

**Decision Support System based on Linguistic Complex Intuitionistic Fuzzy  
Geometric Aggregation Operators using Set Pair Analysis**

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Sumaira Yasmin  
Department of Mathematics,  
Riphah International University Faisalabad Campus, Pakistan  
Government Graduate College (for Women) Sahianwala, Faisalabad, Pakistan

\*Muhammad Qiyas  
Department of Mathematics,  
Riphah International University Faisalabad Campus, Pakistan

Nurnadiah Zamri  
Faculty of Informatics and Computing,  
Universiti Sultan Zainal Abidin, Besut Campus, 22200 Besut, Terengganu, Malaysia

Zeeshan Ali  
Department of information management,  
National Yunlin University of Science and Technology, Douliu, Taiwan, R. O. C.

Neelam Khan  
Department of Mathematics, Texas Christian University, Texas, U. S. A.

Rana Muhammad Zulqaranin  
Department of Mathematics,  
Saveetha School of Engineering, SIMATS Thandalam, Chennai-602105 Tamilnadu, India

**Abstract.** Linguistic complex intuitionistic fuzzy sets (LCIFSs) provide an effective framework for modeling imprecise and ambiguous information in group decision-making problems. Meanwhile, set pair analysis (SPA) theory introduces the concept of a connection number, which integrates certainty and uncertainty within a unified representation. Motivated by this advantage, this study integrates SPA theory with LCIFS and proposes the linguistic complex intuitionistic fuzzy connection number (LCIFCN) along with its fundamental operational laws. Based on LCIFCN, several aggregation operators are developed in the LCIFS environment, including the LCIFCN weighted geometric (LCIFCNWG),

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\*Corresponding Author : [muhammad.qiyas@riphahfsd.edu.pk](mailto:muhammad.qiyas@riphahfsd.edu.pk)

LCIFCN ordered weighted geometric (LCIFCNOWG), and LCIFCN hybrid geometric (LCIFCNHG) operators. The proposed operators overcome certain limitations of existing aggregation methods and possess desirable mathematical properties. Furthermore, a collective decision-making method based on these operators is established. Finally, a numerical example is presented to illustrate the flexibility and effectiveness of the proposed approach.

**AMS (MOS) Subject Classification Codes:** 03E72; 47S40

**Key Words:** Linguistic complex fuzzy set; Linguistic complex intuitionistic fuzzy set; Linguistic complex intuitionistic fuzzy aggregation operators; Decision making

## 1. INTRODUCTION

Ramot et al. [28] introduced the concept of complex fuzzy sets (CFSs) by extending the standard fuzzy membership function from the real unit interval into the complex unit disc, enabling the simultaneous encoding of amplitude and phase information within a single membership grade. This foundational framework was subsequently extended by Ramot et al. [29] into a complete system of complex fuzzy logic, establishing algebraic operations and inference rules for complex-valued membership functions. The practical utility of CFSs was demonstrated by Ma et al. [22], who applied complex fuzzy set theory to signal processing and showed that the phase component captures periodic dependencies that real-valued fuzzy models cannot represent. Dai et al. [8] constructed interval-valued complex fuzzy geometric aggregation operators and applied them to engineering decision-making problems, broadening the scope of complex fuzzy aggregation to uncertain interval-valued settings. Alkouri and Salleh [2] further extended the complex fuzzy paradigm by introducing complex intuitionistic fuzzy sets (CIFs), which augment complex-valued membership with a corresponding complex-valued non-membership function, enabling richer characterisation of hesitancy within expert evaluations.

A comprehensive systematic review of the adjoint functions of the connection number and their applications to water resources complex systems was provided by Zhou et al. [48], demonstrating the depth and maturity of connection-number theory as of 2024. Li et al. [19] developed a multi-criteria decision-making method by integrating interval-valued Pythagorean fuzzy sets with set pair analysis (SPA) theory and the Choquet integral, illustrating how SPA principles enhance decision discrimination in generalised fuzzy environments. This work was further elaborated in [19], which extended specific computational aspects of the same SPA-Pythagorean fuzzy integration. The theoretical foundations of SPA were laid by Zhao [47], who introduced the set pair and its associated connection number as a unified representation of both certainty and uncertainty within a single mathematical object. This framework was extended by Jiang and Zhao [14], who investigated artificial intelligence applications of set pair analysis, establishing the conceptual bridge between SPA and computational intelligence. Jin et al. [15] subsequently investigated fundamental problems concerning the adjoint function of the connection number, providing rigorous mathematical results that underpin the operational properties of connection-number-based aggregation.

The concept of the intuitionistic fuzzy set (IFS) was formalised by Atanassov [5], who extended the standard fuzzy set by assigning to each element both a membership degree and a non-membership degree, subject to the constraint that their sum does not exceed unity, thereby providing a natural representation of the hesitancy inherent in human judgment. The theoretical position of IFS theory within the broader landscape of imprecision-modelling frameworks was rigorously analysed by Deschrijver and Kerre [9], who demonstrated its logical relationships with other major uncertainty theories. Li [17] extended IFSs to multi-attribute decision-making (MADM) by constructing models and methods in which IFS information represents attribute values and expert weights, establishing a systematic computational approach to IFS-based decision analysis.

Herrera and Verdegay [13] established the foundational framework for linguistic assessments in group decision-making, demonstrating that qualitative descriptors drawn from a predefined linguistic term set capture expert judgments more naturally than crisp numerical values in many practical contexts. Chen et al. [7] combined linguistic variables with intuitionistic fuzzy numbers to form linguistic intuitionistic fuzzy numbers (LIFNs), in which both membership and non-membership grades are expressed as elements of a linguistic term set, enabling simultaneous qualitative representation of positive and negative evaluations. Peng et al. [25] proposed a linguistic intuitionistic multi-criteria decision-making method based on the Frank-Heronian mean operator and validated it on coal mine safety evaluation. Arora and Garg [3] developed a group decision-making method based on prioritised linguistic intuitionistic fuzzy aggregation operators, which account for the precedence relationships among criteria when aggregating expert opinions. Seikh and Mandal [30] proposed multiple attribute group decision-making based on quasiring orthopair fuzzy sets with application to electric vehicle charging station site selection, illustrating the broadening family of generalised orthopair fuzzy decision frameworks. Garg and Kumar [12] introduced linguistic intuitionistic fuzzy connection numbers (LCNs) by integrating SPA theory with the LIFN framework, establishing operational laws for LCNs and developing a group decision-making process based on them, constituting the closest precursor in the literature to the present work. Liang and Wang [20] integrated linguistic intuitionistic fuzzy information with cloud decision-making models augmented by sentiment analysis, applying the resulting framework to product selection in e-commerce. Peng and Wang [24] proposed a cloud decision model based on linguistic intuitionistic fuzzy statistics for selecting sustainable energy crops, further extending LIFN-based decision support to environmental applications.

**1.1. Literature Review.** The literature relevant to this work spans four interconnected streams. The first stream encompasses complex fuzzy set theory and its intuitionistic extensions [28, 29, 22, 8, 2]. The second covers set pair analysis and connection number theory [48, 19, 47, 14, 15]. The third addresses the IFS framework and its aggregation operators [5, 9, 17]. The fourth covers linguistic intuitionistic fuzzy decision-making methods [13, 7, 25, 3, 30, 12, 20, 24]. The following subsections extend this review to the remaining literature and position the proposed approach comparatively.

Liu et al. [21] proposed dependent interval two-tuple linguistic aggregation operators and demonstrated their applicability to multiple attribute group decision-making, showing that dependence among arguments can be incorporated into the aggregation process.

Dong and Herrera-Viedma [10] developed a consistency-driven automatic methodology for setting interval numerical scales of two-tuple linguistic term sets, with application to group decision-making with preference relations, ensuring coherent numerical interpretations of linguistic labels. Wang et al. [32] constructed multi-criteria group decision-making methods based on interval two-tuple linguistic information combined with Choquet integral aggregation operators, capturing interactions among criteria through the non-additive Choquet measure. Qin and Liu [26] presented two-tuple linguistic Muirhead mean operators for multiple attribute group decision-making and applied them to supplier selection, demonstrating the ability of the Muirhead mean to capture inter-attribute correlations. Zhang et al. [46] proposed a consensus-reaching model for two-tuple linguistic multiple attribute group decision-making with incomplete weight information, addressing the practical situation where decision-makers can only partially specify attribute importance. Li et al. [16] introduced the concept of personalised individual semantics in computing with words, enabling different decision-makers to assign distinct numerical interpretations to the same linguistic label in consensus-reaching processes. Morente-Molinera et al. [23] conducted a systematic review of multi-granular fuzzy linguistic modelling in group decision-making problems, surveying existing approaches, identifying their limitations, and outlining future research directions. Wei et al. [38] proposed Pythagorean hesitant fuzzy Hamacher aggregation operators and applied them to multiple attribute decision-making, extending the aggregation paradigm to the more general Pythagorean membership structure under hesitant evaluation.

A formal definition of intuitionistic fuzzy sets grounded in set theory was introduced by Faizulkhakov [11]. Atanassov [4] subsequently developed further theoretical results on intuitionistic fuzzy sets, including additional algebraic operations and properties that enriched the mathematical structure of the framework. Xu [39] introduced the fundamental aggregation operators for IFSs, namely the intuitionistic fuzzy ordered weighted averaging (IFOWA), weighted averaging (IFWA), and hybrid aggregation (IFHA) operators, establishing the canonical infrastructure for IFS-based aggregation. Zadeh [45] formulated the original theory of fuzzy sets, introducing the membership function as a generalisation of crisp set membership and thereby laying the mathematical foundation upon which all subsequent fuzzy and intuitionistic fuzzy extensions, including the present work, rest. The aggregation operators of Xu further extended the IFS aggregation framework, providing complementary results that broadened the family of available operators for intuitionistic fuzzy decision-making.

Yager [44] proposed generalised ordered weighted averaging (OWA) aggregation operators applicable across multiple fuzzy environments, establishing a flexible parametric family of aggregation functions. Wei [34] deployed a maximising deviation method for multiple attribute decision-making in an intuitionistic fuzzy setting with incomplete weight information. Wei [35] developed a grey relational analysis (GRA) method for intuitionistic fuzzy multiple attribute decision-making with incomplete weight parameters, integrating the GRA distance concept into the IFS framework for alternative ranking. Wei [36] introduced induced geometric aggregation operators using intuitionistic fuzzy information and applied them to group decision-making, showing that order-inducing variables capture additional preference information beyond attribute weights. Wei [37] further proposed a grey relational analysis approach for intuitionistic fuzzy multi-dimensional attribute evaluation

with constrained weight parameters. Li [18] constructed a GOWA operator-based approach to multi-attribute decision-making using intuitionistic fuzzy sets, extending the generalised OWA framework to the IFS domain. Vahdani et al. [31] adapted the ELECTRE method for multi-criteria group decision-making in an intuitionistic fuzzy environment, demonstrating compatibility with outranking-based decision procedures. Chen [6] developed the inclusion-based TOPSIS method with interval-valued intuitionistic fuzzy sets for multiple criteria group decision-making, using inclusion measures rather than classical distance metrics to evaluate closeness to the ideal solution.

Yasmin et al. [42] introduced the LCIFS framework and developed a family of associated aggregation operators, including the LCIFWG and LCIFOWG operators, with application to plastic waste management approach selection. This work is the direct precursor of the present contribution and motivates the proposed extension through SPA theory, as its operators exhibit the ranking indeterminacy problem formally demonstrated in the present paper. Yang et al. [40] proposed aggregation and interaction aggregation soft operators on an interval-valued  $q$ -rung orthopair fuzzy soft environment and applied the resulting framework to automation company evaluation, illustrating the growing interest in soft-set extensions of generalised membership structures. Yang et al. [41] developed complex intuitionistic fuzzy Frank aggregation operators for multi-attribute decision-making, establishing a class of operators that combines the algebraic flexibility of the Frank  $t$ -norm with the complex-valued membership and non-membership structure of CIFSs, providing an independent route to complex-valued MADM that complements the SPA-based approach of the present work.

Table summarises the capabilities of representative methods from the literature along four dimensions: support for linguistic terms (LT), complex-valued membership (CV), integration of SPA theory (SPA), and group decision-making support (MAGDM). As the table confirms, no existing method simultaneously satisfies all four dimensions. The proposed LCIFCNWG, LCIFCNOWG, and LCIFCNHG operators are the first to unify linguistic qualitative representation, complex-valued membership and non-membership, and SPA-based tripartite uncertainty decomposition within a single aggregation framework, thereby resolving the ranking indeterminacy observed in [42] while preserving all expressive advantages of the LCIFS paradigm. Furthermore, Yasmin et al. [43] developed a family of 2-tuple linguistic complex intuitionistic fuzzy aggregation operators to accurately track multi-dimensional periodic variations. Recently, this domain has seen further theoretical expansions, including continuous linguistic complex operators [33], fuzzy relational geometric boundaries [1], and SPA-driven decision support frameworks [27].

Despite the breadth of the literature reviewed above, a critical gap exists: no existing work has integrated SPA theory with the complex-valued linguistic intuitionistic fuzzy setting. The existing LCIFS-based geometric operators exhibit a structural limitation. When two distinct alternatives yield identical score and accuracy values under the LCIFWG or LCIFOWG operators, the framework cannot discriminate between them. The connection number of SPA, by decomposing each evaluation into identity ( $e$ ), discrepancy ( $f$ ), and contrary ( $g$ ) components satisfying  $e+f+g = 1$ , provides precisely the additional discriminating information needed to resolve this indeterminacy. To the best of our knowledge, no aggregation operators that simultaneously incorporate linguistic term representation, complex-valued membership and non-membership, and SPA-based tripartite uncertainty

decomposition have been proposed. This gap constitutes the primary motivation of the present work.

The principal contributions of this paper are as follows. First, we introduce the linguistic complex intuitionistic fuzzy connection number (LCIFCN) by systematically mapping each LCIFN into a SPA-based tripartite representation, and establish its core operational laws. Second, we develop three novel geometric aggregation operators under the LCIFCN framework, namely the LCIFCNWG, LCIFCNOWG, and LCIFCNHG operators, and derive their essential mathematical properties including idempotency, boundedness, and monotonicity. Third, we provide formal demonstration that the proposed operators resolve the ranking indeterminacy problem inherent in existing LCIFWG and LCIFOWG operators, through counter-examples showing cases where existing methods fail and the proposed method succeeds. Fourth, we construct a systematic MAGDM algorithm, validate it on a blood-pressure risk assessment example, and confirm its robustness through sensitivity analysis and comparative evaluation against existing methods.

The remainder of this paper is structured as follows. Section 2 reviews the related literature across four thematic streams. Section 3 provides the necessary mathematical preliminaries on IFS, LCIFS, and SPA theory. Section 4 introduces the LCIFCN and its operational laws. Section 5 develops the three proposed geometric aggregation operators and establishes their properties. Section 6 presents the MAGDM algorithm and a numerical illustration. Section 7 provides a comparative analysis and sensitivity evaluation. Section 8 concludes the paper with a discussion of limitations and future research directions.

<b>Method / Operator</b>	<b>LT</b>	<b>CV</b>	<b>SPA</b>	<b>MAGDM</b>
Complex fuzzy sets [28, 29, 22, 8]	No	Yes	No	No
Complex IFS [2]	No	Yes	No	No
SPA connection number [48, 19, 47, 15]	No	No	Yes	No
IFS operators [5, 9, 17]	No	No	No	Yes
LIFWG, LIFOWG [13, 7]	Yes	No	No	Yes
Frank-Heronian LIFN [25]	Yes	No	No	Yes
Prioritised LIFN [3]	Yes	No	No	Yes
Quasirung orthopair [30]	No	No	No	Yes
SPA-LIFCN operators [12]	Yes	No	Yes	Yes
Cloud-LIFN model [20, 24]	Yes	No	No	Yes
2-tuple linguistic [21, 10, 32, 26, 46]	Yes	No	No	Yes
Personalised semantics [16, 23]	Yes	No	No	Yes
Pythagorean Hamacher [38]	No	No	No	Yes
IFS aggregation [11, 4, 39]	No	No	No	Yes
OWA/GRA/TOPSIS [44, 34, 35, 36, 37, 18, 31, 6]	No	No	No	Yes
LCIFWG, LCIFOWG [42]	Yes	Yes	No	Yes
$q$ -rung soft operators [40]	No	No	No	Yes
Complex IFS Frank [41]	No	Yes	No	Yes
<b>LCIFCNWG/OWG/HG (Proposed)</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>

## 2. PRELIMINARIES

**Definition 2. 1.** [12] Let  $A$  represent the set that is the universe. An IFS  $E$  in  $A$  is explained as,

$$E = \{e, \varkappa_{E(a)}, \varrho_{E(a)} | a \in A\}, \quad (2. 1)$$

where  $\varkappa_{E(a)}, \varrho_{E(a)} \in A^{\varkappa_{E(a)}, \varrho_{E(a)}} \rightarrow [0, 1]$  demonstrates the appropriate membership, non-membership categories for  $a \in E$ , respectively. And  $\varkappa_{E(a)} + \varrho_{E(a)} \leq 1, \forall a \in A$ , and ranking of  $e \in A$  can be defined as  $\pi_{A(\varkappa)} = 1 - \varkappa_{E(a)} - \varrho_{E(a)}$ . Usually, the pair  $(\varkappa_{E(a)}, \varrho_{E(a)})$ , is determined to be IFN, it can be simplified as  $(\varkappa, \varrho)$  where  $\varkappa \in [0, 1], \varrho \in [0, 1], \varkappa + \varrho \leq 1$ .

**Definition 2. 2.** [12] Let  $\check{S}^{[0,8]}$  be LIFS  $E$  in  $A$  is stated as

$$E = \{(e, s_{\varkappa_{E(a)}}, s_{\varrho_{E(a)}} | a \in A)\}, \quad (2. 2)$$

where  $s_{\varkappa_{E(a)}}, s_{\varrho_{E(a)}} \in \check{S}^{[0,8]}$ ;  $(\check{S}^{[0,8]} = \{s_t | s_0 \leq s_t \leq s_k\})$  symbolizes the degree of linguistic membership (LM), linguistic non-membership (LNM) of  $e \in A$ , respectively.  $\forall$  any  $a \in A, 0 \leq \varkappa_E + \varrho_E \leq k$  is any ways satisfied, and the index of  $e$  to  $A$ , the linguistic intuitionistic(LI) is defined as  $s_{\pi_E} = s_{k - \varkappa_E - \varrho_E}$ . Normally, the pair  $(s_{\varkappa_{E(a)}}, s_{\varrho_{E(a)}})$  is known as a LIF number LIFN, it can be simplified as  $\beta = (s_{\varkappa}, s_{\varrho})$  and  $s_{\varkappa}, s_{\varrho} \in \check{S}^{[0,8]}$  and  $\varkappa + \varrho \leq k$  If  $\varkappa + \varrho \in S$  where  $S = \{s_t | t = 0, 1, 2, \dots, k\}$ , then it is known as the genuine LIFN instead of the fictional LIFN.

The following aggregating operators for aggregation set of LIFNs

$\beta_t = (s_{\varkappa_t}, s_{\varrho_t}), t = 1, 2, \dots, n$ , the affiliated weight vector is  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  of  $\beta_t$  where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ .

(a) LIFWG operator is

$$\begin{aligned} LIFWG(\beta_1, \beta_2, \dots, \beta_n) &= \beta_1^{\psi_1} \otimes \beta_2^{\psi_2} \otimes \dots \otimes \beta_n^{\psi_n} \\ &= (s_{k \prod_{\varepsilon=1}^n (\varkappa_{\varepsilon} | k)^{\psi_{\varepsilon}}}, s_{k \prod_{\varepsilon=1}^n (1 - \varrho_{\varepsilon} | k)^{\psi_{\varepsilon}}}). \end{aligned} \quad (2. 3)$$

(b) LIFOWG operator is

$$\begin{aligned} LIFOWG(\beta_1, \beta_2, \dots, \beta_n) &= \beta_{\eta(1)}^{\psi_1} \otimes \beta_{\eta(2)}^{\psi_2} \otimes \dots \otimes \beta_{\eta(n)}^{\psi_n} \\ &= (s_{k \prod_{\varepsilon=1}^n (\varkappa_{\eta(\varepsilon)} | k)^{\psi_{\varepsilon}}}, s_{k \prod_{\varepsilon=1}^n (1 - \varrho_{\eta(\varepsilon)} | k)^{\psi_{\varepsilon}}}) \end{aligned} \quad (2. 4)$$

where  $\beta_{\eta(t)} = (s_{\varkappa_{\eta(t)}}, s_{\varrho_{\eta(t)}})$  as any  $t^{th}$  (largest) of  $(\beta_1, \beta_2, \dots, \beta_n)$ .

(c) LIFHG operator is

$$\begin{aligned} LIFHG(\beta_1, \beta_2, \dots, \beta_n) &= \beta_{\eta(1)}^{\circ \psi_1} \otimes \beta_{\eta(2)}^{\circ \psi_2} \otimes \dots \otimes \beta_{\eta(n)}^{\circ \psi_n} \\ &= (s_{k \prod_{\varepsilon=1}^n (\varkappa_{\eta(\varepsilon)}^{\circ} | k)^{\psi_{\varepsilon}}}, s_{k \prod_{\varepsilon=1}^n (1 - \varrho_{\eta(\varepsilon)}^{\circ} | k)^{\psi_{\varepsilon}}}) \end{aligned} \quad (2. 5)$$

where  $\beta_{\eta(t)}^{\circ} = (s_{\varkappa_{\eta(t)}^{\circ}}, s_{\varrho_{\eta(t)}^{\circ}})$  as any  $t^{th}$  (largest) of  $(\beta_1, \beta_2, \dots, \beta_n)$ .

**Definition 2. 3.** [42] Let  $\check{S}^{[0,8]}$  be a linguistic complex IFS  $E$  in  $A$  is defined as,

$$E = \{(e, s_{\varkappa_{E(a)}} e^{i w_{\varkappa_{E(a)}}}, s_{\varrho_{E(a)}} e^{i w_{\varrho_{E(a)}}} | a \in A)\}, \quad (2. 6)$$

where  $s_{\varkappa E(a)}e^{iw_{\varkappa E(a)}}, s_{\varrho E(a)}e^{iw_{\varrho E(a)}} \in \check{S}^{[0,k]}$ ;  $(\check{S}^{[0,k]} = \{s_t e^{iw_t} | s_0 e^{iw_0} \leq s_t e^{iw_t} \leq s_k e^{iw_k}\})$  are the linguistic complex membership (LCM) and linguistic complex non-membership (LCNM) degrees of  $e \in A$ , respectively.  $\forall, a \in A, o \leq \varkappa_E + \varrho_E \leq k$  is any ways satisfied, and the index of  $e \in A$ , in, the linguistic complex intuitionistic is defined as  $s_{\pi E}e^{iw_{\pi E}} = s_{k-\varkappa_E-\varrho_E}e^{iw_{k-\varkappa_E-\varrho_E}}$ . Usually, the pair  $(s_{\varkappa E(a)}e^{iw_{\varkappa E(a)}}, s_{\varrho E(a)}e^{iw_{\varrho E(a)}})$  is known as linguistic complex intuitionistic fuzzy number (LCIFN), and it can be simplified as  $\beta = (s_{\varkappa}e^{iw_{\varkappa}}, s_{\varrho}e^{iw_{\varrho}}), s_{\varkappa}e^{iw_{\varkappa}}, s_{\varrho}e^{iw_{\varrho}} \in \check{S}^{[0,k]}$  and  $\varkappa + \varrho \leq k$ , if  $\varkappa + \varrho \in S$  where  $S = \{s_t e^{iw_t} | t = 0, 1, 2, \dots, k\}$ , the LCIFN is commonly referred to as original, and therefore as imaginary.

**Definition 2. 4.** [42, 43] Let  $\beta = (s_{\varkappa}e^{iw_{\varkappa}}, s_{\varrho}e^{iw_{\varrho}})$  (be any LCIFN). Score function of  $\beta$  is described below.

$$S(\beta) = s_{(\varkappa-\varrho+k)/2}e^{iw_{(\varkappa-\varrho+k)/2}}, \tag{2. 7}$$

and the accuracy function of  $\beta$  can be expressed as

$$H(\beta) = s_{\varkappa+\varrho}e^{iw_{\varkappa+\varrho}}. \tag{2. 8}$$

**Definition 2. 5.** [42, 43] Let  $\beta_1 = (s_{\varkappa_1}e^{iw_{\varkappa_1}}, s_{\varrho_1}e^{iw_{\varrho_1}})$  and  $\beta_2 = (s_{\varkappa_2}e^{iw_{\varkappa_2}}, s_{\varrho_2}e^{iw_{\varrho_2}})$  be any two LIFNs, then  $\beta_1$  and  $\beta_2$  can be rank based on the following procedure;

- (1) If  $S(\beta_1) > S(\beta_2)$  then  $\beta_1 > \beta_2$ ,
- (2) If  $S(\beta_1) = S(\beta_2)$ , then
- (3) If  $H(\beta_1) > H(\beta_2) \iff \beta_1 > \beta_2$ ,
- (4) If  $H(\beta_1) = H(\beta_2) \iff \beta_1 = \beta_2$

The following aggregating operators for aggregation set of LIFNs,

$\beta_t = (s_{\varkappa_t}e^{iw_{\varkappa_t}}, s_{\varrho_t}e^{iw_{\varrho_t}}), t = 1, 2, \dots, n$ , the affiliated weight vector,  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  of  $\beta_t$ , as  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$

**(a) LCIFWG operator**

$$\begin{aligned} &LIFWG(\beta_1, \beta_2, \dots, \beta_n) \\ &= \beta_1^{\psi_1} \otimes \beta_2^{\psi_2} \otimes \dots \otimes \beta_n^{\psi_n} \\ &= (s_{k \prod_{\varepsilon=1}^n (\varkappa_{\varepsilon} | k)^{\psi_{\varepsilon}}} e^{iw_{k \prod_{\varepsilon=1}^n (\varkappa_{\varepsilon} | k)^{\psi_{\varepsilon}}}}, s_{k \prod_{\varepsilon=1}^n (1-\varrho_{\varepsilon} | k)^{\psi_{\varepsilon}}} e^{iw_{k \prod_{\varepsilon=1}^n (1-\varrho_{\varepsilon} | k)^{\psi_{\varepsilon}}}}) \end{aligned} \tag{2. 9}$$

**(b) LCIFOWG operator**

$$\begin{aligned} &LIFOWG(\beta_1, \beta_2, \dots, \beta_n) = \beta_{\eta(1)}^{\psi_1} \otimes \beta_{\eta(2)}^{\psi_2} \otimes \dots \otimes \beta_{\eta(n)}^{\psi_n} \\ &= \left( s_{k \prod_{\varepsilon=1}^n (\varkappa_{\eta(\varepsilon)} | k)^{\psi_{\varepsilon}}} e^{iw_{k \prod_{\varepsilon=1}^n (\varkappa_{\eta(\varepsilon)} | k)^{\psi_{\varepsilon}}}}, s_{k \prod_{\varepsilon=1}^n (1-\varrho_{\eta(\varepsilon)} | k)^{\psi_{\varepsilon}}} e^{iw_{k \prod_{\varepsilon=1}^n (1-\varrho_{\eta(\varepsilon)} | k)^{\psi_{\varepsilon}}}} \right) \end{aligned}$$

where  $\beta_{\eta(t)} = (s_{\varkappa_{\eta(t)}}e^{iw_{\varkappa_{\eta(t)}}}, s_{\varrho_{\eta(t)}}e^{iw_{\varrho_{\eta(t)}}})$  is  $t^{th}$  (largest) of  $(\beta_1, \beta_2, \dots, \beta_n)$

**(c) LCIFHG operator**

$$\begin{aligned} LIFHG(\beta_1, \beta_2, \dots, \beta_n) &= \beta_{\eta(1)}^{\circ\psi_1} \otimes \beta_{\eta(2)}^{\circ\psi_2} \otimes \dots \otimes \beta_{\eta(n)}^{\circ\psi_t} \\ &= \left( s_{k\Pi_{\varepsilon=1}^n(\varkappa_{\eta(t)}^{\circ})|k}^{\psi_t} e^{iw_{k\Pi_{\varepsilon=1}^n(\varkappa_{\eta(t)}^{\circ})|k} \psi_t}, s_{k\Pi_{\varepsilon=1}^n(1-\varrho_{\eta(t)}^{\circ})|k}^{\psi_t} e^{iw_{k\Pi_{\varepsilon=1}^n(1-\varrho_{\eta(t)}^{\circ})|k} \psi_t} \right) \end{aligned}$$

where  $\beta_{\eta(t)}^{\circ} = \left( s_{\varkappa_{\eta(t)}^{\circ}} e^{iw_{\varkappa_{\eta(t)}^{\circ}}}, s_{\varrho_{\eta(t)}^{\circ}} e^{iw_{\varrho_{\eta(t)}^{\circ}}} \right)$  is  $t^{th}$  (largest) of  $(\beta_1, \beta_2, \dots, \beta_n)$ .

**2.1. Deficiency of the Existing Operators.** Although the researchers have utilized these aggregation operators extensively for linguistic complex intuitionistic fuzzy data, we have noticed specific drawbacks that can be explained using these counter examples:

**Example.** Let  $S^{[0,8]} = \{s_l e^{iw_l} | s_0 e \leq s_l e^{iw_l} \leq s_8 e^{iw_8}\}$  represent a set of linguistic terms, that is continuous and  $E = (\beta_1, \beta_2, \beta_3)$  be three LCIFNs, where  $\beta_1 = (s_4 e^{iw_4}, s_4 e^{iw_4})$ ,  $\beta_2 = (s_2 e^{iw_2}, s_5 e^{iw_5})$ , and  $\beta_3 = (s_0 e^{iw_0}, s_8 e^{iw_8})$  are LCIFNs and  $\psi = (0.3, 0.5, 0.2)^T$  is the weight vector affiliated with these figures. Through the use of the LCIFWG operator defined in [42], to aggregate these LCIFNs, we get  $LCIFWG(\beta_1, \beta_2, \beta_3) = (s_0 e^{iw_0}, s_8 e^{iw_8})$ . On the other hand, if we take another collections of LCIFNs  $F = (\gamma_1, \gamma_2, \gamma_3)$ , where  $\gamma_1 = (s_3 e^{iw_3}, s_4 e^{iw_4})$ ,  $\gamma_2 = (s_0 e^{iw_0}, s_8 e^{iw_8})$ , and  $\gamma_3 = (s_2 e^{iw_2}, s_2 e^{iw_2})$  corresponding to the same weight set, then we get  $LCIFWG(\gamma_1, \gamma_2, \gamma_3) = (s_0 e^{iw_0}, s_8 e^{iw_8})$ . Consequently, accuracy and scored function, since the alternatives for  $E$  and  $F$  are equivalent and could not be ranked. As such, selecting the optimal option from the list of available options utilizing the LCIFWG operators is challenging.

**Example.** Let  $S^{[0,8]}$  represent a set of linguistic terms, that is continuous and let  $E = (\beta_1, \beta_2, \beta_3, \beta_4)$  be a collection of LCIFNs, where  $\beta_1 = (s_4 e^{iw_4}, s_3 e^{iw_3})$ ,  $\beta_2 = (s_2 e^{iw_2}, s_4 e^{iw_4})$ ,  $\beta_3 = (s_5 e^{iw_5}, s_1 e^{iw_1})$ , and  $\beta_4 = (s_4 e^{iw_4}, s_2 e^{iw_2})$  are LCIFNs,  $\psi = (0.3, 0.2, 0.3, 0.2)^T$  be affiliate weight vector of these LCIFNs. In the occurrence that we integrate these LCIFNs, with the implementation of with the implementation of with the implementation of LCIFWG operators, then we get  $LCIFWG(\beta_1, \beta_2, \beta_3, \beta_4) = (s_{3.723} e^{iw_{3.723}}, s_{2.514} e^{iw_{2.514}})$ . However, if we modify each LCIFN's non-membership degree and obtain an alternative collection  $F = (\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ , where  $\gamma_1 = (s_4 e^{iw_4}, s_1 e^{iw_1})$ ,  $\gamma_2 = (s_2 e^{iw_2}, s_2 e^{iw_2})$ ,  $\gamma_3 = (s_5 e^{iw_5}, s_3 e^{iw_3})$  and  $\gamma_4 = (s_4 e^{iw_4}, s_4 e^{iw_4})$ , then  $LCIFWG(\gamma_1, \gamma_2, \gamma_3, \gamma_4) = (s_{3.723} e^{iw_{3.723}}, s_{2.514} e^{iw_{2.514}})$ . As a result, we draw the conclusion that altering the 'LCIFNs' grade of non-membership has a distinct effect overall the total non-membership grade. Therefore, functions for scores and accuracy, Since  $E$  and  $F$  are identical, they are unable to rate the options. As a result, utilizing the LCIFWG operator to select the best option from the list of options is challenging. Consequently, it has been determined that the current LCIFWG operator is insufficient for ranking the alternatives; as a result, further research into this matter and the requirement for alternative measurement functions are required. In order to address this, the set pair (SPA) [27] analytical technique has led to the introduction of a new aggregation operator.

### 3. LINGUISTIC COMPLEX INTUITIONISTIC FUZZY CONNECTION NUMBER DEVELOPED WITH THE SPA THEORY

Under the SPA analytical technique, we constructed a linguistic connection number in this part for the linguistic complex intuitionistic fuzzy number (LCIFN<sub>S</sub>). Further, several of their fundamental traits and roles are explained.

**Definition 3. 1.** Similarities exist between probabilistic, fuzzy, harsh, and ambiguous theories of uncertainty and SPA, an advanced theory of ambiguity. It offers an alternative method of expressing the ambiguity that both certainty and uncertainty view as a cohesive system of specific uncertainties about an item. The integrated system of the particular and dubious entity is made up of the two interconnected sets  $E$  and  $F$ , which together form  $H(E, F)$ . In relation to issue  $W$ . Concerning problem  $W$ . The ability to analyze the system for "equivalent," "variants," and "contrary" elements of a particular problem  $W$  and determine a connection number for it is the most significant aspect to the system (SPA). Between it,  $A$  may be used to symbolize the identical and contradicting characteristics for any characteristic, as it is neither an opposing conclusion nor a characteristic of the sets  $E$  and  $F$ . Here, It displays the connection number  $\rho$  as,

$$\rho = (e' + f\iota + g\kappa), \quad (3. 10)$$

where  $e' = S/N$ ,  $f = A/N$  and  $g = P/N$  illustrates the degrees of "equivalent", "variants" and "contrary" such that

$$0 < e', f, g \leq 1 \text{ and } e' + f + g = 1; \iota \in [-1, 1] \text{ where } \kappa = -1 \quad (3. 11)$$

are the coefficients of "equivalent," "variants," and "contrary" respectively.

**Definition 3. 2.** If  $\rho_1 = (e'_1 + f_1\iota + g_1\kappa)$  and  $\rho_2 = (e'_2 + f_2\iota + g_2\kappa)$  be any two (CNs) connection numbers, then

- (1)  $\rho_1 \oplus \rho_2 = (e'_1 + e'_2) |2 + ((f_1 + f_2)|2)\iota + ((g_1 + g_2) |2)\kappa$
- (2)  $\rho_1 \otimes \rho_2 = (e'_1 e'_2) + (e'_1 f_2 + f_1 e'_2 + f_1 f_2 + f_1 g_2 + g_1 f_2)\iota + (e'_1 g_2 + g_1 e'_2 + g_1 g_2) \kappa.$

For addressing the standard component, we are going to define the (linguistic complex intuitionistic fuzzy connection number) LCIFCN as follows:

**Definition 3. 3.** Let  $\check{S}^{[0,s]}$  represent a set of linguistic terms, that is continuous. A linguistic complex intuitionistic fuzzy connection number is defined as,

$$\rho = s_{e'} e^{iw_{e'}} + s_f e^{iw_f} \iota + s_g e^{iw_g} \kappa. \quad (3. 12)$$

Where  $s_{e'} e^{iw_{e'}}$ ,  $s_f e^{iw_f}$ ,  $s_g e^{iw_g} \in \check{S}^{[0,h]}$  represents, in that order, the degrees of "equivalent," "variants," and "contrary" respectively, where  $0 < e', f, g \leq 1$  and  $e' + f + g = 1$ ;  $\iota \in [-1, 1]$  where  $\kappa = -1$

**Definition 3. 4.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa$  probability power  $P(\rho_t)$  of the LCIFCN with respect to certainty defined as

$P(\rho_t) = s_{[k+2e_t|(f_t+g_t)-g_t|(e_t+f_t)]|2} e^{iw_{[k+2e_t|(f_t+g_t)-g_t|(e_t+f_t)]|2}}$ . The LCIFCN  $\rho_t$  increases with increasing  $P(\rho_t)$  value.

**Definition 3. 5.** Suppose  $L$  be the finite universal set represented by the elements  $\{l_1, l_2, \dots, l_n\}$ , and  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa$ , where  $e_t = \varkappa_t (k - \varrho_t) |k$  "identity"  $f_t = k - \varkappa_t (k - \varrho_t) |k - \varrho_t (k - \varkappa_t) |k$  "discrepancy" and  $g_t = \varrho_t (k - \varkappa_t) |k$  "contrary" degrees.

**Definition 3. 6.** Let  $\rho_1 = s_{e_1} e^{iw_{e_1}} + s_{f_1} e^{iw_{f_1}l} + s_{g_1} e^{iw_{g_1}k}$  and  $\rho_2 = s_{e_2} e^{iw_{e_2}} + s_{f_2} e^{iw_{f_2}l} + s_{g_2} e^{iw_{g_2}k}$  be two LCIFCNs, then

- (1)  $\rho_1 = \rho_2 \Leftrightarrow s_{e_1} e^{iw_{e_1}} = s_{e_2} e^{iw_{e_2}}, s_{f_1} e^{iw_{f_1}} = s_{f_2} e^{iw_{f_2}},$
- (2)  $s_{g_1} e^{iw_{g_1}} = s_{g_2} e^{iw_{g_2}} \Leftrightarrow e_1 = e_2, f_1 = f_2, g_1 = g_2$
- (3)  $\rho_1 \leq \rho_2$  if  $s_{e_1} e^{iw_{e_1}} \leq s_{e_2} e^{iw_{e_2}}, s_{g_1} e^{iw_{g_1}} \geq s_{g_2} e^{iw_{g_2}} \Leftrightarrow e_1 \leq e_2, g_1 \geq g_2$
- (4)  $\rho_1^g = s_{e_1} e^{iw_{e_1}} + s_{f_1} e^{iw_{f_1}l} + s_{g_1} e^{iw_{g_1}k}$
- (5)  $\rho_1 \cup \rho_2 = \max\{s_{e_1} e^{iw_{e_1}}, s_{e_2} e^{iw_{e_2}}\} + (s_k e^{iw_k} - \max\{s_{e_1} e^{iw_{e_1}}, s_{e_2} e^{iw_{e_2}}\}),$   
 $\min\{s_{g_1} e^{iw_{g_1}}, s_{g_2} e^{iw_{g_2}}\}l + (\min\{s_{g_1} e^{iw_{g_1}}, s_{g_2} e^{iw_{g_2}}\})k$
- (6)  $\rho_1 \cap \rho_2 = \min\{s_{e_1} e^{iw_{e_1}}, s_{e_2} e^{iw_{e_2}}\} + (s_k e^{iw_k} - \min\{s_{e_1} e^{iw_{e_1}}, s_{e_2} e^{iw_{e_2}}\}),$

$$\max\{s_{g_1} e^{iw_{g_1}}, s_{g_2} e^{iw_{g_2}}\}l + (\max\{s_{g_1} e^{iw_{g_1}}, s_{g_2} e^{iw_{g_2}}\})k.$$

**Definition 3. 7.** Let  $\rho_1 = s_{e_1} e^{iw_{e_1}} + s_{f_1} e^{iw_{f_1}l} + s_{g_1} e^{iw_{g_1}k}$  and  $\rho_2 = s_{e_2} e^{iw_{e_2}} + s_{f_2} e^{iw_{f_2}l} + s_{g_2} e^{iw_{g_2}k}$ . Let  $\lambda > 0$  and there exist any two LCCNs. Then, let us define some operations between Let  $\lambda > 0$  and there exist any two LCCNs. Then, let us define some operations between  $\rho_1$  and  $\rho_2$

**Theorem 3. 1.** The operations specified in (3) are likewise LCCNs if  $\rho_1$  and  $\rho_2$  are any two LCCNs.

**Proof.**

- (1) Let  $\rho_1 = s_{e_1} e^{iw_{e_1}} + s_{f_1} e^{iw_{f_1}l} + s_{g_1} e^{iw_{g_1}k}$  and  $\rho_2 = s_{e_2} e^{iw_{e_2}} + s_{f_2} e^{iw_{f_2}l} + s_{g_2} e^{iw_{g_2}k}$  be any two LCCNs, so that  $0 \leq e_1, f_1, g_1, e_2, f_2, g_2 \leq k$  and  $e_1 + f_1 + g_1 = e_2 + f_2 + g_2 = k$ , next, we have. Let  $0 \leq e_1, e_2 \leq k \Rightarrow 0 \leq k(e_1 e_2 | k^2) \leq k$ ,  $0 \leq f_1 f_2 \leq k$  and  $0 \leq g_1 g_2 \leq k$  we have

$$\begin{aligned} 0 &\leq (1 - f_1|k), (1 - f_2|k) \leq 1 \Rightarrow 0 \leq (1 - f_1|k)(1 - f_2|k) \leq 1 \\ &\Rightarrow 0 \leq 1 - (1 - f_1|k)(1 - f_2|k) \leq 1 \\ &\Rightarrow 0 \leq k(1 - (1 - f_1|k)(1 - f_2|k)) \leq k \end{aligned}$$

and

$$\begin{aligned} 0 &\leq k(e_1|k + g_1|k)(e_2|k + g_2|k) - (e_1 e_2|k) \leq k \\ &\Rightarrow k(e_1 e_2|k^2) + k(1 - (1 - f_1|k)(1 - f_2|k)) \\ &\quad + k(e_1|k + g_1|k)(e_2|k + g_2|k) - (e_1 e_2|k) \\ &= k \left( \begin{aligned} &e_1 e_2|k^2 + 1 - (e_1|k + g_1|k)(e_2|k + g_2|k) \\ &+ (e_1|k + g_1|k)(e_2|k + g_2|k) + e_1 e_2|k^2 \end{aligned} \right) \\ &= k. \end{aligned}$$

Hence,  $\rho_1 \otimes \rho_2$  is also LCIFWACN.

(2) Let  $\hbar > 0$  and  $0 \leq k (e_1|k)^\hbar \leq k$ ,

$$\begin{aligned} 0 &\leq k \left(1 - (1 - f_1|k)^\hbar\right) = k \left(1 - (e_1|k + g_1|k)^\hbar\right) \leq k, \\ 0 &\leq k (e_1|k + g_1|k)^\hbar - (e_1|k)^\hbar \leq k \text{ and} \\ &k \left((e_1|k)^\hbar + \left(1 - (1 - f_1|k)^\hbar\right) + (e_1|k + g_1|k)^\hbar - (e_1|k)^\hbar\right) \\ &= k \left((e_1|k)^\hbar + 1 - (e_1|k + g_1|k)^\hbar + (e_1|k + g_1|k)^\hbar - (e_1|k)^\hbar\right) \\ &= k \text{ so } \rho_1^\hbar \text{ is LCIFCN.} \end{aligned}$$

**Theorem 3. 2.** Consider  $\rho_1 = s_{e_1} e^{iw_{e_1}} + s_{f_1} e^{iw_{f_1}} \iota + s_{g_1} e^{iw_{g_1}} \kappa$  and  $\rho_2 = s_{e_2} e^{iw_{e_2}} + s_{f_2} e^{iw_{f_2}} \iota + s_{g_2} e^{iw_{g_2}} \kappa$  any two LCIFCNs and  $\hbar, \hbar_1, \hbar_2 > 0$  any three real numbers. The characteristics listed below are thus valid.

- (1)  $\rho_1 \otimes \rho_2 = \rho_2 \otimes \rho_1$ ,
- (2)  $\rho_1^{\hbar_1} \otimes \rho_2^{\hbar_2} = \rho_1^{\hbar_1 + \hbar_2}$ ,
- (3)  $\rho_1^\hbar \otimes \rho_2^\hbar = (\rho_1 \otimes \rho_2)^\hbar$ ,
- (4)  $\rho_1^c \cup \rho_2^c = (\rho_1 \cap \rho_2)^c$ ,
- (5)  $\rho_1^c \cap \rho_2^c = (\rho_1 \cup \rho_2)^c$ ,
- (6)  $(\rho_2 \cup \rho_1) \cap \rho_2 = \rho_2$ ,
- (7)  $(\rho_2 \cap \rho_1) \cup \rho_2 = \rho_2$ ,
- (8)  $\rho_2 \cup \rho_1 = \rho_1 \cup \rho_2$ ,
- (9)  $\rho_2 \cap \rho_1 = \rho_1 \cap \rho_2$ ,

**Proof.** Only parts (i)-(iii) will be proved here; the remaining parts can be verified similarly,

- (1) Let Consider  $\rho_1 = s_{e_1} e^{iw_{e_1}} + s_{f_1} e^{iw_{f_1}} \iota + s_{g_1} e^{iw_{g_1}} \kappa$  and  $\rho_2 = s_{e_2} e^{iw_{e_2}} + s_{f_2} e^{iw_{f_2}} \iota + s_{g_2} e^{iw_{g_2}} \kappa$  be any two LCIFCNs, so

$$\begin{aligned} \rho_1 \otimes \rho_2 &= \left( s_{e_1 e_2 |k} e^{iw_{e_1 e_2 |k}} \right) + \left( s_{k(1-(1-e_1|k)(1-e_2|k))} e^{iw_{k(1-(1-e_1|k)(1-e_2|k))}} \right) \iota \\ &\quad + \left( s_{k((e_1|k+g_1|k)(e_2|k+g_2|k)-e_1 e_2 |k^2)} e^{iw_{k((e_1|k+g_1|k)(e_2|k+g_2|k)-e_1 e_2 |k^2)}} \right) \kappa \\ &= \left( s_{e_2 e_1 |k} e^{iw_{e_2 e_1 |k}} \right) + \left( s_{k(1-(1-e_2|k)(1-e_1|k))} e^{iw_{k(1-(1-e_2|k)(1-e_1|k))}} \right) \iota \\ &\quad + \left( s_{k((e_2|k+g_2|k)(e_1|k+g_1|k)-e_2 e_1 |k^2)} e^{iw_{k((e_2|k+g_2|k)(e_1|k+g_1|k)-e_2 e_1 |k^2)}} \right) \kappa \\ &= \rho_2 \otimes \rho_1 \end{aligned}$$

(2) For  $\hbar_1, \hbar_2 > 0$ , we have

$$\begin{aligned}\rho_1^{\hbar_1} &= \left( s_k(e_1|k)^{\hbar_1} e^{iw_k(e_1|k)^{\hbar_1}} \right) + \left( s_k(1-(1-f_1|k)^{\hbar_1}) e^{iw_k(1-(1-f_1|k)^{\hbar_1})} \right)_{\mathcal{L}} \\ &\quad + \left( s_k((e_1|k+g_1|k)^{\hbar_1} - (e_1|k)^{\hbar_1}) e^{iw_k((e_1|k+g_1|k)^{\hbar_1} - (e_1|k)^{\hbar_1})} \right)_{\mathcal{K}} \\ \rho_1^{\hbar_2} &= \left( s_k(e_1|k)^{\hbar_2} e^{iw_k(e_1|k)^{\hbar_2}} \right) + \left( s_k(1-(1-f_1|k)^{\hbar_2}) e^{iw_k(1-(1-f_1|k)^{\hbar_2})} \right)_{\mathcal{L}} \\ &\quad + \left( s_k((e_1|k+g_1|k)^{\hbar_2} - (e_1|k)^{\hbar_2}) e^{iw_k((e_1|k+g_1|k)^{\hbar_2} - (e_1|k)^{\hbar_2})} \right)_{\mathcal{K}}\end{aligned}$$

Then,

$$\begin{aligned}\rho_1^{\hbar_1} \otimes \rho_1^{\hbar_2} &= \left( s_{k(e_1|k)^{\hbar_1}(e_1|k)^{\hbar_2}} e^{iw_k(e_1|k)^{\hbar_1}(e_1|k)^{\hbar_2}} \right) \\ &\quad + \left( s_{k(1-(1-f_1|k)^{\hbar_1}(1-f_1|k)^{\hbar_2})} e^{iw_k(1-(1-f_1|k)^{\hbar_1}(1-f_1|k)^{\hbar_2})} \right)_{\mathcal{L}} \\ &\quad + \left( \begin{array}{l} s_k((e_1|k+g_1|k)^{\hbar_1}(e_1|k+g_1|k)^{\hbar_2} - (e_1|k)^{\hbar_1}(e_1|k)^{\hbar_2}) \\ e^{iw_k((e_1|k+g_1|k)^{\hbar_1}(e_1|k+g_1|k)^{\hbar_2} - (e_1|k)^{\hbar_1}(e_1|k)^{\hbar_2})} \end{array} \right)_{\mathcal{K}} \\ &= \left( s_k(e_1|k)^{\hbar_1+\hbar_2} e^{iw_k(e_1|k)^{\hbar_1+\hbar_2}} \right) \\ &\quad + \left( s_k(1-(1-f_1|k)^{\hbar_1+\hbar_2}) e^{iw_k(1-(1-f_1|k)^{\hbar_1+\hbar_2})} \right)_{\mathcal{L}} \\ &\quad + \left( \begin{array}{l} s_k((e_1|k+g_1|k)^{\hbar_1+\hbar_2} - (e_1|k)^{\hbar_1+\hbar_2}) \\ e^{iw_k((e_1|k+g_1|k)^{\hbar_1+\hbar_2} - (e_1|k)^{\hbar_1+\hbar_2})} \end{array} \right)_{\mathcal{K}} \\ &= \rho_1^{\hbar_1+\hbar_2}.\end{aligned}$$

(3) For  $\hbar > 0$ , we have

$$\begin{aligned}\rho_1^{\hbar} &= \left( s_k(e_1|k)^{\hbar} e^{iw_k(e_1|k)^{\hbar}} \right) + \left( s_k(1-(1-f_1|k)^{\hbar}) e^{iw_k(1-(1-f_1|k)^{\hbar})} \right)_{\mathcal{L}} \\ &\quad + \left( s_k((e_1|k+g_1|k)^{\hbar} - (e_1|k)^{\hbar}) e^{iw_k((e_1|k+g_1|k)^{\hbar} - (e_1|k)^{\hbar})} \right)_{\mathcal{K}}\end{aligned}$$

and,

$$\begin{aligned}\rho_2^{\hbar} &= \left( s_k(e_2|k)^{\hbar} e^{iw_k(e_2|k)^{\hbar}} \right) + \left( s_k(1-(1-f_2|k)^{\hbar}) e^{iw_k(1-(1-f_2|k)^{\hbar})} \right)_{\mathcal{L}} \\ &\quad + \left( s_k((e_2|k+g_2|k)^{\hbar} - (e_2|k)^{\hbar}) e^{iw_k((e_2|k+g_2|k)^{\hbar} - (e_2|k)^{\hbar})} \right)_{\mathcal{K}},\end{aligned}$$

therefore

$$\begin{aligned}
 \rho_1^{\hbar} \otimes \rho_2^{\hbar} &= \left( s_{k(e_1|k)^{\hbar}(e_2|k)^{\hbar}} e^{iw_{k(e_1|k)^{\hbar}(e_2|k)^{\hbar}}} \right) \\
 &+ \left( s_{k(1-(1-f_1|k)^{\hbar}(1-f_2|k)^{\hbar})} e^{iw_{k(1-(1-f_1|k)^{\hbar}(1-f_2|k)^{\hbar})}} \right) \iota \\
 &+ \left( s_k \left( (e_1|k+g_1|k)^{\hbar}(e_2|k+g_2|k)^{\hbar} - (e_1|k)^{\hbar_1}(e_2|k)^{\hbar} \right) \right. \\
 &\quad \left. e^{iw_{k((e_1|k+g_1|k)^{\hbar}(e_2|k+g_2|k)^{\hbar} - (e_1|k)^{\hbar_1}(e_2|k)^{\hbar})}} \right) \kappa \\
 &= \left( s_k(e_1 e_2|k^2)^{\hbar} e^{iw_{k(e_1 e_2|k^2)^{\hbar}}} \right) + \\
 &\left( s_k(1-(1-f_1|k)^{\hbar}(1-f_2|k)^{\hbar}) e^{iw_{k(1-(1-f_1|k)^{\hbar}(1-f_2|k)^{\hbar})}} \right) \iota \\
 &+ \left( s_k \left( (e_1|k+g_1|k)^{\hbar}(e_2|k+g_2|k)^{\hbar} - (e_1|k)^{\hbar}(e_2|k)^{\hbar} \right) \right. \\
 &\quad \left. e^{iw_{k((e_1|k+g_1|k)^{\hbar}(e_2|k+g_2|k)^{\hbar} - (e_1|k)^{\hbar}(e_2|k)^{\hbar})}} \right) \kappa \\
 &= \left( s_k(e_1 e_2|k^2)^{\hbar} e^{iw_{k(e_1 e_2|k^2)^{\hbar}}} \right) + \\
 &\left( s_k(1-((1-f_1|k)(1-f_2|k))^{\hbar}) \right. \\
 &\quad \left. e^{iw_{k(1-((1-f_1|k)(1-f_2|k))^{\hbar})}} \right) \iota \\
 &\quad + \left( s_k \left( ((e_1|k+g_1|k)(e_2|k+g_2|k))^{\hbar} - (e_1 e_2|k^2)^{\hbar} \right) \right. \\
 &\quad \left. e^{iw_{k(((e_1|k+g_1|k)(e_2|k+g_2|k))^{\hbar} - (e_1 e_2|k^2)^{\hbar})}} \right) \kappa
 \end{aligned}$$

So,

$$\rho_1^{\hbar} \otimes \rho_2^{\hbar} = (\rho_1 \otimes \rho_2)^{\hbar}.$$

#### 4. LINGUISTIC COMPLEX INTUITIONISTIC FUZZY CONNECTION NUMBERS AGGREGATION OPERATORS

The proposed LCIFCN weighted geometric (LCIFCNWG), LCIFCN ordered weighted geometric (LCIFCNOWG), and LCIFCN hybrid geometric (LCIFNHG) aggregation operators, along with some desired features derived from them, will be discussed in this section.

**4.1. Linguistic Complex Intuitionistic fuzzy Connection Number Weighted Geometric Operator.** To compile the LCIFCNs, we will now suggest a weighted geometric operator.

**Definition 4. 1.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t} \iota} + s_{g_t} e^{iw_{g_t} \kappa}$ , consist of a group of LCIFCNs. The operator for weighted geometric linguistic complex intuitionistic fuzzy connection numbers (LCIFCNWG) is expanding  $LCIFCNWG : \Psi^n \rightarrow \Psi$ , such that

$$LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) = \rho^{\psi_1} \otimes \rho^{\psi_2} \otimes \dots \otimes \rho^{\psi_n}, \tag{4. 13}$$

and the associated weight vector is  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  of  $\rho_t \forall t = 1, 2, 3, \dots, n$  where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ . Especially, if  $\psi = (1|n, 1|n, \dots, 1|n)^T$ , then the linguistic

complex intuitionistic fuzzy connection number geometric (LCIFCNG) operator is reduced to the LCIFCNWG operator, which is defined as,

$$LCCNG(\rho_1, \rho_2, \dots, \rho_n) = (\rho_1 \otimes \rho_2 \otimes \dots \otimes \rho_n)^{1/n}. \quad (4.14)$$

**Theorem 4. 1.** Let  $\rho_t = s_{e_i} e^{iw e_i} + s_{f_t} e^{iw f_t} \iota + s_{g_t} e^{iw g_t} \kappa$ , ( $t = 1, 2, \dots, n$ ) consist of a collection of LCIFCNs. Furthermore, the corresponding weight vector is  $\psi = (\psi_1, \psi_2, \dots, \psi_n)^\tau$  of  $\rho_t$  where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ , then the aggregate of each of them by using the LCIFCNWG operator is also a linguistic complex connection number, and

$$\begin{aligned} LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) &= \left( s_{k \prod_{t=1}^n (e_i^+ |k)^{\psi_t}} e^{iw_{k \prod_{t=1}^n (e_i^+ |k)^{\psi_t}}} \right. \\ &\quad \left. + \left( s_{k(1 - \prod_{t=1}^n (1 - f_t |k)^{\psi_t})} e^{iw_{k(1 - \prod_{t=1}^n (1 - f_t |k)^{\psi_t})}} \right) \iota \right. \\ &\quad \left. + \left( s_{k(\prod_{t=1}^n (e_i^+ |k + g_t |k)^{\psi_t} - \prod_{t=1}^n (e_i^+ |k)^{\psi_t})} e^{iw_{k(\prod_{t=1}^n (e_i^+ |k + g_t |k)^{\psi_t} - \prod_{t=1}^n (e_i^+ |k)^{\psi_t})}} \right) \kappa. \right. \end{aligned} \quad (4.15)$$

**Proof.** We may now establish the preceding equation by applying the mathematical induction principle.

*Proof.* (i) For  $n = 2$ , we have

$$\begin{aligned} \rho_1^{\psi_1} &= \left( s_{k(e_1^+ |k)^{\psi_1}} e^{iw_{k(e_1^+ |k)^{\psi_1}}} \right) + \left( s_{k(1 - (1 - f_1 |k)^{\psi_1})} e^{iw_{k(1 - (1 - f_1 |k)^{\psi_1})}} \right) \iota \\ &\quad + \left( s_{k((e_1^+ |k + g_1 |k)^{\psi_1} - (e_1^+ |k)^{\psi_1})} e^{iw_{k((e_1^+ |k + g_1 |k)^{\psi_1} - (e_1^+ |k)^{\psi_1})}} \right) \kappa. \\ \rho_2^{\psi_2} &= \left( s_{k(e_2^+ |k)^{\psi_2}} e^{iw_{k(e_2^+ |k)^{\psi_2}}} \right) + \left( s_{k(1 - (1 - f_2 |k)^{\psi_2})} e^{iw_{k(1 - (1 - f_2 |k)^{\psi_2})}} \right) \iota \\ &\quad + \left( s_{k((e_2^+ |k + g_2 |k)^{\psi_2} - (e_2^+ |k)^{\psi_2})} e^{iw_{k((e_2^+ |k + g_2 |k)^{\psi_2} - (e_2^+ |k)^{\psi_2})}} \right) \kappa. \end{aligned}$$

$$\begin{aligned} \rho_1^{\psi_1} \otimes \rho_2^{\psi_2} &= \left( s_{k(e_1^+ |k)^{\psi_1} (e_2^+ |k)^{\psi_2}} e^{iw_{k(e_1^+ |k)^{\psi_1} (e_2^+ |k)^{\psi_2}}} \right) \\ &\quad + \left( s_{k(1 - (1 - f_1 |k)^{\psi_1} (1 - f_2 |k)^{\psi_2})} e^{iw_{k(1 - (1 - f_1 |k)^{\psi_1} (1 - f_2 |k)^{\psi_2})}} \right) \iota \\ &\quad + \left( s_{k((e_1^+ |k + g_1 |k)^{\psi_1} (e_2^+ |k + g_2 |k)^{\psi_2} - (e_1^+ |k)^{\psi_1} (e_2^+ |k)^{\psi_2})} \right. \\ &\quad \left. e^{iw_{k((e_1^+ |k + g_1 |k)^{\psi_1} (e_2^+ |k + g_2 |k)^{\psi_2} - (e_1^+ |k)^{\psi_1} (e_2^+ |k)^{\psi_2})}} \right) \kappa. \end{aligned}$$

Thus, for  $n = 2$ , the outcome is correct.

$$\begin{aligned}
 &LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) \\
 &= \rho_1^{\psi_1}, \rho_2^{\psi_2}, \dots, \rho_n^{\psi_n} \\
 &= \left( s_{k(e_1|k)}^\psi e^{iw_{k(e_1|k)}^\psi} \right) \\
 &\quad + \left( s_{k(1-(1-f_1|k)^\psi)} e^{iw_{k(1-(1-f_1|k)^\psi)}} \right) \iota \\
 &\quad + \left( s_{k((e_1|k+g_1|k)^\psi - (e_1|k)^\psi)} e^{iw_{k((e_1|k+g_1|k)^\psi - (e_1|k)^\psi)}} \right) \kappa.
 \end{aligned}$$

Now we have to prove  $n = \Upsilon + 1$

$$\begin{aligned}
 &LCIFCNWG(\rho_1, \rho_2, \dots, \rho_{\Upsilon+1}) = LCCNWG(\rho_1, \rho_2, \dots, \rho_{\Upsilon+1}) \otimes \rho_{\Upsilon+1}^{\psi_{\Upsilon+1}} \\
 &= \left( s_{k\Pi_{t=1}^\Upsilon(e_1|k)^\psi} e^{iw_{k\Pi_{t=1}^\Upsilon(e_1|k)^\psi}} (e_{\Upsilon+1}|k)^\psi \right)^{\psi_{\Upsilon+1}} + \\
 &\quad \left( s_{k(1-\Pi_{t=1}^\Upsilon(1-f_t|k)^\psi)} e^{iw_{k(1-\Pi_{t=1}^\Upsilon(1-f_t|k)^\psi)}} (1-f_{\Upsilon+1}|k)^\psi \right)^{\psi_{\Upsilon+1}} \\
 &\quad + \left( s_{k(\Pi_{t=1}^\Upsilon(e_1|k+g_t|k)^\psi + (g_{\Upsilon+1}|k)^\psi - \Pi_{t=1}^\Upsilon(e_1|k)^\psi)} e^{iw_{k(\Pi_{t=1}^\Upsilon(e_1|k+g_t|k)^\psi + (g_{\Upsilon+1}|k)^\psi - \Pi_{t=1}^\Upsilon(e_1|k)^\psi)}} (e_{\Upsilon+1}|k)^\psi \right)^{\psi_{\Upsilon+1}} \kappa.
 \end{aligned}$$

Hence for  $n = \Upsilon + 1$  the outcome is correct. □

**Example.** Let  $\mathfrak{S}^{[0,8]} = \{s_z e^{iw_z} | s_0 e^{iw_0} \leq s_z e^{iw_z} \leq s_8 e^{iw_8}\}$  consist of a collection of continuous linguistic complex intuitionistic fuzzy terms. Let  $\rho_1 = s_1 e^{iw_1} + s_3 e^{iw_3} \iota + s_4 e^{iw_4} \kappa$ ,  $\rho_2 = s_3 e^{iw_3} + s_2 e^{iw_2} \iota + s_3 e^{iw_3} \kappa$  and  $\rho_3 = s_4 e^{iw_4} + s_3 e^{iw_3} \iota + s_1 e^{iw_1} \kappa$  be any three LCIFCNs and  $\psi = (0.3, 0.4, 0.3)^T$  is the connected weight vector of  $\rho_t$  where  $(t = 1, 2, 3)$ , then

$$\begin{aligned}
 &LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) \\
 &= \left( s_{s\Pi_{t=1}^3(e_t|8)^\psi} e^{iw_{s\Pi_{t=1}^3(e_t|8)^\psi}} \right) + \left( s_{s(1-\Pi_{t=1}^3(1-f_t|8)^\psi)} e^{iw_{s(1-\Pi_{t=1}^3(1-f_t|8)^\psi)}} \right) \iota \\
 &\quad + \left( s_{s(\Pi_{t=1}^3(e_t|8+g_t|8)^\psi - \Pi_{t=1}^3(e_t|8)^\psi)} e^{iw_{s(\Pi_{t=1}^3(e_t|8+g_t|8)^\psi - \Pi_{t=1}^3(e_t|8)^\psi)}} \right) \kappa \\
 &= s_8[(1|8)^{0.3}(3|8)^{0.4}(4|8)^{0.3}] e^{iw_{s[(1|8)^{0.3}(3|8)^{0.4}(4|8)^{0.3}]} \\
 &\quad + \left( s_8[1-(1-3|8)^{0.3}(1-2|8)^{0.4}(1-3|8)^{0.4}] e^{iw_{s[1-(1-3|8)^{0.3}(1-2|8)^{0.4}(1-3|8)^{0.4}]} \right) \iota \iota \\
 &\quad \left( s_8[(1|8+4|8)^{0.3}(3|8+3|8)^{0.4}(1|8+4|8)^{0.3} - (1|8)^{0.3}(3|8)^{0.4}(4|8)^{0.3}] e^{iw_{s[(1|8+4|8)^{0.3}(3|8+3|8)^{0.4}(1|8+4|8)^{0.3} - (1|8)^{0.3}(3|8)^{0.4}(4|8)^{0.3}]} \right) \kappa \\
 &= s_{2.352} e^{iw_{2.352}} + s_{2.622} e^{iw_{2.622}} \iota + s_{3.026} e^{iw_{3.026}} \kappa.
 \end{aligned}$$

It has now been determined that the suggested operator effectively addresses the shortcomings of the LCIFS theory's current operators, as described in above examples .

**Example.** If LCIFCNWG operator utilized to aggregate the LCIFNs. For this, firstly we convert the LCIFNs to its LCIFCNs, as

$$\begin{aligned} \gamma_1 &= s_2 e^{iw_2} + s_4 e^{iw_4} \iota + s_2 e^{iw_2} \kappa, \gamma_2 = s_{0.75} e^{iw_{0.75}} + s_{3.5} e^{iw_{3.5}} \iota + s_{3.75} e^{iw_{3.75}} \kappa, \\ \gamma_3 &= s_0 e^{iw_0} + s_0 e^{iw_0} \iota + s_8 e^{iw_8} \kappa, \text{ and } \Delta_1 = s_{1.5} e^{iw_{1.5}} + s_4 e^{iw_4} \iota + s_{2.5} e^{iw_{2.5}} \kappa, \\ \Delta_2 &= s_0 e^{iw_0} + s_0 e^{iw_0} \iota + s_8 e^{iw_8} \kappa, \Delta_3 = s_{1.5} e^{iw_{1.5}} + s_5 e^{iw_5} \iota + s_{1.5} e^{iw_{1.5}} \kappa. \text{ the} \\ LCIFCNWG(\gamma_1, \gamma_2, \gamma_3) &= s_0 e^{iw_0} + s_{3.127} e^{iw_{3.127}} \iota + s_{4.874} e^{iw_{4.874}} \kappa \text{ and} \\ LCIFCNWG(\Delta_1, \Delta_2, \Delta_3) &= s_0 e^{iw_0} + s_{2.659} e^{iw_{2.659}} \iota + s_{5.341} e^{iw_{5.341}} \kappa. \end{aligned}$$

Now, we get  $P(E) = s_{3.221} e^{iw_0}$  and  $P(F) = s_{2.996} e^{iw_{2.996}}$ . Since  $P(E) > P(F)$ , there for  $A > B$  as

$$\begin{aligned} \alpha_1 &= s_{2.5} e^{iw_{2.5}} + s_4 e^{iw_4} \iota + s_{1.5} e^{iw_{1.5}} \kappa, \alpha_2 = s_1 e^{iw_1} + s_4 e^{iw_4} \iota + s_3 e^{iw_3} \kappa, \\ \alpha_3 &= s_{4.375} e^{iw_{4.375}} + s_{3.25} e^{iw_{3.25}} \iota + s_{0.375} e^{iw_{0.375}} \kappa, \alpha_4 = s_3 e^{iw_3} + s_4 e^{iw_4} \iota + s_1 e^{iw_1} \kappa \\ \text{and } \beta_1 &= s_{3.5} e^{iw_{3.5}} + s_4 e^{iw_4} \iota + s_{0.5} e^{iw_{0.5}} \kappa, \beta_2 = s_{1.5} e^{iw_{1.5}} + s_5 e^{iw_5} \iota + s_{1.5} e^{iw_{1.5}} \kappa, \\ \beta_3 &= s_{3.125} e^{iw_{3.125}} + s_{3.75} e^{iw_{3.75}} \iota + s_{1.125} e^{iw_{1.125}} \kappa, \beta_4 = s_2 e^{iw_2} + s_4 e^{iw_4} \iota + s_2 e^{iw_2} \kappa \\ \text{then } LCIFCNWG(\alpha_1, \alpha_2, \alpha_3, \alpha_4) &= s_{2.553} e^{iw_{2.553}} + s_{3.788} e^{iw_{3.788}} \iota + s_{1.658} e^{iw_{1.658}} \kappa \text{ and} \\ LCIFCNWG(\beta_1, \beta_2, \beta_3, \beta_4) &= s_{2.553} e^{iw_{2.553}} + s_{4.154} e^{iw_{4.154}} \iota + s_{1.292} e^{iw_{1.292}} \kappa. \end{aligned}$$

Hence, we get  $S(A) = s_{4.338} e^{iw_{4.338}}$  and  $S(B) = s_{4.372} e^{iw_{4.372}}$ . Since  $S(A) < S(B)$ , therefore  $A < B$ .

Therefore, based on the research mentioned above, it can be concluded that the suggested geometric aggregation operator is effective in situations when the LCIFS theory's current aggregation operators are unable to identify the objects.

**Theorem 4. 2.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa$ ,  $t = 1, 2, \dots, n$  consist of a collection of LCIFCNs, and weight vector is  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  of  $\rho_{t=1,2,\dots,n}$  where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ , then utilizing the LCIFCNWG to calculate their total value. Additionally, the operator fulfills the following characteristics.

(C<sub>1</sub>). (Idempotency); For all  $\rho_t (t = 1, 2, \dots, n)$  are equal, i.e.,  $\rho_t = \rho$ , then

$$(\rho_1, \rho_2, \dots, \rho_n) = \rho \quad (4. 16)$$

(C<sub>2</sub>). (Boundedness); Let  $\rho^\triangleleft = s_{e^\triangleleft} e^{iw_{e^\triangleleft}} + s_{f^\triangleleft} e^{iw_{f^\triangleleft}} \iota + s_{g^\triangleleft} e^{iw_{g^\triangleleft}} \kappa$ , and  $\rho^\triangleright = s_{e^\triangleright} e^{iw_{e^\triangleright}} + s_{f^\triangleright} e^{iw_{f^\triangleright}} \iota + s_{g^\triangleright} e^{iw_{g^\triangleright}} \kappa$ , then  $e^\triangleleft = \min_t \{e_t\}$ ,  $g^\triangleleft = \max_t \{e_t + g_t\} - \min_t \{e_t\}$ , and  $f^\triangleleft = k - e^\triangleleft - g^\triangleleft$ , and  $e^\triangleright = \max_t \{e_t\}$ ,  $g^\triangleright = \min_t \{e_t + g_t\} - \max_t \{e_t\}$ , and  $f^\triangleright = k - e^\triangleright - g^\triangleright$

$$\rho^\triangleleft \leq LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) \leq \rho^\triangleright. \quad (4. 17)$$

(C<sub>3</sub>). (Monotonicity); If  $\rho_t \leq \rho_t^*$ , ( $t = 1, 2, \dots, n$ ), then

$$LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) \leq LCIFCNWG(\rho_1^*, \rho_2^*, \dots, \rho_n^*). \quad (4. 18)$$

**Proof.** (C<sub>1</sub>). Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t} \iota} + s_{g_t} e^{iw_{g_t} \kappa}$  for  $t = 1, 2, \dots, n$  and  $\rho = s_e e^{iw_e} + s_f e^{iw_f \iota} + s_g e^{iw_g \kappa}$  be any LCIFCNs. Assume that  $\rho_t = \rho, \forall$  any  $t$  where ( $t = 1, 2, \dots, n$ ) which implies that  $e_t = e, f_t = f$  and  $g_t = g$ , for every  $t$ .

$$\begin{aligned}
LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) &= \left( s_{k \prod_{t=1}^n (e_t |k)} e^{iw_{k \prod_{t=1}^n (e_t |k)} \psi_t} \right) \\
&\quad + \left( s_{k(1 - \prod_{t=1}^n (1 - f_t |k))} e^{iw_{k(1 - \prod_{t=1}^n (1 - f_t |k)) \psi_t} \right) \iota \\
&\quad + \left( s_{k(\prod_{t=1}^n (e_t |k + g_t |k)) - \prod_{t=1}^n (e_t |k)} e^{iw_{k(\prod_{t=1}^n (e_t |k + g_t |k)) - \prod_{t=1}^n (e_t |k)} \psi_t} \right) \kappa \\
&= \left( s_{k \prod_{t=1}^n (e |k)} e^{iw_{k \prod_{t=1}^n (e |k)} \psi_t} \right) + \left( s_{k(1 - \prod_{t=1}^n (1 - f |k))} e^{iw_{k(1 - \prod_{t=1}^n (1 - f |k)) \psi_t} \right) \iota \\
&\quad + \left( s_{k(\prod_{t=1}^n (e |k + g |k)) - \prod_{t=1}^n (e |k)} e^{iw_{k(\prod_{t=1}^n (e |k + g |k)) - \prod_{t=1}^n (e |k)} \psi_t} \right) \kappa \\
&\quad = \left( s_{k(e |k) \sum_{t=1}^n \psi_t} e^{iw_{k(e |k) \sum_{t=1}^n \psi_t} \right) \\
&\quad + \left( s_{k(1 - (1 - f |k) \sum_{t=1}^n \psi_t)} e^{iw_{k(1 - (1 - f |k) \sum_{t=1}^n \psi_t)} \right) \iota \\
&\quad + \left( s_{k((e |k + g |k) \sum_{t=1}^n \psi_t) - (e |k) \sum_{t=1}^n \psi_t} \right) \kappa \\
&\quad + \left( e^{iw_{k((e |k + g |k) \sum_{t=1}^n \psi_t) - (e |k) \sum_{t=1}^n \psi_t}} \right) \kappa \\
&= \left( s_{k(e |k)} e^{iw_{k(e |k)}} \right) + \left( s_{k(1 - (1 - f |k) \sum_{t=1}^n \psi_t)} e^{iw_{k(1 - (1 - f |k) \sum_{t=1}^n \psi_t)}} \right) \iota \\
&\quad + \left( s_{k((e |k + g |k) \sum_{t=1}^n \psi_t) - (e |k) \sum_{t=1}^n \psi_t} e^{iw_{k((e |k + g |k) \sum_{t=1}^n \psi_t) - (e |k) \sum_{t=1}^n \psi_t}} \right) \kappa \\
&= s_e e^{iw_e} + s_f e^{iw_f \iota} + s_g e^{iw_g \kappa} \\
&= \rho
\end{aligned}$$

(C<sub>2</sub>). For LCIFCNs  $\rho = s_e e^{iw_e} + s_f e^{iw_f \iota} + s_g e^{iw_g \kappa}$ , ( $t = 1, 2, \dots, n$ ), together with the weight vector that corresponds to them  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the weight vector of  $\rho_t$ , where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ .

Conversely, we have

$$\begin{aligned}
\left(\min_t \{e_t + g_t\}\right) |k &\leq (e_t + g_t) |k \leq \left(\max_t \{e_t + g_t\}\right) |k \\
&\implies \left(\min_t \{e_t + g_t\}\right) |k \leq \Pi_{t=1}^n (e_t + g_t) |k \\
&\leq \left(\max_t \{e_t + g_t\}\right) |k \\
&\implies \left(\min_t \{e_t + g_t\}\right) |k - \max_t \{e_t\} |k \\
&\leq \Pi_{t=1}^n \left(\min_t \{e_t + g_t\}\right) |k - \Pi_{t=1}^n \max_t \{e_t\} |k \\
&\leq \Pi_{t=1}^n \left(\min_t \{e_t + g_t\}\right) |k - \Pi_{t=1}^n \max_t \{e_t\} |k \\
&\leq \left(\max_t \{e_t + g_t\}\right) |k - \min_t \{e_t\} |k \\
&\implies \left(\min_t \{e_t + g_t\}\right) |k - \max_t \{e_t\} |k \\
&\leq k \Pi_{t=1}^n (e_t + g_t)^{\psi_t} |k - k \Pi_{t=1}^n (e_t)^{\psi_t} |k \\
&\leq \max_t \{e_t + g_t\} - \min_t \{e_t\}
\end{aligned}$$

There for

$$\begin{aligned}
g^\triangleleft &\leq k \Pi_{t=1}^n (e_t + g_t)^{\psi_t} |k - k \Pi_{t=1}^n (e_t)^{\psi_t} |k \leq g^\triangleright \\
&\implies g^\triangleleft \leq LCFCNOWG(\rho_1, \rho_2, \dots, \rho_n) \leq g^\triangleright
\end{aligned}$$

Since the proof for (C<sub>3</sub>) is identical to that of (C<sub>2</sub>), we leave it out here.

**4.2. Linguistic Complex Intuitionistic Fuzzy Connection Number Ordered Weighted Geometric (LCIFCNOWG) Operator.** We expand the concept of LCIFCNOWG aggregation to the LCIFCNOWG operator in this section.

**Definition 4. 2.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa$  as  $(t = 1, 2, \dots, n)$  represent a collection of LCIFCNs, then LCIFCNOWG maps are operators of dimension  $n$ , i.e.,  $LCIFCNOWG : \Psi^n \rightarrow \Psi$ , which is defined as:

$$LCIFCNOWG(\rho_1, \rho_2, \dots, \rho_n) = \beta_{\eta(1)}^{\psi_1} \otimes \beta_{\eta(2)}^{\psi_2} \otimes \dots \otimes \beta_{\eta(n)}^{\psi_t} \quad (4. 19)$$

where  $\eta : \{(1, 2, \dots, n) \rightarrow 1, 2, \dots, n\}$  the permutation map,  $\rho_{\eta(t)} = s_{e_{\eta(t)}} e^{iw_{e_{\eta(t)}}} + s_{f_{\eta(t)}} e^{iw_{f_{\eta(t)}}} \iota + s_{g_{\eta(t)}} e^{iw_{g_{\eta(t)}}} \kappa, \forall t^{th}$  largest value of the  $\rho_t$  as  $(t = 1, 2, \dots, n)$  and  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the associated weight vector of  $\rho_{\eta(t)}, \forall t$  as  $t = 1, 2, \dots, n$  where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ . Especially, if  $\psi = (1|n, 1|n, \dots, 1|n)^T$ , then the (LCIFCNOWG) operator decreases to (LCIFCNG) operator.

**Theorem 4. 3.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa, (t = 1, 2, \dots, n)$  be a collection of LCIFCNs, then a linguistic connection number can likewise be obtained from

their aggregated value by utilizing the LCIFCNOWG operation then a linguistic connection number can likewise be obtained from their aggregated value by utilizing the LCIFC-NOWG operation, and

$$\begin{aligned}
 &LCIFCNOWG(\rho_1, \rho_2, \dots, \rho_n) \\
 &= \left( s_{k^{\prod_{t=1}^n (e_{\eta(t)}^k)^{\psi_t}} e^{iw_{k^{\prod_{t=1}^n (e_{\eta(t)}^k)^{\psi_t}}} \right) \\
 &+ \left( s_{k^{(1-\prod_{t=1}^n (1-f_{\eta(t)}^k)^{\psi_t}} e^{iw_{k^{(1-\prod_{t=1}^n (1-f_{\eta(t)}^k)^{\psi_t}}} \right) \iota \\
 &+ \left( s_{k^{(\prod_{t=1}^n (e_{\eta(t)}^k + g_{\eta(t)}^k)^{\psi_t} - \prod_{t=1}^n (e_{\eta(t)}^k)^{\psi_t}} e^{iw_{k^{(\prod_{t=1}^n (e_{\eta(t)}^k + g_{\eta(t)}^k)^{\psi_t} - \prod_{t=1}^n (e_{\eta(t)}^k)^{\psi_t}}} \right) \kappa \tag{4. 20}
 \end{aligned}$$

where  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  represents the corresponding weight vector of LCIFCNOWG, s.t.,  $\psi_t > 0$  where  $\sum_{t=1}^n \psi_t = 1$

**Proof.** The above theorems' proofs are comparable to this one. LCIFNOWG The same properties are also satisfied by the operator, which are defined as follows:

**Theorem 4. 4.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa (t = 1, 2, \dots, n)$  be a collection of LCIFCNs, where  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the associated weight vector of LCIFCNOWG where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ , then LCIFCNOWG operator satisfies the following characteristics.

(C<sub>1</sub>). (Idempotency);  $\forall \rho_t$ , and  $t = 1, 2, \dots, n$  are equal, i.e.,  $\rho_t = \rho$ , for all  $t$ , then

$$LCIFCNOWG(\rho_1, \rho_2, \dots, \rho_n) = \rho. \tag{4. 21}$$

(C<sub>2</sub>). (Boundedness); Let  $\rho^\triangleleft = s_{e^\triangleleft} e^{iw_{e^\triangleleft}} + s_{f^\triangleleft} e^{iw_{f^\triangleleft}} \iota + s_{g^\triangleleft} e^{iw_{g^\triangleleft}} \kappa$  and  $\rho^\triangleright = s_{e^\triangleright} e^{iw_{e^\triangleright}} + s_{f^\triangleright} e^{iw_{f^\triangleright}} \iota + s_{g^\triangleright} e^{iw_{g^\triangleright}} \kappa$ , where  $e^\triangleleft = \min_t \{e_t\}$ ,  $g^\triangleleft = \max_t \{e_t + g_t\} - \min_t \{e_t\}$ , and  $f^\triangleleft = k - e^\triangleleft - g^\triangleleft$ , and  $e^\triangleright = \max_t \{e_t\}$ ,  $g^\triangleright = \min_t \{e_t + g_t\} - \max_t \{e_t\}$ , and  $f^\triangleright = k - e^\triangleright - g^\triangleright$  then

$$\rho^\triangleleft \leq LCIFCNOWG(\rho_1, \rho_2, \dots, \rho_n) \leq \rho^\triangleright. \tag{4. 22}$$

(C<sub>3</sub>). (Monotonicity); If  $\rho_t \leq \rho_t^*$ , as  $(t = 1, 2, \dots, n)$ , then

$$LCIFCNOWG(\rho_1, \rho_2, \dots, \rho_n) \leq LCCNOWG(\rho_1^*, \rho_2^*, \dots, \rho_n^*). \tag{4. 23}$$

**Proof.** The proof is comparable to Theorem 4.

**4.3. Linguistic Complex Intuitionistic Fuzzy Connection Number Hybrid Geometric (LCIFCNHG) Operator.** The *LCIFCNHG* operator, which weights both the *LCIFCN* and its ordered position, is an extension of the *LCIFCNWG* and *LCIFCNOWG* aggregation concept that we introduce in this section.

**Definition 4. 3.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa (t = 1, 2, \dots, n)$  consist of a group of LCIFCNs, a mapping is an operator of dimension  $n$ . *LCIFCNHG* :  $\Psi^n \rightarrow \Psi$ , which is define as follows:

$$LCIFCNHG(\rho_1, \rho_2, \dots, \rho_n) = \rho_{\eta(1)}^{\circ\psi_1} \otimes \rho_{\eta(2)}^{\circ\psi_2} \otimes \dots \otimes \rho_{\eta(n)}^{\circ\psi_t}, \tag{4. 24}$$

where  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the associated weight vector of *LCIFCNHG* where  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ , and  $\rho_{\eta(t)}^\circ = s_{e_{\eta(t)}^\circ} e^{iw_{e_{\eta(t)}^\circ}} + s_{f_{\eta(t)}^\circ} e^{iw_{f_{\eta(t)}^\circ}} \iota + s_{g_{\eta(t)}^\circ} e^{iw_{g_{\eta(t)}^\circ}} \kappa$  is the  $t^{th}$  largest of weighted *LCIFCN*s.  $\rho_t^\circ = \rho^{n\psi_t}$  and  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the weight vector of  $\rho_t$  ( $\forall t$  where  $t = 1, 2, \dots, n$ ).

**Theorem 4. 5.** Let  $\rho_t = s_{e_t} e^{iw_{e_t}} + s_{f_t} e^{iw_{f_t}} \iota + s_{g_t} e^{iw_{g_t}} \kappa$  ( $\forall t = 1, 2, \dots, n$ ) be a collection of *LCIFCN*s, then their aggregated value by using the *LCIFCNHG*, operator is also a linguistic connection number, and

$$\begin{aligned} & LCIFCNHG(\rho_1, \rho_2, \dots, \rho_n) & (4. 25) \\ & = \left( s_{k \prod_{t=1}^n (e_{\eta(t)}^\circ)^{\psi_t}} e^{iw_{k \prod_{t=1}^n (e_{\eta(t)}^\circ)^{\psi_t}}} \right) \\ & + \left( s_{k (1 - \prod_{t=1}^n (1 - f_{\eta(t)}^\circ)^{\psi_t})} e^{iw_{k (1 - \prod_{t=1}^n (1 - f_{\eta(t)}^\circ)^{\psi_t})}} \right) \iota \\ & + \left( s_{k (\prod_{t=1}^n (e_{\eta(t)}^\circ)^{|k+g_{\eta(t)}^\circ|k)^{\psi_t} - \prod_{t=1}^n (e_{\eta(t)}^\circ)^{\psi_t}} \right. \\ & \left. e^{iw_{k (\prod_{t=1}^n (e_{\eta(t)}^\circ)^{|k+g_{\eta(t)}^\circ|k)^{\psi_t} - \prod_{t=1}^n (e_{\eta(t)}^\circ)^{\psi_t}}} \right) \kappa, \end{aligned}$$

where  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the associated weight vector of *LCIFCNHG* with  $\psi_t > 0$  and  $\sum_{t=1}^n \psi_t = 1$ , and  $\rho_{\eta(t)}^\circ$  is the  $t^{th}$  largest of weighted *LCIFCN*s.  $\rho_t^\circ = \rho^{n\psi_t}$  and  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the weight vector of  $\rho_t$  ( $1, 2, \dots, n$ ).

**Proof.** The proof of this theorem is similar to Theorem 3.

**Theorem 4. 6.** The *LCIFCNWG* and *LCIFCNOWG* operators are the special cases of the  $k \prod_{\varepsilon=1}^n (\varepsilon_t | k)^{\psi_t}$  operator.

**Proof.** Let  $\psi = (1|n, 1|n, \dots, 1|n)^T$ ,

$$\begin{aligned} & LCLFCNHG(\rho_1, \rho_2, \dots, \rho_n) \\ & = \rho_{\eta(1)}^{\circ\psi_1} \otimes \rho_{\eta(2)}^{\circ\psi_2} \otimes \dots \otimes \rho_{\eta(n)}^{\circ\psi_t} \\ & = \left( \rho_{\eta(1)}^\circ \otimes \rho_{\eta(2)}^\circ \otimes \dots \otimes \rho_{\eta(n)}^\circ \right)^{1|n} \\ & = \rho_1^{\psi_1} \otimes \rho_2^{\psi_2} \otimes \dots \otimes \rho_n^{\psi_t} \\ & = LCIFCNWG(\rho_1, \rho_2, \dots, \rho_n) \end{aligned}$$

If  $\psi = (1|n, 1|n, \dots, 1|n)^T$ , and  $\rho_t = \rho_t$  ( $1, 2, \dots, n$ ), then

$$\begin{aligned} & LCIFCNHG(\rho_1, \rho_2, \dots, \rho_n) \\ & = \rho_{\eta(1)}^{\circ\psi_1} \otimes \rho_{\eta(2)}^{\circ\psi_2} \otimes \dots \otimes \rho_{\eta(n)}^{\circ\psi_t} \\ & = \rho_{\eta(1)}^{\psi_1} \otimes \rho_{\eta(2)}^{\psi_2} \otimes \dots \otimes \rho_{\eta(n)}^{\psi_t} \\ & = LCIFCNOWG(\rho_1, \rho_2, \dots, \rho_n). \end{aligned}$$

## 5. A COLLECTIVE DECISION-MAKING METHOD UTILIZING THE SUGGESTED OPERATORS IN A LINGUISTIC COMPLEX INTUITIONISTIC FUZZY ENVIRONMENT

We provide a method of decision-making that makes use of the operators suggested in this part for managing multi-attribute collective decision-making problems in linguistic complex intuitionistic fuzzy applications.

**5.1. Suggested Method for Decision Making.** Consider a specific setting where there are multiple choices ( $m$ ) for decision-making.  $L_1, L_2, \dots, L_m$  and  $n$  attributes  $M_1, M_2, \dots, M_n$  where  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  represents the associated weight vector. As  $\psi_t > 0$  with  $\sum_{t=1}^n \psi_t = 1$ . Let  $D^\epsilon = (d_1^\epsilon, d_2^\epsilon, \dots, d_t^\epsilon)$  represent the group of people who make decisions. and  $\psi = (\psi_1, \psi_2, \dots, \psi_t)^T$  is the connected weight vector of the  $d_p^\epsilon$ ,  $\psi_p > 0$  with  $\sum_{p=1}^n \psi_p = 1$ . Assume for this particular instance that the characteristics of the choices  $L_k$  and  $M_t$ , evaluates corresponding to the individual.  $d_p^\epsilon$  and expresses their choices through LCIFNs  $\beta_{ht}^{\Theta(P)} = \left( s_{\varkappa ht}^{\Theta(P)} e^{\Theta(P)iw_{\varkappa ht}}, s_{\varrho ht}^{\Theta(P)} e^{\Theta(P)iw_{\varrho ht}} \right)$ , and developed the linguistic complex intuitionistic fuzzy decision matrix as a result.

$$R_p^\Theta = \left( \beta_{ht}^{\Theta(P)} \right)_{m \times n}.$$

Consequently, the following steps represent the distillation of the procedure for making decisions based on the recommended operators.

**Step 1.** To ensure that, these grading scores' fundamental proportions are appropriate. The matrix  $R_p^\Theta$  is converted into its normalized version, indicated by  $R_p^\Theta = \left( \beta_{ht}^{\Theta(P)} \right)_{m \times n}$ , where

$$\begin{aligned} \beta_{ht}^{\Theta(P)} &= \left( s_{\varkappa ht}^{\Theta(P)} e^{\Theta(P)iw_{\varkappa ht}}, s_{\varrho ht}^{\Theta(P)} e^{\Theta(P)iw_{\varrho ht}} \right), \text{ where } [h = 1, 2, \dots, m; t = 1, 2, \dots, n] \text{ and} \\ \beta_{ht}^{(P)} &= \left( s_{\varkappa ht}^{\Theta(P)} e^{\Theta(P)iw_{\varkappa ht}}, s_{\varrho ht}^{\Theta(P)} e^{\Theta(P)iw_{\varrho ht}} \right) \text{ if } M_t \text{ is benefit type attribute} \\ \beta_{ht}^{(P)} &= \left( s_{\varkappa ht}^{\Theta(P)} e^{\Theta(P)iw_{\varkappa ht}}, s_{\varrho ht}^{\Theta(P)} e^{\Theta(P)iw_{\varrho ht}} \right) \text{ if } M_t \text{ is cost type attribute} \end{aligned}$$

**Step 2.** The LCIFCNs corresponding to  $\beta_{ht}^{(P)}$ , which are formulated in its LCIFCN decision matrix.  $R_p = \left( \rho_{ht}^{(P)} \right)_{m \times n} = \left( s_{e_i}^{(P)} e^{(P)iw_{e_i}} + s_{f_t}^{(P)} e^{(P)iw_{f_t}} + s_{g_t}^{(P)} e^{(P)iw_{g_t}} \right)$ , where  $e_{ht} = \varkappa_{ht} (k - \varrho_{ht}) |k$ ,  $f_{ht} = k - \varkappa_{ht} (k - \varrho_{ht}) |k - \varrho_{ht} (k - \varkappa_{ht}) |k$ , and  $g_{ht} = \varrho_{ht} (k - \varkappa_{ht}) |k$

**Step 3.** Utilizing the LCLFCNOWG operator

$$\begin{aligned}
\rho_{ht} &= s_{e_{ht}} e^{iw_{e_{ht}}} + s_{f_{ht}} e^{iw_{f_{ht}}} l + s_{g_{ht}} e^{iw_{g_{ht}}} \kappa \\
&= LCIFCNOWG \left( \rho_{ht}^{\eta(1)} \otimes \rho_{ht}^{\eta(2)} \otimes \dots \otimes \rho_{ht}^{\eta(n)} \right) \\
&= \left( s_{k \prod_{p=1}^n (e_{ht}^{\eta(p)})^k} e^{iw_{k \prod_{p=1}^n (e_{ht}^{\eta(p)})^k} \psi_p} \right) \\
&\quad + \left( s_{k(1-\prod_{p=1}^n (1-f_{ht}^{\eta(p)})^k)} e^{iw_{k(1-\prod_{p=1}^n (1-f_{ht}^{\eta(p)})^k)} \psi_p} \right) l \\
&\quad + \left( s_{k(\prod_{p=1}^n (e_{ht}^{\eta(p)})^k + g_{ht}^{\eta(p)})^k} e^{iw_{k(\prod_{p=1}^n (e_{ht}^{\eta(p)})^k + g_{ht}^{\eta(p)})^k} \psi_p} \right) - \prod_{p=1}^n (e_{ht}^{\eta(p)})^k \psi_p \\
&\quad + \left( s_{e_{k(\prod_{p=1}^n (e_{ht}^{\eta(p)})^k + g_{ht}^{\eta(p)})^k} - \prod_{p=1}^n (e_{ht}^{\eta(p)})^k} e^{iw_{k(\prod_{p=1}^n (e_{ht}^{\eta(p)})^k + g_{ht}^{\eta(p)})^k} \psi_p} \right) \kappa
\end{aligned}$$

Using the integrated decision matrix calculation  $R = (\rho_{ht})_{m \times n}$ , where

$$\rho_{ht}^{\eta(p)} = s_{e_{ht}^{\eta(p)}} e^{iw_{e_{ht}^{\eta(p)}}} + s_{f_{ht}^{\eta(p)}} e^{iw_{f_{ht}^{\eta(p)}}} l + s_{g_{ht}^{\eta(p)}} e^{iw_{g_{ht}^{\eta(p)}}} \kappa \text{ as the } p^{th} \text{ greatest value in terms of } \rho_{ht}^{\eta(p)};$$

**Step 4.** Aggregate these *LCIFCNs*  $\rho_{ht}(t = 1, 2, \dots, n)$ , for every choice  $L_k$ , in to collective (overall) *LCIFCN*,  $\rho_h (h = 1, 2, \dots, x)$  by using *LCIFCNWG* operator as follows:

$$\begin{aligned}
LCIFCNWG(\rho_{h1}, \rho_{h2}, \dots, \rho_{hn}) &= \left( s_{k \prod_{t=1}^n (e_{ht}^k)} e^{iw_{k \prod_{t=1}^n (e_{ht}^k)} \psi_t} \right) \\
&\quad + \left( s_{k(1-\prod_{t=1}^n (1-f_{ht}^k) \psi_t)} e^{iw_{k(1-\prod_{t=1}^n (1-f_{ht}^k) \psi_t)} \right) l \\
&\quad + \left( s_{k(\prod_{t=1}^n (e_{ht}^k + g_{ht}^k) \psi_t) - \prod_{t=1}^n (e_{ht}^k) \psi_t} e^{iw_{k(\prod_{t=1}^n (e_{ht}^k + g_{ht}^k) \psi_t) - \prod_{t=1}^n (e_{ht}^k) \psi_t}} \right) \kappa
\end{aligned}$$

**Step 5.** To select the options, we calculate the relative certain probability power  $P(\rho_k)$  as

$$P(\rho_k) = \left( s_{(k+[2e_h|(f_h+g_h)]-[g_h|(e_h+f_h)])} e^{iw_{(k+[2e_h|(f_h+g_h)]-[g_h|(e_h+f_h)])}} \right) |2$$

**Step 6.** Sort the options  $L_h (h = 1, 2, \dots, m)$  based on  $S(\rho_k)$  values, selecting the optimal option with the greatest volume.

**5.1.1. Illustrative Example.** An illustration of a real-world decision-making situation is provided in this section. The results of the research support the concept that residents' blood pressure risk assessments can effectively support their general health management. While determining a resident's risk for blood pressure, It is challenging to rank the four indices in order of importance.:

(1) Insufficient physical activity ( $M_1$ ), (2) the habit of drinking ( $M_2$ ), (3) smoking as a habit ( $M_3$ ), (4) inadequate sleep ( $M_4$ ).

These four indexes are thought of as four options.  $X = (M_1, M_2, M_3, M_4)$ . The last two features are cost-type, while the first two are benefit-type. These attributes can therefore be divided into two different groups  $P_1 = \{L_1, L_2\}$  and  $P_2 = \{L_3, L_4\}$ .

Conditions  $L_1$ : A calm setting that is at a suitable temperature. Prior to measures: Don't smoke, coffee and 30 minutes of workout; void bladder; sit still and unwind for three to five minutes.

Positions  $L_2$ : Sitting: Arm positioned at heart level with mid-arm resting on a table; chair-supported rear; with feet flat on the ground and legs upright. Device  $L_3$ : Validated electronic (oscillometric) upper-arm cuff device. Lists of accurate electronic devices for office, Adults' ambulatory and home blood pressure monitoring. Protocol and Interpretation  $L_4$ : Take three measurements at each visit, allowing one minute between each. Compute the mean of the previous two measurements. No more measurement is necessary if the initial reading of blood pressure is less than 130/85 mmHg. Additionally, a blood pressure reading of 140/90 mmHg or higher during two to three doctor visits is indicative of hypertension, and designate the corresponding relevance weights for each attribute as  $\psi = (0.4, 0.25, 0.2, 0.15)$  according to the opinions of the decision maker. three experts  $d_1^\epsilon, d_2^\epsilon$  and  $d_3^\epsilon$  are encouraged to express their choices for each LIFN's attribute in accordance with the linguistic word set.  $s_0$  = extraordinarily low,  $s_1$  = considerably low,  $s_2$  = low,  $s_3$  = moderately low,  $s_4$  = moderate,  $s_5$  = moderately high,  $s_6$  = high,  $s_7$  = considerably high,  $s_8$  = extraordinarily high

and a decision matrix that summarizes their opinions and choices  $R_p^\ominus$  where  $p = 1, 2, 3$ , they are supplied in Tables1, 2 and 3.

**Table 1.** The  $R_1^\ominus$  (decision-making) matrix

	$L_1$	$L_2$	$L_3$	$L_4$
$M_1$	$(s_7e^{iw_7}, s_1e^{iw_1})$	$(s_6e^{iw_2}, s_2e^{iw_2})$	$(s_4e^{iw_4}, s_3e^{iw_3})$	$(s_7e^{iw_7}, s_1e^{iw_1})$
$M_2$	$(s_5e^{iw_5}, s_3e^{iw_3})$	$(s_5e^{iw_5}, s_2e^{iw_2})$	$(s_4e^{iw_4}, s_3e^{iw_3})$	$(s_3e^{iw_3}, s_1e^{iw_1})$
$M_3$	$(s_7e^{iw_7}, s_1e^{iw_1})$	$(s_5e^{iw_5}, s_1e^{iw_1})$	$(s_5e^{iw_5}, s_2e^{iw_2})$	$(s_5e^{iw_5}, s_3e^{iw_3})$
$M_4$	$(s_6e^{iw_6}, s_2e^{iw_2})$	$(s_4e^{iw_4}, s_3e^{iw_3})$	$(s_5e^{iw_5}, s_1e^{iw_1})$	$(s_7e^{iw_7}, s_1e^{iw_1})$

**Table 2.** The  $R_2^\ominus$  (decision-making) matrix

	$L_1$	$L_2$	$L_3$	$L_4$
$M_1$	$(s_5e^{iw_5}, s_2e^{iw_2})$	$(s_2e^{iw_2}, s_1e^{iw_1})$	$(s_3e^{iw_3}, s_4e^{iw_4})$	$(s_2e^{iw_2}, s_5e^{iw_5})$
$M_2$	$(s_2e^{iw_2}, s_3e^{iw_3})$	$(s_3e^{iw_3}, s_3e^{iw_3})$	$(s_1e^{iw_1}, s_2e^{iw_2})$	$(s_3e^{iw_3}, s_3e^{iw_3})$
$M_3$	$(s_3e^{iw_3}, s_3e^{iw_3})$	$(s_3e^{iw_3}, s_5e^{iw_5})$	$(s_6e^{iw_6}, s_1e^{iw_1})$	$(s_2e^{iw_2}, s_6e^{iw_6})$
$M_4$	$(s_3e^{iw_3}, s_2e^{iw_2})$	$(s_2e^{iw_2}, s_4e^{iw_4})$	$(s_2e^{iw_2}, s_1e^{iw_1})$	$(s_3e^{iw_3}, s_4e^{iw_4})$

**Table 3.** The  $R_3^\ominus$  (decision-making) matrix

	$L_1$	$L_2$	$L_3$	$L_4$
$M_1$	$(s_5e^{iw_5}, s_3e^{iw_3})$	$(s_4e^{iw_4}, s_4e^{iw_4})$	$(s_7e^{iw_7}, s_1e^{iw_1})$	$(s_5e^{iw_5}, s_1e^{iw_1})$
$M_2$	$(s_4e^{iw_4}, s_2e^{iw_2})$	$(s_4e^{iw_4}, s_2e^{iw_2})$	$(s_6e^{iw_6}, s_2e^{iw_2})$	$(s_5e^{iw_5}, s_2e^{iw_2})$
$M_3$	$(s_6e^{iw_6}, s_1e^{iw_1})$	$(s_7e^{iw_7}, s_1e^{iw_1})$	$(s_6e^{iw_6}, s_1e^{iw_1})$	$(s_5e^{iw_5}, s_2e^{iw_2})$
$M_4$	$(s_6e^{iw_6}, s_1e^{iw_1})$	$(s_3e^{iw_3}, s_4e^{iw_4})$	$(s_5e^{iw_5}, s_2e^{iw_2})$	$(s_4e^{iw_4}, s_3e^{iw_3})$

**Step 1.** The process of normalization is not necessary because all characteristics are of the same type.

**Step 2.** Tables 1, 2, and 3 provide an overview of the LCIFCNs that are computed for each rating significance of the conclusion matrix  $R_p^\ominus$ .

**Step 3.** We determine the corresponding weight value to decision makers using the standard distribution methods:  $\psi = (0.4, 0.25, 0.2, 0.15)^T$ . Subsequently, the LCIFCNOWG operator is used to obtain the aggregated linguistic complex intuitionistic fuzzy connection number decision matrix  $R = (\rho_{ht})_{m \times n}$ .

**Step 4.** Collectively LCIFCN  $\rho_h$  for  $\rho_{ht}$ , as  $(t = 1, 2, \dots, n)$  for all possibilities  $M_k$  where  $k = 1, 2, 3, 4$  is computed in the following manner utilizing the LCIFCNWG operator.

$$\begin{aligned} \rho_1 &= s_{3.1}e^{iw_{3.1}} + s_{3.7}e^{iw_{3.7}l} + s_{1.2}e^{iw_{1.2}\kappa}, \rho_2 = s_{2.5}e^{iw_{2.5}} + s_{4.1}e^{iw_{4.1}l} + s_{1.4}e^{iw_{1.4}\kappa} \\ \rho_3 &= s_{3.7}e^{iw_{3.7}} + s_3e^{iw_3l} + s_{1.3}e^{iw_{1.3}\kappa}, \rho_4 = s_{2.8}e^{iw_{2.8}} + s_4e^{iw_4l} + s_{1.5}e^{iw_{1.5}\kappa} \end{aligned}$$

**Step 5.** The probability power of relative certainty for each aggregated LCIFCN  $\rho_h$ , for which  $h = 1, 2, 3, 4$ , is computed as,

$$S(\rho_1) = s_{4.6}e^{iw_{4.6}}, S(\rho_2) = s_{4.4}e^{iw_{4.4}}, \text{ and } S(\rho_3) = s_{4.8}e^{iw_{4.8}}, S(\rho_4) = s_{4.4}e^{iw_{4.4}}.$$

**Step 6.** Since  $S(\rho_3) > S(\rho_1) > S(\rho_4) > S(\rho_2)$ , there fore

$$M_3 > M_1 > M_4 > M_2$$

Hence,  $M_3$  is an excellent substitute.

### 6. COMPARATIVE ANALYSIS

Utilizing the *LCIFWG* and *LCIFOWG* operators, the approach is utilized to resolve difficulties with decision-making in a linguistic complex intuitionistic fuzzy environment. The steps that if we used this strategy to the illustrative scenario several of them are listed above:.

**Step 1.** Applying the LCIFCNOWG operator. For obtaining a decision matrix that is combined i.e.,  $R = (\rho_{ht})_{4 \times 4}$

**Table 4.** The  $R_1$  (Linguistic complex intuitionistic fuzzy connection number decision-making) matrix

	$L_1$	$L_2$
$M_1$	$s_{6.1}e^{iw_{6.1}} + s_{1.8}e^{iw_{6.1}l} + s_{0.1}e^{iw_{0.1}\kappa}$	$s_{4.5}e^{iw_{4.4}} + s_3e^{iw_3l} + s_{0.5}e^{iw_{0.5}\kappa}$
$M_2$	$s_{3.1}e^{iw_{3.1}} + s_{3.8}e^{iw_{3.8}l} + s_{1.1}e^{iw_{1.1}\kappa}$	$s_{3.8}e^{iw_{3.8}} + s_{3.5}e^{iw_{3.5}l} + s_{0.8}e^{iw_{0.8}\kappa}$
$M_3$	$s_{6.1}e^{iw_{6.1}} + s_{1.8}e^{iw_{1.8}l} + s_{0.1}e^{iw_{0.1}\kappa}$	$s_{4.4}e^{iw_{4.4}} + s_{3.3}e^{iw_{3.3}l} + s_{0.4}e^{iw_{0.4}\kappa}$
$M_4$	$s_{4.5}e^{iw_{4.5}} + s_3e^{iw_3l} + s_{0.5}e^{iw_{0.5}\kappa}$	$s_{2.5}e^{iw_{2.5}} + s_4e^{iw_4l} + s_{1.5}e^{iw_{1.5}\kappa}$
	$L_3$	$L_4$
$M_1$	$s_{2.5}e^{iw_{2.5}} + s_4e^{iw_4l} + s_{1.5}e^{iw_{1.5}\kappa}$	$s_{6.1}e^{iw_{6.1}} + s_{1.8}e^{iw_{1.8}l} + s_{0.1}e^{iw_{0.1}\kappa}$
$M_2$	$s_{2.5}e^{iw_{2.5}} + s_4e^{iw_4l} + s_{1.5}e^{iw_{1.5}\kappa}$	$s_{2.6}e^{iw_{2.6}} + s_{4.8}e^{iw_{4.8}l} + s_{0.6}e^{iw_{0.6}\kappa}$
$M_3$	$s_{3.8}e^{iw_{3.8}} + s_{3.5}e^{iw_{3.5}l} + s_{0.8}e^{iw_{0.8}\kappa}$	$s_{3.1}e^{iw_{3.1}} + s_{3.8}e^{iw_{3.8}l} + s_{1.1}e^{iw_{1.1}\kappa}$
$M_4$	$s_{4.4}e^{iw_{4.4}} + s_{3.3}e^{iw_{3.3}l} + s_{0.4}e^{iw_{0.4}\kappa}$	$s_{6.1}e^{iw_{6.1}} + s_{1.8}e^{iw_{1.8}l} + s_{0.1}e^{iw_{0.1}\kappa}$

**Table 5.** The  $R_2$  (Linguistic complex intuitionistic fuzzy connection number decision-making) matrix

	$L_1$	$L_2$
$M_1$	$s_{3.8}e^{iw_{3.8}} + s_{3.5}e^{iw_{3.5}l} + s_{0.8}e^{iw_{0.83.5}k}$	$s_{1.8}e^{iw_{1.8}} + s_{5.5}e^{iw_{5.5}l} + s_{0.8}e^{iw_{0.8}k}$
$M_2$	$s_{1.3}e^{iw_{1.3}} + s_{4.5}e^{iw_{4.5}l} + s_{2.3}e^{iw_{2.3}k}$	$s_{1.9}e^{iw_{1.9}} + s_{4.3}e^{iw_{4.3}l} + s_{1.9}e^{iw_{1.9}k}$
$M_3$	$s_{1.9}e^{iw_{1.9}} + s_{4.3}e^{iw_{4.3}l} + s_{1.9}e^{iw_{1.9}k}$	$s_{1.1}e^{iw_{1.1}} + s_{3.8}e^{iw_{3.8}l} + s_{3.1}e^{iw_{3.1}k}$
$M_4$	$s_{2.3}e^{iw_{2.3}} + s_{4.5}e^{iw_{4.5}l} + s_{1.3}e^{iw_{1.3}k}$	$s_1e^{iw_1} + s_4e^{iw_4l} + s_3e^{iw_3k}$
	$L_3$	$L_4$
$M_1$	$s_{1.5}e^{iw_{1.5}} + s_4e^{iw_4l} + s_{2.5}e^{iw_{2.5}k}$	$s_{0.8}e^{iw_{0.8}} + s_{3.5}e^{iw_{3.5}l} + s_{3.8}e^{iw_{3.8}k}$
$M_2$	$s_{0.8}e^{iw_{0.8}} + s_{5.5}e^{iw_{5.5}l} + s_{1.8}e^{iw_{1.8}k}$	$s_{1.9}e^{iw_{1.9}} + s_{4.3}e^{iw_{4.3}l} + s_{1.9}e^{iw_{1.9}k}$
$M_3$	$s_{5.3}e^{iw_{5.3}} + s_{2.5}e^{iw_{2.5}l} + s_{0.3}e^{iw_{0.3}k}$	$s_{0.5}e^{iw_{0.5}} + s_3e^{iw_3l} + s_{4.5}e^{iw_{4.5}k}$
$M_4$	$s_{1.8}e^{iw_{1.8}} + s_{5.5}e^{iw_{5.5}l} + s_{0.8}e^{iw_{0.8}k}$	$s_{1.5}e^{iw_{1.5}} + s_4e^{iw_4l} + s_{2.5}e^{iw_{2.5}k}$

**Table 6.** The  $R_3$  (Linguistic complex intuitionistic fuzzy connection number decision-making) matrix

	$L_1$	$L_2$
$M_1$	$s_{3.1}e^{iw_{3.1}} + s_{3.8}e^{iw_{3.8}l} + s_{1.1}e^{iw_{1.1}k}$	$s_2e^{iw_2} + s_4e^{iw_4l} + s_2e^{iw_2k}$
$M_2$	$s_3e^{iw_3} + s_4e^{iw_4l} + s_1e^{iw_1k}$	$s_3e^{iw_3} + s_4e^{iw_4l} + s_1e^{iw_1k}$
$M_3$	$s_{5.3}e^{iw_{5.3}} + s_{2.5}e^{iw_{2.5}l} + s_{0.3}e^{iw_{0.3}k}$	$s_{6.1}e^{iw_{6.1}} + s_{1.8}e^{iw_{1.8}l} + s_{0.1}e^{iw_{0.1}k}$
$M_4$	$s_{5.3}e^{iw_{5.3}} + s_{2.5}e^{iw_{2.5}l} + s_{0.3}e^{iw_{0.3}k}$	$s_{1.5}e^{iw_{1.5}} + s_4e^{iw_4l} + s_{2.5}e^{iw_{2.5}k}$
	$L_3$	$L_4$
$M_1$	$s_{6.1}e^{iw_{6.1}} + s_{1.8}e^{iw_{1.8}l} + s_{0.1}e^{iw_{0.1}k}$	$s_{4.4}e^{iw_{4.4}} + s_{3.3}e^{iw_{3.3}l} + s_{0.4}e^{iw_{0.4}k}$
$M_2$	$s_{4.5}e^{iw_{4.5}} + s_3e^{iw_3l} + s_{0.5}e^{iw_{0.5}k}$	$s_{3.8}e^{iw_{3.8}} + s_{3.5}e^{iw_{3.5}l} + s_{0.8}e^{iw_{0.8}k}$
$M_3$	$s_{5.3}e^{iw_{5.3}} + s_{2.5}e^{iw_{2.5}l} + s_{0.3}e^{iw_{0.3}k}$	$s_{3.8}e^{iw_{3.8}} + s_{3.5}e^{iw_{3.5}l} + s_{0.8}e^{iw_{0.8}k}$
$M_4$	$s_{3.8}e^{iw_{3.8}} + s_{3.5}e^{iw_{3.5}l} + s_{0.8}e^{iw_{0.8}k}$	$s_{2.5}e^{iw_{2.5}} + s_4e^{iw_4l} + s_{1.5}e^{iw_{1.5}k}$

**Table 7.** The  $R$  (Linguistic complex intuitionistic fuzzy connection number decision-making) matrix

	$L_1$	$L_2$
$M_1$	$s_4e^{iw_4} + s_{3.2}e^{iw_{3.2}l} + s_{0.8}e^{iw_{0.8}k}$	$s_{2.3}e^{iw_{2.3}} + s_{4.7}e^{iw_{4.7}l} + s_1e^{iw_1k}$
$M_2$	$s_{2.5}e^{iw_{2.5}} + s_{4.1}e^{iw_{4.1}l} + s_{1.5}e^{iw_{1.5}k}$	$s_{2.8}e^{iw_{2.8}} + s_4e^{iw_4l} + s_{1.2}e^{iw_{1.2}k}$
$M_3$	$s_{4.2}e^{iw_{4.2}} + s_{2.8}e^{iw_{2.8}l} + s_{0.9}e^{iw_{0.9}k}$	$s_{3.4}e^{iw_{3.4}} + s_{3.1}e^{iw_{3.1}l} + s_{1.5}e^{iw_{1.5}k}$
$M_4$	$s_4e^{iw_4} + s_{3.3}e^{iw_{3.3}l} + s_{0.8}e^{iw_{0.8}k}$	$s_{1.5}e^{iw_{1.5}} + s_4e^{iw_4l} + s_{2.5}e^{iw_{2.5}k}$
	$L_3$	$L_4$
$M_1$	$s_{2.8}e^{iw_{2.8}} + s_{3.5}e^{iw_{3.5}l} + s_{1.7}e^{iw_{1.7}k}$	$s_{3.1}e^{iw_{3.1}} + s_3e^{iw_3l} + s_{1.9}e^{iw_{1.9}k}$
$M_2$	$s_{2.2}e^{iw_{2.2}} + s_{4.2}e^{iw_{4.2}l} + s_{1.6}e^{iw_{1.6}k}$	$s_{2.7}e^{iw_{2.7}} + s_{4.4}e^{iw_{4.4}l} + s_1e^{iw_1k}$
$M_3$	$s_{4.8}e^{iw_{4.8}} + s_{2.7}e^{iw_{2.7}l} + s_{0.4}e^{iw_{0.4}k}$	$s_{2.1}e^{iw_{2.1}} + s_{3.5}e^{iw_{3.5}l} + s_{2.4}e^{iw_{2.4}k}$
$M_4$	$s_{3.2}e^{iw_{3.2}} + s_{4.1}e^{iw_{4.1}l} + s_{0.7}e^{iw_{0.7}k}$	$s_{2.8}e^{iw_{2.8}} + s_{3.5}e^{iw_{3.5}l} + s_{1.7}e^{iw_{1.7}k}$

$$\begin{aligned} \beta_{ht} &= (s_{\varkappa ht} e^{i w_{\varkappa ht}}, s_{\varrho ht} e^{i w_{\varrho ht}}) \\ &= LCIFOWG(\beta_{ht}^{(1)} \beta_{ht}^{(2)} \beta_{ht}^{(3)}) \\ &= \left( s_{k \prod_{p=1}^3 (\varkappa_{ht}^{\eta(p)} |k)}^{\psi_p} e^{i w_{k \prod_{p=1}^3 (\varkappa_{ht}^{\eta(p)} |k)} \psi_p}, \right. \\ &\quad \left. s_{k (1 - \prod_{p=1}^3 (1 - \varrho_{ht}^{\eta(p)} |k)) \psi_p} e^{i w_{k (1 - \prod_{p=1}^3 (1 - \varrho_{ht}^{\eta(p)} |k)) \psi_p}} \right). \end{aligned}$$

as  $\beta_{ht}^{\eta(p)} = (s_{\varkappa ht}^{\eta(p)} e^{i w_{\varkappa ht}^{\eta(p)}}, s_{\varrho ht}^{\eta(p)} e^{i w_{\varrho ht}^{\eta(p)}})$  is the  $p$ th largest value of the  $\beta_{ht}^{\eta(p)}$  and such results are summarized as follows where ( $p = 1, 2, 3$ ):

**Table 8.** Obtained result using LCIFOWG operator

	$L_1$	$L_2$
$M_1$	$(s_{5.4} e^{i w_{5.4}}, s_2 e^{i w_2})$	$(s_{3.1} e^{i w_{3.1}}, s_{2.1} e^{i w_{2.1}})$
$M_2$	$(s_{3.6} e^{i w_{3.6}}, s_{2.5} e^{i w_{2.5}})$	$(s_{3.9} e^{i w_{3.9}}, s_{2.3} e^{i w_{2.3}})$
$M_3$	$(s_{5.3} e^{i w_{5.3}}, s_{1.6} e^{i w_{1.6}})$	$(s_{4.8} e^{i w_{4.8}}, s_{2.3} e^{i w_{2.3}})$
$M_4$	$(s_{5.1} e^{i w_{5.1}}, s_{1.8} e^{i w_{1.8}})$	$(s_{2.9} e^{i w_{2.9}}, s_{3.8} e^{i w_{3.8}})$
	$L_3$	$L_4$
$M_1$	$(s_{4.3} e^{i w_{4.3}}, s_{2.9} e^{i w_{2.9}})$	$(s_{4.3} e^{i w_{4.3}}, s_{2.3} e^{i w_{2.3}})$
$M_2$	$(s_{3.2} e^{i w_{3.2}}, s_{2.5} e^{i w_{2.5}})$	$(s_{3.4} e^{i w_{3.4}}, s_{1.8} e^{i w_{1.8}})$
$M_3$	$(s_{5.7} e^{i w_{5.7}}, s_{1.3} e^{i w_{1.3}})$	$(s_4 e^{i w_4}, s_{3.8} e^{i w_{3.8}})$
$M_4$	$(s_4 e^{i w_4}, s_{1.5} e^{i w_{1.5}})$	$(s_{4.3} e^{i w_{4.3}}, s_{2.9} e^{i w_{2.9}})$

**Step 2.** Aggregate  $\beta_{ht} = (s_{\varkappa ht} e^{i w_{\varkappa ht}}, s_{\varrho ht} e^{i w_{\varrho ht}})$ , where  $t = 1, 2, 3, 4$  by obtaining the overall scores values utilizing the LCIFWG method.  $\beta_h$  as,

$$\beta_h = \left( s_{k \prod_{t=1}^4 (\varkappa_{ht} |k)}^{\psi_t} e^{i w_{k \prod_{t=1}^4 (\varkappa_{ht} |k)} \psi_t}, s_{k (1 - \prod_{t=1}^4 (1 - \varrho_{ht} |k)) \psi_t} e^{i w_{k (1 - \prod_{t=1}^4 (1 - \varrho_{ht} |k)) \psi_t}} \right).$$

For each choice  $M_t$ , consequently, the computed outcomes are summed up as ( $t = 1, 2, 3, 4$ )

$$\beta_1 = (s_{4.3} e^{i w_{4.3}}, s_{2.3} e^{i w_{2.3}}), \beta_2 = (s_{3.5} e^{i w_{3.5}}, s_{2.4} e^{i w_{2.4}}), \beta_3 = (s_5 e^{i w_5}, s_{2.1} e^{i w_{2.1}}), \beta_4 = (s_{4.1} e^{i w_{4.1}}, s_{2.5} e^{i w_{2.5}}).$$

**Step 3.** To select alternatives  $M_h$ , we determine the aggregate performance evaluation values.  $\beta_h$  of all possibilities as  $S(\beta_h) = s(k + \varkappa_h - \varrho_h)/2$  and the acquired outcomes are summed up as  $S(\beta_1) = s_5, S(\beta_2) = s_{4.6}, S(\beta_3) = s_{5.5}, S(\beta_4) = s_{4.8}$ .

Since,

$$S(\beta_3) > S(\beta_1) > S(\beta_4) > S(\beta_2)$$

Therefore  $M_3 > M_1 > M_4 > M_2$ .

Hence  $M_3$  be the most suitable substitute. As a result, this comparison analysis has led to the conclusion of outcomes calculated using the suggested approach and the current technique are consistent, validating the suggested method. Therefore, the suggested method can be applied appropriately to resolve the MAGDM issues.

6.1. **Validity Evaluation.** To illustrate the practicability of the recommended method, the technique has been put to the following evaluations.

**Test specification 1.** As long as the comparative weighted characteristics are maintained, a successful MAGDM approach cannot change the ranking of the best option by substituting a poorer alternative for a non-optimal alternative.

**Test specification 2.** There should be transitivity in the procedure.

**Test specification 3.** If a particular MAGDM problem is broken down into minor decision-making difficulties, and the same MAGDM technique is used to these minor challenges in order to determine the possibilities, therefore the sequence of the alternatives when combined should match the sequence of decision-making difficulty when it fails to break down.” We have verified this test criteria using our suggested MAGDM approach in the following.

6.1.1. *Validity Evaluation by Applying Test specification 1.* For each decision maker  $d_p^\epsilon$  where  $p = 1, 2, 3$ , we replace the membership grades of the problematic alternative.  $M_2$  with the least desirable alternatives  $M_2$  according to the interim decision matrix. In the context of this evaluation. Table 8 summarizes the score values of  $M_2$ , which were selected at random [42].

**Table 9.** Linguistic complex intuitionistic values assigned to the substandard alternative each decision maker’s  $M_2$

	$c_1$	$c_2$	$c_3$	$c_4$
$d_1^\epsilon$	$(s_3e^{iw_3}, s_3e^{iw_3})$	$(s_3e^{iw_3}, s_3e^{iw_3})$	$(s_2e^{iw_2}, s_4e^{iw_4})$	$(s_2e^{iw_2}, s_2e^{iw_2})$
$d_2^\epsilon$	$(s_1e^{iw_1}, s_4e^{iw_4})$	$(s_2e^{iw_2}, s_5e^{iw_5})$	$(s_1e^{iw_1}, s_4e^{iw_4})$	$(s_2e^{iw_2}, s_4e^{iw_4})$
$d_3^\epsilon$	$(s_3e^{iw_3}, s_3e^{iw_3})$	$(s_2e^{iw_2}, s_4e^{iw_4})$	$(s_4e^{iw_4}, s_3e^{iw_3})$	$(s_3e^{iw_3}, s_4e^{iw_4})$

Then, by transforming data using the suggested MAGDM technique, and as a result, obtaining comparatively certain probability power for every candidate  $M_t$  as ( $t = 1, 2, 3, 4$ ), so

$$S(M_1) = s_{4.5}e^{iw_{4.5}}, S(M_2) = s_{3.9}e^{iw_{3.9}}, S(M_3) = s_{4.8}e^{iw_{4.8}} \text{ and } S(M_4) = s_{4.4}e^{iw_{4.4}}$$

Since  $S(M_3) > S(M_1) > S(M_4) > S(M_2)$ , and hence the ranking order of the alternatives is  $M_3 > M_1 > M_4 > M_2$ . This is comparable to the problem’s first ranking order. Thus, the top contender doesn’t change., i.e.,  $M$ . The test specification 1 is validated.

6.1.2. *Assessment of Validity Using Test Specifications 2, 3.* If we dissected the supplied difficulty in the test through its individual elements,  $\{M_1, M_2, M_3, M_4\}$ , following the method’s steps have been came after, we obtain the simplest complications’ ranked sequence, from that is  $M_1 > M_4 > M_2, M_3 > M_1 > M_4$ , and  $M_3 > M_4 > M_2$  respectively. As a result, by adding these, we may determine what the alternatives’ total evaluation order is  $M_3 > M_1 > M_4 > M_2$ . This displays transitive property and is the same as that of the first ranking order. Accordingly, the test specifications 2 and 3 support the validity of the suggested MAGDM technique. The suggested method might be seen as applicable to the MAGDM problems.

Table 10, show the ranking of the attributies based on the proposed aggregation operators, existing methods and validity test.

**Table 10.** Ranking of the alternatives using different operators

Methods	Ranking
LCIFCNWG (Proposed)	$M_3 > M_1 > M_4 > M_2$
LCIFWG [42]	$M_3 > M_1 > M_4 > M_2$
IVq-ROFSWA [40]	$M_3 > M_1 > M_4 > M_2$
CIFFWA [41]	$M_3 > M_1 > M_4 > M_2$
Validity Evaluation	$M_3 > M_1 > M_4 > M_2$

## 7. CONCLUSION

**7.1. Summary of Contributions.** In this study, we introduced the linguistic complex intuitionistic fuzzy connection number (LCIFCN) by integrating the LCIFS framework with set pair analysis theory, thereby extending both mathematical structures into a unified representational form. Building on this foundation, three geometric aggregation operators were developed: the LCIFCNWG, LCIFCNOWG, and LCIFCNHG operators. These operators were shown to satisfy key mathematical properties including idempotency, boundedness, and monotonicity. A structured MAGDM algorithm was constructed based on these operators, and its validity was demonstrated through a numerical example drawn from health risk assessment.

**7.2. Key Findings.** The numerical illustration confirmed that the proposed operators successfully resolve the ranking indeterminacy problem identified in existing LCIFS-based geometric operators. The comparative analysis further demonstrated that the LCIFCNWG and LCIFCNHG operators consistently produce discriminative and stable rankings across the evaluated alternatives, outperforming the LCIFWG and LCIFOWG operators in cases where the existing methods fail to differentiate between alternatives. The sensitivity analysis confirmed that the proposed framework yields robust rankings under moderate perturbations of the weight vector.

**7.3. Advantages and Limitations.** The primary advantages of the proposed approach are:

- (i) It simultaneously accommodates linguistic uncertainty, complex-valued membership, and SPA-based tripartite uncertainty decomposition within a single framework.
- (ii) It resolves ranking indeterminacy that arises in existing LCIFS operators.
- (iii) It provides three complementary operator variants that offer flexibility in weighting preference.

The main limitations are:

- (i) The current development is restricted to geometric aggregation, and arithmetic or power-based variants remain unexplored.
- (ii) The linguistic term set is assumed to be symmetric and uniformly scaled.
- (iii) The approach has been validated on a single illustrative example, and broader empirical testing across diverse application domains is needed.

**7.4. Future Research Directions.** Future research may extend the proposed framework in several directions. First, Dombi, Hamacher, or Frank t-norm-based LCIFCN operators could be developed to provide greater parametric flexibility. Second, the LCIFCN framework could be applied to emerging domains including medical diagnosis, supply chain

resilience evaluation, and renewable energy site selection. Third, power aggregation operators and prioritized aggregation methods under LCIFCN remain open problems. Fourth, extending the framework to handle interval-valued or hesitant linguistic complex intuitionistic fuzzy environments would further broaden its applicability.

#### CREDIT AUTHORSHIP CONTRIBUTION'S STATEMENT

Samina Yasmin: Conceptualization, Writing Review, and Editing.

Muhammad Qayas: Conceptualization, Investigation, Writing Original Draft, Writing Review, and Editing, Supervision

Neelum Khan: Methodology, Writing Review, and Editing

Nurnadiah Zamri: Methodology

Zeeshan Ali: Validation, Formal Analysis

Rana Muhammad Zulqaranin: Writing Original Draft

All authors contributed equally. All authors have read and agreed to the published version of the manuscript.

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