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Derivation of Reduced Reverse Degree Based $\mathbb{RRDM}$-Polynomials, Operators and Topological Indices of Metal-Organic Framework Utilizing Python Programming

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Abstract. Metal-organic frameworks (MOFs) stand out because to their highly customizable surface areas, pore sizes, and internal surface characteristics. Modern scientific attention has focused on a class of molecules called metal-organic frameworks (MOFs). They are the subject of extensive study because of the growing amount of information about them and their potential uses. MOFs with promising uses in renewable energy, particularly as gas storage and as adsorbents with a large capacity, satisfy a range of requirements for disconnection. More and more, it is being used for important purposes in biological imaging, thin film devices, catalysis, and membranes. There is a yearly increase in the representations of MOFs because scientists are continually finding new MOFs. In addition, MOFs are finding new uses, which shows how versatile these materials are. In this study Python programming is utilized to compute the reduced

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reverse degree-based $\mathbb{RRDM}-$Polynomials, Differential and Integral Operators, for the computation of topological descriptors and Python Code for $\mathbb{RRDM}-$Polynomials, Differential and Integral Operators for the two metal-organic frameworks, FeTPyP-Co and CoBHT (CO) lattice. Following relation between $\mathbb{RRD}$ $\mathbb{TD}s$ is obtained for FeTPyP-Co lattice MoF:

$$\begin{aligned}\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{FTC}) &> \mathbb{RRDM}_2(\mathbb{FTC}) > \mathbb{RRDHM}_1(\mathbb{FTC}) > \mathbb{RRDM}_1(\mathbb{FTC})\\ &> \mathbb{RRDSDD}(\mathbb{FTC}) > \mathbb{RRD}\sigma(\mathbb{FTC}) > \mathbb{RRDI}(\mathbb{FTC}) > \mathbb{RRDH}(\mathbb{FTC})\\ &> \mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC}). \qquad (0.\ 1)\end{aligned}$$

and relation between $\mathbb{RRD}$ $\mathbb{TD}s$ is obtained for CoBHT (CO) lattice MoF:

$$\begin{aligned}\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{CBL}) &> \mathbb{RRDHM}_1(\mathbb{CBL}) > \mathbb{RRDM}_2(\mathbb{CBL}) > \mathbb{RRDM}_1(\mathbb{CBL})\\ &> \mathbb{RRDSDD}(\mathbb{CBL}) > \mathbb{RRD}\sigma(\mathbb{CBL}) > \mathbb{RRDI}(\mathbb{CBL}) > \mathbb{RRDH}(\mathbb{CBL})\\ &> \mathbb{RRD}^m\mathbb{M}_2(\mathbb{CBL}). \qquad (0.\ 2)\end{aligned}$$

In addition, we provide a numerical and graphical analysis of the topological descriptors calculated.

AMS (MOS) Subject Classification Codes: 05C99, 05C31, 47E05, 05C09, 47G10, 47N50, 05C10

Key Words: Python Programming, Metal Organic Frameworks, Molecular Structure, M-Polynomial, Reduced Reverse Degree, Integral Operator, Differential Operator, Topological Descriptor, Derivation, Analysis.

1. Introduction

Metal-organic frameworks have recently garnered significant attention, particularly as newly developed porous materials. Among their many possible uses are materials for medication delivery, gas storage, gas/vapour separation, catalysis, and luminescence[10]. Metal-organic frameworks (MOFs) are complexes of a core metal ion or atom encircled by a few natural ligands [3]. These ligands are generally natural compounds, including a useful group capable of binding to metal centre via covalent or coordinate bonds. The resultant structure is a complex wherein the metal ion or atom is coordinated with the ligands and encompassed by a coordination sphere [18].

Metal–organic frameworks (MOFs), alternatively referred to as porous coordination polymers, are synthesised through the assembly of metal ions with natural ligands. This process has led to the emergence of a novel class of crystalline porous materials. The ability to design and achieve fine-tunable, uniform pore structures positions these materials as promising candidates for a range of applications. The integration of metal-organic frameworks (MOFs) and functional materials is being controlled to develop new multifunctional composites and hybrids. These materials demonstrate enhanced properties that surpass those of the individual components, resulting from the synergistic interactions of the functional units involved. This field is evolving quickly and involves multiple disciplines [19].

Among the many uses for metal-organic frameworks are molecular identification, sensing, and catalysis. For example, certain metalloenzyme active sites are made up of metal-organic frameworks, and the enzyme's ability to operate depends on the metal ion or atom's coordination with the ligands. [17]. MOFs are examined not just for their practical applications but also for their intrinsic chemical features and as models for more intricate systems [16]. The architectures of MOFs can be elucidated using methodology like X-ray crystallography, while their reactivity and stability can be examined by several chemical and spectroscopic methodologies [9].

Recently Aqib et al. presented structural analysis of metal organic networks through topological indices in [1]. Recently, Saeed et al. developed structural modeling of plant regulators in [14], Bhatti et al. calculated structural properties of Hex-Derived Networks in [2], and Virk et al. conducted QSPR analysis of antiplatelet drugs and dual antiplatelet therapy in [15]. Gowtham et al. recently conducted a QSPR analysis for a few benzenoid hydrocarbons using topological indices in [5], and in [11], they looked into a new class of cacti from those examined in earlier research. In [12], Rafiullah et al. investigate relationships between a few graphs and classify them using the Sombor index.

2. Material and Methodology

In order to learn how the coordination setting of the iron center impacts the magnetic characteristics of FeTPyPCo MOFs, these materials have been studied. The Co ligand determines the strength of the antiferromagnetic interaction that TPyP ligands induce between iron centers. The optical properties of FeTPyPCo metal-organic frameworks (MOFs) have been studied, which are ascribed to the TPyP ligands. The photocatalytic potential of reactive intermediates produced by TPyP ligands is enhanced by their capacity to absorb visible light and promote photoinduced electron transfer. Potentially useful in many fields, including optics, magnetism, electrocatalysis, and catalysis, FeTPyPCo MOFs are a class of molecular organic frameworks. The graph of FeTPyPCo is shown by the $\mathbb{FTC}$, where TPyP stands for Tetrakis pyridyl porphyrin. The cells that make up this organic-metal structure are neatly organized into rows and columns. Pictured in Figure 1 is the molecular blueprint of FeTPyPCo. There are $74kb$ vertices and $88kb - 2k - 2b + 1$ edges [4]. This article clarifies $\mathbb{FTC}$, which possesses a total atom count of $74kb$, as illustrated in Figure 1.

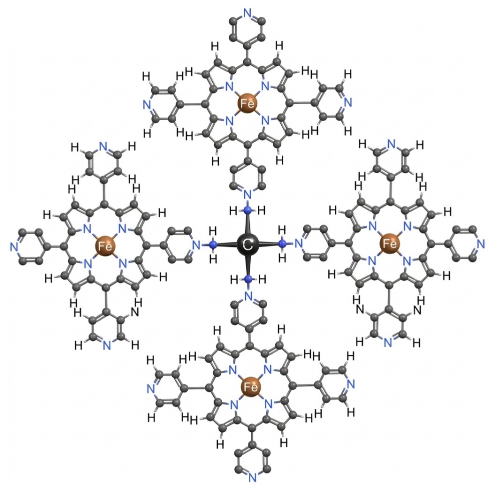

FIGURE 1. Molecular Structure of FeTPyP-Co ($\mathbb{FTC}$)

The CoBHT (CO) lattice stands out as a noteworthy molecular organic framework due to its exceptional magnetic characteristics and potential uses in several fields. A graph of the CoBHT (CO) lattice is shown by the CBT, where the unit cell is in the column and k is the unit cell in the row. The CoBHT (CO) lattice's molecular graph structure is shown in Figure 2, where the square portion represents the lattice's unit structure. The CBT contains $27kb$ vertices and $36kb - 2(k + b)$ edges [4]. Figure 2 depicts a two-dimensional 3×3 CoBHT(CO) lattice configuration [4].

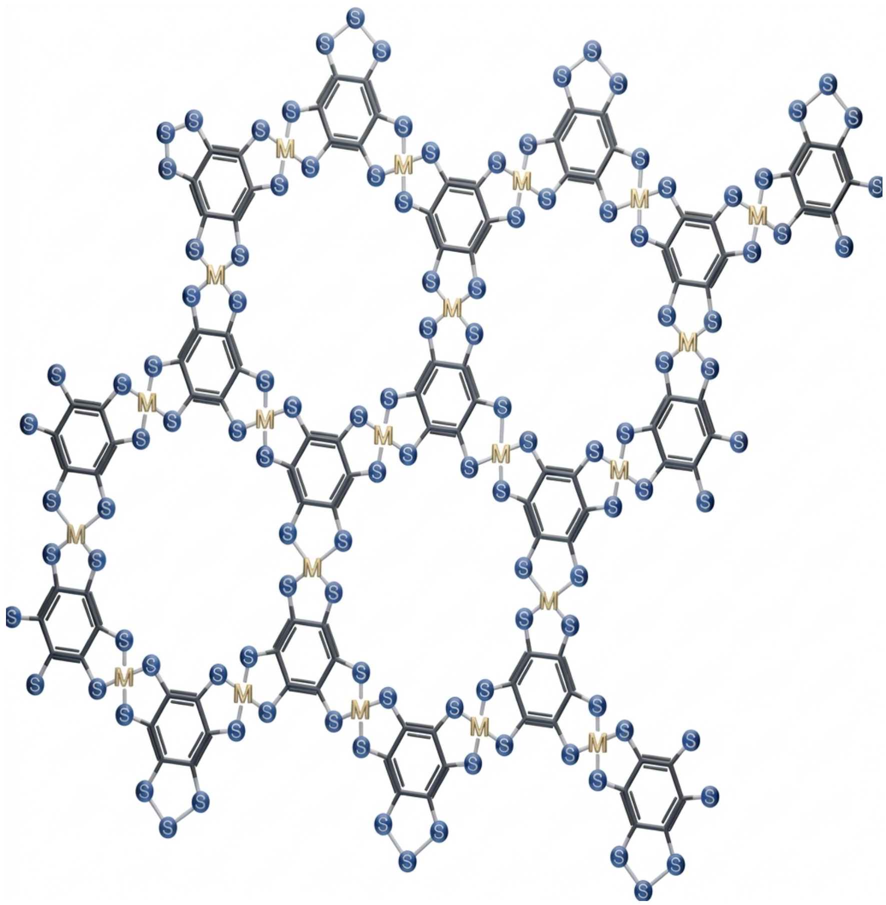

FIGURE 2. Molecular Structure of CoBHT MoF ($\mathbb{CBL}$)

In 2024, Khan et al. initiated a study on reduced reverse degree-based polynomials for a graph [7]. They utilized the $\mathcal{RR}_{\mathcal{D}}\mathcal{M}$-Polynomial to examine differential and integral operators. A graph $\mathbb{X}$ with an edge set $\mathbb{E}(\mathbb{G})$ and a collection of vertices $\mathbb{V}(\mathbb{G})$ can be used to represent a chemical structure. This study employs the $\mathrm{RR_D M}$-Polynomial to generate twelve reduced reverse degree-based topological descriptors. Topological descriptors, or $\mathrm{TD}s$, are constants that offer essential information regarding the molecular topology of a molecular graph [7]. Kulli in [8] introduced the concept of reverse degree vertex, defined as $\mathbb{R}_{\mathbb{D}}(\sigma) = \delta(\mathbb{X}) - d(\sigma) + 1$, which inspired Ravi to establish the reduced reverse degree $\mathbb{RRD}(\sigma) = \delta(\mathbb{X}) - d(\sigma) + 2$ in [13]. Hakami et al. [6] recently developed reverse degree-based polynomials, differential and integral operations, and topological descriptors. $d(\sigma)$ and $\mathbb{RRD}(\sigma)$ denote the degree and reduced reverse degree of an atom for the vertex $\sigma \in \mathbb{V}(\mathbb{F})$, whereas $\delta(\mathbb{X})$ signifies the maximum degree of Graph $\mathbb{X}$, and β represents the reduced reverse degree of $\mathbb{X}$. Examine the collection $\mathbb{RRD} = \{(\gamma, \mu) \in \mathbb{N} \times \mathbb{N} : 1 \leq \gamma \leq \mu \leq \beta\}$ We define $\mathbb{RRD}(w, y) = \{\sigma = wy \in \mathbb{E}(\mathbb{X}) : \ \mathbb{RRD}(s) = \gamma, \mathbb{RRD}(t) = \mu\}$. Using the variables w and y, we define our novel idea of a polynomial, termed the Reduced Reverse Degree based $\mathbb{RRD}\,\mathbb{M}$ - polynomial, according to previous research on $\mathbb{RRDM}$ - polynomials [7].

$$\mathbb{RRD}\,\mathbb{M}(\mathbb{X}; w, y) = \sum_{\gamma \leq \mu} \nu(\gamma, \mu) w^{\gamma} y^{\mu} \tag{2. 3}$$

$\nu(r, u)$ are used to represent the number of edges $\tau\tau \in \mathbb{E}(\mathbb{X})$ such that $\{\mathbb{RRD}(\tau), \mathbb{RRD}(\tau)\} = \{c, v\}$

The formulae connecting the $\mathbb{RRD}\,\mathbb{M}$-Polynomial to the reduced reverse degree-based topological descriptors are found in Table 1. In Table 1, operators are defined as below

Topological index	**Formula From $\mathbb{RRDM}(\mathbb{X})$**
$\mathbb{RRDM}_1$	$(\mathbb{D}_c + \mathbb{D}_v)(\mathbb{RRD}\,\mathbb{M}(\mathbb{X}))\big\vert_{c=v=1}$
$\mathbb{RRDM}_2$	$(\mathbb{D}_c\mathbb{D}_v)(\mathbb{RRD}\,\mathbb{M}(\mathbb{X})\big\vert_{c=v=1}$
$\mathbb{RRDHM}_1$	$(\mathbb{D}_c + \mathbb{D}_v)^2(\mathbb{RRD}\,\mathbb{M}(\mathbb{X})\big\vert_{c=v=1}$
$\mathbb{RRD}\sigma$	$(\mathbb{D}_c - \mathbb{D}_v)^2(\mathbb{RRD}\,\mathbb{M}(\mathbb{X}))\big\vert_{c=v=1}$
$\mathbb{RRD}^m\mathbb{M}_2$	$(\mathbb{I}_c\mathbb{I}_v)(\mathbb{RRD}\,\mathbb{M}(\mathbb{X})\big\vert_{c=v=1}$
$\mathbb{RRDR}e\mathbb{ZG}_3$	$(\mathbb{D}_c\mathbb{D}_v)(\mathbb{D}_c + \mathbb{D}_v)(\mathbb{RRD}\,\mathbb{M}(\mathbb{X})\big\vert_{c=v=1}$
$\mathbb{RRDSDD}$	$(\mathbb{D}_c\mathbb{I}_v + \mathbb{I}_c\mathbb{D}_v)(\mathbb{RRD}\,\mathbb{M}(\mathbb{X})\big\vert_{c=v=1}$
$\mathbb{RRDH}$	$2J\mathbb{I}_c(\mathbb{RRD}\,\mathbb{M}(\mathbb{X})\big\vert_{c=v=1}$
$\mathbb{RRDI}$	$\mathbb{I}_c J\mathbb{D}_c\mathbb{D}_v(\mathbb{RRD}\,\mathbb{M}(\mathbb{X})\big\vert_{c=v=1}$

TABLE 1. Topological indices derivation formula from $\mathbb{RRD}\,\mathbb{M}$-Polynomial

:

$$\mathbb{D}_c = c\frac{\partial(\mathbb{RRDM}(\mathbb{X}; c, v))}{\partial c} \tag{2. 4}$$

$$\mathbb{D}_v = v\frac{\partial(\mathbb{RRD}\,\mathbb{M}(\mathbb{X}; c, v))}{\partial v} \tag{2. 5}$$

$$\mathbb{I}_v = \int_0^v \frac{1}{z}(\mathbb{RRDM}(\mathbb{X}; z, v))dz \tag{2. 6}$$

$$\mathbb{I}_c = \int_0^c \frac{1}{z}(\mathbb{RRDM}(\mathbb{X}; c, z))dz \tag{2. 7}$$

$$\mathbb{J}(g(c, v)) = g(c, c)$$

$$\mathbb{Q}_\alpha = c^\alpha g(c, v)$$

3. RESULTS

This section contains the findings of this study.

3.1. $\mathbb{RRDM}-$Polynomials, Operators and $\mathcal{TD}s$ of MoF FeTPyP-Co ($\mathbb{FTC}$).

Atom Bond Partition	(5,3)	(4,3)	(3,3)	(3,2)
Cardinality	$24kb+1$	$6k+6b-6$	$56kb-4k-4b+2$	$8kb-4k-4b+4$

TABLE 2. $\mathbb{RRD}$ Based Edge Partition of FeTPyP-Co MOFs Structure

Reduced Reverse Degree-Based $\mathbb{RRDM}$-Polynomial for $\mathbb{FTC}$

Employing Equation (2. 3) and Table 2, we get

$$\begin{aligned}\mathbb{RRDM}(\mathbb{FTC};x,y) &= \sum_{i\leq j}\nu_{(r,u)}x^iy^j\\ &= \nu_{(5,3)}x^5y^3+\nu_{(4,3)}x^4y^3\nu_{(3,3)}x^3y^3+\nu_{(3,2)}x^3y^2\end{aligned}$$

This implies

$$\begin{aligned}\mathbb{RRDM}(\mathbb{FTC};x,y) &= (24kb+1)x^5y^3+[6k+6b-6]x^4y^3\\ &+ [56kb-4k-4b+2]x^3y^3\\ &+ [8kb-4k-4b+4]x^3y^2\end{aligned} \tag{3. 8}$$

Differential Operators for $\mathbb{FTC}$

Equation (3. 8) & Equation (2. 4) gives $\mathbb{D}_x(\mathbb{RRDM}(\mathbb{FTC};x,y))$ for the structure of $\mathbb{FTC}$.

$$\begin{aligned}\mathbb{D}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= x\frac{\partial \mathbb{RRDM}\mathbb{FTC};x,y)}{\partial x}\\ &= x\frac{\partial}{\partial x}\Big[(24kb+1)x^5y^3+[6k+6b-6]x^4y^3\\ &+ [56kb-4k-4b+2]x^3y^3\\ &+ [8kb-4k-4b+4]x^3y^2\Big]\end{aligned}$$

$$\begin{aligned}\mathbb{D}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= x\frac{\partial}{\partial x}(24kb+1)x^5y^3+x\frac{\partial}{\partial x}[6k+6b-6]x^4y^3\\ &+ x\frac{\partial}{\partial x}[56kb-4k-4b+2]x^3y^3\\ &+ x\frac{\partial}{\partial x}[8kb-4k-4b+4]x^3y^2\end{aligned}$$

Utilizing differential operator (2. 4), we have

$$\begin{aligned}\mathbb{D}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= [120kb+5]x^5y^3+(24k+24b-24)x^4y^3\\ &+ [168kb-12k-12b+6]x^3y^3\\ &+ [24kb-12k-12b+12]x^3y^2\end{aligned} \tag{3. 9}$$

Similarly, using Equation (2. 5) and (3. 22) :

$$
\begin{aligned}
\mathbb{D}_y(M(\mathbb{FTC};x,y)) &= y\frac{\partial \mathbb{RRDM}\mathbb{FTC};x,y)}{\partial y} \\
&= y\frac{\partial}{\partial y}\Big[(24kb+1)x^5y^3+[6k+6b-6]x^4y^3 \\
&+ [56kb-4k-4b+2]x^3y^3 \\
&+ [8kb-4k-4b+4]x^3y^2\Big]
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{D}_y(M(\mathbb{FTC};x,y)) &= y\frac{\partial}{\partial y}[24kb+1]x^5y^3+y\frac{\partial}{\partial y}(6k+6b-6)x^4y^3 \\
&+ y\frac{\partial}{\partial y}[56kb-4k-4b+2]x^3y^3 \\
&+ y\frac{\partial}{\partial y}[8kb-4k-4b+4]x^3y^2
\end{aligned}
$$

Utilizing differential operator (2. 5) , we have

$$
\begin{aligned}
\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= [72kb+3]x^5y^3+(18k+18b-18)x^4y^3 \\
&+ [168kb-12k-12b+6]x^3y^3 \\
&+ [16kb-8k-8b+8]x^3y^2 \qquad (3.\ 10)
\end{aligned}
$$

The integral operators for $\mathbb{FTC}$

Equation (3. 8) & Equation (2. 6) implies $\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)$ for the structure of $\mathbb{FTC}$.

$$
\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) = \int_0^x \frac{\mathbb{RRDM}(\mathbb{FTC};z,y)}{z}dz
$$

Utilizing Integral operator (2. 6), we obtain

$$
\begin{aligned}
\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x,y) &= \int_0^x \frac{\mathbb{RRDM}(\mathbb{FTC};z,y)}{z}dz \\
\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \int_0^x [\frac{1}{5}(24kb+1)z^5y^3+[\frac{1}{4}(6kb+6b-6)z^4y^3] \\
&+ \frac{1}{3}(56kb-4k-4b+2)z^3y^3] \\
&+ [\frac{1}{3}(8kb-4k-4b-4)z^3y^2]dz
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x,y) &= [\frac{1}{5}(24kb+1)z^5y^3]\big|_0^x+[6k+6b-6)z^3y^3]\big|_0^x \\
&+ [\frac{1}{3}(56kb-4k-4b+2)z^3y^3]\big|_0^x \\
&+ [\frac{1}{3}(8kb-4k-4b-4)z^3y^2]\big|_0^x
\end{aligned}
$$

Simplifying last relation gives

$$\begin{aligned}\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{1}{5}(24kb+1)x^5y^3+[\frac{1}{4}(6k+6b-4)x^4y^3]\\ &+ [\frac{1}{3}(56kb-4k-4b+2)x^3y^3]\\ &+ [\frac{1}{3}(8kb-4k-4b-4)x^3y^2]\end{aligned}$$

Similarly, using Equation (2. 7) and (3. 22) we gain

$$\begin{aligned}\mathbb{I}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \int_0^y \frac{\mathbb{RRDM}(\mathbb{FTC};x,z)}{z}dz\\ \mathbb{I}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \int_0^y (24kb+1)x^5z^3\\ &+ [6k+6b-6)x^4z^2]\\ &+ [(56kb-4k-4b+2)x^3z^2]\\ &+ [(8kb-4k-4b+4)x^3z^2]dz\end{aligned}$$

Utilizing Integral operator, we obtain

$$\begin{aligned}\mathbb{I}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \int_0^y \frac{1}{3}(24kb+1)x^5z^3dz+\int_0^y \frac{1}{3}[6k+6b-6]x^2z^3dz\\ &+ \int_0^y \frac{1}{3}[56kb-4k-4b+2]x^3z^3dz\\ &+ \int_0^y \frac{1}{2}[8kb-4k-4b+4]x^3z^2dz\end{aligned}$$

$$\begin{aligned}\mathbb{I}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{1}{3}[24kb+1]x^5y^3\big|_0^y+\frac{1}{3}(6k+6b-6)x^4z^2\big|_0^y\\ &+ \frac{1}{3}(56kb-4k-4b+2)x^3z^2\big|_0^y\\ &+ \frac{1}{2}(8kb-4k-4b+4)x^3z^2\big|_0^y\end{aligned}$$

Simplifying last relation gives

$$\begin{aligned}\mathbb{I}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{1}{3}(24kb+1)x^5y^3\\ &+ \frac{1}{3}(6k+6b-6)x^4y^3\\ &+ \frac{1}{3}(56kb-4k-4b+2)x^3y^3\\ &+ \frac{1}{2}(8kb-4k-4b+4)x^3y^2 \qquad (3.\ 11)\end{aligned}$$

Reduced Reverse First Zagreb $\mathbb{RRDM}$-Polynomial for $\mathbb{FTC}$

Employing Equation (2. 3), Table 1 and Equations (2. 4) and (2. 5) provides a Reduced

Reverse First Zagreb $\mathbb{RRDM}$-Polynomial.

$$\begin{aligned}\mathbb{RRDM}_1(\mathbb{FTC};x,y) &= (\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y))\\ &= \mathbb{D}_x(\mathbb{RRDM}(\mathbb{FTC};x,y))+\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x,y))\\ &= [120kb+15]x^5y^3+(24k+24b-24)x^4y^3\\ &\quad+[168kb-12k-12b+6]x^3y^3\\ &\quad+[24kb-12k-12b+12]x^3y^2\\ &\quad+[72kb+3]x^5y^3+(18k+18b-18)x^4y^3\\ &\quad+[168kb-12k-12b+6]x^3y^3\\ &\quad+[16kb-8k-8b+8]x^3y^2.\end{aligned} \tag{3. 13}$$

This gives

$$\begin{aligned}\mathbb{RRDM}_1(\mathbb{FTC};x,y)) &= (192kb+9)x^5y^3+(42kb+42k-42)x^4y^3\\ &+ (336kb-24k-24b+12)x^3y^3\\ &+ (40kb-20k-20b+20)x^3y^2\end{aligned} \tag{3. 14}$$

Reduced Reverse Second Zagreb $\mathbb{RRDM}$-Polynomial for $\mathbb{FTC}$

Employing Equations (2. 3), (2. 4) and (2. 5) we get

$$\begin{aligned}\mathbb{RRDM}_2(\mathbb{FTC},x,y) &= \mathbb{D}_x[72kb+3]x^5y^3+(18k+18b-18)x^4y^3\\ &+ [168kb-12k-12b+6]x^3y^3\\ &+ [16kb-8k-8b+8]x^3y^2\\ \mathbb{RRDM}_2(\mathbb{FTC},x,y) &= x\frac{\partial}{\partial x}[72kb+3]x^5y^3+x\frac{\partial}{\partial x}[18k+18b+18]x^4y^3\\ &+ x\frac{\partial}{\partial x}[168kb-12k-12b+6]x^3y^3\\ &+ x\frac{\partial}{\partial x}[16kb-8k-8b+8]x^3y^2\end{aligned}$$

This gives

$$\begin{aligned}\mathbb{RRDM}_2(\mathbb{FTC};x,y)) &= (360kb+15)x^5y^3\\ &+ (72kb+72b-72)x^4y^3\\ &+ (504kb-36k-36b+18)x^3y^3\\ &+ (48kb-25k-24b+25)x^3y^2\end{aligned} \tag{3. 15}$$

Reduced Reverse First Zagreb Index of $\mathbb{FTC}$
By adding (3. 9) and (3. 10), we receive

$$\begin{aligned}
&\mathbb{D}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) + \mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x,y))\\
&\quad = [120kb+5]x^5y^3 + (24k+24b-24)x^4y^3\\
&\qquad + [168kb-12k-12b+6]x^3y^3\\
&\qquad + [24b-12k-12b+12]x^3y^2 + [72kb+3]x^5y^3\\
&\qquad + (18k+18b-18)x^4y^3\\
&\qquad + [168kb-12k-12b+6]x^3y^3\\
&\qquad + [16kb-8k-8b+8]x^3y^2,
\end{aligned}$$

$$\begin{aligned}
&(\mathbb{D}_x + \mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y))\\
&\quad = (192kb+8)x^5y^3 + (42b+42k-42)x^4y^3\\
&\qquad + (336kb-24k-24b+12)x^3y^3\\
&\qquad + (40kb-20k-20b+20)x^3y^2,
\end{aligned}$$

$$\begin{aligned}
&(\mathbb{D}_x + \mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y))\\
&\quad = (192kb+8)x^5y^3 + (42b+42k-42)x^4y^3\\
&\qquad + (336kb-24k-24b+12)x^3y^3\\
&\qquad + (40kb-20k-20b+20)x^3y^2,
\end{aligned}$$

$$\mathbb{RRDM}_1(\mathbb{FTC}) = 568kb - 2k - 2b - 2.$$

Reduced Reverse Second Zagreb Index for $\mathbb{FTC}$

Employing Table 1 along with Equation (3. 15), we get

$$\begin{aligned}
\mathbb{RRDM}(\mathbb{FTC};x,y) &= (24kb+1)x^5y^3+[6k+6b-6]x^4y^3\\
&+ [56kb-4k-4b+2]x^3y^3\\
&+ [8kb-4k-4b+4]x^3y^2\\
\mathbb{D}_x[(\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x,y))] &= \mathbb{D}_x[72kb+3]x^5y^3+(18k+18b-18)x^4y^3\\
&+ [168kb-12k-12b+6]x^3y^3\\
&+ [16kb-8k-8b+8]x^3y^2\\
(\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (360kb+15)x^5y^3\\
&+ (72kb+72b-72)x^4y^3\\
&+ (504kb-36k-36b+18)x^3y^3\\
&+ (48kb-25k-24b+25)x^3y^2\\
(\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (360kb+15)x^5y^3\\
&+ (72k+72b-72)x^4y^3\\
&+ (504kb-36k-36b+18)x^3y^3\\
&+ (48kb-24k-24b+24)x^3y^2\\
\mathbb{RRDM}_2(\mathbb{FTC}) &= 912kb+12k+12b-15
\end{aligned}$$

Reduced Reverse Hyper 1^{st} Zagreb Index of $\mathbb{FTC}$

Emplying of Table 1 and Equation (3. 14), we have

$$\begin{aligned}
\mathbb{D}_X^2(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (600kb+25)x^5y^3+(96k+96b-96)x^4y^3\\
&\quad+[504kb-36k-36b+36]x^3y^3\\
&\quad+[72kb-36k-36b+36]x^3y^2,
\end{aligned}$$

$$\begin{aligned}
\mathbb{D}_y^2(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (216kb+9)x^5y^3+[54k+54b-54]x^4y^3\\
&\quad+[504kb-36k-36b+18]x^3y^3\\
&\quad+[32kb-16k-16b+16]x^4y^4,
\end{aligned}$$

$$\begin{aligned}
(2\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (720kb+30)x^5y^3+[144k+144b-144]x^4y^3\\
&\quad+[1008kb-72k-72b+36]x^3y^3\\
&\quad+[96kb-48k-48b+48]x^3y^2.
\end{aligned}$$

Employing limits gives

$$\mathbb{RRDHM}_1(\mathbb{FTC}) = 3752kb+50k+50b-58$$

Reduced Reverse Sigma Index for $\mathbb{RRDM}$-Polynomial of $\mathbb{FTC}$

Using Table 1 and applying the differential operator to Equations (2. 4) and (2. 5), we obtain

$$\begin{aligned}
\mathbb{D}_x^2(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (600kb+3)x^5y^3 + (96kb+96h-96)x^4y^3 \\
&+ [504kb-36k-36b+36]x^3y^3 \\
&+ [72kb-36k-36b+36]x^3y^2,
\end{aligned}$$

$$\begin{aligned}
\mathbb{D}_y^2(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (216kb+9)x^5y^3 + [54k+54b-54]x^4y^3 \\
&+ [504kb-36k-36b+18]x^3y^3 \\
&+ [32kb-16k-16b+16]x^3y^2,
\end{aligned}$$

$$\begin{aligned}
(2\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (720kb+30)x^5y^3 + [144k+144b-144]x^4y^3 \\
&+ [1008kb-72k-72b+36]x^3y^3 \\
&+ (96kb-144k-144b+48)x^3y^2,
\end{aligned}$$

$$\begin{aligned}
(\mathbb{D}_x^2+\mathbb{D}_y^2-2\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= (96kb-18)x^5y^3 + (6k+6b-6)x^4y^3 \\
&+ (8kb-4k-4b+4)x^3y^2.
\end{aligned}$$

Employing limits gives

$$\mathbb{RRD}\sigma(\mathbb{FTC}) \quad = \quad 104kb+118k+118b-4$$

Reduced Reverse Second Modified Zagreb Index for $\mathbb{RRD}^m\mathbb{M}_2$-Polynomial of $\mathbb{FTC}$

On Table 1, apply $\mathbb{I}_x$ on (3. 11)

$$\begin{aligned}
(\mathbb{I}_x\mathbb{I}_y)(\mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC}:x,y)) &= \mathbb{I}_x\left[\mathbb{I}_y(\mathbb{RRDM}(\mathbb{FTC}:x,y))\right] \\
&= \mathbb{I}_x\left[\frac{1}{3}(24kb+1)x^5y^3 + \frac{1}{3}(6k+6b-6)x^4y^3\right. \\
&\quad + \frac{1}{3}(56kb-4k-4b+2)x^3y^3 \\
&\quad \left. + \frac{1}{2}(8kb-4k-4b+4)x^3y^2\right].
\end{aligned} \tag{3. 16}$$

Simplifying last relation gives

$$\begin{aligned}
(\mathbb{I}_x\mathbb{I}_y)(M(G:x,y)) \quad &= \quad \frac{1}{15}(24kb+1)x^5y^3 \\
&+ \quad \frac{1}{12}(6k+6b-6)x^4y^3 \\
&+ \quad \frac{1}{9}(56kb-4k-4b+2)x^3y^3 \\
&+ \quad \frac{1}{6}(8kb-4k-4b+4)x^3y^2
\end{aligned}$$

$$\begin{aligned}(\mathbb{I}_x\mathbb{I}_y)(\mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC}:x,y)) &= \frac{1}{15}(24kb+1)x^5y^3\\ &+ \frac{1}{12}(6k+6b-6)x^4y^3\\ &+ \frac{1}{9}(56kb-4k-4b+2)x^3y^3\\ &+ \frac{1}{6}(8kb-4k-4b+4)x^3y^2\end{aligned}$$

Employing limits gives

$$\mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC}) = \frac{412}{90}kb-\frac{11}{18}k-\frac{11}{18}b+\frac{41}{90}$$

Reduced Reverse Redefined Third Zagreb Index of $\mathbb{FTC}$

From Table 1, by utilizing Equation (2. 3) gives

$$\begin{aligned}\mathbb{RRDM}(\mathbb{FTC};x,y) = (24kb+1)x^5y^3+(6k+6b-6)x^4y^3\\ +(56kb-4k-4b+2)x^3y^3\\ +(8kb-4k-4b+4)x^3y^2.\end{aligned} \tag{3. 17}$$

$$\begin{aligned}(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) = (192kb+18)x^5y^3\\ +(42kb+42k-42)x^4y^3\\ +(336kb-24k-24b+12)x^3y^3\\ +(40kb-20k-20b+20)x^3y^2.\end{aligned} \tag{3. 18}$$

$$\begin{aligned}\mathbb{D}_y(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) = \mathbb{D}_y\Big[(192kb+9)x^5y^3\\ +(42kb+42k-42)x^4y^3\\ +(336kb-24k-24b+12)x^3y^3\\ +(40kb-20k-20b+20)x^3y^2\Big].\end{aligned} \tag{3. 19}$$

$$\begin{aligned}\mathbb{D}_y(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) = (576kb+54)x^5y^3\\ +(126b+126h-126)x^4y^3\\ +(1008kb-72k-72b+36)x^3y^3\\ +(80kb-40g-40h+40)x^3y^2.\end{aligned} \tag{3. 20}$$

$$\begin{aligned}(\mathbb{D}_x\mathbb{D}_y)(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) = (2880kb+270)x^5y^3\\ +(504k+504b-504)x^4y^3\\ +(6048kb-216k-2016b+108)x^3y^3\\ +(240kb-120k-120b+120)x^3y^2.\end{aligned} \tag{3. 21}$$

Employing limits gives

$$\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{FTC}) = 3724kb + 168k + 168b - 6$$

Reduced Reverse Symmetric Division Degree Index of $\mathbb{FTC}$

Employing Table 1 along with Equation (3. 11), we have

$$\begin{aligned}(\mathbb{I}_y)(\mathbb{RRDM}(\mathbb{FTC}: x, y)) &= [\frac{1}{3}(24kb+1)x^5y^3] + [\frac{1}{3}(6k+6b-6)]x^4y^3 \\ &+ [\frac{1}{3}(56kb-4k-4b+2)]x^3y^3 \\ &+ \frac{1}{2}[8kb-4k-4b+4]x^3y^2\end{aligned}$$

Utilizing operator $\mathbb{D}_y$, we have

$$\begin{aligned}(\mathbb{D}_x\mathbb{I}_y)(\mathbb{RRDM}(\mathbb{FTC}: x, y)) &= \frac{5}{3}(24kb+1)x^5y^3 + [\frac{4}{3}(6k+6b-6)]x^4y^3 \\ &+ [56kb-4k-4b+2]x^3y^3 \\ &+ [\frac{3}{2}(8kb-4k-4b+4)]x^3y^2\end{aligned}$$

From Equation (3. 10), we have

$$\begin{aligned}\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC}; x, y)) &= (72kb+3)x^5y^3 + [18k+18b-18]x^4y^3 \\ &+ [168kb-12k-12b+6]x^3y^3 \\ &+ [16kb-8k-8b+6]x^3y^2\end{aligned}$$

Utilizing integral operator $\mathbb{D}_x$, we have

$$\begin{aligned}
\mathbb{I}_x\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{1}{5}(72kb+3)x^5y^3+[\frac{1}{2}(9k+9b-9)]x^4y^3\\
&+ [56kb-4k-4b+2]x^3y^3\\
&+ [\frac{1}{3}(16kb-8k-8b+8]x^3y^2\\
(\mathbb{D}_x\mathbb{I}_y+\mathbb{I}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{5}{3}(24kb+1)x^5y^3+[\frac{4}{3}(6k+6b-6)]x^4y^3\\
&+ [56kb-4k-4b+2]x^3y^3\\
&+ [\frac{3}{2}(8kb-4k-4b+4)]x^3y^2+(72kb+3)x^5\\
&+ [18k+18b-18]x^4y^3\\
&+ [168kb-12k-12b+6]x^3y^3\\
&+ [16kb-8k-8b+6]x^3y^2\\
(\mathbb{D}_x\mathbb{I}_y+\mathbb{I}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{5}{3}(24kb+1)x^5y^3+[\frac{4}{3}(6k+6b-6)]x^4y^3\\
&+ [56kb-4k-4b+2]x^3y^3\\
&+ [\frac{3}{2}(8kb-4k-4b+4)]x^3y^2+(72kb+3)x^5\\
&+ [18k+18b-18]x^4y^3\\
&+ [168kb-12k-12b+6]x^3y^3\\
&+ [16kb-8k-8b+6]x^3y^2
\end{aligned}$$

$$\mathbb{RRDSDD}(\mathbb{FTC}) = \frac{1148}{6}kb+\frac{31}{6}k+\frac{31}{6}b-\frac{153}{6}$$

Reduced Reverse Harmonic Index for $\mathbb{RRDM}$-Polynomial of $\mathbb{FTC}$

Employing Equation (2. 3) and Table 1, we have

$$\begin{aligned}
(\mathbb{RRDM}(\mathbb{FTC};x;y)) &= (24kb+1)x^5y^3+[6k+6b-6]x^4y^3\\
&+ [56kb-4k-4b+2]x^3y^3\\
&+ [8kb-4k-4b+4]x^3y^2
\end{aligned}$$

This gives

$$\begin{aligned}
\mathbb{I}_xJ(\mathbb{RRDM}(\mathbb{FTC};x;y)) &= (24kb+1)x^8+[6k+6b-6]x^7\\
&+ (56kb-4k-4b+2)x^6\\
&+ [8kb-4k-4b+4]x^5
\end{aligned}$$

Utilizing integral operator $\mathbb{I}_x$, we have

$$\begin{aligned}
2J\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x;y)) &= \frac{2}{8}(24kb+1)x^8 \\
&+ [\frac{2}{7}(6k+6b-6)x^7 + \frac{2}{6}[56kb_4k_4b+2]]x^6 \\
&+ \frac{2}{5}(8kb-4k-4b+4)x^5 \\
2J\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x;y))\big|_{x=1} &= \frac{2}{8}(24kb+1)x^8 + [\frac{2}{7}(6k+6b-6)x^7 \\
&+ \frac{2}{6}[56kb_4k_4b+2]]x^6 \\
&+ \frac{2}{5}(8kb-4k-4b+4)x^5\big|_{x=1}
\end{aligned}$$

$$\mathbb{RRD}H(\mathbb{FTC}) = \frac{418}{15}kb - \frac{128}{105}k - \frac{128}{105}b + \frac{337}{420}$$

Reduced Reverse Inverse Sum Index for $\mathbb{RRDM}$-Polynomial of $\mathbb{FTC}$

Employing Equation (3. 15) and Table 1, we get

$$\begin{aligned}
\mathbb{D}_x\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x;y)) &= (360kb+15)x^5y^3 + [72k+72b-72]x^4y^3 \\
&+ [504kb-36k-36b+18]x^3y^3 \\
&+ [48kb-24k-24b+24]x^3y^2
\end{aligned}$$

This gives

$$\begin{aligned}
J\mathbb{D}_x\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x;y)) &= (360kb+15)x^8 + [72k+72b-72]x^7 \\
&+ [504kb-36k-36b+18]x^6 \\
&+ [48kb-24k-24b+24]x^5
\end{aligned}$$

Utilizing the integral operator $\mathbb{I}_x$, we have

$$\begin{aligned}
\mathbb{I}_xJ\mathbb{D}_x\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x;y)) &= \frac{1}{8}[(360kb+15)]x^8 + \frac{1}{7}[72k+72b-72]x^7 \\
&+ \frac{1}{6}[504kb-36k-36b+18]x^6 \\
&+ \frac{1}{5}[48kb-24k-24b+24]x^5 \\
\mathbb{I}_xJ\mathbb{D}_x\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x;y))\big|_{x=1} &= \frac{1}{8}[(360kb+15)] + \frac{1}{7}[72k+72b-72] \\
&+ \frac{1}{6}[504kb-36k-36b+18] \\
&+ \frac{1}{5}[48kb-24k-24b+24]\big|_{x=1}
\end{aligned}$$

$$\mathbb{RRD}I(\mathbb{FTC}) = \frac{10566}{210}kb - \frac{1608}{210}k - \frac{1608}{210}b + \frac{2232}{630}$$

3.2. **Python Code for $\mathbb{RRDM}-$Polynomial and Differential and Integral Operators for $\mathbb{FTC}$.** For the sake of the accuracy of the results, the Python programming is utilized, and code for $\mathbb{RRDM}-$Polynomial for $\mathbb{FTC}$ is given in Figure 3 and Code for the Differential and Integral Operators for $\mathbb{FTC}$ is given in Figure 4.

```
import sympy as sp
# Define symbols
x, y, k, b = sp.symbols('x y k b')

# Given edge partition with frequencies
edge_partition = {
   (1, 3): 24*k*b + 1,
   (2, 3): 6*k+ 6*b - 6,
   (3, 3): 56*k*b - 4*k - 4*b + 2,
   (4, 4): 8*k*b - 4*k - 4*b + 4
}

# Construct reduced reverse degree M-polynomial
RRDM = sum(freq * x**i * y**j for (i, j), freq in edge_partition.items())

# Expand polynomial
RRDM_expanded = sp.expand(RRDM)

RRDM_expanded
```

FIGURE 3. Python Code for $\mathbb{RRDM}-$Polynomial for $\mathbb{FTC}$

```
import sympy as sp

# Define symbols
x, y = sp.symbols('x y')

# Example: Reduced reverse degree M-polynomial
RRDM = (
    (24*sp.symbols('k')*sp.symbols('b') + 1)*x*y**3
    + (6*sp.symbols('k') + 6*sp.symbols('b') - 6)*x**2*y**3
    + (56*sp.symbols('k')*sp.symbols('b') - 4*sp.symbols('k')
      - 4*sp.symbols('b') + 2)*x**3*y**3
    + (8*sp.symbols('k')*sp.symbols('b') - 4*sp.symbols('k')
      - 4*sp.symbols('b') + 4)*x**4*y**4
)

# ---------------------------
# Differential Operators
# ---------------------------

def D_x(F):
    """ Differential operator D_x """
    return sp.simplify(x * sp.diff(F, x))

def D_y(F):
    """ Differential operator D_y """
    return sp.simplify(y * sp.diff(F, y))

# ---------------------------
# Integral Operators
# ---------------------------

def I_x(F):
    """ Integral operator I_x """
    t = sp.symbols('t', positive=True)
    return sp.simplify(sp.integrate(F.subs(x, t)/t, (t, 0, x)))

def I_y(F):
    """ Integral operator I_y """
    t = sp.symbols('t', positive=True)
    return sp.simplify(sp.integrate(F.subs(y, t)/t, (t, 0, y)))
# ---------------------------
# Apply Operators
# ---------------------------
Dx_RRDM = D_x(RRDM)
Dy_RRDM = D_y(RRDM)

Sx_RRDM = S_x(RRDM)
Sy_RRDM = S_y(RRDM)

# Display results
Dx_RRDM, Dy_RRDM, Ix_RRDM, Iy_RRDM
```

FIGURE 4. Python Code for Differential and Integral Operators for $\mathbb{FTC}$

3.3. **Graphical Analysis of Numerical Values of computed TDs of $\mathbb{FTC}$.** The computational values of TDs are shown in Tables 3-7. For numerical computation purposes, we use the same values ranging from 1 to 9 for k and b. We also presented a graphical representation of these computational values in Figure 5. The $\mathbb{TD}s$ values gradually increased as the values of k and b increased. Based on our findings, we have obtained the following relation between $\mathbb{RRD}$ TDs:

$\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{FTC}) > \mathbb{RRDM}_2(\mathbb{FTC}) > \mathbb{RRDHM}_1(\mathbb{FTC}) > \mathbb{RRDM}_1(\mathbb{FTC}) > \mathbb{RRDSDD}(\mathbb{FTC}) > \mathbb{RRD}\sigma(\mathbb{FTC}) > \mathbb{RRDI}(\mathbb{FTC}) > \mathbb{RRDH}(\mathbb{FTC}) > \mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC})$.

$k=b$	$\mathbb{RRDM}_1(\mathbb{FTC})$	$\mathbb{RRDM}_2(\mathbb{FTC})$	$\mathbb{RRDHM}_1(\mathbb{FTC})$	$\mathbb{RRD}\sigma(\mathbb{FTC})$
1	562	921	794	88
2	2262	3681	3150	404
3	5098	8265	7010	928
4	9070	14673	12375	1660
5	14178	20905	19242	2600
6	20422	32961	27614	3748
7	27802	44841	37490	5104
8	36318	58545	48870	6668
9	45970	74073	61754	8440

TABLE 3. Numerical values of $\mathbb{RRDM}_1(\mathbb{FTC})$, $\mathbb{RRDM}_2(\mathbb{FTC})$, $\mathbb{RRDHM}_1(\mathbb{FTC})$ and $\mathbb{RRD}\sigma(\mathbb{FTC})$

$k=b$	$\mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC})$	$\mathbb{RRD}H(\mathbb{FTC})$	$\mathbb{RRDSDD}(\mathbb{FTC})$	$\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{FTC})$	$\mathbb{RRD}I(\mathbb{FTC})$
1	3.81111	26.23	176.167	4054	38.542857
2	16.322222	10.39	760.5	15562	174.17171429
3	37.98889	244.28	1727.5	34518	410.428571
4	68.8111	436.92	3077.17	60922	747.314286
5	108.788899	685.28	4809.5	94774	1184.828571
6	157.922	989.37	6924.5	136074	1722.971429
7	216.21111	1379.20	9422.17	184822	2361.742857
8	283.6555	1764.76	12302.5	241018	3101.142857
9	360.255556	2236.10	15565.5	304662	3941.171429

TABLE 4. Computational values of $\mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC})$, $\mathbb{RRDH}(\mathbb{FTC})$, $\mathbb{RRDSDD}(\mathbb{FTC})$, $\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{FTC})$, $\mathbb{RRD}I(\mathbb{FTC})$

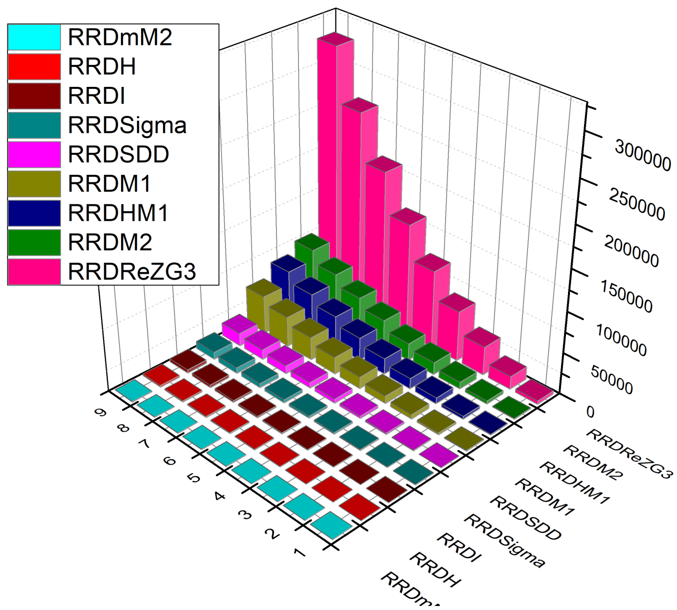

FIGURE 5. Comparing numerical values of $\mathbb{RRDTD}s$ of $\mathbb{FTC}$ graphically

Atom Bond Partition	(5,3)	(4,4)	(4,3)	(3,3)	(4,2)
Cardinality	$2(k+b)$	$2(k+b)$	$12kb-2(k+b)$	$12kb$	$12kb-2(k+b)$

TABLE 5. $\mathbb{RRD}$ Based Edge Partition of $\mathbb{CBL}$

3.4. $\mathbb{RRDM}$**- Polynomials, Differential, Integral Operators and Topological Descriptors of MoF CoBHT (CO) Lattice** $(\mathbb{CBL})$**.** **The** $\mathbb{RRDM}$**-polynomial for** $\mathbb{CBL}$
Employing Equation (2. 3) and Table 5, we have

$$\begin{aligned}\mathbb{RRDM}(\mathbb{CBL};x,y) &= \sum_{i\le j}\mu_{(r,u)}x^iy^j\\ &= \mu_{(5,3)}x^5y^3+\mu_{(4,4)}x^4y^4\mu_{(4,3)}x^4y^3+\mu_{(3,3)}x^3y^3+\mu_{(4,2)}x^4y^2\end{aligned}$$

This implies

$$\begin{aligned}\mathbb{RRDM}(\mathbb{CBL};x,y) &= 2(k+b)x^5y^3+2[k+b]x^4y^4 \\ &+ [12kb-2(k+b)]x^4y^3\\ &+ [12kb]x^3y^3\\ &+ [12kb-2(k+b)]x^4y^2\end{aligned} \tag{3. 22}$$

Differential operators for $\mathbb{CBL}$

Employing Equation (3. 22) and Equation (2. 4), we obtain

$$\begin{aligned}
\mathbb{D}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= x\frac{\partial \mathbb{RRDM}(\mathbb{CBL};x,y)}{\partial x} \\
&= x\frac{\partial}{\partial x}2(k+b)x^5y^3 + 2[k+b]x^4y^4 \\
&+ [12kb-2(k+b)]x^4y^3 \\
&+ [12kb]x^3y^3 \\
&+ [12kb-2(k+b)]x^4y^2
\end{aligned}$$

$$\begin{aligned}
\mathbb{D}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= x\frac{\partial}{\partial x}2(k+b)x^5y^3 + x\frac{\partial}{\partial x}2[k+b]x^4y^4 \\
&+ x\frac{\partial}{\partial x}[12kb-2(k+b)]x^4y^3 \\
&+ x\frac{\partial}{\partial x}[12kb]x^3y^3 \\
&+ x\frac{\partial}{\partial x}[12kb-2(k+b)]x^4y^2
\end{aligned}$$

Utilizing differential operator (2. 4), we have

$$\begin{aligned}
\mathbb{D}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 10(k+b)x^5y^3 + 8[k+b]x^4y^4 \\
&+ [48kb-8(k+b)]x^4y^3 \\
&+ [36kb]x^3y^3 \\
&+ [48kb-8(k+b)]x^4y^2 \qquad (3.\ 23)
\end{aligned}$$

Similarly, employing Equation (2. 5) and (3. 22) :

$$\begin{aligned}
\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= y\frac{\partial \mathbb{RRDM}(\mathbb{CBL};x,y)}{\partial y} \\
&= y\frac{\partial}{\partial y}2(k+b)x^5y^3 + 2[k+b]x^4y^4 \\
&+ [12kb-2(k+b)]x^4y^3 \\
&+ [12kb]x^3y^3 \\
&+ [12kb-2(k+b)]x^4y^2
\end{aligned}$$

$$\begin{aligned}\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= x\frac{\partial}{\partial x}2(k+b)x^5y^3 + x\frac{\partial}{\partial x}2[k+b]x^4y^4\\ &+ x\frac{\partial}{\partial x}[12kb-2(k+b)]x^4y^3\\ &+ x\frac{\partial}{\partial x}[12kb]x^3y^3\\ &+ x\frac{\partial}{\partial x}[12kb-2(k+b)]x^4y^2\end{aligned}$$

Utilizing differential operator (2. 5), we have

$$\begin{aligned}\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 6(k+b)x^5y^3 + 8[k+b]x^4y^4\\ &+ [36kb-6(k+b)]x^4y^3\\ &+ [36kb]x^3y^3\\ &+ [24kb-4(k+b)]x^4y^2\end{aligned} \tag{3. 24}$$

Integral operators for $\mathbb{CBL}$

Employing Equation (3. 22) and using (2. 6), for the required outcome $\mathbb{I}_x(\mathbb{RRDM}(\mathbb{CBL};x,y))$.

$$\mathbb{I}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) = \int_0^x \frac{\mathbb{RRDM}(\mathbb{CBL};z,y)}{z}dz$$

$$\begin{aligned}\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 2(k+b)x^5y^3 + 2[k+b]x^4y^4\\ &+ [12kb-2(k+b)]x^4y^3\\ &+ [12kb]x^3y^3\\ &+ [12kb-2(k+b)]x^4y^4\end{aligned}$$

Utilizing Integral operator (2. 6), we obtain

$$\begin{aligned}\mathbb{I}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \int_0^x \frac{\mathbb{RRDM}(\mathbb{CBL};z,y)}{z}dz\\ \mathbb{I}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \int_0^x 2(k+b)z^5y^3 + 2[k+b]z^4y^4\\ &+ [12kb-2(k+b)]z^4y^3\\ &+ [12kb]z^3y^3\\ &+ [12kb-2(k+b)]z^4y^4\end{aligned}$$

$$\begin{aligned}
\mathbb{I}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= [\frac{2}{5}(k+b)z^5y^3]\big|_0^x + [\frac{1}{2}(k+b)z^4y^4]\big|_0^x \\
&+ [3kb - \frac{1}{2}(k+b)z^4y^3]\big|_0^x \\
&+ (4kb)z^3y^3\big|_0^x \\
&+ [3kb - \frac{1}{2}(k+b)z^4y^2]\big|_0^x
\end{aligned}$$

Simplifying last relation gives

$$\begin{aligned}
\mathbb{I}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \frac{2}{5}(k+b)x^5y^3 + \frac{1}{2}(k+b)x^4y^4 \\
&+ [3kb - \frac{1}{2}(k+b)x^4y^3 \\
&+ (4kb)x^4y^3 \\
&+ [3kb - \frac{1}{2}(k+b)]x^4y^2 \qquad (3.\ 25)
\end{aligned}$$

Similarly, by Equations (2. 7) and (3. 22) we have $\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL};x,y))$.

$$\begin{aligned}
\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \int_0^y \frac{\mathbb{RRDM}(\mathbb{CBL};x,z)))}{z}dz \\
\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= [2(k+b)x^5z^3 + 2[k+b]x^4z^4 \\
&+ [12kb - 2(k+b)]x^4z^3 \\
&+ [12kb]x^3z^3 \\
&+ [12kb - 2(k+b)]x^4z^4]dz
\end{aligned}$$

Utilizing Integral operator, we obtain

$$\begin{aligned}
\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \int_0^y 2(k+b)x^5z^3dz + \int_0^y 2(k+b)x^4z^4dz \\
&+ \int_0^y [12kb - 2(k+b)]x^4z^3dz \\
&+ \int_0^y +[12kb]x^3z^3dz \\
&+ \int_0^y [12kb - 2(k+b)]x^4z^2dz
\end{aligned}$$

$$\begin{aligned}
\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= [\frac{2}{3}(k+b)x^5z^3]\big|_0^x + [\frac{1}{2}(k+b)x^4z^4]\big|_0^x \\
&+ [4kb - \frac{2}{3}(k+b)x^4z^3]\big|_0^x \\
&+ (4kb)x^3z^3\big|_0^x \\
&+ [6kb - (k+b)x^4z^2]\big|_0^x
\end{aligned}$$

Simplifying the last relation gives

$$\begin{aligned}
\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \frac{2}{3}(k+b)x^5y^3+\frac{1}{2}(k+b)x^4y^4\\
&+ [4kb-\frac{1}{2}(k+b)x^4y^3\\
&+ (4kb)x^4y^3\\
&+ [6kb-(k+b)]x^4y^2 \qquad (3.\ 26)
\end{aligned}$$

Reduced Reverse First Zagreb $\mathbb{RRDM}$-Polynomial for $\mathbb{CBL}$

Employing Equation (3. 22), and Table 1, we get

$$\begin{aligned}
\mathbb{RRDM}_1(\mathbb{CBL};x,y) &= (\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y))\\
&= \mathbb{D}_x(\mathbb{RRDM}(\mathbb{CBL};x,y))+\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y))\\
&= 10(k+b)x^5y^3+8[k+b]x^4y^4\\
&+ [48kb-8(k+b)]x^4y^3\\
&+ [36kb]x^3y^3\\
&+ [48kb-8(k+b)]x^4y^2\\
&\\
&+ 6(k+b)x^5y^3+8[k+b]x^4y^4\\
&+ [36kb-6(k+b)]x^4y^3\\
&+ [36kb]x^3y^3\\
&+ [24kb-4(k+b)]x^4y^2
\end{aligned}$$

This gives

$$\begin{aligned}
\mathbb{RRDM}_1(\mathbb{CBL};x,y) &= 16(k+b)x^5y^3+16[k+b]x^4y^4\\
&+ [84kb-14(k+b)]x^4y^3\\
&+ [72kb]x^3y^3\\
&+ [72kb-10(k+b)]x^4y^2
\end{aligned}$$

Reduced Reverse Second Zagreb $\mathbb{RRDM}$-Polynomial for $\mathbb{CBL}$

Employing Equation (3. 22) and Table 1, we get

$$\begin{aligned}
\mathbb{RRDM}_2(\mathbb{CBL},x,y) &= \mathbb{D}_x[6(k+b)x^5y^3+8[k+b]x^4y^4\\
&+ [36kb-6(k+b)]x^4y^3\\
&+ [36kb]x^3y^3\\
&+ [24kb-4(k+b)]x^4y^2]\\
\mathbb{RRDM}_2(\mathbb{CBL},x,y) &= x\frac{\partial}{\partial x}6(k+b)x^5y^3+x\frac{\partial}{\partial x}8[k+b]x^4y^4\\
&+ x\frac{\partial}{\partial x}[36kb-6(k+b)]x^4y^3\\
&+ x\frac{\partial}{\partial x}[36kb]x^3y^3\\
&+ x\frac{\partial}{\partial x}[24kb-4(k+b)]x^4y^2
\end{aligned}$$

This gives

$$\begin{aligned}
\mathbb{RRDM}_2(\mathbb{CBL};x,y) &= 30(k+b)x^5y^3+32[k+b]x^4y^4\\
&+ [144kb-24(k+b)]x^4y^3\\
&+ [108kb]x^3y^3\\
&+ [96kb-8(k+b)]x^4y^2
\end{aligned}$$

Reduced Reverse General Randic $\mathbb{RRDM}$-polynomial for $\mathbb{CBL}$

Using Equation (3. 22), Table 1 through Equation (3. 23) and (3. 24)

$$\begin{aligned}
\mathbb{RRDM}_{R_\alpha}(\mathbb{CBL},x,y) &= \mathbb{D}_x^\alpha\mathbb{D}_y^\alpha[2(k+b)x^5y^3+2[k+b]x^4y^4\\
&+ [12kb-2(k+b)]x^4y^3\\
&+ [12kb]x^3y^3\\
&+ [12kb-2(k+b)]x^4y^2]
\end{aligned}$$

$$\begin{aligned}
\mathbb{RRDM}_{R_\alpha}(\mathbb{CBL},x,y) &= (\mathbb{D}_x^\alpha)(3)^\alpha.2(k+b)x^5y^3+(4)^\alpha[2(k+b)]x^4y^4\\
&+ (3)^\alpha[12kb-2(k+b)]x^4y^3\\
&+ (3)^\alpha[12kb]x^3y^3\\
&+ (2)^\alpha[12kb-2(k+b)]x^4y^2
\end{aligned}$$

$$
\begin{aligned}
\mathbb{RRDM}_{R_\alpha}(\mathbb{CBL};x,y) &= (15)^\alpha.2(k+b)x^5y^3+(16)^\alpha[2(k+b)]x^4y^4 \\
&+ (12)^\alpha[12kb-2(k+b)]x^4y^3 \\
&+ (9)^\alpha[12kb]x^3y^3 \\
&+ (8)^\alpha[12kb-2(k+b)]x^4y^2
\end{aligned}
$$

Reduced Reverse First Zagreb Index of $\mathbb{CBL}$

Adding Equation (3. 23) and (3. 24), we obtain

$$
\begin{aligned}
\mathbb{D}_x(\mathbb{RRDM}(\mathbb{CBL};x,y))+\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 10(k+b)x^5y^3+8[k+b]x^4y^4 \\
&+ [48kb-8(k+b)]x^4y^3 \\
&+ [36kb]x^3y^3 \\
&+ [48kb-8(k+b)]x^4y^2 \\
&+ 6(k+b)x^5y^3+8[k+b]x^4y^4 \\
&+ [36kb-6(k+b)]x^4y^3 \\
&+ [36kb]x^3y^3 \\
&+ [24kb-4(k+b)]x^4y^2 \\
(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 16(k+b)x^5y^3+16[k+b]x^4y^4 \\
&+ [84kb-14(k+b)]x^4y^3 \\
&+ [72kb]x^3y^3 \\
&+ [72kb-10(k+b)]x^4y^2 \\
(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 16(k+b)x^5y^3+16[k+b]x^4y^4 \\
&+ [84kb-14(k+b)]x^4y^3 \\
&+ [72kb]x^3y^3 \\
&+ [72kb-10(k+b)]x^4y^2 \\
\mathbb{RRDM}_1(\mathbb{CBL}) &= 36(k+b)+156kb
\end{aligned}
$$

Reduced Reverse Second Zagreb Index for $\mathbb{CBL}$

Making use of Table 1 with Equation (3. 24), we get

$$\begin{aligned}
\mathbb{D}_x[(\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y))] &= \mathbb{D}_x[6(k+b)x^5y^3 + 8[k+b]x^4y^4 \\
&+ [36kb - 6(k+b)]x^4y^3 \\
&+ [36kb]x^3y^3 \\
&+ [24kb - 4(k+b)]x^4y^2 \\
(\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 30(k+b)x^5y^3 + 32[k+b]x^4y^4 \\
&+ [144kb - 24(k+b)]x^4y^3 \\
&+ [108kb]x^3y^3 \\
&+ [96kb - 8(k+b)]x^4y^2 \\
(\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 30(k+b)x^5y^3 + 32[k+b]x^4y^4 \\
&+ [144kb - 24(k+b)]x^4y^3 \\
&+ [108kb]x^3y^3 \\
&+ [96kb - 8(k+b)]x^4y^2 \\
\mathbb{RRDM}_2(\mathbb{CBL}) &= 60(k+b) + 732kb
\end{aligned}$$

Reduced Reverse Hyper 1^{st} Zagreb Index of $\mathbb{CBL}$

Emplying of Table 1, we have

$$\begin{aligned}
\mathbb{D}_x^2(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 50(k+b)x^5y^3 + 32[k+b]x^4y^4 \\
&+ [192kb - 32(k+b)]x^4y^3 \\
&+ [108kb]x^3y^3 \\
&+ [192kb - 32(k+b)]x^4y^2 \\
\mathbb{D}_y^2(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 18(k+b)x^5y^3 + 32[k+b]x^4y^4 \\
&+ [108kb - 18(k+b)]x^4y^3 \\
&+ [108kb]x^3y^3 \\
&+ [96kb - 16(k+b)]x^4y^2 \\
(2\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 60(k+b)x^5y^3 + 64[k+b]x^4y^4 \\
&+ [288kb - 48(k+b)]x^4y^3 \\
&+ [216kb]x^3y^3 \\
&+ [192kb - 16(k+b)]x^4y^2 \\
(\mathbb{D}_x^2 + \mathbb{D}_y^2 + 2\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 128(k+b)x^5y^3 + 128[k+b]x^4y^4 \\
&+ [588kb - 98(k+b)]x^4y^3 \\
&+ [432kb]x^3y^3 \\
&+ [480kb - 64(k+b)]x^4y^2
\end{aligned}$$

Employing limits gives

$$\mathbb{RRDHM}_1(\mathbb{CBL}) \quad = \quad 94(k+b) + 1500kb$$

Reduced Reverse Sigma Index for $\mathbb{RRDM}$-Polynomial of $\mathbb{CBL}$

Emplying Table 1 and differential operator on Equation (3. 23) and Equation (3. 24), we have

$$\begin{aligned}
\mathbb{D}_x^2(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 50(k+b)x^5y^3 + 32[k+b]x^4y^4 \\
&+ [192kb - 32(k+b)]x^4y^3 \\
&+ [108kb]x^3y^3 \\
&+ [192kb - 32(k+b)]x^4y^2 \\
\mathbb{D}_y^2(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 18(k+b)x^5y^3 + 32[k+b]x^4y^4 \\
&+ [108kb - 18(k+b)]x^4y^3 \\
&+ [108kb]x^3y^3 \\
&+ [96kb - 16(k+b)]x^4y^2 \\
(2\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 60(k+b)x^5y^3 + 64[k+b]x^4y^4 \\
&+ [288kb - 48(k+b)]x^4y^3 \\
&+ [216kb]x^3y^3 \\
&+ [192kb - 16(k+b)]x^4y^2 \\
(\mathbb{D}_x^2 + \mathbb{D}_y^2 - 2\mathbb{D}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 128(k+b)x^5y^3 + 128[k+b]x^4y^4 \\
&+ [588kb - 98(k+b)]x^4y^3 \\
&+ [432kb]x^3y^3 \\
&+ [480kb - 64(k+b)]x^4y^2
\end{aligned}$$

Employing limits gives

$$\mathbb{RRD}\sigma(\mathbb{CBL}) \quad = \quad -26(k+b) + 108kb$$

Reduced Reverse Second Modified Zagreb Index for $\mathbb{RRDM}$-Polynomial of $\mathbb{CBL}$

Emplying Table 1 and $\mathbb{I}_x$ on Equation (3. 26)

$$\begin{aligned}
(\mathbb{I}_x\mathbb{I}_y)(\mathbb{RRDM}(\mathbb{CBL}:x,y)) &= \mathbb{I}_x[\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL}:x,y)] \\
&= \mathbb{I}_x[\frac{2}{3}(k+b)x^5y^3 + \frac{1}{2}(k+b)x^4y^4 \\
&+ [4kb - \frac{2}{3}(k+b)x^4y^3 \\
&+ (4kb)x^3y^3 \\
&+ [6kb - (k+b)]x^4y^2]
\end{aligned}$$

Simplifying last relation gives

$$\begin{aligned}(\mathbb{I}_x\mathbb{I}_y)(M(G:x,y)) &= \frac{2}{15}(k+b)x^5y^3+\frac{1}{8}(k+b)x^4y^4\\ &+ [kb-\frac{1}{6}(k+b)x^4y^3\\ &+ (\frac{4}{3}kb)x^3y^3\\ &+ [\frac{3}{2}kb-\frac{1}{4}(k+b)]x^4y^2\end{aligned}$$

$$\begin{aligned}(\mathbb{I}_x\mathbb{I}_y)(\mathbb{RRDM}(\mathbb{CBL}:x,y)) &= \frac{2}{15}(k+b)x^5y^3+\frac{1}{8}(k+b)x^4y^4\\ &+ [kb-\frac{1}{6}(k+b)x^4y^3\\ &+ (\frac{4}{3}kb)x^3y^3\\ &+ [\frac{3}{2}kb-\frac{1}{4}(k+b)]x^4y^2\end{aligned}$$

Employing limits gives

$$\mathbb{RRD}^m\mathbb{M}_2(\mathbb{CBL}) = \frac{13}{20}(k+b)+\frac{23}{6}(kb)$$

Reduced Reverse Redefined Third Zagreb Index of $\mathbb{CBL}$

From Table 1, Redefined Third Zagreb Index for $\mathbb{RRDM}$-Polynomial is

$$\begin{aligned}\mathbb{RRDM}(\mathbb{CBL};x,y) &= 2(k+b)x^5y^3+2[k+b]x^4y^4\\ &+ [12kb-2(k+b)]x^4y^3\\ &+ [12kb]x^3y^3\\ &+ [12kb-2(k+b)]x^4y^2\\ (\mathbb{D}_x+\mathbb{D}_y)(M(G;x,y)) &= 16(k+b)x^5y^3+16[k+b]x^4y^4\\ &+ [84kb-14(k+b)]x^4y^3\\ &+ [72kb]x^3y^3\\ &+ [72kb-10(k+b)]x^4y^2\end{aligned}$$

$$\begin{aligned}
\mathbb{D}_y(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x;y)) &= \mathbb{D}_y 16(k+b)x^5y^3+16[k+b]x^4y^4\\
&+ [84kb-14(k+b)]x^4y^3\\
&+ [72kb]x^3y^3\\
&+ [72kb-10(k+b)]x^4y^2\\
\mathbb{D}_y(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x;y)) &= 48(k+b)x^5y^3+64[k+b]x^4y^4\\
&+ [252kb-42(k+b)]x^4y^3\\
&+ [216kb]x^3y^3\\
&+ [144kb-10(k+b)]x^4y^2\\
(\mathbb{D}_x\mathbb{D}_y)(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x;y)) &= 240(k+b)x^5y^3+256[k+b]x^4y^4\\
&+ [1008kb-168(k+b)]x^4y^3\\
&+ [648kb]x^3y^3\\
&+ [567kb-80(k+b)]x^4y^2\\
(\mathbb{D}_x\mathbb{D}_y)(\mathbb{D}_x+\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x;y)) &= 240(k+b)x^5y^3+256[k+b]x^4y^4\\
&+ [1008kb-168(k+b)]x^4y^3\\
&+ [648kb]x^3y^3\\
&+ [567kb-80(k+b)]x^4y^2
\end{aligned}$$

Employing limits gives

$$\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{CBL}) = 248(k+b)+2232$$

Reduced Reverse Symmetric Division Degree Index of $\mathbb{CBL}$

Employing Table 1 and Equation (3. 26), we have

$$\begin{aligned}
(\mathbb{I}_y)(\mathbb{RRDM}(\mathbb{CBL}:x,y)) &= \frac{2}{3}(k+b)x^5y^3+\frac{1}{2}(k+b)x^4y^4\\
&+ [4kb-\frac{2}{3}(k+b)x^4y^3\\
&+ (4kb)x^3y^3\\
&+ [6kb-(k+b)]x^4y^2
\end{aligned}$$

Utilizing the operator $\mathbb{D}_y$, we have

$$\begin{aligned}
(\mathbb{D}_x\mathbb{I}_y)(\mathbb{RRDM}(\mathbb{CBL}:x,y)) &= \frac{10}{3}(k+b)x^5y^3+\frac{4}{2}(k+b)x^4y^4\\
&+ [16kb-\frac{8}{3}(k+b)x^4y^3\\
&+ (12kb)x^3y^3\\
&+ [24kb-4(k+b)]x^4y^2
\end{aligned}$$

From Equation (3. 24), we have

$$\begin{aligned}
\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 6(k+b)x^5y^3+8[k+b]x^4y^4\\
&+ [36kb-6(k+b)]x^4y^3\\
&+ [36kb]x^3y^3\\
&+ [24kb-4(k+b)]x^4y^2
\end{aligned}$$

Utilizing integral operator $\mathbb{D}_x$, we have

$$\begin{aligned}
\mathbb{I}_x\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \frac{6}{5}(k+b)x^5y^3+2(k+b)x^4y^4\\
&+ [9kb-\frac{3}{2}(k+b)x^4y^3\\
&+ (12kb)x^3y^3\\
&+ [6kb-(k+b)]x^4y^2\\
(\mathbb{D}_x\mathbb{I}_y+\mathbb{I}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \frac{10}{3}(k+b)x^5y^3+\frac{4}{2}(k+b)x^4y^4\\
&+ [16kb-\frac{8}{3}(k+b)x^4y^3\\
&+ (12kb)x^3y^3\\
&+ [24kb-4(k+b)]x^4y^2\\
&+ \frac{6}{5}(k+b)x^5y^3+2(k+b)x^4y^4\\
&+ [9kb-\frac{3}{2}(k+b)x^4y^3\\
&+ (12kb)x^3y^3\\
&+ [6kb-(k+b)]x^4y^2\\
(\mathbb{D}_x\mathbb{I}_y+\mathbb{I}_x\mathbb{D}_y)(\mathbb{RRDM}(\mathbb{CBL};x,y)) &\\
&= \frac{10}{3}(k+b)x^5y^3+\frac{4}{2}(k+b)x^4y^4\\
&+ [16kb-\frac{8}{3}(k+b)x^4y^3\\
&+ (12kb)x^3y^3\\
&+ [24kb-4(k+b)]x^4y^2\\
&+ \frac{6}{5}(k+b)x^5y^3+2(k+b)x^4y^4\\
&+ [9kb-\frac{3}{2}(k+b)x^4y^3\\
&+ (12kb)x^3y^3\\
&+ [6kb-(k+b)]x^4y^2
\end{aligned}$$

$$\mathbb{RRDSDD}(\mathbb{CBL}) = \frac{161}{30}(k+b) + 166kb$$

Reduced Reverse Inverse Sum Index for $\mathbb{RRDM}$-Polynomial of $\mathbb{CBL}$

$$\mathbb{RRD}I(\mathbb{CBL}) = \frac{251}{84}(k+b) + \frac{382}{7}(kb)$$

Employing Table 1 and Equation (3. 22), we have

$$\begin{aligned}
\mathbb{D}_x\mathbb{D}_y\mathbb{RRDM}(\mathbb{CBL};x;y) &= 30(k+b)x^5y^3 + 32[k+b]x^4y^4 \\
&+ [144kb - 24(k+b)]x^4y^3 \\
&+ [108kb]x^3y^3 \\
&+ [96kb - 8(k+b)]x^4y^2
\end{aligned}$$

Apply $J(f(x,y)) = f(x,x)$

$$\begin{aligned}
J(\mathbb{RRDM}(\mathbb{CBL};x;y)) &= 30(k+b)x^8 + 32[k+b]x^8 \\
&+ [144kb - 24(k+b)]x^7 \\
&+ [108kb]x^6 \\
&+ [96kb - 8(k+b)]x^6
\end{aligned}$$

Utilizing integral operator $\mathbb{I}_x$, we have

$$\begin{aligned}
\mathbb{I}_xJ(\mathbb{RRDM}(\mathbb{CBL};x;y)) &= \frac{30}{8}(k+b)x^8 + \frac{32}{8}(k+b)x^8 \\
&+ \frac{1}{7}[144kb - 24(k+b)x^7 \\
&+ \frac{1}{6}(108kb)x^6 \\
&+ \frac{1}{6}[96kb - 8(k+b)]x^6 \\
\mathbb{I}_xJ(\mathbb{RRDM}(\mathbb{CBL};x;y))\big|_{x=1} &= \frac{30}{8}(k+b)x^8 + \frac{32}{8}(k+b)x^8 \\
&+ \frac{1}{7}[144kb - 24(k+b)x^7 \\
&+ \frac{1}{6}(108kb)x^6 \\
&+ \frac{1}{6}[96kb - 8(k+b)]x^6\big|_{x=1}
\end{aligned}$$

$$\mathbb{RRD}I(\mathbb{CBL}) = \frac{251}{84}(k+b) + \frac{382}{7}(kb)$$

Reduced Reverse Harmonic Index for $\mathbb{RRDM}$-Polynomial of $\mathbb{CBL}$

Employing Table 1 and Equation (3. 22), we have

$$\mathbb{RRDM}(\mathbb{CBL};x,y) = -\frac{5(k+b)}{21} + \frac{80kb}{7}$$

Apply $J(f(x,y)) = f(x,x)$

$$
\begin{aligned}
J(\mathbb{RRDM}(\mathbb{CBL};x;y)) &= 2(k+b)x^8 + 2[k+b]x^8 \\
&+ [12kb - 2(k+b)]x^7 \\
&+ [12kb]x^6 \\
&+ [12kb - 2(k+b)]x^6
\end{aligned}
$$

Utilizing $\mathbb{I}_x$, we find

$$
\begin{aligned}
\mathbb{I}_x J(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \frac{2}{8}(k+b)x^8 + \frac{2}{8}(k+b)x^7 \\
&+ \frac{1}{7}[12kb - 2(k+b)]x^7 \\
&+ \frac{12}{6}kb\,x^6 \\
&+ \frac{1}{6}[12kb - 2(k+b)]x^6, \\
2\mathbb{I}_x J(\mathbb{RRDM}(\mathbb{CBL};x,y))\Big|_{x=1} &= \frac{1}{2}(k+b) + \frac{2}{7}[12kb - 2(k+b)] \\
&+ \frac{24}{6}kb + \frac{2}{6}[12kb - 2(k+b)]\Big|_{x=1}.
\end{aligned}
$$

RRDH(CBL)=-5(k+b)$\frac{}{21+\frac{80kb}{7}}$

3.5. **Python Code for $\mathbb{RRDM}-$Polynomial and Differential and Integral Operators for $\mathbb{CBL}$.** For the sake of the accuracy of the results, the Python programming is utilized, and code for $\mathbb{RRDM}-$Polynomial for $\mathbb{FTC}$ is given in Figure 6 and Code for the Differential and Integral Operators for $\mathbb{CBL}$ is given in Figure 7.

```
import sympy as sp
# Define symbols
x, y, k, b = sp.symbols('x y k b')

# Given edge partition with frequencies
edge_partition = {
   (1, 3): 2*k+2*b,
   (2, 2): 2*k+2*b,
   (2, 3): 12*k*b- 2*k – 2*b,
   (3, 3): 12*k*b,
   (2, 4): 12*k*b- 2*k – 2*b
}

# Construct reduced reverse degree M-polynomial
RRDM = sum(freq * x**i * y**j for (i, j), freq in edge_partition.items())

# Expand polynomial
RRDM_expanded = sp.expand(RRDM)

RRDM_expanded
```

FIGURE 6. Python Code for $\mathbb{RRDM}-$Polynomial for $\mathbb{CBL}$

```
import sympy as sp

# Define symbols
x, y = sp.symbols('x y')

# Example: Reduced reverse degree M-polynomial
RRDM = (
   (2*sp.symbols('k')+2*sp.symbols('b'))*x*y**3
   + (2*sp.symbols('k') + 2*sp.symbols('b'))*x**2*y**2
   + (12*sp.symbols('k')*sp.symbols('b') - 2*sp.symbols('k')
     - 2*sp.symbols('b'))*x**2*y**3
   + (12*sp.symbols('k')*sp.symbols('b'))*x**3*y**3
  + (12*sp.symbols('k')*sp.symbols('b') - 2*sp.symbols('k')
     - 2*sp.symbols('b'))*x**2*y**4
)
# ---------------------------
# Differential Operators
# ---------------------------
def D_x(F):
   """ Differential operator D_x """
   return sp.simplify(x * sp.diff(F, x))

def D_y(F):
   """ Differential operator D_y """
   return sp.simplify(y * sp.diff(F, y))
# ---------------------------
# Integral Operators
# ---------------------------
def I_x(F):
   """ Integral operator I_x """
   t = sp.symbols('t', positive=True)
   return sp.simplify(sp.integrate(F.subs(x, t)/t, (t, 0, x)))

def I_y(F):
   """ Integral operator I_y """
   t = sp.symbols('t', positive=True)
   return sp.simplify(sp.integrate(F.subs(y, t)/t, (t, 0, y)))
# ---------------------------
# Apply Operators
# ---------------------------
Dx_RRDM = D_x(RRDM)
Dy_RRDM = D_y(RRDM)

Sx_RRDM = S_x(RRDM)
Sy_RRDM = S_y(RRDM)

# Display results
Dx_RRDM, Dy_RRDM, Ix_RRDM, Iy_RRDM
```

FIGURE 7. Python Code for Differential and Integral Operators for $\mathbb{CBL}$

3.6. **Graphical analysis of the numerical outcomes of $\mathcal{TD}s$ of $\mathbb{CBL}$.** Tables 3-7 display the numerical values of $\mathcal{TD}s$. For numerical computation, we use the same values for k and b, which range from 1 to 9. A graphical depiction of these numerical values is also shown in Figure 8. As the values of k and b increased, the TD values also increased gradually. We derive the subsequent relationship among calculated $\mathbb{RRDTD}s$

$\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{CBL}) > \mathbb{RRDHM}_1(\mathbb{CBL}) > \mathbb{RRDM}_2(\mathbb{CBL}) > \mathbb{RRDM}_1(\mathbb{CBL}) > \mathbb{RRDSDD}(\mathbb{CBL}) > \mathbb{RRD}\sigma(\mathbb{CBL}) > \mathbb{RRDI}(\mathbb{CBL}) > \mathbb{RRDH}(\mathbb{CBL}) > \mathbb{RRD}^m\mathbb{M}_2(\mathbb{CBL})$.

$k=b$	$\mathbb{RRDM}_1(\mathbb{CBL})$	$\mathbb{RRDM}_2(\mathbb{CBL})$	$\mathbb{RRDHM}_{\not{k}}(\mathbb{CBL})$	$\mathbb{RRD}\sigma(\mathbb{CBL})$
1	228	852	1668	56
2	768	3168	6376	328
3	1620	6948	14064	816
4	2784	12192	24752	1520
5	4260	18900	38440	2440
6	6048	27072	55128	3576
7	8148	36708	74816	4928
8	10560	47808	97504	6496
9	13284	60372	123192	8280

TABLE 6. Numerical values of $\mathbb{RRDM}_1(\mathbb{CBL})$, $\mathbb{RRDM}_2(\mathbb{CBL})$, $\mathbb{RRDHM}_{\not{k}}(\mathbb{CBL})$ and $\mathbb{RRD}\sigma(\mathbb{CBL})$

$k=b$	$\mathbb{RRDSDD}(\mathbb{CBL})$	$\mathbb{RRDH}(\mathbb{CBL})$	$\mathbb{RRD}I(\mathbb{CBL})$	$\mathbb{RRD}^m\mathbb{M}_2(\mathbb{CBL})$	$\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{CBL})$
1	176.73333	10.952381	60.547619	5.133333	2728
2	685.466667	44.761905	230.238095	17.933333	9920
3	1526.2	101.428571	509.071429	38.4	21576
4	2698.93333	180.952381	897.047619	66.533333	37696
5	4203.666667	283.3333333	1394.16666667	102.333333	58280
6	6040.4	408.75142429	2000.428751	145.8	83328
7	8209.133	556.6666666	2715.8333333	196.933333	112840
8	10709.866667	727.619048	3540.380952	255.733333	146816
9	13542.6	921.4285	4474.071429	322.2	185256

TABLE 7. Computational values of $\mathbb{RRDSDD}(\mathbb{CBL})$, $\mathbb{RRDH}(\mathbb{CBL})$, $\mathbb{RRD}I(\mathbb{CBL})$, $\mathbb{RRD}^m\mathbb{M}_2(\mathbb{CBL})$, and $\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{CBL})$.

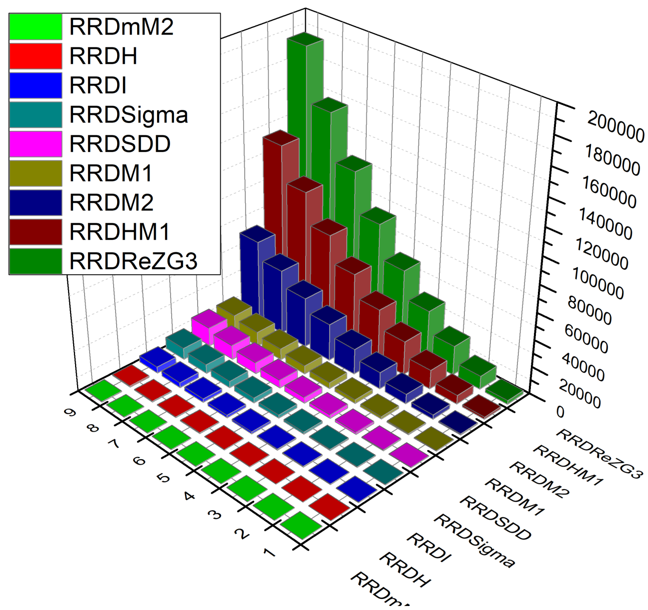

FIGURE 8. Illustration of the numerical values of $\mathbb{RRDTD}s$ associated with $\mathbb{CBL}$

CONCLUSION

Two-dimensional metal-organic frameworks (2D MOFs) exhibit variable topological structures, abundant open active sites, and elevated specific surface areas, indicating their potential in gas storage, adsorption and separation, energy conversion, and other fields. Recently, researchers have devised innovative strategies to mitigate the detrimental effects of traditional methods on the synthesis of high-quality 2D MOFs. Metal–organic frameworks (MOFs) have emerged as promising possibilities for applications in nanomedicine, with the inaugural example currently undergoing a phase II human clinical study. Metal-organic frameworks (MOFs) provide a variety of advantageous features that render them appropriate for diverse applications, including drug administration, imaging, and novel therapeutic modalities, frequently in conjunction.

We computed the $\mathbb{RRDM}$-Polynomial for structure of a $\mathbb{FTC}$

$$
\begin{aligned}
\mathbb{RRDM}(\mathbb{FTC}; x, y) &= (24kb+1)x^5y^3 + [6k+6b-6]x^4y^3 \\
&+ [56kb-4k-4b+2]x^3y^3 \\
&+ [8kb-4k-4b+4]x^3y^2
\end{aligned}
$$

and differential operators

$$\begin{aligned}\mathbb{D}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= [120kb+5]x^5y^3+(24k+24b-24)x^4y^3\\ &+ [168kb-12k-12b+6]x^3y^3\\ &+ [24kb-12k-12b+12]x^3y^2\end{aligned}$$

and

$$\begin{aligned}\mathbb{D}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= [72kb+3]x^5y^3+(18k+18b-18)x^4y^3\\ &+ [168kb-12k-12b+6]x^3y^3\\ &+ [16kb-8k-8b+8]x^3y^2\end{aligned}$$

Integral operators

$$\begin{aligned}\mathbb{I}_x(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{1}{5}(24kb+1)x^5y^3+[\frac{1}{4}(6k+6b-4)x^4y^3]\\ &+ [\frac{1}{3}(56kb-4k-4b+2)x^3y^3]\\ &+ [\frac{1}{3}(8kb-4k-4b-4)x^3y^2]\end{aligned}$$

and

$$\begin{aligned}\mathbb{I}_y(\mathbb{RRDM}(\mathbb{FTC};x,y)) &= \frac{1}{3}(24kb+1)x^5y^3\\ &+ \frac{1}{3}(6k+6b-6)x^4y^3\\ &+ \frac{1}{3}(56kb-4k-4b+2)x^3y^3\\ &+ \frac{1}{2}(8kb-4k-4b+4)x^3y^2\end{aligned}$$

Moreover, We computed the $\mathbb{RRDM}$-Polynomial for structure of a MoF $\mathbb{CBL}$

$$\begin{aligned}\mathbb{RRDM}(\mathbb{CBL};x,y) &= 2(k+b)x^5y^3+2[k+b]x^4y^4\\ &+ [12kb-2(k+b)]x^4y^3\\ &+ [12kb]x^3y^3\\ &+ [12kb-2(k+b)]x^4y^2\end{aligned}$$

and the differential operators for $\mathbb{CBL}(x,y)$ are

$$\begin{aligned}\mathbb{D}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 10(k+b)x^5y^3+8[k+b]x^4y^4\\ &+ [48kb-8(k+b)]x^4y^3\\ &+ [36kb]x^3y^3\\ &+ [48kb-8(k+b)]x^4y^2\end{aligned}$$

and

$$\begin{aligned}\mathbb{D}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= 6(k+b)x^5y^3+8[k+b]x^4y^4\\ &+ [36kb-6(k+b)]x^4y^3\\ &+ [36kb]x^3y^3\\ &+ [24kb-4(k+b)]x^4y^2\end{aligned}$$

Integral operators

$$\begin{aligned}\mathbb{I}_x(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \frac{2}{5}(k+b)x^5y^3+\frac{1}{2}(k+b)x^4y^4\\ &+ [3kb-\frac{1}{2}(k+b)x^4y^3\\ &+ (4kb)x^4y^3\\ &+ [3kb-\frac{1}{2}(k+b)]x^4y^2\end{aligned}$$

and

$$\begin{aligned}\mathbb{I}_y(\mathbb{RRDM}(\mathbb{CBL};x,y)) &= \frac{2}{3}(k+b)x^5y^3+\frac{1}{2}(k+b)x^4y^4\\ &+ [4kb-\frac{1}{2}(k+b)x^4y^3\\ &+ (4kb)x^4y^3\\ &+ [6kb-(k+b)]x^4y^2\end{aligned}$$

Additionally, we used the $\mathbb{RRDM}$-Polynomial, the differential, and integral operators to calculate the nine topological indices of two MOFs $\mathbb{FTC}$ and $\mathbb{CBL}$. These indices are also compared numerically and graphically. The relationships between $\mathbb{RRDTD}s$ for two MoF structures, $\mathbb{FTC}$ and $\mathbb{CBL}$, are shown by the following inequalities:

$$\begin{aligned}\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{FTC}) &> \mathbb{RRDM}_2(\mathbb{FTC}) > \mathbb{RRDHM}_1(\mathbb{FTC}) > \mathbb{RRDM}_1(\mathbb{FTC})\\ &> \mathbb{RRDSDD}(\mathbb{FTC}) > \mathbb{RRD}\sigma(\mathbb{FTC}) > \mathbb{RRDI}(\mathbb{FTC}) > \mathbb{RRDH}(\mathbb{FTC})\\ &> \mathbb{RRD}^m\mathbb{M}_2(\mathbb{FTC}),\end{aligned}$$

$$\begin{aligned}\mathbb{RRDR}e\mathbb{ZG}_3(\mathbb{CBL}) &> \mathbb{RRDHM}_1(\mathbb{CBL}) > \mathbb{RRDM}_2(\mathbb{CBL}) > \mathbb{RRDM}_1(\mathbb{CBL})\\ &> \mathbb{RRDSDD}(\mathbb{CBL}) > \mathbb{RRD}\sigma(\mathbb{CBL}) > \mathbb{RRDI}(\mathbb{CBL}) > \mathbb{RRDH}(\mathbb{CBL})\\ &> \mathbb{RRD}^m\mathbb{M}_2(\mathbb{CBL}).\end{aligned}$$

To enhance the efficiency and accuracy of the computed results, Python programming is employed, and Python Codes for $\mathbb{RRDM}-$Polynomials and differential and integral operators for both MOFs are provided in Figures 3, 4, 6 and 7. This study will provide a new avenue for researchers to employ this methodology for similar studies.

CREDIT AUTHORSHIP CONTRIBUTION'S STATEMENT

K. H. Hakami, and A. R. Khan ontributed to the investigation, analyzing the data curation, designing the experiments, conceptualization, and Methodology. S. Sadaf and M. N. Husin contributed to data analysis, computation, and calculation verifications. All authors read and approved the final version.

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