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A Multi-Criteria Decision-Making Technique Based on Generalized Bipolar Fuzzy Prioritized Operators and Their Application in Supply Chain Management

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Abstract. The evaluation and assessment of AI-driven models for supply chain management require complex decision-making frameworks that can cope with uncertainty, indecision, and conflicting expert opinions. The expressiveness of traditional fuzzy decision-making models is limited and is not able to represent both positive and negative sides of the criteria. In addition, existing generalized aggregation operators (AOs) rely on arbitrary weights of criteria that introduce subjectivity and reduce the accuracy of the findings. This paper addresses these shortcomings by suggesting a new multi-criteria decision-making (MCDM) method based on generalized bipolar fuzzy prioritized operators (G-BFPR). We propose four new operators, namely, generalized bipolar fuzzy prioritized average (G-BFPRA), generalized bipolar fuzzy prioritized weighted average (G-BFPRWA), generalized bipolar fuzzy prioritized geometric (G-BFPRG), and generalized bipolar fuzzy prioritized weighted geometric (G-BFPRWG) operators. This framework, in a systematic manner, computes weights using priority relations, eliminating subjectivity and reflecting on positive and negative preferences in uncertain settings. Our approach provides a context-specific, balanced assessment mechanism of AI-based supply chain models, which is demonstrated by a case study that validates the superiority of our approach to the current theories. The

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proposed bipolar fuzzy MCDM method offers a holistic solution to the ranking of optimization techniques in supply chain management, which enhances operational performance, minimizes costs, and enhances sustainability. Furthermore, the needs and benefits of the proposed work are disclosed in this article, as it includes a comparative analysis

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1. Introduction

The optimization of the supply chain is now primarily based on data-driven decisions to enhance operational performance, flexibility, and operational stability by introducing AI technologies. The existing organizational management practices fail to work as companies experience a number of challenges that involve unstable demands, broken supply chains, fluctuating costs, and logistical inefficiencies. The use of real-time analytics and automation, and intelligent decision-support systems helps businesses to be more accurate in their forecasts and make operations easier and less risky. The AI aspects of supply chain optimization enable the organization to constantly modify its structure by changing it depending on market shifts and operational uncertainty. The use of data-driven insights can help organizations to make strategic decisions that can lead to sustainable results and waste reduction, and resource management optimization. The existing developments in AI-based supply chain solutions still demand that organizations solve several issues when choosing the most appropriate optimization strategies. The need to have a formal assessment framework is due to the fact that the available techniques are of a different nature, and their data points are unpredictable and conflicting. The organization-wide search of ranking data-centric models assists businesses in choosing solutions that meet operational requirements and scalability objectives in addition to particular objectives. A structured decision-making procedure helps find the most powerful optimization method that also balances factors such as expenses and implementation risks with performance results.

The proposed research uses the multi-criteria decision-making (MCDM) method along with bipolar fuzzy generalized prioritized operators to develop a decision-support system for ranking optimization methods. These operators deliver superior performance in assessment when decision-makers experience doubts about exact criterion value assignments because they create a framework that supports the realistic interpretation of information. This research delivers an organized method for supply chain managers to identify, evaluate, and rank AI-based optimization methods to optimize operational performance and reduce costs while improving sustainability.

Why is it necessary to evaluate the AI-driven supply chain models? How do the generalized bipolar fuzzy prioritized (G-BFPR) operators enhance this process? Supply chains are getting more data-driven, dynamic, and complex in the era of AI. Several AI-based models exist to address a particular supply chain issue, but the most suitable model needs to be determined with conflicting criteria, professional views, and ambiguity. G-BFPR operators allow decision-makers to systematically analyze and prioritize these models, considering the positive and negative sides of performance, as well as the relative weight associated with each attribute. Such an approach offers more realistic, balanced, and context-specific decision-making in the real-world supply chain environment. G-BFPR operators provide a powerful decision-making process to handle both positive and negative information simultaneously to make more discriminating and realistic judgments about alternatives. Applied to AI-based supply chain models, G-BFPR supports contradictory expert views, ambiguous performance measures, and uncertain data using bipolar membership degrees. This improves the process of prioritization through the integration of both supportive and conflicting judgments within an organized manner, resulting in a balanced and trustworthy model ranking under challenging decision conditions.

What is the importance of bipolar fuzzy sets (BFSs) to handle MCDM problems? BFSs [32] are a generalization of the classical fuzzy sets where, in addition to the positive membership degree, the negative membership degree is also considered, which allows to expression of the opposite criteria (e.g., satisfaction and dissatisfaction) in the same format. This bipolarity improves the MCDM models by taking into account the vagueness and conflicts that are characteristic of most practical problems. BFSs enable decision-makers to use dual evaluations (positive and negative) to model trade-offs and enhance the level of criteria discrimination. For instance, they help to generate consensus solutions by summing up experts' judgments and criteria weights, at the same time taking into account supportive and opposing stakeholders' opinions. Moreover, the methods based on the concept of BFS use score functions to reconcile these two types of evaluation and to allow for the systematic ordering of alternatives even in the case of conflict or lack of information. Ali et al. [3] discussed the bipolar fuzzy relation. Ali et al. [4] analyzed the parameter reduction within the framework of bipolar fuzzy soft set, and Alcantud [2] discussed the relation among fuzzy soft and soft topologies. Riaz and Tehrim [21] devised the bipolar fuzzy soft topology via Q-neighborhood.

1.1. **Motivation and justification for the proposed framework.** The choice of AI-based supply chain optimization models has its own peculiarities and requires the application of generalized bipolar fuzzy prioritized aggregation operators within the framework of BFS.

- Bipolarity is required in supply chain decisions: In practice, supply chain management decision-makers need to consider both positive (benefits, strengths, opportunities) and negative (risks, weaknesses, limitations) sides of AI models. As an example, in the case of assessing a demand forecasting model based on machine learning, specialists must be able to capture both the positive and negative sides. The traditional fuzzy sets and their variants (intuitionistic fuzzy sets (IFS), hesitant fuzzy sets (HFS), etc.) are only able to model the membership degrees within the range [0,1], and do not explicitly represent the negative/opposing assessments that are important in technology assessment decisions. To address this weakness, BFSs offer two membership levels: $K^P \epsilon [0,1]$of satisfaction/agreement and $K^N \epsilon [-1,0]$ of dissatisfaction/disagreement.

- Requirement of prioritized AOs: In the evaluation of supply chain models, criteria are naturally priority-related. The evaluation of ”Forecast Accuracy and Performance” should precede that of ”Responsiveness to Market Disruptions” since, without accurate forecasting, the responsiveness is meaningless. Likewise, Ease of Integration relies on performance measures, and Data Requirements relies on the complexity of integration. Current generalized AOs use arbitrary weights, and this makes them subjective. Weights are calculated systematically by our prioritized operators. This makes sure that the significance of each criterion is based on the satisfaction of all the previous criteria, which is a true decision-making logic.
- Superiority in the supply chain context These three features combined render our framework exceptionally appropriate to the evaluation of AI supply chain models since:
  (1) It is explicit in strengths and weaknesses (bipolarity).
  (2) It honors natural criteria dependencies (prioritization).
  (3) It removes subjective weighting (systematic weighting).

1.2. **Research gap.** The rapid development of AI and intelligent systems has drastically reshaped supply chain activities, rendering them more autonomous, data-centric, and strategically sophisticated. Such a transition requires advanced decision-making tools that can tackle uncertainty, indecisiveness, and conflicting opinions of experts. Although conventional fuzzy decision-making models like fuzzy sets, hesitant fuzzy sets, and intuitionistic fuzzy sets have been extensively used in MCDM in supply chain management, they have limited expressiveness. These models fail to cope with fixed membership values, do not possess the capacity to express bipolar (positive and negative) preferences at the same time, and tend to underperform in effectively expressing decision-makers' indecision. Generalized aggregation operators (AOs) are useful tools in information fusion and have been widely investigated in different types of fuzzy sets. One of the main drawbacks of the generalized aggregation operators is that the criteria weights are assigned arbitrarily by the decision-makers. This random assignment of weights brings subjectivity into the decision-making process and hence reduces the reliability of the results. Such subjective weighting mechanisms do not reflect the nature of the problem and the interconnections between the criteria. We summarize some research gaps as follows.

- The current bipolar fuzzy MCDM techniques employ arbitrary weight assignment, which is against the objective of dealing with subjective uncertainty.
- In IFS, there are priority aggregation operators [30], [17], however, IFS is unable to express negative evaluations explicitly, and the non-membership degree is a measure of hesitation, not of opposition.
- There is no current framework that incorporates all three features: bipolarity, generalization, and prioritization. This is a combination that is necessary in complex technology evaluation in supply chains.

To fill this research gap, there is a need to incorporate prioritized techniques with bipolar fuzzy generalized aggregation operators. This integration would systematically decide weights in accordance with the priority relations between the criteria and eliminate subjectivity. However, this methodological advancement has not been advanced in the existing literature.

1.3. **Contributions and innovation.** This study proposes a new decision-making model that embeds G-BFPR operators in evaluating and ranking AI-based supply chain models. Its primary contribution is to overcome the shortcomings of conventional MCDM methods using bipolar fuzzy logic to better capture both positive and negative preferences in uncertain environments. This research introduces a novel MCDM framework by using G-BFPR operators in the evaluation and ranking of AI-driven supply chain models. The key contribution lies in addressing the limitations of traditional MCDM approaches by incorporating G-BFPR operators within BFSs to effectively represent both positive and negative preferences under uncertainty. The key contributions of the proposed study are;

- Development of G-BFPR AOs to overcome the limitations of existing fuzzy frameworks.
- Th e newly developed operators are: Generalized bipolar fuzzy prioritized average (G-BFPA) operators, generalized bipolar fuzzy prioritized weighted average (G-BFPWA) operators, generalized bipolar fuzzy prioritized geometric (G-BFPG) operators, generalized bipolar fuzzy prioritized weighted geometric (G-BFPWG) operators.
- Development of the BF-MCDM technique and its algorithm using the proposed operators.
- Case study evaluation and ranking of AI-driven supply chain models based on proposed operators.
- Comparative analysis with existing theories to validate the superiority of the proposed framework in supply chain management.

AI-based supply chain model assessment is bound to have both positive and negative features. Conventional fuzzy-based decision models are mostly concerned with positive assessments and do not explicitly represent these conflicting facets. The bipolar fuzzy representation used in this paper allows modeling supportive and conflicting expert opinions at the same time, which results in a more realistic and complete evaluation process. Additionally, the use of prioritized aggregation is a real-world decision situation in supply chain management where some criteria (e.g., model performance) inherently outweigh others (e.g., ease of deployment or scalability). The proposed solution provides a more credible, context-sensitive, and practically significant decision-support system to rank AI-based supply chain optimization models in the face of uncertainty by integrating bipolarity and priority-based weighting in a generalized aggregation model. Further, although the current research is on AI-based supply chain optimization models, the suggested work can be applied to a wider range of complex decision-making problems with conflicting criteria and dual assessments. An example is the choice of renewable energy technology, sustainable supply chain design, and green supplier evaluation, where there is a need to balance between the positive and negative factors.

1.4. **Study Framework.** This article is designed to present a clear and organized discussion of the suggested method. This article starts with an introduction and sets the entire research. The detailed study framework is explained as follows: Section 2 is on the literature review or background study. Section 3 discusses the basic concepts of the suggested theory. Section 4 constructs some new aggregation operators. Section 5 discusses an MCDM approach based on the BF framework and information. Section 6 elaborates on the comparative analysis and results. Section 7 concludes the entire manuscript and provides key findings.

2. Literature Review

Several academic studies show widespread academic interest in data-driven supply chain management approaches that emerged in recent years. The review section examines various studies on data-driven supply chain management strategies and their application. Gumte et al. [13] deal with uncertainties that influence planning models in supply chains through their strong optimization strategy. The combination of predictive analytics and real-time data can be used to make better decisions in volatile settings due to their research, which offers a sophisticated approach to supply chain stability. The authors Hewitt and Frejinger [15] show how optimization models can be made more efficient by using customized data-driven methods that result in improved supply chain operational responsiveness in dynamic environments. Fattahi [10] builds on the uncertainty-based studies by providing data-driven approaches to building supply chain networks with social responsibilities. Their study reveals the reason why the inclusion of social responsibility measures in supply chain decisions is used to combine both operational effectiveness and ethical standards. A global research by Tsai et al. [26] shows that data-driven analytics is one of the fundamental drivers of establishing supply chains that utilize resources effectively and safeguard the environment. Li and Liu [16] state that big data analytics assessment improves supply chain operations at all levels. Firms that apply insights based on big data have superior operational capabilities alongside reduced inefficiencies based on their research results. The study by Chavez et al. [9] shows the integration of data-driven supply chains to enhance better manufacturing capacities and high customer satisfaction, and explains the benefits of analytics application in supply chain operations. Biswas and Sen [7] suggest a big data-based supply chain analytics architecture since it may enhance real-time decision-making and forecasting. Tseng et al. [27] examine data-driven sustainable supply chain management measures. The authors examine the way industrial disruptions require organizations to pursue dual-minded approaches to efficiency and sustainability objectives. Zhang [31] shows a statistical optimization system of supply chain demand forecasting that shows how predictive computational models can enhance financial recordkeeping and stock control activities. Sundarakani et al. [24] carry out a study on the integration of blockchain in supply chains. The application of blockchain and big data through research methodology resulted in evidence that was validated to demonstrate improved security alongside transparency in supply chain management networks. In addition, MCDM has become an essential instrument in the resolution of complex decision-making issues in different fields, such as supply chain management, infrastructure development, and education technology. The growing use of data-driven methods in MCDM has also increased its applicability, especially in the management of uncertainty and the combination of various evaluation criteria. Beinabadi et al. [6] focus on the importance of sustainable decision-making in the supply chain of the automotive industry. Their research introduces a data-driven MCDM model that takes into account economic, environmental, and social sustainability aspects. Nguyen [20] uses spherical fuzzy sets in an MCDM method to rank global augmented reality providers in education. The study is able to deal with uncertainty and imprecise information by using an advanced fuzzy logic system, which guarantees a more detailed evaluation of augmented reality applications. Sharma and Anand [23] suggest an MCDM model that is specifically created to manage the supply chain. Their contribution combines different decision-making methods to assess the performance of the supply chain according to the efficiency, resilience, and sustainability criteria. Alkharasani [5] introduces a data-driven MCDM model of infrastructure development planning in developing nations. The research is aimed at ranking infrastructure projects in terms of economic viability, social effects, and sustainability. Buyukozkan et al. [8] introduce an integrated fuzzy MCDM approach for evaluating supply chain analytics. Their study combines fuzzy logic with classical MCDM techniques to assess the effectiveness of various analytical tools in supply chain management. Mehdiabadi et al. [19] developed a fuzzy hybrid SWARA-MABAC model to assess the sustainability service chain capabilities of the oil and gas industry, which offers a systematic approach to prioritizing the main aspects of operations and improving the accuracy of decisions in an uncertain environment. Tufan and Ulutas [28] created an integrated approach to the selection of suppliers, which combines the LODECI and CORASO techniques to enhance decision-making in the food industry, which also indicates the practical use of MCDM techniques in supply chain management. Garg et al. [11] proposed a new method based on Aczel-Alsina power AOs under bipolar fuzzy information. They revealed practical uses of bipolar fuzzy models in quantum computing, which could help solve the complex uncertainties in quantum systems. Akram et al. [1] used bipolar fuzzy TOPSIS and ELECTRE-I methods, particularly for diagnostic problems. Rehman and Mahmood [22] studied the MCDM approach by using bipolar fuzzy Yager operators. Gul [12]expanded the VIKOR approach by combining bipolar fuzzy preference d-covering and fuzzy rough set theory to provide a new framework of MCDM in the face of uncertainty. Hakim et al. [14] discussed fuzzy bipolar soft quasi-ideals in ordered semigroups. The concept of T-bipolar soft modules was discussed by Mahmood and Rehman [18].

3. Preliminaries

This section contains the basic concepts and mathematical tools that will be needed in the development of the proposed framework. These preliminaries make the paper self-contained and easy to understand the generalized bipolar fuzzy prioritized aggregation operators that are presented in the following sections. Bipolar fuzzy sets (BFSs) are the extension of classical fuzzy sets, which can represent positive and negative information at the same time. In most real-life decision-making situations, decision-makers usually indicate satisfaction and dissatisfaction about an alternative in relation to a particular criterion. Fuzzy sets (FSs) and their generalizations, including intuitionistic and hesitant FSs, are primarily concerned with positive membership data and are unable to explicitly represent this duality. The definition of BFS is as below

Definition 1 [32]. The following form

$$\mathcal{E} = \left\{ \left(\chi, \mathcal{K}_{\mathcal{E}}^{P}(\chi), \mathcal{K}_{\mathcal{E}}^{N}(\chi)\right) \mid \chi \in X \right\}$$

is diagnosed as a *bipolar fuzzy set* (BFS), in which the positive degree of membership of an element $\chi \in X$ is represented by $\mathcal{K}_{\mathcal{E}}^{P}(\chi)$ and the negative degree of membership of an element $\chi \in X$ is represented by $\mathcal{K}_{\mathcal{E}}^{N}(\chi)$. The bipolar fuzzy number (BFN) is of the form

$$\mathcal{E} = \left(\mathcal{K}_{\mathcal{E}}^{P}, \mathcal{K}_{\mathcal{E}}^{N}\right)$$

in this article.

In order to be able to aggregate and compare bipolar fuzzy information, proper operational laws should be established for BFNs. These operations allow combining several BFNs while maintaining their bipolar structure. Addition, multiplication, scalar operations, and power operations are common operations that are necessary in building aggregation operators in decision-making models.

Definition 2 [11]. By taking two BFNs

$$\mathcal{E}_1 = \left(\mathcal{K}^P_{\mathcal{E}_1}, \mathcal{K}^N_{\mathcal{E}_1}\right) \quad \text{and} \quad \mathcal{E}_2 = \left(\mathcal{K}^P_{\mathcal{E}_2}, \mathcal{K}^N_{\mathcal{E}_2}\right),$$

along with $g \geq 0$, the operations for these numbers are devised as follows:

(1)

$$\mathcal{E}_1 \oplus \mathcal{E}_2 = \left(\mathcal{K}^P_{\mathcal{E}_1} + \mathcal{K}^P_{\mathcal{E}_2} - \mathcal{K}^P_{\mathcal{E}_1}\mathcal{K}^P_{\mathcal{E}_2},\ -\left(\mathcal{K}^N_{\mathcal{E}_1}\mathcal{K}^N_{\mathcal{E}_2}\right)\right)$$

(2)

$$\mathcal{E}_1 \otimes \mathcal{E}_2 = \left(\mathcal{K}^P_{\mathcal{E}_1}\mathcal{K}^P_{\mathcal{E}_2},\ \mathcal{K}^N_{\mathcal{E}_1} + \mathcal{K}^N_{\mathcal{E}_2} + \mathcal{K}^N_{\mathcal{E}_1}\mathcal{K}^N_{\mathcal{E}_2}\right)$$

(3)

$$g\mathcal{E}_1 = \left(1 - \left(1 - \mathcal{K}^P_{\mathcal{E}_1}\right)^g,\ -\left|\mathcal{K}^N_{\mathcal{E}_1}\right|^g\right)$$

(4)

$$\mathcal{E}_1^g = \left(\left(\mathcal{K}^P_{\mathcal{E}_1}\right)^g,\ -1 + \left(1 + \mathcal{K}^N_{\mathcal{E}_1}\right)^g\right)$$

To rank the alternatives expressed in the form of BFNs, BFNs need to be converted into similar scalar values. The functions of *score* and *accuracy* are important in this transformation, as they both take into account positive and negative membership degrees.

Definition 3 [29]. For finding the score and accuracy values of a bipolar fuzzy number

$$\mathcal{E} = \left(\mathcal{K}^P_{\mathcal{E}}, \mathcal{K}^N_{\mathcal{E}}\right),$$

the following equations will be used:

$$\mathcal{S}(\mathcal{E}) = \frac{1}{2}\left(1 + \mathcal{K}^P_{\mathcal{E}} + \mathcal{K}^N_{\mathcal{E}}\right), \qquad \mathcal{S}(\mathcal{E}) \in [0,1], \tag{3. 1}$$

$$\mathcal{H}(\mathcal{E}) = \frac{\mathcal{K}^P_{\mathcal{E}} - \mathcal{K}^N_{\mathcal{E}}}{2}, \qquad \mathcal{H}(\mathcal{E}) \in [0,1]. \tag{3. 2}$$

4. Generalized bipolar fuzzy prioritized AOs

This part presents a new category of AOs, which are the generalized bipolar fuzzy prioritized AOs that are generalized bipolar fuzzy prioritized average (G-BFPRA), generalized bipolar fuzzy prioritized weighted average (G-BFPRWA), generalized bipolar fuzzy prioritized geometric (G-BFPRG), and generalized bipolar fuzzy prioritized weighted geometric (G-BFPRWG) operators. The bipolar fuzzy term implies that the operators are designed in the bipolar fuzzy environment, which enables the aggregation of positive and negative membership degrees simultaneously. The fact that the significance of criteria is established based on predetermined priority relations and not arbitrary weight assignment is reflected in the term prioritized. The operators are called generalized since they are extensions of a number of existing aggregation models as special cases. Specifically, the proposed operators can be simplified to standard bipolar fuzzy AOs when priority relations are disregarded; they can be simplified to conventional fuzzy AOs when bipolar information is ignored. Therefore, the suggested framework is a generalization of the current averaging and geometric operators, which include bipolar information and priority-based weighting mechanisms. Besides the definition of these operators, the basic mathematical characteristics of these operators, such as idempotency, monotonicity, and boundedness, are also explored in this section, which guarantees the rationality and reliability of the aggregation process in the application of decision-making.

Definition 4 Assume there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \ldots, \theta,$$

then the G–BFPR? operator is implied as follows:

$$G\text{–}BFPR?(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) = \bigoplus_{\sigma=1}^{\theta} \left(\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta} \tau_\sigma} (\mathcal{E}_\sigma)^\varepsilon \right)^{\frac{1}{\varepsilon}}. \tag{4. 3}$$

Note that $\tau_1 = 1$, $\tau_\sigma = \prod_{\xi=1}^{\sigma-1} \mathcal{S}(\mathcal{E}_\xi)$, where $\sigma = 1, 2, \ldots, \theta$, and $\mathcal{S}(\mathcal{E}_\sigma)$ is the score value of BFN $\mathcal{E}_\sigma$.

Theorem 1. If there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \ldots, \theta,$$

then after aggregating BFNs by employing the G–BFPR? operator, a BFN will be obtained:

$$G\text{–}BFPR?(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) = \left(1 - \prod_{\sigma=1}^{\theta} \left(1 - \left(\mathcal{K}^P_{\mathcal{E}_\sigma}\right)^\varepsilon \right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta} \tau_\sigma}}, \right.$$
$$\left. -1 + \left(1 - \prod_{\sigma=1}^{\theta} \left[-1 + \left(1 + \mathcal{K}^N_{\mathcal{E}_\sigma} \right)^\varepsilon \right]^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta} \tau_\sigma}} \right) \right). \tag{4. 4}$$

Proof. Assume that $\theta = 2$, then

$$(\mathcal{E}_1)^\varepsilon = \left(\left(\mathcal{K}^P_{\mathcal{E}_1}\right)^\varepsilon, \ -1 + \left(1 + \mathcal{K}^N_{\mathcal{E}_1}\right)^\varepsilon \right),$$

$$(\mathcal{E}_2)^\varepsilon = \left(\left(\mathcal{K}^P_{\mathcal{E}_2}\right)^\varepsilon, \ -1 + \left(1 + \mathcal{K}^N_{\mathcal{E}_2}\right)^\varepsilon \right).$$

After this, we have

$$\frac{\tau_1}{\sum_{\sigma=1}^{\theta} \tau_\sigma} (\mathcal{E}_1)^\varepsilon = \left(1 - \left(1 - \left(\mathcal{K}^P_{\mathcal{E}_1}\right)^\varepsilon \right)^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta} \tau_\sigma}}, - \left[-1 + \left(1 + \mathcal{K}^N_{\mathcal{E}_1}\right)^\varepsilon \right]^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta} \tau_\sigma}} \right),$$

$$\frac{\tau_2}{\sum_{\sigma=1}^{\theta} \tau_\sigma} (\mathcal{E}_2)^\varepsilon = \left(1 - \left(1 - \left(\mathcal{K}^P_{\mathcal{E}_2}\right)^\varepsilon \right)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta} \tau_\sigma}}, - \left[-1 + \left(1 + \mathcal{K}^N_{\mathcal{E}_2}\right)^\varepsilon \right]^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta} \tau_\sigma}} \right).$$

Hence,

$$\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}(\mathcal{E}_1)^\varepsilon \oplus \frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}(\mathcal{E}_2)^\varepsilon = \left(1-\left(1-(\mathcal{K}_{\mathcal{E}_1}^P)^\varepsilon\right)^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}, -\left[-1+(1+\mathcal{K}_{\mathcal{E}_1}^N)^\varepsilon\right]^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right) \oplus \left(1-\left(1-(\mathcal{K}_{\mathcal{E}_2}^P)^\varepsilon\right)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}, -\left[-1+(1+\mathcal{K}_{\mathcal{E}_2}^N)^\varepsilon\right]^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right)$$

$$= \left(1-\prod_{\sigma=1}^{2}\left(1-(\mathcal{K}_{\mathcal{E}_\sigma}^P)^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}, -1+\prod_{\sigma=1}^{2}\left[-1+(1+\mathcal{K}_{\mathcal{E}_\sigma}^N)^\varepsilon\right]^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right).$$

Consequently, Eq. (1) is true for $\theta = 2$. After that, assume that Eq. (1) is true for $\theta = \upsilon$, i.e.,

$$\bigoplus_{\sigma=1}^{\upsilon}\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}(\mathcal{E}_\sigma)^\varepsilon = \left(1-\prod_{\sigma=1}^{\upsilon}\left(1-(\mathcal{K}_{\mathcal{E}_\sigma}^P)^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}}, -1+\prod_{\sigma=1}^{\upsilon}\left[-1+(1+\mathcal{K}_{\mathcal{E}_\sigma}^N)^\varepsilon\right]^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}}\right).$$

In the last, to prove that Eq. (1) is true for $\theta = \upsilon + 1$, we have

$$\bigoplus_{\sigma=1}^{\upsilon+1}\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}(\mathcal{E}_\sigma)^\varepsilon = \left(\bigoplus_{\sigma=1}^{\upsilon}\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}(\mathcal{E}_\sigma)^\varepsilon\right) \oplus \frac{\tau_{\upsilon+1}}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}(\mathcal{E}_{\upsilon+1})^\varepsilon$$

$$= \left(1-\prod_{\sigma=1}^{\upsilon+1}\left(1-(\mathcal{K}_{\mathcal{E}_\sigma}^P)^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}}, -1+\prod_{\sigma=1}^{\upsilon+1}\left[-1+(1+\mathcal{K}_{\mathcal{E}_\sigma}^N)^\varepsilon\right]^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}}\right).$$

Consequently, Eq. (1) is true for all $\upsilon + 1$. Hence, it implies that it is true for θ.

G-BFPRA operator has satisfied the underneath properties.

Idempotency: Assume there is an assembly of BFNs $\mathcal{E}_\sigma = (\mathcal{K}_{\mathcal{E}_\sigma}^P, \mathcal{K}_{\mathcal{E}_\sigma}^N)$, $\sigma = 1, 2, \ldots, \theta$, then if $\mathcal{E}_\sigma = \mathcal{E}$, then

$$G\text{–}BFPR?(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) = \mathcal{E}.$$

Monotonicity: Assume there are two assemblies of BFNs $\mathcal{E}_\sigma = (\mathcal{K}_{\mathcal{E}_\sigma}^P, \mathcal{K}_{\mathcal{E}_\sigma}^N)$ and $\mathcal{E}'_\sigma = (\mathcal{K}_{\mathcal{E}'_\sigma}^P, \mathcal{K}_{\mathcal{E}'_\sigma}^N)$, $\sigma = 1, 2, \ldots, \theta$, and if $\mathcal{K}_{\mathcal{E}_\sigma}^P \le \mathcal{K}_{\mathcal{E}'_\sigma}^P$, $\mathcal{K}_{\mathcal{E}_\sigma}^N \ge \mathcal{K}_{\mathcal{E}'_\sigma}^N$, then

$$G\text{–}BFPR?(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) \le G\text{–}BFPR?(\mathcal{E}'_1, \mathcal{E}'_2, \ldots, \mathcal{E}'_\theta).$$

Boundedness: If there is an assembly of BFNs $\mathcal{E}_\sigma = (\mathcal{K}_{\mathcal{E}_\sigma}^P, \mathcal{K}_{\mathcal{E}_\sigma}^N)$, $\sigma = 1, 2, \ldots, \theta$, if

$$\mathcal{E}^- = \left(\min_\sigma\{\mathcal{K}_{\mathcal{E}_\sigma}^P\}, \min_\sigma\{\mathcal{K}_{\mathcal{E}_\sigma}^N\}\right) \quad \text{and} \quad \mathcal{E}^+ = \left(\max_\sigma\{\mathcal{K}_{\mathcal{E}_\sigma}^P\}, \max_\sigma\{\mathcal{K}_{\mathcal{E}_\sigma}^N\}\right),$$

then

$$\mathcal{E}^{-} \leq G\text{–}BFPR?(\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_\theta) \leq \mathcal{E}^{+}.$$

Definition 5. Assume there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \dots, \theta,$$

then the G–BFPRWA operator is implied as follows:

$$G\text{–}BFPRWA(\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_\theta) = \bigoplus_{\sigma=1}^{\theta} \left(\frac{w_\sigma \tau_\sigma}{\sum_{\sigma=1}^{\theta} w_\sigma \tau_\sigma} (\mathcal{E}_\sigma)^{\varepsilon} \right)^{\frac{1}{\varepsilon}}. \tag{4. 5}$$

Keep in mind that $\varepsilon > 0$, and $\mathbf{w} = (w_1, w_2, \dots, w_\theta)$ is a weight vector along with the condition that $0 \leq w_\sigma \leq 1$ and $\sum_{\sigma=1}^{\theta} w_\sigma = 1$.

Theorem 2. Assume there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \dots, \theta,$$

then after aggregating BFNs by employing the G–BFPRWA operator, a BFN will be obtained:

$$G\text{–}BFPRWA(\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_\theta) = \left(1 - \prod_{\sigma=1}^{\theta} \left(1 - (\mathcal{K}^P_{\mathcal{E}_\sigma})^{\varepsilon}\right)^{\frac{w_\sigma \tau_\sigma}{\sum_{\sigma=1}^{\theta} w_\sigma \tau_\sigma}}, \right.$$
$$\left. -1 + \prod_{\sigma=1}^{\theta} \left[-1 + (1 + \mathcal{K}^N_{\mathcal{E}_\sigma})^{\varepsilon}\right]^{\frac{w_\sigma \tau_\sigma}{\sum_{\sigma=1}^{\theta} w_\sigma \tau_\sigma}} \right). \tag{4. 6}$$

Proof. Same as proof of Theorem 1.

The G–BFPRWA operator holds the monotonicity, idempotency, and boundedness.

Definition 6. Assume there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \dots, \theta,$$

then the G–BFPRG operator is implied as follows:

$$G\text{–}BFPRG(\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_\theta) = \left(\bigotimes_{\sigma=1}^{\theta} (\mathcal{E}_\sigma)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta} \tau_\sigma}} \right)^{\frac{1}{\varepsilon}}. \tag{4. 7}$$

Note that $\tau_1 = 1$, $\tau_\sigma = \prod_{\xi=1}^{\sigma-1} \mathcal{S}(\mathcal{E}_\xi)$, where $\sigma = 1, 2, \dots, \theta$, and $\mathcal{S}(\mathcal{E}_\sigma)$ is the score value of BFN $\mathcal{E}_\sigma$.

Theorem 3. Assume there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \dots, \theta,$$

then after aggregating BFNs by employing the G–BFPRG operator, a BFN will be obtained:

$$G\text{–}BFPRG(\mathcal{E}_1,\mathcal{E}_2,\ldots,\mathcal{E}_\theta)=\left(1-\left[\prod_{\sigma=1}^{\theta}\left(1-(\mathcal{K}^P_{\mathcal{E}_\sigma})^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right]^{\frac{1}{\varepsilon}},\right.$$
$$\left.-\left[-1+\prod_{\sigma=1}^{\theta}\left(1-|\mathcal{K}^N_{\mathcal{E}_\sigma}|^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right]^{\frac{1}{\varepsilon}}\right). \quad (4.\ 8)$$

Proof. Assume that $\theta=2$, then

$$\mathcal{E}_1^\varepsilon=\left(1-(1-\mathcal{K}^P_{\mathcal{E}_1})^\varepsilon,-|\mathcal{K}^N_{\mathcal{E}_1}|^\varepsilon\right),$$
$$\mathcal{E}_2^\varepsilon=\left(1-(1-\mathcal{K}^P_{\mathcal{E}_2})^\varepsilon,-|\mathcal{K}^N_{\mathcal{E}_2}|^\varepsilon\right).$$

After this, we have

$$\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}(\mathcal{E}_1)^\varepsilon=\left(1-(1-\mathcal{K}^P_{\mathcal{E}_1})^\varepsilon\right)^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}},-\left(|\mathcal{K}^N_{\mathcal{E}_1}|^\varepsilon\right)^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}$$

$$(\mathcal{E}_2)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}=\left((1-\mathcal{K}^P_{\mathcal{E}_2})^\varepsilon\right)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}},\ -1+\left(|\mathcal{K}^N_{\mathcal{E}_2}|^\varepsilon\right)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}$$

Hence,

$$(\mathcal{E}_1)^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\otimes(\mathcal{E}_2)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}=\left(\left(1-\mathcal{K}^P_{\mathcal{E}_1})^\varepsilon\right)^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}},\ -1+\left(|\mathcal{K}^N_{\mathcal{E}_1}|^\varepsilon\right)^{\frac{\tau_1}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right)$$
$$\otimes\left(\left(1-\mathcal{K}^P_{\mathcal{E}_2})^\varepsilon\right)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}},\ -1+\left(|\mathcal{K}^N_{\mathcal{E}_2}|^\varepsilon\right)^{\frac{\tau_2}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right)$$
$$=\left(\prod_{\sigma=1}^{2}\left(1-(\mathcal{K}^P_{\mathcal{E}_\sigma})^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta}\tau_\sigma}},\ -1+\prod_{\sigma=1}^{2}\left(1-|\mathcal{K}^N_{\mathcal{E}_\sigma}|^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\theta}\tau_\sigma}}\right).$$

Consequently, Eq. (2) is true for $\theta=2$. After that, assume that Eq. (2) is true for $\theta=\upsilon$, i.e.,

$$\bigotimes_{\sigma=1}^{\upsilon}(\mathcal{E}_\sigma)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}}=\left(\prod_{\sigma=1}^{\upsilon}\left(1-(\mathcal{K}^P_{\mathcal{E}_\sigma})^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}},\ -1+\prod_{\sigma=1}^{\upsilon}\left(1-|\mathcal{K}^N_{\mathcal{E}_\sigma}|^\varepsilon\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}}\right).$$

In the last, to prove that Eq. (2) is true for $\theta = \upsilon + 1$, we have

$$\begin{aligned}\bigotimes_{\sigma=1}^{\upsilon+1}(\mathcal{E}_\sigma)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}} &= \left(\bigotimes_{\sigma=1}^{\upsilon}(\mathcal{E}_\sigma)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon}\tau_\sigma}}\right)\otimes(\mathcal{E}_{\upsilon+1})^{\frac{\tau_{\upsilon+1}}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}}\\ &= \left(\prod_{\sigma=1}^{\upsilon+1}\left(1-(\mathcal{K}^P_{\mathcal{E}_\sigma})^{\varepsilon}\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}},\right.\\ &\qquad \left. -1+\prod_{\sigma=1}^{\upsilon+1}\left(1-|\mathcal{K}^N_{\mathcal{E}_\sigma}|^{\varepsilon}\right)^{\frac{\tau_\sigma}{\sum_{\sigma=1}^{\upsilon+1}\tau_\sigma}}\right).\end{aligned}$$

Consequently, Eq. (2) is true for all $\upsilon + 1$. Hence, it implies that it is true for θ.

G–BFPRG operator has satisfied the underneath properties.

Idempotency: Assume there is an assembly of BFNs $\mathcal{E}_\sigma = (\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma})$, $\sigma = 1, 2, \ldots, \theta$. If $\mathcal{E}_\sigma = \mathcal{E}$, then

$$G\text{–}BFPRG(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) = \mathcal{E}.$$

Monotonicity: Assume there are two assemblies of BFNs $\mathcal{E}_\sigma = (\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma})$ and $\mathcal{E}'_\sigma = (\mathcal{K}^P_{\mathcal{E}'_\sigma}, \mathcal{K}^N_{\mathcal{E}'_\sigma})$, $\sigma = 1, 2, \ldots, \theta$. If $\mathcal{K}^P_{\mathcal{E}_\sigma} \leq \mathcal{K}^P_{\mathcal{E}'_\sigma}$ and $\mathcal{K}^N_{\mathcal{E}_\sigma} \geq \mathcal{K}^N_{\mathcal{E}'_\sigma}$, then

$$G\text{–}BFPRG(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) \leq G\text{–}BFPRG(\mathcal{E}'_1, \mathcal{E}'_2, \ldots, \mathcal{E}'_\theta).$$

Boundedness: Assume there is an assembly of BFNs $\mathcal{E}_\sigma = (\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma})$, $\sigma = 1, 2, \ldots, \theta$. If

$$\mathcal{E}^- = \left(\min_\sigma\{\mathcal{K}^P_{\mathcal{E}_\sigma}\}, \min_\sigma\{\mathcal{K}^N_{\mathcal{E}_\sigma}\}\right), \quad \mathcal{E}^+ = \left(\max_\sigma\{\mathcal{K}^P_{\mathcal{E}_\sigma}\}, \max_\sigma\{\mathcal{K}^N_{\mathcal{E}_\sigma}\}\right),$$

then

$$\mathcal{E}^- \leq G\text{–}BFPRG(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) \leq \mathcal{E}^+.$$

$$\mathcal{E}^- \leq G\text{–}BFPRG(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) \leq \mathcal{E}^+.$$

Definition 7. Assume there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \ldots, \theta,$$

then the G–BFPRWG operator is implied as follows:

$$G\text{–}BFPRWG(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) = \left(\bigotimes_{\sigma=1}^{\theta}(\mathcal{E}_\sigma)^{\frac{w_\sigma\tau_\sigma}{\sum_{\sigma=1}^{\theta} w_\sigma\tau_\sigma}}\right)^{\frac{1}{\varepsilon}}. \tag{4. 9}$$

Keep in mind that $\varepsilon > 0$, and $\mathbf{w} = (w_1, w_2, \ldots, w_\theta)$ is a weight vector along with the condition that $0 \leq w_\sigma \leq 1$ and $\sum_{\sigma=1}^{\theta} w_\sigma = 1$.

Theorem 4. Assume there is an assembly of BFNs

$$\mathcal{E}_\sigma = \left(\mathcal{K}^P_{\mathcal{E}_\sigma}, \mathcal{K}^N_{\mathcal{E}_\sigma}\right), \quad \sigma = 1, 2, \ldots, \theta,$$

then after aggregating BFNs by employing the G–BFPRWG operator, a BFN will be obtained:

$$G\text{–}BFPRWG(\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_\theta) = \left(1 - \left[\prod_{\sigma=1}^{\theta} \left(1 - (\mathcal{K}^P_{\mathcal{E}_\sigma})^\varepsilon\right)^{\frac{w_\sigma \tau_\sigma}{\sum_{\sigma=1}^{\theta} w_\sigma \tau_\sigma}} \right]^{\frac{1}{\varepsilon}}, \right.$$
$$\left. - \left[-1 + \prod_{\sigma=1}^{\theta} \left(1 - |\mathcal{K}^N_{\mathcal{E}_\sigma}|^\varepsilon\right)^{\frac{w_\sigma \tau_\sigma}{\sum_{\sigma=1}^{\theta} w_\sigma \tau_\sigma}} \right]^{\frac{1}{\varepsilon}} \right). \tag{4. 10}$$

Proof. Same as proof of Theorem 3.
The G–BFPRWG operator holds the monotonicity, idempotency, and boundedness.

5. Bipolar Fuzzy MCDM Technique Based on G–BFPR AOs

MCDM problems are those that are concerned with the selection or ranking of alternatives based on a number of criteria, which are usually conflicting. These issues are also complicated in practice, including uncertainty, imprecision, and conflicting expert views. In real-world applications like AI-driven supply chain management, in order to overcome these difficulties, this paper constructs a bipolar fuzzy MCDM model, which incorporates generalized prioritized AOs. The suggested MCDM framework calculates the importance of criteria based on priority relations and uses generalized operators to aggregate expert appraisals in the form of bipolar fuzzy numbers to derive the overall performance scores. In contrast to the traditional MCDM methods, the given model does not presuppose the subjective weight allocation and clearly represents both positive and negative appraisals, which results in more realistic and credible decision results.

Let there be a MCDM problem where the set of alternatives is denoted by

$$\mathcal{F} = \{F_1, F_2, \ldots, F_n\},$$

and the set of criteria is

$$\mathcal{O} = \{O_1, O_2, \ldots, O_m\}.$$

The aim is to choose the most appropriate option from the given set, based on judgments according to the specified criteria. Judgments are supplied by experts or decision makers as BFNs, specified as

$$\mathcal{E}_{ij} = \left(\mathcal{K}^P_{\mathcal{E}_{ij}}, \mathcal{K}^N_{\mathcal{E}_{ij}} \right), \quad (0 \leq i \leq n,\ 0 \leq j \leq m).$$

In this method, to avoid subjectivity and bias in the weight assignment, a prioritized-based weighting scheme is employed. The following are the step-by-step procedures for solving the MCDM problem using the bipolar fuzzy MCDM approach.

Step 1: Normalization. Since the criteria may be of varying types, i.e., benefit or cost, the values of evaluation need to be normalized. Normalization is done to ensure that all values fall within a similar range, allowing unbiased comparison. Normalization is carried out by applying the following formula:

$$\mathcal{D}_{ij}^{N} = \begin{cases} \left(\mathcal{K}_{\mathcal{E}_{ij}}^{P}, \mathcal{K}_{\mathcal{E}_{ij}}^{N}\right), & \text{for benefit kind,} \\ \left(1 - \mathcal{K}_{\mathcal{E}_{ij}}^{P},\ -1 - \mathcal{K}_{\mathcal{E}_{ij}}^{N}\right), & \text{for cost kind.} \end{cases} \tag{5. 11}$$

Step 2: Get the Prioritization τ_σ. After the normalization of the decision matrix, get the prioritization by using the following formula:

$$\tau_\sigma = \prod_{\xi=1}^{\sigma-1} \mathcal{S}(\mathcal{F}_\xi)\,, \tag{5. 12}$$

where $\sigma = 1, 2, \ldots, n$, $\xi = 1, 2, \ldots, n$, and $\tau_1 = 1$ for $\sigma = 1, 2, \ldots, n$.

Step 3: Aggregate the Decision Matrix. Aggregate the normalized evaluations using one of the proposed operators from the generalized class G–BFPR?, G–BFPRWA, G–BFPRG, and G–BFPRWG operators. Further, if the expert or decision maker also wants to give weights to the criteria, then one must use G–BFPRWA or G–BFPRWG operators.

Step 4: Find the Score and Accuracy Values. For every alternative, calculate the score value from the aggregated bipolar fuzzy number. If several alternatives have the same score value, then calculate the accuracy values to differentiate among them more accurately.

Step 5: Rank the Alternatives. Rank all alternatives by their score or, if necessary, accuracy values. The best choice under the criteria is the alternative with the highest score (and accuracy in the event of a tie).

Step 6: Final Decision. Select the top-ranked alternative as the most suitable option in the given decision-making scenario.

The flowchart of this method is devised in Fig. 1.

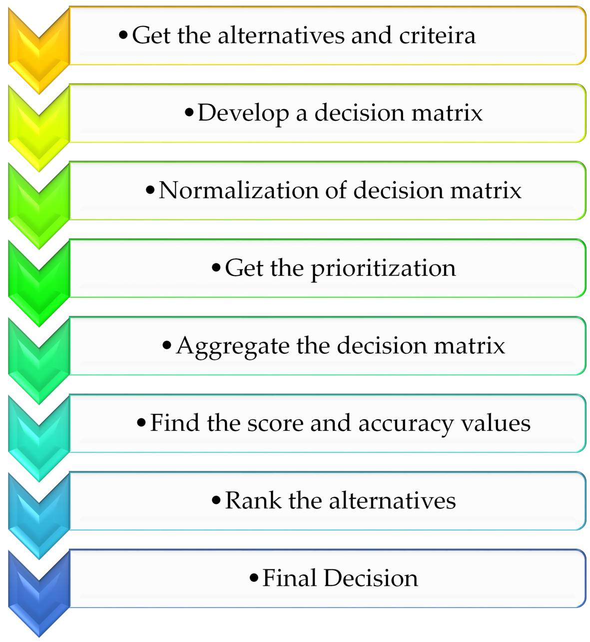

FIGURE 1. The flowchart of the devised MCDM approach

5.1. **Case study.** A mid-sized consumer goods company faces challenges in optimizing its supply chain due to growing demand variability, supplier unpredictability, and customer expectations for faster deliveries. To address these complexities, the company is evaluating advanced AI-driven supply chain models to enhance forecasting accuracy, operational agility, and resilience. However, multiple conflicting criteria and expert opinions make the selection process difficult. To solve this, the company employs the developed MCDM approach to evaluate and rank the most suitable models under uncertain and imprecise environments. The company shortlisted the following AI-driven supply chain models, which are listed in Table 1.

TABLE 1. The AI-driven supply chain models, along with an explanation.

Notation	Models	Explanation
$\mathcal{F}_1$	Machine learning–based demand forecasting model	The model draws its predictions from historical data, including sales reports and weather information, along with promotional activities and external influences. Through preventive planning, the model helps businesses prevent both product shortages and product surpluses. The model might require occasional retraining when sales patterns experience unexpected directions.
$\mathcal{F}_2$	AI-powered inventory optimization model	The inventory optimization model uses AI algorithms to establish stock levels and reorder points, and safety stock using current supply and demand signals for optimal results. Using this model, organizations cut their inventory expenses yet still maintain their designated service standards. The model might encounter difficulties when adapting to fast-changing market situations.
$\mathcal{F}_3$	Real-time AI logistics optimization system	This model uses AI to process routing functions as well as schedule shipments while allocating transportation resources. This system enhances delivery performance while reducing costs through different types of constraints. The system demonstrates considerable capability but becomes expensive when deployed, while requiring high-quality data flow.
$\mathcal{F}_4$	Intelligent Supplier Risk Assessment Model	AI technology evaluates supplier risks through analyses of delivery history, alongside financial health indicators, geopolitical components, and compliance records. This system helps organizations make better selections of suppliers and implement risk reduction measures. External unstructured data sources used by this system may lead to variations in system consistency.

To evaluate these models, the company has selected the criteria, which are interpreted in Table 2.

TABLE 2. The criteria of the considered models, along with an explanation.

Notation	Criteria	Explanation
δ_1	Forecast Accuracy and Performance	This criterion determines how well the model uses actual market data for forecasting supply chain and demand patterns. The level of performance directly affects both operational planning accuracy and unexpected operational situations. The effectiveness of service quality and cost-efficiency depends on this performance criterion.
δ_2	Responsiveness to Market Disruptions	The metric determines the model's speed at which it responds to immediate alterations affecting supply timings, regulatory requirements, or sudden changes in demand. Resilient operations that can adapt quickly emerge from models with high responsiveness features. Pandemics and uncertain environments demand the implementation of this system.
δ_3	Ease of Integration and Deployment	Ease of integration with existing supply chain software, as well as Electronic Resource Planning systems and workflows, is factored into this quality assessment. Integration of AI technology leads to faster time-to-value while fostering improved system acceptance by users. Deploying complex systems to the system can cause delays in return on investment needs and demands technicians with specific abilities.
δ_4	Data Requirements and Scalability	The factor evaluates both the needed data quantity as well as its diversity and standard alongside the model's capacity for expansion to multiple business units or regional deployments. The performance of a model should remain stable through partial input data while simultaneously expanding with the company's expansion. The overreliance on big data could limit its applications within settings with limited availability of data.

After the assessment of these models based on the selected criteria, the expert provides their assessment values in the form of BFNs, given in Table 3.

TABLE 3. The evaluation values for models.

	δ_1	δ_2	δ_3	δ_4
$\mathcal{F}_1$	$(0.823, -0.247)$	$(0.761, -0.336)$	$(0.634, -0.428)$	$(0.706, -0.219)$
$\mathcal{F}_2$	$(0.778, -0.198)$	$(0.945, -0.317)$	$(0.819, -0.244)$	$(0.836, -0.093)$
$\mathcal{F}_3$	$(0.688, -0.292)$	$(0.842, -0.175)$	$(0.973, -0.263)$	$(0.986, -0.115)$
$\mathcal{F}_4$	$(0.981, -0.138)$	$(0.913, -0.166)$	$(0.725, -0.319)$	$(0.667, -0.217)$

These assessments represent both the positive and negative aspects associated with each model under uncertainty.

Step 1: As all the criteria are of benefit type, after normalization, the same decision matrix will be obtained as given in Table 3.

Step 2: The interpreted matrix

$$\boldsymbol{\tau}_{\mathcal{F}} = \begin{bmatrix} 1 & 0.788 & 0.561 & 0.339 \\ 1 & 0.979 & 0.643 & 0.506 \\ 1 & 0.698 & 0.582 & 0.505 \\ 1 & 0.922 & 0.805 & 0.566 \end{bmatrix} \tag{5. 13}$$

Step 3: The aggregated values of the models after using the proposed G–BFPR?, G–BFPRWA, G–BFPRG, and G–BFPRWG operators are derived in Table 4.

TABLE 4. The aggregated values of the assessment values, where the weight for prioritized weighted operators is $(0.2, 0.3, 0.1, 0.4)$ and $\varepsilon = 4$.

Operators	$\mathcal{F}_1$	$\mathcal{F}_2$	$\mathcal{F}_3$	$\mathcal{F}_4$
G–BFPR?	$(0.92, -0.292)$	$(0.957, -0.199)$	$(0.972, -0.202)$	$(0.965, -0.221)$
G–BFPRWA	$(0.807, -0.279)$	$(0.939, -0.182)$	$(0.965, -0.179)$	$(0.948, -0.238)$
G–BFPRG	$(0.721, -0.334)$	$(0.814, -0.252)$	$(0.756, -0.244)$	$(0.756, -0.239)$
G–BFPRWG	$(0.737, -0.312)$	$(0.822, -0.254)$	$(0.765, -0.230)$	$(0.746, -0.213)$

These outcomes are obtained by employing the devised operators one by one. For instance,

$$G\text{–}BFPRA(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4) = \begin{pmatrix} 1 - \Big[(1-0.823^4)^{0.372}(1-0.761^4)^{0.293} \\ \qquad (1-0.634^4)^{0.209}(1-0.706^4)^{0.126}\Big], \\ -1 + \Big[(1-0.247^4)^{0.372}(1-0.336^4)^{0.293} \\ \qquad (1-0.428^4)^{0.209}(1-0.219^4)^{0.126}\Big] \end{pmatrix}$$
$$= (0.92, -0.292).$$

Step 4: Score Values. The score values of the models are interpreted in Table 5.

TABLE 5. The score values of the AI-driven supply chain models.

Operators	$\mathcal{S}(\mathcal{F}_1)$	$\mathcal{S}(\mathcal{F}_2)$	$\mathcal{S}(\mathcal{F}_3)$	$\mathcal{S}(\mathcal{F}_4)$
G–BFPR?	0.814	0.879	0.885	0.872
G–BFPRWA	0.807	0.879	0.893	0.855
G–BFPRG	0.693	0.781	0.756	0.759
G–BFPRWG	0.713	0.784	0.768	0.766

These score values are obtained by employing the score function. For instance,

$$\mathcal{S}(\mathcal{F}_1) = \frac{1}{2}(1 + 0.92 - 0.292) = 0.814. \qquad (5.\ 14)$$

Step 5: Ranking of the Models. The ranking of the models is presented in Table 6.

TABLE 6. The ranking of the models.

Operators	**Ranking**
G–BFPR?	$\mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_1)$
G–BFPRWA	$\mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_1)$
G–BFPRG	$\mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_1)$
G–BFPRWG	$\mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_1)$

Step 6: Final Decision. Based on the score values and rankings interpreted in Steps 4 and 5, we can see that by employing G–BFPR? and G–BFPRWA operators, $\mathcal{F}_3$ is the finest model. While employing G–BFPRG and G–BFPRWG operators, we have that $\mathcal{F}_2$ is a good model.

The graphical interpretation of the result can be seen in Fig. 2.

6. Comparative Analysis

To illustrate the effectiveness and benefits of our proposed methodology, we have performed a comparative analysis with several existing theoretical frameworks in the field. Zhao et al. [33] proposed a generalized averaging AOs in intuitionistic fuzzy sets (IFSs), and Tan et al. [25] proposed a MCDM method based on generalized geometric AOs in the IFS context. In addition, Yu [30] developed a MCDM approach based on generalized prioritized AOs in IFSs, and Liang et al. [17] developed a MCDM methodology based on generalized prioritized AOs in intuitionistic trapezoidal fuzzy sets (ITFSs). We have used these existing operators and MCDM approaches together with our proposed method to solve a particular problem that we have in our case study. The comparative results are vividly presented in Table 7 and Fig. 3, which give a clear picture of the performance differences.

Table 7. The result of employing existing and proposed theories.

Approaches and Operators	$\mathcal{S}(\mathcal{F}_1)$	$\mathcal{S}(\mathcal{F}_2)$	$\mathcal{S}(\mathcal{F}_3)$	$\mathcal{S}(\mathcal{F}_4)$	**Ranking**
Zhao et al. [33]	***	***	***	***	***
Tan et al. [25]	***	***	***	***	***
Yu [30]	***	***	***	***	***
Liang et al. [17]	***	***	***	***	***
G–BFPR?	0.814	0.879	0.885	0.872	$\mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_1)$
G–BFPRWA	0.807	0.879	0.893	0.855	$\mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_1)$
G–BFPRG	0.693	0.781	0.756	0.759	$\mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_1)$
G–BFPRWG	0.713	0.784	0.768	0.766	$\mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_3) > \mathcal{S}(\mathcal{F}_4) > \mathcal{S}(\mathcal{F}_1)$

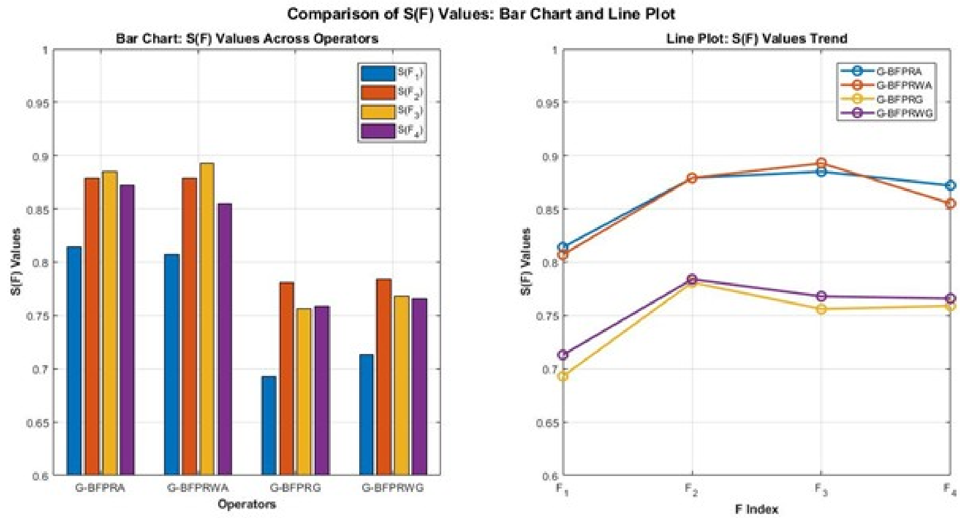

FIGURE 2. The geometrical interpretation of result.

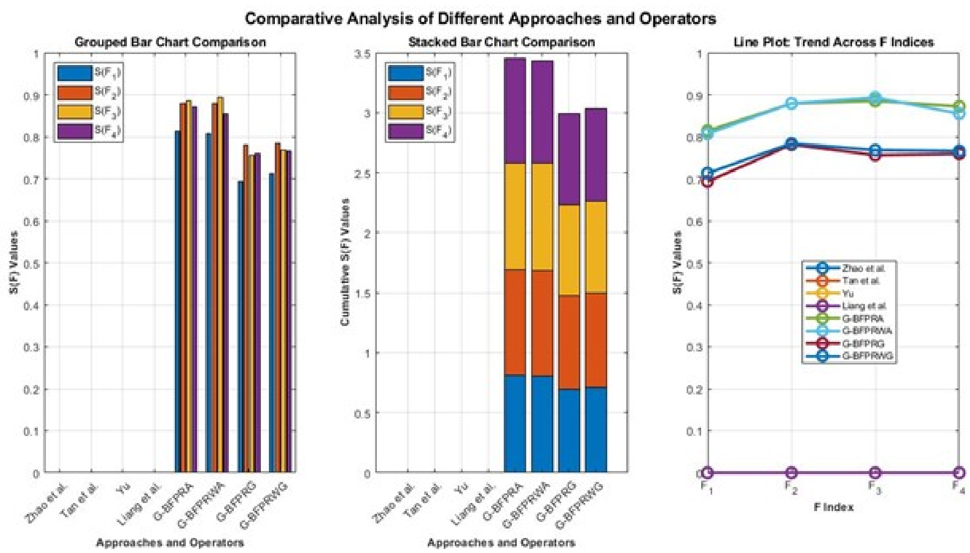

FIGURE 3. The graph of the comparison.

All the existing methods and the suggested G-BFPR-based MCDM framework are used on the same case study with the same criteria, alternatives, and evaluation data to provide a fair and meaningful comparison. As it is presented in Table 7 and Fig. 3, it is possible to notice the significant differences in ranking outcomes between the methods under consideration. The current intuitionistic fuzzy and intuitionistic trapezoidal fuzzy-based methods mainly combine the positive membership data and use the predetermined or subjective criteria weights. As a result, such techniques have poor discrimination ability in the processing of conflicting expert views and uncertain data. Conversely, the suggested framework also integrates bipolar fuzzy information and priority-based weight calculation, which enables it to integrate supportive and opposing evaluations in a single framework. Consequently, the suggested G-BFPR operators produce more sensitive and stable rankings across the aggregation models (average and geometric, weighted and unweighted). This increased discrimination ability gives analytical support to the excellence of the suggested method, especially in complicated decision situations where uncertainty, bipolarity, and criteria dependency are present.

6.1. **Robustness and stability analysis of the proposed method.** MCDM Robustness is the stability and reliability of the ranking results when there is a change in modeling assumptions and aggregation behavior. To test the strength of the proposed framework, several generalized bipolar fuzzy prioritized operators, i.e., G-BFPRA, G-BFPRWA, G-BFPRG, and G-BFPRWG, are used on the same decision matrix. Table 6 demonstrates that the ranking results are highly consistent, despite the fact that various aggregation mechanisms (average versus geometric, weighted versus unweighted) are used. Specifically, the highest-ranking alternatives are the same or in the neighboring positions in all the suggested operators. It means that the final decision does not depend on the selection of a particular aggregation structure, which proves the stability of the suggested approach.

7. Conclusion

This research presented a new MCDM framework based on G-BFPR operators to solve the complex issues of assessing AI-driven supply chain models. The study successfully filled a critical gap in the literature by creating four new operators: G-BFPRA, G-BFPRWA, G-BFPRG, and G-BFPRWG operators. These operators have systematically determined criteria weights based on priority relationships, eliminating subjectivity, and have been able to capture positive and negative preferences in uncertain decision situations. We found that the proposed BF-MCDM technique offered significant advantages over the conventional fuzzy decision-making techniques since it offered a more balanced and context-specific evaluation mechanism. The implementation of the case study proved that this framework enabled supply chain managers to make superior decisions in selecting optimization methodologies, which led to superior operational performance, reduced costs, and improved sustainability. The comparative analysis also helped to prove the superiority of our approach to the complex decision situations characterized by uncertainty, indecision, and contradictory expert opinions. The practical implications of this study were important to organizations that work in the dynamic world of AI-based supply chain optimization. The fact that our framework allowed businesses to select solutions that best fit their particular operational requirements, scalability objectives, and strategic goals was made possible by providing a robust decision support system that was capable of managing the dual nature of evaluation criteria and systematically ranking them. This moderated decision-making was particularly helpful in the dynamic supply chain settings where numerous stakeholders with different preferences had to reach an agreement on significant technological investments.

Moreover, the comparative analysis in Table 7 shows that in the case the same dataset is considered with the help of the current intuitionistic fuzzy and intuitionistic trapezoidal fuzzy-based methods, significant differences and less discriminative rankings are observed. Conversely, the suggested G-BFPR-based framework is always able to generate clear and stable ranking patterns because it is capable of integrating both bipolar information and priority-based weights at the same time. These findings affirm that the suggested approach provides strong and dependable outcomes in the presence of uncertainty and different aggregation levels, which is why it applies to complex real-world decision-making issues like the evaluation of AI-based supply chain models.

CREDIT AUTHORSHIP CONTRIBUTION'S STATEMENT

All authors contributed equally to the conception, analysis, and writing of the manuscript. All authors have read and approved the final version.

DECLARATION

Conflict of Interest. The authors declare no conflict of interest.

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