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Partial Γ-Semimodules over Partial Γ-Semirings

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Abstract. In this paper, we study the concepts of left (right) partial Γ-semimodules over a partial Γ-semirings by the illustrations of several examples. Also, we obtain the characterisation of partial Γ-subsemimodule generated by a nonempty subset in terms of its elements.

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1. Introduction

In 1995, Rao[6] developed the theory of Γ-semirings and showed that this class is a common extension of semirings and Γ-rings. In 2014, Mala[8] defined the concept of partial Γ-semiring by replacing the binary addition in Γ-semirings to infinitary partial addition and showed that this class is a common extension of partial semirings introduced by Arbib, Manes[2] and Benson[5] and Rao[6] Γ-semirings. Further, Mala[9]and [10] studied theory of ideals for the Γ-so-rings.

In this paper, we study the concepts of left (right) partial Γ-semimodules over a partial Γ-semirings by the illustrations of several examples. Also, we obtain the characterisation of partial Γ-subsemimodule generated by a nonempty subset in terms of its elements.

2. Notations and Preliminaries

In the preliminaries, we recollect the necessary concepts from the literature.

Notations: Throughout this paper, we use the following notations.
(1) **PM** stands for partial monoid.
(2) **PΓSR** stands for partial Γ-semiring.
(3) **PΓI** stands for partial Γ-ideal.
(4) **PΓSM** stands for partial Γ-semimodule.
(5) **PΓSSM** stands for partial Γ-subsemimodule.

Definition 2.1. *A mapping $a : \Delta \to G$ from a set Δ to a nonempty set G is called a $\Delta - family$ in G. It is denoted by $(a_l : l \in \Delta)$, where $a_l = a(l)\ \forall\ l \in \Delta$.*

Definition 2.2. *A $sub\ family$ of $(a_l : l \in \Delta)$ is a family $(a_k : k \in K)$ where $K \subseteq \Delta$. The family $(a_l : l \in \emptyset)$ is called an $empty\ family$.*

Now let us consider an infinitary operation Σ which takes families in G to elements of G, but which may not be defined for all families in G. By "infinitary", we mean that Σ may be applied to a family $(a_l : l \in \Delta)$ in G, for which the cardinality of the index set Δ is infinite. Since $\Sigma(a_l : l \in \Delta)$ need not be defined for an arbitrary family $(a_l : l \in \Delta)$ in G, Σ is said to be *partially-defined*. A family $(a_l : l \in \Delta)$ in G is said to be *summable* if $\Sigma(a_l : l \in \Delta)$ is defined and is in G.

Definition 2.3. [5] *A G be nonempty set and Σ be an infinitary partial addition on G. Then the structure (G, Σ) is called a* **PM** *if it satisfies the following conditions:*
(M1) Unary Sum Axiom: If $(g_l : l \in \Delta)$ is in G and $\Delta = \{k\}$, then $\Sigma_{l\in\Delta} g_l = g_k \in G$.
(M2) Partition-Associativity Axiom: If $(g_l : l \in \Delta)$ is in G and $(\Delta_k : k \in K)$ is a partition of Δ, then $\Sigma_{l\in\Delta} g_l \in G \iff \Sigma_{l\in\Delta_k} g_l \in G\ \forall\ k \in K$ and $\Sigma_{k\in K}(\Sigma_{l\in\Delta_k} g_l) \in G$, and $\Sigma_{l\in\Delta} g_l = \Sigma_{k\in K}(\Sigma_{l\in\Delta_k} g_l)$.

In a **PM** (G, Σ), the empty family is summable. Its sum, denoted by 0_G, is such that the sum of an arbitrary number of 0_G's is itself equal to 0_G. furthermore, 0_G acts as an additive zero in **PM** (G, Σ).

Example 2.4. [5] *Let $Pfn(A, B)$ be the set of all partial functions from a set A to a set B. Define Σ on $Pfn(A, B)$ as follows: Let $(f_l : l \in \Delta)$ be a family in $Pfn(A, B)$. Then $\Sigma_{l\in\Delta} f_l \in Pfn(A, B) \iff$ for $l,\ k$ in Δ such that $l \neq k$, $dom(f_l) \bigcap dom(f_k) = \emptyset$ and for any $a \in A$,*

$$a(\Sigma_l f_l) = \begin{cases} af_l,\ if\ a \in dom(f_l)\ for\ some\ l \in \Delta; \\ undefined,\ otherwise. \end{cases}$$

Then $(Pfn(A, B), \Sigma)$ is a **PM**.

Definition 2.5. [8] *Let (S, Σ) and (Γ, Σ^*) be two* **PM***s. Then S is called a* **PΓSR** *if there is an operation $S \times \Gamma \times S \longrightarrow S : (a, \mu, b) \mapsto a\mu b\ \forall\ a,\ b \in S$ and $\mu \in \Gamma$ subject to the following conditions*
$\forall\ a,\ b,\ c,\ (a_l : l \in \Delta) \in S$ and $\mu, \gamma, (\mu_l : l \in \Delta) \in \Gamma$
(S1) $a\mu(b\gamma c) = (a\mu b)\gamma c$,
(S2) $\Sigma_{l\in\Delta} a_l \in S$ implies that $\Sigma_{l\in\Delta}(a\mu a_l) \in S$ and $a\mu[\Sigma_{l\in\Delta} a_l] = \Sigma_{l\in\Delta}(a\mu a_l)$,
$[\Sigma_{l\in\Delta} a_l]\mu a = \Sigma_{l\in\Delta}(a_l \mu a)$,
(S3) $\Sigma^*_{l\in\Delta} \mu_l \in \Gamma$ implies that $\Sigma_{l\in\Delta}(a\mu_l b) \in S$ and $a(\Sigma^*_{l\in\Delta} \mu_l)b = \Sigma_{l\in\Delta}(a\mu_l b)$.

Example 2.6. [8] *Consider the* **PM***s $(Pfn(A, B), \Sigma)$ and $(Pfn(B, A), \Sigma^*)$ as defined in the Example 2.4. Now define an operation $Pfn(A, B) \times Pfn(B, A) \times Pfn(A, B) \longrightarrow Pfn(A, B) : (g, \mu, h) \mapsto g\mu h$ where*

$a(g\mu h) = (((ag)\mu)h)$, for any $a \in A$. Then $Pfn(A,B)$ is a **PΓSR** *where $\Gamma = Pfn(B,A)$.*

In general $Pfn(A,B)$ need not be a Γ-semiring, because an arbitrary family in the **PΓSR** $Pfn(A,B)$ need not be summable. Here $\Gamma = Pfn(B,A)$.

Definition 2.7. [8] *A* **PΓSR** *S is said to have a* left (right) unity *if there exists a family $(e_l : l \in \Delta)$ of elements of S and a family $(\mu_l : l \in \Delta)$ of elements of Γ such that $\Sigma_l e_l \mu_l s = s$ ($\Sigma_l s \mu_l e_l = s$) for any s in S.*

Definition 2.8. [9] *Let S be a* **PΓSR**, *K be a nonempty subset of S and Ω be a nonempty subset of Γ. Then the pair (K,Ω) of (S,Γ) is said to be a* left (right) **PΓI** *of S if it satisfies the following:*
(i) $(a_l : l \in \Delta)$ is a summable family in S and $a_l \in K\ \forall l \in \Delta$ implies $\Sigma_l a_l \in K$,
(ii) $(\mu_l : l \in \Delta)$ is a summable family in Γ and $\mu_l \in \Omega\ \forall l \in \Delta$ implies $\Sigma_l \mu_l \in \Omega$, and
(iii) for all $s \in S,\ a \in K$ and $\mu \in \Omega$, $s\mu a \in K$ ($a\mu s \in K$).

3. Partial Γ-Semimodules over S

In this section we define left (right) **PΓSM** N over S and **PΓSSM** of N and several examples are studied.

Definition 3.1. *Let S be a* **PΓSR** *and (N,Σ') be a* **PM**. *Then N is called a left (right)* **PΓSM** *over S if $\exists$ an operation $S \times \Gamma \times N \to N : (s,\mu,n) \mapsto s\mu n$ ($N \times \Gamma \times S \to N : (n,\mu,s) \mapsto n\mu s$) which satisfies the following axioms:*
(SM1) if $\Sigma'_l n_l \in N$ then $\Sigma'_l(s\mu n_l) \in N$ and $s\mu(\Sigma'_l n_l) = \Sigma'_l(s\mu n_l)$,
(SM2) if $\Sigma^*_l \mu_l \in \Gamma$ then $\Sigma'_l(s\mu_l n) \in N$ and $s(\Sigma^*_l \mu_l)n = \Sigma'_l(s\mu_l n)$
(where Σ^* is the partial addition in Γ),
(SM3) if $\Sigma_l s_l \in S$ then $\Sigma'_l(s_l\mu n) \in N$ and $(\Sigma_l s_l)\mu n = \Sigma'_l(s_l \mu n)$
(where Σ is the partial addition in S),
(SM4) $(s\mu t)\alpha n = s\mu(t\alpha n)$,
(SM5) $0_S\mu n = s0_\Gamma n = s\mu 0_N = 0_N$ for every $n,\ n_i \in N,\ \mu,\ \mu_i,\ \alpha \in \Gamma,\ s,\ s_i,\ t \in S$.

Definition 3.2. *Let S be a* **PΓSR** *with left (right) unity and (N,Σ') be a* **PM**. *Then N is called a left (right)* **PΓSM** *with left (right) unity over S if $\exists$ an operation $S \times \Gamma \times N \to N : (s,\mu,n) \mapsto s\mu n$ ($N \times \Gamma \times S \to N : (n,\mu,s) \mapsto n\mu s$) which satisfies the following axioms:*
(SM1) if $\Sigma'_l n_l \in N$ then $\Sigma'_l(s\mu n_l) \in N$ and $s\mu(\Sigma'_l n_l) = \Sigma'_l(s\mu n_l)$,
(SM2) if $\Sigma^*_l \mu_l \in \Gamma$ then $\Sigma'_l(s\mu_l n) \in N$ and $s(\Sigma^*_l \mu_l)n = \Sigma'_l(s\mu_l n)$
(where Σ^* is the partial addition in Γ),
(SM3) if $\Sigma_l s_l \in S$ then $\Sigma'_l(s_l\mu n) \in N$ and $(\Sigma_l s_l)\mu n = \Sigma'_l(s_l \mu n)$
(where Σ is the partial addition in S),
(SM4) $(s\mu t)\alpha n = s\mu(t\alpha n)$,
(SM5) $0_S\mu n = s0_\Gamma n = s\mu 0_N = 0_N$ for every $n,\ n_i \in N,\ \mu,\ \mu_i,\ \alpha \in \Gamma,\ s,\ s_i,\ t \in S$,
(SM6) $\Sigma'_l e_l \mu_l n = n$ ($\Sigma'_l n \mu_l e_l = n$) for all $n \in N$.

For the convenience of study the symbol Σ is used hereafter instead of the partial additions Σ in S, Σ^* in Γ and Σ' in N irrespective of the context.

Following are some examples of a **PΓSM** over a **PΓSR** S.

Example 3.3. *Every* **P**Γ**SR** S *is a left (right)* **P**Γ**SM** *over* S *by the operation* $S\times\Gamma\times S\to S:(s,\mu,t)\mapsto s\mu t$ *for any* $s,\ t\in S$ *and* $\mu\in\Gamma$. *Also every left (right)* **P**Γ**I** *of* S *is a left (right)* **P**Γ**SM** *over* S.

Example 3.4. *Let* S *be* **P**Γ**SR**. *Take* $N:=S^n$ *for any positive integer* n. *Define* Σ *on* N *as follows: Let* $(a_l:l\in\Delta)$ *be elements of* n*-tuples in* N. *Then each* $a_l=[a_{l1},a_{l2},...,a_{ln}]$. *Now* $\Sigma_l a_l\in N\Longleftrightarrow\Sigma_l a_{lj}\in S\ \forall\ 1\leq j\leq n$ *and* $\Sigma_l a_l=[\Sigma_l a_{l1},\Sigma_l a_{l2},...,\Sigma_l a_{ln}]$. *Then* (N,Σ) *is a* **PM**. *Now define an operation* $S\times\Gamma\times N\to N$ *by* $(s,\mu,[a_1,a_2,...,a_n])\mapsto[s\mu a_1,s\mu a_2,...,s\mu a_n]$ *(*$N\times\Gamma\times S\to N:([a_1,a_2,...,a_n],\mu,s)\mapsto[a_1\mu s,a_2\mu s,...,a_n\mu s]$*) for any* $s\in S$, $\mu\in\Gamma$ *and* $[a_1,a_2,...,a_n]\in N$. *Then it can be verified that* N *is a left (right)* **P**Γ**SM** *over* S.

Example 3.5. *Let* $Z^-:=\{x\mid x$ *a nonpositive integer* $\}$. *Let* $S:=\Gamma:=\{\begin{pmatrix} a & b\\ c & d\end{pmatrix}\mid a,b,c,d\in Z^-\bigcup\{0\}\}$. *Then* S *and* Γ *are* **PM***s with finite support of usual matrix addition. Moreover,* S *is a* **P**Γ**SR** *with usual matrix multiplication. Let* $N_1=\{\begin{pmatrix} a & 0\\ c & 0\end{pmatrix}\mid a,c\in Z^-\bigcup\{0\}\}$. *Then* N_1 *is a left* **P**Γ**SM** *over* S *with the usual matrix multiplication. Let* $N_2=\{\begin{pmatrix} a & b\\ 0 & 0\end{pmatrix}\mid a,b\in Z^-\bigcup\{0\}\}$. *Then* N_2 *is a right* **P**Γ**SM** *over* S *with the usual matrix multiplication.*

Example 3.6. *Consider the* **PM***s* $S:=Pfn(A,B)$, $\Gamma:=Pfn(B,A)$ *and* $N:=Pfn(A,A)$ *as defined in the Example* 2.2. *Moreover* S *is a* **P**Γ**SR** *by the operation as defined in the Example* 2.4. *Now the operation* $S\times\Gamma\times N\to N:(f,\mu,n)\mapsto f\mu n$ *where* $a(f\mu n)=(((af)\mu)n)$, *for any* $a\in A$, $f\in S$, $\mu\in\Gamma$*) and* $n\in N$. *Then* $N=Pfn(A,A)$ *is a left* **P**Γ**SM** *over* S.

Definition 3.7. *Let* S *be a* **P**Γ**SR**, N *be a left (right)* **P**Γ**SM** *over* S *and* $K\subseteq N$ *(*$K\neq\emptyset$*). Then* K *is called a* **P**Γ**SSM** *of* N *if the following holds in K:*
(SSM1) if $\Sigma_l a_l\in N$ *and* $a_l\in K\ \forall\ l\in\Delta$ *then* $\Sigma_l a_l\in K$, *and*
(SSM2) if $s\in S,\ \mu\in\Gamma,\ a\in K$ *then* $s\mu a\in K$ *(*$a\mu s\in K$*).*

Example 3.8. *Let* S *be a* **P**Γ**SR**. *Then from the definition* 2.6, *trivially every left (right)* **P**Γ**I** *of* S *is a* **P**Γ**SSM** *of the left (right)* **P**Γ**SM** N *over* S.

Example 3.9. *Consider the* **P**Γ**SM** N *over* S *as in the Example* 3.4. *Take* $K=\{[a,0,...,0]\mid a\in S\}$. *Then* K *is a* **P**Γ**SSM** *of* N.

3.10. Remark. Let N be a left (right) **P**Γ**SM** over a **P**Γ**SR** S. Then
(i) $\{0\}$ and N are **P**Γ**SSM** of N, called trivial **P**Γ**SSM**s, and
(ii) if $\{K_l\mid l\in\Delta\}$ be a family of **P**Γ**SSM**s of N then $\bigcap\limits_{l\in\Delta}K_l$ is a **P**Γ**SSM** of N.

Definition 3.10. *Let* N *be a left (right)* **P**Γ**SM** N *over* S *and* $B\subseteq N$ *(*$B\neq\emptyset$*). Then the* **P**Γ**SSM** *of* N *generated by* B *is denoted by* $S\Gamma B$ *(*$B\Gamma S$*) and is defined as* $S\Gamma B=\bigcap\{K\mid K$ *is a left (right)* **P**Γ**SSM** *of* N

and $B \subseteq K\}$.

Theorem 3.11. *Let S be a* **PΓSR** *with unity and N be a left (right)* **PΓSM** *over S. Then for any $B \subseteq N$ ($B \neq \emptyset$), $S\Gamma B = \{\Sigma_l s_l \mu_l b_l \mid s_l \in S,\ \mu_l \in \Gamma,\ b_l \in B$ and $\Sigma_l s_l \mu_l b_l \in N\}$ ($B\Gamma S = \{\Sigma_l b_l \mu_l s_l \mid b_l \in B,\ \mu_l \in \Gamma,\ s_l \in S$ and $\Sigma_l b_l \mu_l s_l \in N\}$).*

Proof. Take $T^* = \{\Sigma_l s_l \mu_l b_l \mid s_l \in S,\ \mu_l \in \Gamma,\ b_l \in B$ and $\Sigma_l s_l \mu_l b_l \in N\}$. First we claim that T^* is a **PΓSSM** of N containing B: Let $\Sigma_l a_l \in N$ and $a_l \in T^*$, $\forall\ l \in \Delta$. Then each $a_l = \Sigma_j s_{lj} \mu_{lj} b_{lj}$, $s_{lj} \in S,\ \mu_{lj} \in \Gamma,\ b_{lj} \in B$. $\Rightarrow \Sigma_l a_l = \Sigma_l(\Sigma_j s_{lj} \mu_{lj} b_{lj})$, and so $\Sigma_l a_l \in T^*$. Let $t \in S,\ \beta \in \Gamma$ and $a \in T^*$. Then $t \in S,\ \beta \in \Gamma$ and $a = \Sigma_l s_l \mu_l b_l$, $s_l \in S,\ \mu_l \in \Gamma,\ b_l \in B$. $\Rightarrow t\beta a = t\beta(\Sigma_l s_l \mu_l b_l) = \Sigma_l (t\beta s_l)\mu_l b_l$, $t\beta s_l \in S,\ \mu_l \in \Gamma,\ b_l \in B$ and so $t\beta a \in T^*$. Hence T^* is a **PΓSSM** of N. Since S has left unity, there exists $(e_l : l \in \Delta)$ in S, $(\mu_l : l \in \Delta)$ in Γ such that $\Sigma_l e_l \mu_l s = s$. Now let $b \in B \subseteq N$. Then $\Sigma_l e_l \mu_l b = b$. $\Rightarrow b \in T^*$ and hence $B \subseteq T^*$.

Now it is enough to prove T^* is the smallest **PΓSSM** of N containing B: Let M be a **PΓSSM** of N containing B and $a \in T^*$. Then $a = \Sigma_l s_l \mu_l b_l$, $s_l \in S,\ \mu_l \in \Gamma,\ b_l \in B$. Since $B \subseteq M$, $b_l \in M$, $l \in \Delta$. $\Rightarrow s_l \mu_l b_l \in M$, $l \in \Delta$. $\Rightarrow \Sigma_l s_l \mu_l b_l \in M$. $\Rightarrow a \in M$ and so $T^* \subseteq M$. Therefore T^* is the smallest **PΓSSM** of N containing B. Hence the theorem. □

Definition 3.12. *Let N be a left* **PΓSM** *over a* **PΓSR** *S, K be a* **PΓSSM** *of N and $n^* \in N$. Then $(K : n^*) = \{a \in S \mid a\mu n^* \in K\ \forall \mu \in \Gamma\}$.*

Theorem 3.13. *Let N be a left* **PΓSM** *over a* **PΓSR** *S, K be a* **PΓSSM** *of N and $n^* \in N$. Then $(K : n^*)$ is a left* **PΓI** *of S.*

Proof. Note that $(K : n^*) = \{a \in S \mid a\mu n^* \in K\ \forall \mu \in \Gamma\}$. Let $\Sigma_l a_l \in S$ and $a_l \in (K : n^*)\ \forall l \in \Delta$. Then $a_l \mu n^* \in K\ \forall \mu \in \Gamma,\ l \in \Delta$. $\Rightarrow \Sigma_l (a_l \mu n^*) \in K\ \forall \mu \in \Gamma$ and so $\Sigma_l a_l \in (K : n^*)$. Let $s \in S$, $\beta \in \Gamma$ and $a \in (K : n^*)$. Then $s \in S,\ \beta \in \Gamma$ and $a\mu n^* \in K\ \forall \mu \in \Gamma$. Since K is a **PΓSSM** of N, $(s\beta a)\mu n^* = s\beta(a\mu n^*) \in K\ \forall \mu \in \Gamma$ and so $s\beta a \in (K : n^*)$. Hence $(K : n^*)$ is a left **PΓI** of S. □

Definition 3.14. *If K is a* **PΓSSM** *of N and $C \subseteq N$ ($C \neq \emptyset$) then $(K : C) = \bigcap\{(K : c) \mid c \in C\}$.*

Theorem 3.15. *If K and K^* are* **PΓSSM***s of a left* **PΓSM** *N over S and $C,\ D$ are nonempty subsets of N. Then*
(i) if $C \subseteq D$ then $(K : D) \subseteq (K : C)$,
(ii) $(K \bigcap K^* : C) = (K : C) \bigcap (K^* : C)$, and
(iii) if $\Sigma(c, d) \in N\ \forall\ c \in C$ and $d \in D$ then $(K : C) \bigcap (K : D) \subseteq (K : C + D)$ and $(K : C) \bigcap (K : D) = (K : C + D)$ if $0_N \in C \bigcap D$ where $\Sigma(c, d) = \Sigma_l (c_l, d_l), l \in \Delta$ and $C + D = \{\Sigma(c, d) \mid c \in C, d \in D\}$.

Proof. (i) Assume that $C \subseteq D$ and let $p \in (K : D)$. $\Rightarrow p \in (K : d)\ \forall d \in D$. Since $C \subseteq D$, $p \in (K : c)$ $\forall c \in C$. $\Rightarrow p \in (K : C)$ and so $(K : D) \subseteq (K : C)$.
(ii) $p \in (K \bigcap K^* : C) \Leftrightarrow p \in (K \bigcap K^* : c)\ \forall c \in C \Leftrightarrow p\mu c \in K \bigcap K^*\ \forall \mu \in \Gamma,\ c \in C \Leftrightarrow p\mu c \in K$ and $p\mu c \in K^*\ \forall \mu \in \Gamma,\ c \in C \Leftrightarrow p \in (K : c)$ and $p \in (K^* : c)\ \forall c \in C \Leftrightarrow p \in (K : C) \bigcap (K^* : C)$. Therefore $(K \bigcap K^* : C) = (K : C) \bigcap (K^* : C)$.
(iii) Assume that $\Sigma(c, d) \in N\ \forall\ c \in C$ and $d \in D$. $\Rightarrow C + D \subseteq N$ and $C + D \neq \emptyset$. Let $p \in (K : C) \bigcap (K : D)$. Then $p \in (K : c)$ and $p \in (K : d)\ \forall c \in C,\ d \in D$. $\Rightarrow p\mu c \in K$ and $p\mu d \in K$ $\forall \mu \in \Gamma,\ c \in C,\ d \in D$. $\Rightarrow p\mu\Sigma(c, d) = \Sigma(p\mu c, p\mu d) \in K\ \forall \mu \in \Gamma,\ c \in C,\ d \in D$. $\Rightarrow p \in (K : C + D)$. Therefore $(K : C) \bigcap (K : D) \subseteq (K : C + D)$.

Now assume that $0_N \in C \bigcap D$ and let $p \in (K : C + D)$. Then $p\mu\Sigma(c, d) \in K\ \forall\mu \in \Gamma,\ c \in C,\ d \in D$. $\Rightarrow p\mu\Sigma(c, 0) \in K$ and $p\mu\Sigma(0, d) \in K\ \forall\mu \in \Gamma,\ c \in C,\ d \in D$. $\Rightarrow p\mu c \in K$ and $p\mu d \in K\ \forall\mu \in \Gamma,\ c \in C,\ d \in D$. $\Rightarrow p \in (K : c)$ and $p \in (K : d)\ \forall c \in C,\ d \in D$. $\Rightarrow p \in (K : C) \bigcap (K : D)$. Therefore $(K : C) \bigcap (K : D) = (K : C + D)$.

□

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