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New Subclasses of Analytic Univalent Functions Involving Poisson Distribution Series Bounded by Generalized Pascal Snail

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Abstract. The present paper defines a new subclass of analytic functions by means of the generalized polylogarithm equipped with subordination. The defined new class engages Poisson distribution series and generalized Pascal snail. The early first few coefficients are obtained and their relevance to classical inequality by Fekete-Szego are discussed. The consequences of our parametric equation are pointed out.

AMS (MOS) Subject Classification Codes: 30C45; 30C50; 30C80
Key Words: Analytic Coefficient bound, univalent, Poisson, polylogarithm, Pascal snail, Fekete-Szego inequality.

1. Introduction

Let $\mathcal{A}$ be the collection of all functions f that are complex-differentiable on the open set ($\mathbb{U} = \{z : z \in \mathbb{C}, |z| < 1\}$) around every point in its domain with form (1.1) below and with normalization condition $f(0) = f'(0) - 1 = 0$.

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1. 1}$$

In addition, let S be the collection of all univalent functions $f \in \mathcal{A}$ such that $S \in \mathcal{A}$ is univalent in $\mathbb{U}$.

Let $f(z)$ and $g(z)$ be analytic functions in $\mathbb{U}$, then $f(z)$ is said to be subordinate to $g(z)$, symbolically expressed as $f(z) \prec g(z),\ z \in \mathbb{U}$, provided there exists an analytic function $w(z)$ such that $f(z) = g(w(z))$. Suppose $g(z)$ is univalent in $\mathbb{U}$, then the earlier condition is equivalent to $f(z) \prec g(z)$ if and only if $f(0) = g(0)$ and $f(\mathbb{U}) \subset g(\mathbb{U})$.

Let $p \in P$ be the class of functions with positive real part (Caratheodory functions), such that $p(0) = 1$ and $Rep(z) > 0$ with $|p_n| \leq 2$, [11].
In recent times, there have been many functions unified by subordination using various forms of functions with positive real parts, see [6,8] as an example.
Fekete-Szego in [4] showed that

$$|a_2^2 - \mu a_3| \leq \begin{cases} 4\mu - 3 \,, & \mu \geq 1 \\ 1 + exp^{-\frac{2\mu}{1-\mu}} \,, & 0 \leq \mu \leq 1 \\ 3 - 4\mu \,, & \mu \leq 0 \end{cases}$$

holds for function $f \in S$ and the result is sharp, see [4,10-12] for further detail.
The Hadamard product (or convolution) of $f(z)$ given by (1.1) and

$$\varphi(z) = z + \sum_{n=2}^{\infty} \varphi_n z^n$$

is defined by

$$(f * \varphi)(z) = z + \sum_{n=2}^{\infty} a_n \varphi_n z^n = (\varphi * f)(z). \tag{1. 2}$$

Kanas and Masih [6] presented a unified method for the analytic representation of the domain bounded by a generalized Pascal snail as follows.
For $\alpha, \beta \in [-1, 1], \alpha\beta \neq 1, \gamma \in [0, 1)$, Let $L_{\alpha,\beta,\gamma} : \mathbb{U} \longrightarrow \mathbb{C}$ denote the function defined by

$$L_{\alpha,\beta,\gamma}(z) = \frac{(2 - 2\gamma)z}{(1 - \alpha z)(1 - \beta z)} = \sum_{n=1}^{\infty} B_n z^n = 2(1 - \gamma) \sum_{n=1}^{\infty} \frac{\alpha^n - \beta^n}{\alpha - \beta} z^n, \tag{1. 3}$$

for $\alpha \neq \beta$ and $L_{\alpha,\beta,\gamma}(z) = \sum_{n=1}^{\infty} B_n z^n = 2(1 - \gamma) \sum_{n=1}^{\infty} n\alpha^{n-1} z^n$. For detail information see [2,5,6,10] and the relevant literature therein.

Poisson distribution has recently attracted the attention of a few researchers in geometric function theory, but we have little literature available on the subject matter. Therefore, researchers in the area of geometric function theory must do a more in-depth study of the Poisson distribution series. The few available information can be obtained in [2,9,14,16] and relevant literature cited therein.
Poisson distribution is defined by

$$L(m, z) = z + \sum_{n=2}^{\infty} \frac{m^{n-1}}{(n - 1)!} e^{-m} z^n, \tag{1. 4}$$

with the radius of convergence at infinity.
Let $|z| < 1$ and let there exist $|p_n| \geq 2$, the classical polylogarithm $L_{\lambda,k}(z)$ of Leibniz with Bernoulli in 1969 is absolutely convergent and it is of the form

$$L_{\lambda,\gamma}(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^p}.$$

See [7,13] for details.
Let $f \in A$, then the generalized polylogarithms $D_\lambda^k f(z) : A \longrightarrow A$ is defined as

$$D_\lambda^k f(z) = z + \sum_{n=2}^{\infty} \frac{n^k (n + \lambda - 1)!}{\lambda!(n - 1)!} z^n \tag{1. 5}$$

where $k \in N_0 = \{0, 1, 2, \ldots\}, z \in U$. D_λ^k comprises of both the Salagean and Ruscheweyh derivative operators, see [1,11] for details.

Moreover, by employing the concept of convolution defined earlier in (1.2) and using (1.4) and (1.5) we obtain

$$D_\lambda^k(f*\varphi)*k(m,z)=z+\sum_{n=2}^{\infty}\frac{n^k(n+\lambda-1)!m^{n-1}}{\lambda!((n-1)!)^2}e^{-m}a_n\varphi_n z^n, \qquad (1.\ 6)$$

where $m\geq 0, k, \lambda, \in N_0$.

The present investigation wishes to present a unified subclass of starlike and convex function $T_{k,h}^{\rho;g}(b,\lambda,k(m,z),L_{\alpha,\beta,\gamma}(z))$ to determine the early few coefficient bounds and its associated Fekete-Szego classical inequality, and the relationship with Poisson distribution series bounded by generalized pascal snail, equipped with generalized polylogarithm.

2. Notations and Preliminaries

Lemma 2.1. $[1,10]$ *Let* $p(z)=p_1z+p_2z^2+\ldots\in\Omega$ *be that* $|w(z)|<1$ *in* $\mathbb{U}$*. If* $\mu\in\mathbb{C}$*, then* $|p_2-\mu c_1^2|\leq 2max\{1,|t|\}$*, where t is a complex number.*

Definition 2.2. *If* $\alpha,\beta\in[-1,1],\alpha\beta\neq 0$*, let* $m>0$*,* $k,\lambda\in N_0=\{0,1,2,\ldots\},\alpha\neq\beta,b\in\mathbb{C},\gamma\in[0)$*, and the class* $T_{\lambda,h}^{k,g}(b,\lambda,L(m,z),G_{\alpha,\beta,\gamma}(z))\in\mathcal{A}$ *consisting of the function* f *of the form (1.1) and*

$$g(z)=z+\sum_{n=2}^{\infty}g_nz^n, h(z)=z+\sum_{n=2}^{\infty}h_nz^n,$$

where $g_n>0,h_n>0,g_n-h_n>0$ *and L(m,z) is as earlier defined,then the following subordination condition is satisfied*

$$1+\frac{1}{b}\left[(1-\rho)\frac{D_\lambda^k(f*g)(z)*(L(m,z))}{D_\lambda^k(f*h)(z)}+\rho\frac{D_\lambda^k(f*g)(z)'*(L(m,z))}{D_\lambda^k(f*h)(z)}-1\right]\prec G_{\alpha,\beta,\gamma}(z), \qquad (2.\ 7)$$

where $G_{\alpha,\beta,\gamma}(z)=1+L_{\alpha,\beta,\gamma}(z)$ *and* $L_{\alpha,\beta,\gamma}(z)$ *is as earlier defined.*

3. Results

Theorem 3.1. *Let* $G_{\alpha,\beta,\gamma}(z),g(z),h(z),L(m,z)$ *be as earlier defined and* $G_{\alpha,\beta,\gamma}(z)$ *is a modified generalized pascal snail, If* $f(z)\in T_{\lambda,h}^{k,g}(b,\rho,G_{\alpha,\beta,\gamma}(z),L(m,z)),n\geq 2,g_n>0,h_n>0,g_n-h_n>0$*, then*

$$|a_2|\leq\frac{4|b|(1-\gamma)}{2^k(\lambda+1)\left[(1+\rho)me^{-m}(g_2-h_2)\right]}, \qquad (3.\ 8)$$

$$|a_3|\leq\frac{2|b|}{3^k(\lambda+2)(\lambda+1)(m^2e^{-m}g_3-2h_3)}\max\{1,|t|\}, \qquad (3.\ 9)$$

where

$$t=\frac{2(1-\gamma)(\lambda^2!(\alpha+\beta)((1+\rho)me^{-m}(g_2-h_2))^2-(1-\gamma)(h_2^2-me^{-m}g_2h_2))}{\lambda^2!((1+\rho)me^{-m}(g_2-h_2))^2}.$$

Proof. Suppose $f(z)\in T_{\lambda,h}^{k,g}(b,e,G_{\alpha,\beta,\gamma}(z),L(m,z))$, then there is a function $w(z)$ in $\mathbb{U}$ with $w(0)=0$ and $|w(z)<1|$ such that from (2.1) we have

$$1+\frac{1}{b}\left[(1-\rho)\frac{D_\lambda^k(f*g)(z)*(L(m,z))}{D_\lambda^k(f*h)(z)}+\rho\frac{D_\lambda^k(f*g)(z)'*(L(m,z))}{D_\lambda^k(f*h)(z)}-1\right]=G_{\alpha,\beta,\gamma}(w(z))$$

since $w(z) = w_1z + w_2z^2 + \cdots$ is a Schwarz function then

$$G_{\alpha,\beta,\gamma}(w(z)) = 1 + (2-2\gamma)w_1z + (2-2\gamma)((\alpha+\beta)w_1^2 + w_2)z^2 + \cdots .$$

Simplifying further, we have,

$$\begin{aligned} &1 + \frac{2^k(\lambda+1)!}{b\lambda!}\Bigg[(1+\rho)me^{-m}(g_2-h_2)\Bigg]a_2z \\ &+\Bigg[\frac{2^{2k}(\lambda+1)^2}{b\lambda^2!}a_2^2(h_2^2 - me^{-m}g_2h_2) + \frac{3^k(\lambda+2)!}{b\lambda!2!}\left(\frac{m^2e^{-m}}{2}g_3 - h_3\right)\Bigg]z^2 + \cdots \\ &= 1 + (2-2\gamma)w_1z + (2-2\gamma)((\alpha+\beta)w_1^2 + w_2)z^2 + \cdots . \end{aligned}$$

Comparing the coefficient of z and z^2, we have

$$a_2 \leq \frac{4|b|(1-\gamma)}{2^k(\lambda+1)\Bigg[(1+\rho)me^{-m}(g_2-h_2)\Bigg]}$$

$$a_3 \leq \frac{2|b|\lambda!\Bigg[(2-2\gamma)((\alpha+\beta)w_1^2 + w_2) - q_1w_1^2\Bigg]}{3^K(\lambda+2)!(m^2e^{-m}g_3 - 2h_3)}.$$

The proof is complete. □

Corollary 3.2. *Let $G_{\alpha,\beta,\gamma}(z), g(z), h(z), L(m,z)$ be as earlier defined and $G_{\alpha,\beta,\gamma}(z)$ is a modified generalized pascal snail, If $f(z) \in T_{1,h}^{1;g}(b, 0, G_{\alpha,\beta,0}(z), L(m,z)), n \geq 2, g_n > 0, h_n > 0, g_n - h_n > 0$, then*

$$|a_2| \leq \frac{|b|}{me^{-m}(g_2-h_2)}$$

$$|a_3| \leq \frac{|b|}{9(m^2e^{-m}g_3 - 2h_3)}\max\{1, |t|\}$$

where

$$t = \frac{2(\alpha+\beta)(me^{-m}(g_2-h_2))^2 - (h_2^2 - me^{-m}g_2h_2)}{(me^{-m}(g_2-h_2))^2}.$$

Corollary 3.3. *Let $G_{\alpha,\beta,\gamma}(z), g(z), h(z), L(m,z)$ be as earlier defined and $G_{\alpha,\beta,\gamma}(z)$ is a modified generalized pascal snail, If $f(z) \in T_{1,h}^{1;g}(b, 0, G_{\alpha,\beta,0}(z), L(1,z)), n \geq 2, g_n > 0, h_n > 0, g_n - h_n > 0$, then*

$$|a_2| \leq \frac{|b|}{0.37(g_2-h_2)}$$

$$|a_3| \leq \frac{|b|}{9(0.37g_3 - 2h_3)}\max\{1, |t|\}$$

where

$$t = \frac{0.28(\alpha+\beta)(g_2-h_2)^2 - (h_2^2 - 0.37g_2h_2)}{(0.14(g_2-h_2))^2}.$$

Theorem 3.4. *If $f(z)$ given by (1.1) is in the class $T^{k,g}_{\lambda,h}(b,\rho,G_{\alpha,\beta,\gamma}(z),L(m,z))$ and $\mu\in\mathbb{C}$ then*

$$|a_3-\mu a_2^2|\leq\frac{4|b|(1-\gamma)}{3^k(\lambda+2)(\lambda+1)(m^2e^{-m}g_3-2h_3)}\max\{1,|t|\}$$

where

$$t=\alpha+\beta+\frac{2^kb(\lambda+1)^2(h_2^2-me^{-m}g_2h_2)-3^k\times 2^2\mu b(\lambda+2)!(1-\gamma)(m^2e^{-m}g_3-2h_3)}{2^k(\lambda+1)^2((1+\rho)me^{-m}(g_2-h_2))^2}.$$

Proof 3.5. *Substitute the value of a_2 and a_3 in Theorem 3.1,we have*

$$a_3-\mu a_2^2=\frac{2!b\lambda!}{3^k(\lambda+2)!(\frac{m^2}{2}e^{-m}g_3-h_3)}\left[(2-2\gamma)w_2+((2-2\gamma)(\alpha+\beta)-q_1)w_1^2\right]$$

$$-\mu\frac{b^2(2-2\gamma)^2w_1^2}{2^2k(\lambda+1)^2![(1+\rho)me^{-m}g_2-h_2]^2}$$

simplifying further, we obtain

$$|a_3-\mu a_2^2|\leq\frac{2!b\lambda!}{3^k(\lambda+2)!(\frac{m^2}{2}e^{-m}g_3-h_3)}[w_2-tw_1^2] \qquad (3.\ 10)$$

where

$$t=\frac{\mu b(2-2\alpha)3^k(\lambda+2)!(\frac{m^2}{2}e^{-m}g_3-h_3)}{2^{2k+1}\lambda!(\lambda+1)^2![(1+\rho)me^{-m}g_2-h_2]^2}-\frac{q_1}{(2-2\gamma)}$$

. Using Lemma 2 in (3.1) we obtain

$$|a_3-\mu a_2^2|\leq\frac{4|b|\lambda!(2-2\gamma)}{3^k(\lambda+2)!(\frac{m^2}{2}e^{m}g_3-h_3)}\max\left(1,\left|\frac{\mu b(2-2\gamma)3^k(\lambda+2)!(\frac{m^2}{2}e^{-m}g_3-h_3)}{2^{2k+1}\lambda!(\lambda+1)^2![(1+\rho)me^{m}g_2-h_2]^2}-\frac{q_1}{(2-2\gamma)}\right|\right).$$

The proof is complete.

Corollary 3.6. *If $f(z)$ given by (1.1) is in the class $T^{1,g}_{1,h}(b,0,G_{\alpha,\beta,0}(z),L(m,z))$ and $\mu\in\mathbb{C}$ then*

$$|a_3-\mu a_2^2|\leq\frac{2|b|}{9(m^2e^{-m}g_3-2h_3)}\max\{1,|t|\}$$

where

$$t=\alpha+\beta+\frac{b(h_2^2-me^{-m}g_2h_2)-3^2\mu b(m^2e^{-m}g_3-2h_3)}{(me^{-m}(g_2-h_2))^2}.$$

Corollary 3.7. *If $f(z)$ given by (1.1) is in the class $T^{1,g}_{1,h}(b,0,G_{\alpha,\beta,0}(z),L(1,z))$ and $\mu\in\mathbb{C}$ then*

$$|a_3-\mu a_2^2|\leq\frac{2|b|}{9(0.37g_3-2h_3)}\max\{1,|t|\}$$

where

$$t=\alpha+\beta+\frac{b(h_2^2-0.37g_2h_2)-9\mu b(0.37g_3-2h_3)}{0.37(g_2-h_2)^2}.$$

The value of Poisson (me^{-m}) can be deduced from the earlier stated theorems as follows

Theorem 3.8. *Let $G_{\alpha,\beta,\gamma}(z), g(z), h(z), L(m,z)$ be as earlier defined and $G_{\alpha,\beta,\gamma}(z)$ is a modified generalized pascal snail, If $f(z) \in T^{k,g}_{\lambda,h}(b,\rho,G_{\alpha,\beta,\gamma}(z), L(m,z)), n \geq 2, g_n > 0, h_n > 0, g_n - h_n > 0$, then*

$$me^{-m} \leq \frac{4|b|(1-\gamma)}{2^k(\lambda+1)\Bigg[(1+\rho)(g_2-h_2)|a_2|\Bigg]},$$

$$m^2e^{-m} \leq \frac{2|b|}{3^k(\lambda+2)(\lambda+1)(|a_3|g_3-2h_3)} \max\{1,|t|\},$$

where

$$t = \frac{2(1-\gamma)(\lambda^2!(\alpha+\beta)((1+\rho)|a_2|(g_2-h_2))^2-(1-\gamma)(h_2^2-|a_2|g_2h_2))}{\lambda^2!((1+\rho)|a_2|(g_2-h_2))^2}.$$

Corollary 3.9. *Let $G_{\alpha,\beta,\gamma}(z), g(z), h(z), L(m,z)$ be as earlier defined and $G_{\alpha,\beta,\gamma}(z)$ is a modified generalized pascal snail, If $f(z) \in T^{1,g}_{1,h}(b,0,G_{\alpha,\beta,0}(z), L(m,z)), n \geq 2, g_n > 0, h_n > 0, g_n - h_n > 0$, then*

$$me^{-m} \leq \frac{|b|}{|a_2|(g_2-h_2)}$$

$$m^2e^{-m} \leq \frac{|b|}{9(|a_3|g_3-2h_3)} \max\{1,|t|\}$$

where

$$t = \frac{2(\alpha+\beta)(|a_2|(g_2-h_2))^2-(h_2^2-|a_2|g_2h_2)}{(|a_2|(g_2-h_2))^2}.$$

Let f be a starlike function then $|a_n| \leq 2$ and we have

Corollary 3.10. *Let $G_{\alpha,\beta,\gamma}(z), g(z), h(z), L(m,z)$ be as earlier defined and $G_{\alpha,\beta,\gamma}(z)$ is a modified generalized pascal snail, If $f(z) \in T^{1,g}_{1,h}(b,0,G_{\alpha,\beta,0}(z), L(m,z)), n \geq 2, g_n > 0, h_n > 0, g_n - h_n > 0$, then*

$$me^{-m} \leq \frac{|b|}{2(g_2-h_2)}$$

$$m^2e^{-m} \leq \frac{|b|}{9(2g_3-2h_3)} \max\{1,|t|\}$$

where

$$t = \frac{2(\alpha+\beta)(g_2-h_2)^2-(h_2^2-2g_2h_2)}{(g_2-h_2)^2}.$$

Theorem 3.11. *If $f(z)$ given by (1.1) is in the class $T^{k,g}_{\lambda,h}(b,\rho,G_{\alpha,\beta,\gamma}(z), L(m,z))$ and $\mu \in \mathbb{C}$ then*

$$m^2e^{-m}(1-\mu e^{-m}) \leq \frac{4|b|(1-\gamma)}{3^k(\lambda+2)(\lambda+1)(|a_3|g_3-2h_3)} \max\{1,|t|\}$$

where

$$t = \alpha+\beta+\frac{2^kb(\lambda+1)^2(h_2^2-|a_2|g_2h_2)-3^k\times 2^2\mu b(\lambda+2)!(1-\gamma)(|a_3|g_3-2h_3)}{2^k(\lambda+1)^2((1+\rho)|a_2|(g_2-h_2))^2}.$$

Corollary 3.12. *If $f(z)$ given by (1.1) is in the class $T^{1;g}_{1,h}(b,0,G_{\alpha,\beta,0}(z),L(m,z))$ and μ is a complex number then*

$$m^2e^{-m}(1-\mu e^{-m}) \leq \frac{2|b|(1-\gamma)}{9(|a_3|g_3-2h_3)}\max\{1,|t|\}$$

where

$$t=\alpha+\beta+\frac{b(h_2^2-|a_2|g_2h_2)-9\mu b(|a_3|g_3-2h_3)}{(|a_2|(g_2-h_2))^2}.$$

Suppose f is starlike, then $|a_n|\leq 2$ and we have

Corollary 3.13. *If $f(z)$ given by (1.1) is in the class $T^{1;g}_{1,h}(b,0,G_{\alpha,\beta,0}(z),L(m,z))$ and μ is a complex number then*

$$m^2e^{-m}(1-\mu e^{-m}) \leq \frac{2|b|(1-\gamma)}{9(2g_3-2h_3)}\max\{1,|t|\}$$

where

$$t=\alpha+\beta+\frac{b(h_2^2-2g_2h_2)-9\mu b(2g_3-2h_3)}{4(g_2-h_2)^2}.$$

4. Applications

The involvement of the Poisson series and Pascal snail in this research makes its real-life application interesting, particularly, in the areas of telephone calls and radioactive decay events and in the lift generated on airplane wings which is based on Pascal principle.

5. Conclusion

The author can define a subclass of analytic univalent function class involving Poisson distribution series and bounded by generalized Pascal snail polynomials using subordination. Early coefficients of the defined function class are obtained and its analogue of Fekete-Szego classical inequality is derived. Several consequences of the obtained results are pointed out using corollaries .
To the best of our knowledge the results obtained are new.

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