of their respective performances. The ranks for different criteria are well aligned, suggesting that the AA-algorithm verifies against TOPSIS with reasonable accuracy. Figure 7 and Figure 7 present the graphical comparison of the AA algorithm and TOPSIS for clarity.

TABLE 31. Ranking comparison of AA-algorithm with TOPSIS for Recruitment system for universities

	AA-algorithm	Rank	TOPSIS	Rank
Ą_1	-1.319	4	0.437	4
Ą_2	-2.502	5	0.391	5
Ą_3	2.921	1	0.612	1
Ą_4	1.043	2	0.507	2
Ą_5	-0.140	3	0.442	3

TABLE 32. Ranking comparison of AA-algorithm with TOPSIS for Attractiveness of cars for purchase

	AA-algorithm	Rank	TOPSIS	Rank
$\underset{\sim}{A}_1$	0.220	4	0.494	4
$\underset{\sim}{A}_2$	-2.37	5	0.439	6
$\underset{\sim}{A}_3$	2.036	2	0.512	2
$\underset{\sim}{A}_4$	0.427	3	0.505	3
$\underset{\sim}{A}_5$	2.555	1	0.515	1
$\underset{\sim}{A}_6$	-2.94	6	0.485	5

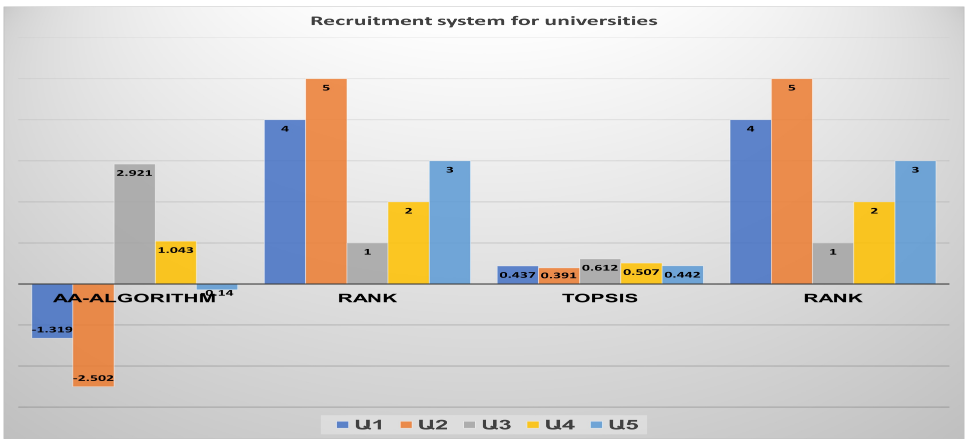

FIGURE 3. Ranking comparison of AA-algorithm and TOPSIS for recruitment system of universities.

8. CONCLUSIONS

This paper discusses two different parameter reduction techniques: the S-normal and I-S-normal parameters. The calculation process of the S-normal parameter technique involves higher computational complexity, while the I-S-normal parameter reduces complexity by decreasing the number of attributes. The numerical results indicate that, although both approaches achieve effective parameter reduction, they may produce indistinguishable or less discriminative decision scores when score-based criteria are applied. In contrast, the rankings obtained from the proposed approach are stable and comparable with those generated by established methods such as TOPSIS, while providing clearer score differentiation.

To overcome this limitation, a new AA-algorithm is introduced. Compared with the S-normal and I-S-normal parameter reduction algorithms, the proposed AA-algorithm requires fewer computational steps, produces more reliable and discriminative scores, and is easier to implement. The reliability and accuracy of the proposed algorithm are demonstrated through several real-life decision-making examples. Consequently, the proposed

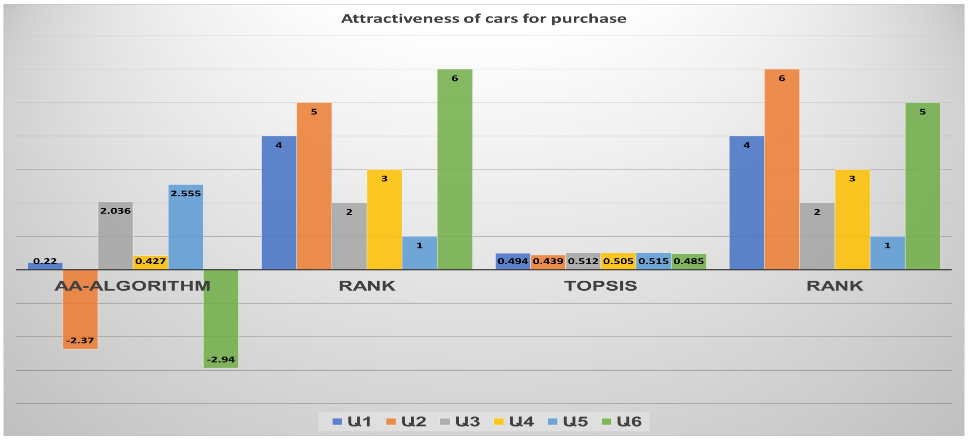

FIGURE 4. Ranking comparison of AA-algorithm and TOPSIS for attractiveness of cars for purchase.

method represents an improved and efficient parameter reduction technique with enhanced performance for score-based decision criteria. Tables 31 and 32 highlight the comparable performance of the AA-algorithm with TOPSIS, reaffirming its effectiveness as a reliable multi-criteria decision-making approach in real-world applications. The proposed AA-algorithm performs effectively in typical decision-making scenarios; however, in extremely high-dimensional or highly noisy datasets, computational cost may increase and score differentiation could be less pronounced. Exploring these conditions and extending the method to a wider range of decision scenarios represents a promising direction for future research.

CREDIT AUTHORSHIP CONTRIBUTION'S STATEMENT

Tabasam Rashid: Investigation, Supervision, Validation. **Muhammad Amman:** Conceptualization, Methodolgy, Data curation, Writing - original draft. **Asif Ali:** Writing - review and editing, Supervision.

DECLARATIONS

Conflict of interest On behalf of all co-authors, I confirm that all authors have agreed to submit this article solely to this journal. We also declare that there are no conflicts of interest related to the publication of this work.

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