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**Navigating Decision Making with Generalized Temporal Intuitionistic Fuzzy Sets and Soft Sets**

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**Abstract.** The study addresses the challenges posed by evolving data within generalized intuitionistic fuzzy sets. Traditional methods often fall short in handling such complexity. To overcome this, we introduce the concept of a generalized temporal intuitionistic fuzzy set, extending the traditional framework to incorporate temporal dynamics. Additionally, we define a generalized temporal intuitionistic fuzzy soft set, integrating temporal aspects into the soft set framework. Recognizing the need for advanced operations like union and intersection to merge opinions across different periods, we propose practical solutions for decision-making in this dynamic context. Two novel multi-criteria decision-making methods are introduced, specifically designed to address decision-making problems within generalized temporal intuitionistic fuzzy soft sets. We develop Scilab codes for these methods, enabling the creation of a multiple-input single-output system. This system is applied to complex real-life examples, involving numerous parameters, time moments, and decision-makers. This comprehensive approach aims to provide robust tools and methodologies for decision-making processes amidst the intricate dynamics of temporal data within the framework of generalized temporal intuitionistic fuzzy sets.

**AMS (MOS) Subject Classification Codes: 03E72; 90C70; 68T37**

**Key Words:** Intuitionistic fuzzy set, Temporal dynamics, Generalized temporal intuitionistic fuzzy soft set, Multi-criteria Decision Making.

## 1. Introduction

The limitations of classical set theory in addressing uncertainties paved the way for the introduction of innovative concepts by pioneering researchers. In 1965, Zadeh [26] introduced fuzzy sets, recognizing the need for a framework capable of handling uncertainty. Fuzzy sets allowed for the representation of vague or imprecise information by assigning membership grades to elements, thereby providing a more flexible approach to modeling uncertainty. Recently, many authors have worked on fuzzy sets and their extensions [13, 2, 19, 23, 20, 21, 14]. Subsequently, Molodtsov [18] proposed soft sets in 1999, further expanding the horizon of uncertainty modeling. Soft sets offered a versatile framework for dealing with uncertainty by allowing elements to belong to multiple sets simultaneously, without the strict boundaries of traditional set theory. While fuzzy sets primarily focus on membership values, Atanassov [3] extended this notion by introducing intuitionistic fuzzy sets, incorporating non-membership values. Intuitionistic fuzzy sets introduced the concept of hesitation, allowing for a more nuanced representation of uncertainty where elements could simultaneously have varying degrees of membership, non-membership, and hesitation. This extension enriched the expressive power of fuzzy set theory, enabling more accurate modeling of complex real-world phenomena with uncertain or incomplete information. The evolution of uncertainty modeling continued with the integration of time dynamics, resulting in the development of temporal intuitionistic fuzzy sets by Atanassov [4] in 1991, enriching the modeling capabilities with temporal aspects. Temporal intuitionistic fuzzy sets extended the framework of intuitionistic fuzzy sets to accommodate temporal variations, enabling the representation of uncertainty that evolves. This integration facilitated more dynamic and realistic modeling of systems and phenomena subject to temporal changes and fluctuations.

Building upon these foundational concepts, Maji [15] and Maji et al. [16] further advanced the field by incorporating fuzzy logic and intuitionistic fuzzy logic into soft sets. Maji introduced fuzzy soft sets, which combined the flexibility of fuzzy sets with the tolerance for ambiguity inherent in soft sets, allowing for a more robust handling of uncertainty in decision-making processes. Subsequently, Maji et al. extended this concept by introducing intuitionistic fuzzy soft sets, which provided a comprehensive framework for modeling uncertainty that encompasses both the indeterminacy of soft sets and the hesitation of intuitionistic fuzzy sets. These developments significantly expanded the repertoire of tools available for uncertainty modeling, offering practitioners more sophisticated and adaptable approaches to address real-world problems with incomplete or uncertain information.

The exploration of the interplay between membership and non-membership values in intuitionistic fuzzy sets gave rise to a nuanced substructure, as demonstrated by Jamkhaneh and Nadarajah [11] in 2002. These advancements collectively signify a profound motivation to address the complexities of uncertainty, offering versatile tools for modeling real-world phenomena with enhanced precision and flexibility. In recent years, decision-making in uncertain environments has emerged as a pivotal challenge across various domains. The evolution of fuzzy set and soft set theories has significantly enhanced our ability to address decision-making complexities in diverse fields. These refined frameworks have proven instrumental in tackling intricate decision-making problems, offering adaptable solutions tailored to specific contexts. The integration of fuzzy logic, soft sets, and intuitionistic fuzzy sets has empowered decision-makers to navigate uncertain and ambiguous situations more effectively, enabling informed and robust decision-making processes even in the face of incomplete or imprecise information. As a result, these theories continue to drive innovation and provide valuable insights into managing uncertainty and complexity in decision-making across a wide range of applications. For instance, fuzzy set theory has found application in critical areas such as supply chain management [6] and statistical data analysis [8], providing robust methodologies to navigate complex decision landscapes. Similarly, the intuitionistic fuzzy set theory has made impactful strides in domains like the aviation industry [7] and medical diagnosis for cancer [1], offering nuanced insights into uncertain scenarios.

Furthermore, the versatility of soft set theory has been demonstrated in diverse applications ranging from design concept evaluation [9] to computational biology [22], empowering decision-makers with flexible tools to explore and evaluate alternatives effectively. Building upon these foundations, the fusion of fuzzy set and soft set theories has led to the development of fuzzy soft sets, which have been applied in areas such as patent quality evaluation [27] and optimization of public health centers [24]. Moreover, integrating intuitionistic fuzzy soft sets has opened new avenues in group medical diagnosis [10] and risk analysis [17], providing sophisticated frameworks to navigate complex decision scenarios with heightened precision and insight. Collectively, these advancements underscore the transformative potential of modified and refined theories in addressing decision-making challenges across a spectrum of applications, driving innovation and progress in diverse fields.

In many real-world scenarios, making decisions solely based on current data may prove insufficient for achieving precise and accurate outcomes. Oftentimes, overlooking the temporal aspect of data can lead to decision-making processes that lack precision. Incorporating historical information and conducting thorough analyses allows for more informed and insightful decision-making. By integrating temporal dynamics into decision models, decision-makers gain a comprehensive understanding of the evolution of phenomena over time, enabling them to anticipate trends, identify patterns, and make more effective decisions. This temporal perspective enhances the robustness and accuracy of decision-making processes, particularly in dynamic and evolving environments where historical context plays a crucial role in shaping outcomes. Data collected across various fields such as economics, medicine, and environmental science are subject to fluctuations over time. Analyzing data from different periods becomes imperative in certain situations to ensure robust decision-making. However, traditional set theories, which do not account for temporal dynamics, fall short of addressing such complex real-world problems.

Recognizing this limitation, Atanassov [4] introduced temporal intuitionistic fuzzy sets in 1991, laying the groundwork for subsequent research on their operations and properties. Despite theoretical advancements, the practical application of these sets in daily life problem-solving remained largely unexplored until recently. In 2019, Khalaf et al. [12] introduced the concept of temporal complex intuitionistic fuzzy sets, shedding light on their utility in decision-making tasks such as pattern recognition and medical diagnosis. This underscored the potential of integrating intuitionistic fuzzy sets with time moments in decision-making processes. By incorporating temporal dynamics into the framework of intuitionistic fuzzy sets, decision-makers can more effectively capture the evolving nature of uncertainties and make informed judgments based on comprehensive temporal information.

However, it's important to note that the traditional constraint of the sum of membership and non-membership values in intuitionistic fuzzy sets not exceeding 1 may not always hold in real-world scenarios. Generalized intuitionistic fuzzy sets, as discussed in [11], offer a more flexible framework to accommodate such cases. By relaxing this constraint, generalized intuitionistic fuzzy sets allow for a more nuanced representation of uncertainty, enabling decision-makers to capture subtleties that may be overlooked in traditional models. This enhanced flexibility is particularly valuable in complex decision-making scenarios where uncertainties exhibit varying degrees of ambiguity and imprecision.

In this paper, we aim to bridge this gap by addressing issues involving generalized intuitionistic fuzzy values at different time moments within a period in the parametric classification of objects. To this end, we propose a novel concept: the generalized temporal intuitionistic fuzzy set, and explore its integration with soft sets. Our study encompasses theoretical developments and practical applications, offering novel approaches to decision-making problems based on generalized temporal intuitionistic fuzzy soft sets. Through fictive real-life examples, we demonstrate the efficacy of our proposed methodologies in real-world decision-making contexts. By introducing this innovative framework, we provide decision-makers with powerful tools to navigate the complexities of uncertain environments, enabling them to make informed and effective decisions with confidence. The main advantages of our proposed method over existing approaches include:

- Incorporation of temporal dynamics into generalized intuitionistic fuzzy sets, allowing for the representation and analysis of evolving uncertainties.
- Development of generalized temporal intuitionistic fuzzy soft sets, integrating the principles of soft sets with temporal intuitionistic fuzzy logic for more robust decision-making.
- Introduction of two novel multi-criteria decision-making methods tailored for decision-making in the context of generalized temporal intuitionistic fuzzy soft sets, with Scilab codes provided for implementation.

- Application of the proposed methodologies to complex real-life decision-making problems, demonstrating enhanced precision and flexibility in handling uncertain and dynamic data.

The subsequent sections of this paper are structured as follows: In Section 2, we revisit fundamental concepts such as fuzzy set, intuitionistic fuzzy set, temporal intuitionistic fuzzy set, generalized intuitionistic fuzzy set, and soft set. Section 3 is dedicated to the definition and various operations of the generalized temporal intuitionistic fuzzy set. Moving forward, Section 4 introduces the concept of the generalized temporal intuitionistic fuzzy soft set, which integrates the principles of generalized temporal intuitionistic fuzzy logic into the soft set framework. In Section 5, we delve into the discussion of two multi-criteria decision-making methods and analyze decision-making problems through the lens of generalized temporal intuitionistic fuzzy soft sets. Finally, Section 6, the concluding section, summarizes the key insights gleaned from our study and outlines avenues for future research in this domain.

## 2. Preliminaries

This section provides a foundational overview of key concepts underpinning the subsequent discussions. Throughout this paper, unless explicitly stated otherwise, $X$ represents the universal set, $E$ denotes the parameter set, $\mu$ and $\nu$ are functions mapping elements of $X$ to the interval $[0,1]$, and $T$ signifies a non-empty set containing time moments.

Zadeh's pioneering work [26] introduced the concept of fuzzy sets, a pivotal tool for modeling uncertainty in real-world applications.

2.1. **Definition.** [26] A fuzzy set $\mathcal{F}$ defined over the universal set $X$ is expressed as $\mathcal{F} = \{\frac{(x)}{\prec \mu_{\mathcal{I}}(x) \succ} : x \in X\}$, where $\mu_{\mathcal{F}} : X \to [0,1]$ denotes the membership function of $\mathcal{F}$. Here, $\mu_{\mathcal{F}}(x)$ signifies the degree to which an element $x \in X$ belongs to the fuzzy set $\mathcal{F}$, offering a flexible framework to model imprecise or uncertain information in real-world contexts.

Zadeh's seminal work introduced fuzzy sets, assigning membership degrees to elements of a universe $X$ to quantify uncertainty. Atanassov's intuitionistic fuzzy sets [3] extended this concept by incorporating non-membership degrees, providing a more nuanced representation of uncertainty and imprecision.

2.2. **Definition.** [3] An intuitionistic fuzzy set, denoted as $\mathcal{I}$ or briefly as $if$-set, defined on a universal set $X$ as $\mathcal{I} = \{\frac{x}{\prec \mu_{\mathcal{I}}(x), \nu_{\mathcal{I}}(x) \succ} : x \in X\}$, where $\mu_{\mathcal{I}}, \nu_{\mathcal{I}} : X \to [0,1]$ represent the membership and non-membership functions of $\mathcal{I}$, respectively. Here, $\mu_{\mathcal{I}}(x)$ and $\nu_{\mathcal{I}}(x)$ denote the membership and non-membership degrees of an element $x \in X$ in $\mathcal{I}$, subject to the condition $0 \le \mu_{\mathcal{I}}(x) + \nu_{\mathcal{I}}(x) \le 1$.

Atanassov introduced intuitionistic fuzzy sets [3], characterizing uncertainty through membership and non-membership degrees within a universe $X$. Atanassov extended this concept to temporal intuitionistic fuzzy sets [4], capturing the evolution of uncertainty over time within the space $X \times T$.

2.3. **Definition.** [4] The temporal intuitionistic fuzzy set $\mathcal{I}^{(T)}$ over the universal set $X$ is defined as $\mathcal{I}^{(T)} = \{\frac{x^{(t)}}{\prec \mu_{\mathcal{I}}(x,t), \nu_{\mathcal{I}}(x,t) \succ} : (x,t) \in X \times T\}$, where $I \subset X$ is a fixed set. Here, $\mu_{\mathcal{I}}(x,t)$ and $\nu_{\mathcal{I}}(x,t)$ represent the membership and non-membership degrees of the element $x \in X$ at the time-moment $t \in T$, respectively, subject to the condition $0 \le \mu_{\mathcal{I}}(x,t) + \nu_{\mathcal{I}}(x,t) \le 1$.

In the description of temporal $if$-set in the literature, it is usually taken $x$ instead of $x^{(t)}$. Throughout this study, we use the notation $x^{(t)}$ to highlight the time-moment.

2.4. **Example.** Consider a real-life scenario where a team of researchers is conducting a study on the air quality of a city over some time. Let $X$ denote the set of pollutants being monitored, including carbon monoxide ($x_1$), sulfur dioxide ($x_2$), nitrogen dioxide ($x_3$), and particulate matter ($x_4$). The researchers collect data at two time points, $t_1$, and $t_2$, representing different periods of the day.

The temporal intuitionistic fuzzy set $\mathcal{I}^{(T)}$ representing the air quality data at these time points is defined as follows: $\mathcal{I}^{(T)} = \{\frac{x_1^{(t_1)}}{\prec 0.5,0.4\succ}, \frac{x_2^{(t_1)}}{\prec 0.6,0.2\succ}, \frac{x_3^{(t_1)}}{\prec 0.3,0.5\succ}, \frac{x_4^{(t_1)}}{\prec 0.4,0.4\succ}, \frac{x_1^{(t_2)}}{\prec 0.4,0.5\succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.4\succ}, \frac{x_3^{(t_2)}}{\prec 0.5,0.3\succ}, \frac{x_4^{(t_2)}}{\prec 0.4,0.4\succ}\}$.

Here, $\frac{x_1^{(t_1)}}{\prec 0.5,0.4\succ}$ represents the intuitionistic fuzzy value of carbon monoxide ($x_1$) at time $t_1$, indicating a moderate level of presence with a membership degree of 0.5 and a non-membership degree of 0.4. Similarly, $\frac{x_1^{(t_2)}}{\prec 0.4,0.5\succ}$ signifies the air quality at time $t_2$, reflecting a slightly lower membership degree for carbon monoxide and a slightly higher non-membership degree, suggesting a decrease in its presence compared to $t_1$.

An intuitionistic fuzzy set is generalized into the intuitionistic fuzzy set of the second type (IFSST) by Atanassov [5] in 1989 as follows.

2.5. **Definition.** [5] The intuitionistic fuzzy set of the second type (IFSST) $\mathcal{G}$, defined on the universal set $X$, is formulated as $\mathcal{G} = \left\{\frac{x}{\prec \mu_{\mathcal{G}}(x),\nu_{\mathcal{G}}(x)\succ} : x \in X\right\}$, where $\mu_{\mathcal{G}}(x)$ and $\nu_{\mathcal{G}}(x)$ denote the membership and non-membership degrees of an element $x \in X$. These degrees adhere to the condition $0 \leq [\mu_{\mathcal{G}}(x)]^2 + [\nu_{\mathcal{G}}(x)]^2 \leq 1$ for each $x \in X$. The collection of all intuitionistic fuzzy sets of the second type on $X$ is denoted by $IFSST(X)$.

An intuitionistic fuzzy set is generalized into the intuitionistic fuzzy set of root type (IFSRT) by Srinivasan [25] as follows.

2.6. **Definition.** [25] The intuitionistic fuzzy set of root type (IFSRT) $\mathcal{G}$, defined on the universal set $X$, is formulated as $\mathcal{G} = \left\{\frac{x}{\prec \mu_{\mathcal{G}}(x),\nu_{\mathcal{G}}(x)\succ} : x \in X\right\}$, where $\mu_{\mathcal{G}}(x)$ and $\nu_{\mathcal{G}}(x)$ denote the membership and non-membership degrees of an element $x \in X$. These degrees adhere to the condition $0 \leq \sqrt{\mu_{\mathcal{G}}(x)} + \sqrt{\nu_{\mathcal{G}}(x)} \leq 1$ for each $x \in X$. The collection of all intuitionistic fuzzy sets of root type on $X$ is denoted by $IFSRT(X)$.

To generalize the IFSST and IFSRT, Jamkhaneh et al. [11] introduced the generalized $if$-set in the following manner.

2.7. **Definition.** [11] The generalized $if$-set $\mathcal{G}$, defined on the universal set $X$, is formulated as $\mathcal{G} = \{\frac{x}{\prec \mu_{\mathcal{G}}(x),\nu_{\mathcal{G}}(x)\succ} : x \in X\}$, where $\mu_{\mathcal{G}}(x)$ and $\nu_{\mathcal{G}}(x)$ denote the membership and non-membership degrees of an element $x \in X$. These degrees adhere to the condition $0 \leq [\mu_{\mathcal{G}}(x)]^{\kappa} + [\nu_{\mathcal{G}}(x)]^{\kappa} \leq 1$ for each $x \in X$, where $\kappa = n$ or $\kappa = \frac{1}{n}$, and $n = 1, 2, ..., N$. The collection of all generalized $if$-sets on $X$ (with $\kappa$) is denoted by $GIFS(X)$ ($GIFS_{\kappa}(X)$).

2.8. **Example.** Consider a real-life scenario where $X$ represents the ages of young women ranging from 0 to 25 years. Let $\mathcal{G}$ denote a group of teenagers whose ages fall between 10 and 15 years. We define the membership and non-membership functions as follows:

$$\mu_{\mathcal{G}}(x) = \begin{cases} \left(\frac{x-5}{5}\right)^{\frac{1}{3}}, & \text{if } 5 \leq x \leq 10 \\ 1, & \text{if } 10 \leq x \leq 15 \\ \left(\frac{20-x}{5}\right)^{\frac{1}{3}}, & \text{if } 15 \leq x \leq 20 \\ 0, & \text{otherwise} \end{cases}, \quad \nu_{\mathcal{G}}(x) = \begin{cases} \left(\frac{10-x}{6}\right)^{\frac{1}{3}}, & \text{if } 4 \leq x \leq 10 \\ 0, & \text{if } 10 \leq x \leq 15 \\ \left(\frac{x-15}{6}\right)^{\frac{1}{3}}, & \text{if } 15 \leq x \leq 21 \\ 0, & \text{otherwise} \end{cases}.$$

Then, since $0 \leq [\mu_{\mathcal{G}}(x)]^3 + [\nu_{\mathcal{G}}(x)]^3 \leq 1$, we denote $\mathcal{G} = \{\frac{x}{\prec \mu_{\mathcal{G}}(x),\nu_{\mathcal{G}}(x)\succ} : x \in X\}$ as a Generalized Intuitionistic Fuzzy Set ($GIFS_3(X)$). It's worth noting that $\mathcal{G}$ differs from an intuitionistic fuzzy set because, for instance, at $x = 8$, we have $\mu_{\mathcal{G}}(x) = 0.8434$ and $\nu_{\mathcal{G}}(x) = 0.6933$, resulting in $\mu_{\mathcal{G}}(x) + \nu_{\mathcal{G}}(x) \nleq 1$.

Molodtsov [18] presented the concept of soft sets in the following manner.

2.9. **Definition.** [18] Consider a universal set $X$ and a set of parameters $E$. Let $\mathcal{P}(X)$ denote the power set of $X$, and $A \subseteq E$. A soft set $\mathcal{S}$ over $X$ is defined as $\mathcal{S}_A = \{(e, \mathcal{S}(e)) : e \in A,\ \mathcal{S}(e) \in \mathcal{P}(X)\}$, where $\mathcal{S} : A \to \mathcal{P}(X)$ is a mapping.

## 3. Generalized Temporal Intuitionistic Fuzzy Set

This section introduces the concept of a generalized temporal intuitionistic fuzzy set (GTIFS). To represent uncertainty and its evolution over time, a GTIFS assigns membership and non-membership degrees to elements of a universe $X$ at specific time instances within a set $T$. We formally define a GTIFS as follows:

3.1. **Definition.** The concept of a generalized temporal intuitionistic fuzzy set ($gtif$-set), denoted as $\mathcal{G}^{(T)}$, on a set $X$, is defined as follows: $\mathcal{G}^{(T)} = \{\frac{x^{(t)}}{\prec \mu_{\mathcal{G}}(x,t), \nu_{\mathcal{G}}(x,t) \succ} : (x,t) \in X \times T\}$. Here, $\mathcal{G} \subset X$ is a fixed set, and $\mu_{\mathcal{G}}(x,t)$ and $\nu_{\mathcal{G}}(x,t)$ represent the membership and non-membership degrees of an element $x \in X$ at the time-moment $t \in T$. The conditions $0 \leq [\mu_{\mathcal{G}}(x,t)]^{\kappa} + [\nu_{\mathcal{G}}(x,t)]^{\kappa} \leq 1$ hold for, $\kappa = n$ or $\kappa = \frac{1}{n}$, where $n = 1, 2, ..., N$. The collection of all $gtif$-sets on $X$ (with parameter $\kappa$) is denoted by $GTIFS(X^{(T)})$ ($GTIFS_{\kappa}(X^{(T)})$).

3.2. **Definition.** The indeterminacy degree of an element $x \in X$ concerning the generalized temporal intuitionistic fuzzy set $\mathcal{G}$ is defined as $\pi_{\mathcal{G}}(x,t) = (1 - [\mu_{\mathcal{G}}(x,t)]^{\kappa} - [\nu_{\mathcal{G}}(x,t)]^{\kappa})^{\frac{1}{\kappa}}$. It is evident that $[\pi_{\mathcal{G}}(x,t)]^{\kappa} + [\mu_{\mathcal{G}}(x,t)]^{\kappa} + [\mu_{\mathcal{G}}(x,t)]^{\kappa} = 1$.

3.3. **Example.** Pea plants thrive in specific temperature ranges, with cold-resistant varieties typically cultivated in autumn and heat-resistant ones in spring. Let $X$ denote the range of temperatures conducive to pea seed growth (from October to March), typically falling between [0°C, 30°C]. Consider $\mathcal{G}$ as the set of temperatures optimal for leek pea seed growth, ranging between 10°C and 20°C. Temperature variations may occur between different months, represented by $T = \{t_1 = October,\ t_2 = March\}$. Here are the membership and non-membership functions for $t_1 = October$:

$$\mu_{\mathcal{G}}(x) = \begin{cases} (\frac{x-5}{5})^{\frac{1}{5}}, & \text{if } 5°C \leq x \leq 10°C \\ 1, & \text{if } 10°C \leq x \leq 20°C \\ (\frac{25-x}{5})^{\frac{1}{5}}, & \text{if } 20°C \leq x \leq 25°C \\ 0, & \text{otherwise} \end{cases}, \quad \nu_{\mathcal{G}}(x) = \begin{cases} (\frac{10-x}{5})^{\frac{1}{5}}, & \text{if } 5°C \leq x \leq 10°C \\ 0, & \text{if } 10°C \leq x \leq 20°C \\ (\frac{x-20}{5})^{\frac{1}{5}}, & \text{if } 20°C \leq x \leq 25°C \\ 1, & \text{otherwise} \end{cases},$$

and the membership and non membership functions for $t_2 = March$ as follows:

$$\mu_{\mathcal{G}}(x) = \begin{cases} (\frac{x^2-25}{75})^{\frac{1}{5}}, & \text{if } 5°C \leq x \leq 10°C \\ 1, & \text{if } 10°C \leq x \leq 20°C \\ (\frac{625-x^2}{225})^{\frac{1}{5}}, & \text{if } 20°C \leq x \leq 25°C \\ 0, & \text{otherwise} \end{cases}, \quad \nu_{\mathcal{G}}(x) = \begin{cases} (\frac{100-x^2}{75})^{\frac{1}{5}}, & \text{if } 5°C \leq x \leq 10°C \\ 0, & \text{if } 10°C \leq x \leq 20°C \\ (\frac{x^2-400}{225})^{\frac{1}{5}}, & \text{if } 20°C \leq x \leq 25°C \\ 1, & \text{otherwise} \end{cases},$$

Hence, with the condition $0 \leq [\mu_{\mathcal{G}}(x)]^5 + [\nu_{\mathcal{G}}(x)]^5 \leq 1$, we classify $\mathcal{G}^{(T)} = \{\frac{x^{(t)}}{\prec \mu_{\mathcal{G}}(x,t), \nu_{\mathcal{G}}(x,t) \succ} : (x,t) \in X \times T\}$ as a $GTIFS_5(X^{(T)})$.

3.4. **Definition.** Let $\mathcal{G}_1^{(T)} = \{\frac{x^{(t)}}{\prec \mu_{\mathcal{G}_1}(x,t), \nu_{\mathcal{G}_1}(x,t) \succ} : (x,t) \in X \times T\}$ and $\mathcal{G}_2^{(T)} = \{\frac{x^{(t)}}{\prec \mu_{\mathcal{G}_2}(x,t), \nu_{\mathcal{G}_2}(x,t) \succ} : (x,t) \in X \times T\}$ be two $gtif$-sets. Then,

**a):** $\mathcal{G}_1^{(T)}$ is a $gtif$-subset of $\mathcal{G}_2^{(T)}$, denoted by $\mathcal{G}_1^{(T)} \widetilde{\subseteq} \mathcal{G}_2^{(T)}$, if $\mu_{\mathcal{G}_1}(x,t) \leq \mu_{\mathcal{G}_2}(x,t)$ and $\nu_{\mathcal{G}_1}(x,t) \geq \nu_{\mathcal{G}_2}(x,t)$ for all $(x,t) \in X \times T$.

**b):** $\mathcal{G}_1^{(T)}$ and $\mathcal{G}_2^{(T)}$ are equal $gtif$-sets, denoted by $\mathcal{G}_1^{(T)} = \mathcal{G}_2^{(T)}$, if $\mu_{\mathcal{G}_1}(x,t) = \mu_{\mathcal{G}_2}(x,t)$ and $\nu_{\mathcal{G}_1}(x,t) = \nu_{\mathcal{G}_2}(x,t)$ for all $(x,t) \in X \times T$.

3.5. **Definition.** Let $T'$ and $T''$ represent sets of time moments. Consider two generalized temporal intuitionistic fuzzy (GTIF) sets: $\mathcal{G}_1^{(T')} = \{\frac{x^{(t)}}{\prec\mu_{\mathcal{G}_1}(x,t),\nu_{\mathcal{G}_1}(x,t)\succ} : (x,t) \in X \times T'\}$ and $\mathcal{G}_2^{(T'')} = \{\frac{x^{(t)}}{\prec\mu_{\mathcal{G}_2}(x,t),\nu_{\mathcal{G}_2}(x,t)\succ} : (x,t) \in X \times T''\}$. The union of $T'$ and $T''$ is denoted as $T^{\cup}$. The fundamental operations "complement," "intersection," and "union" are defined as follows:

**a):** $\widetilde{c}(\mathcal{G}_1^{(T')}) = \{\frac{x^{(t)}}{\prec\nu_{\mathcal{G}_1}(x,t),\mu_{\mathcal{G}_1}(x,t)\succ} : (x,t) \in X \times T'\}$.

**b):** $\mathcal{G}_1^{(T')} \widetilde{\cap}\, \mathcal{G}_2^{(T'')} = \{\frac{x^{(t)}}{\prec\min\{\widetilde{\mu}_{\mathcal{G}_1}(x,t),\widetilde{\mu}_{\mathcal{G}_2}(x,t)\},\max\{\widetilde{\nu}_{\mathcal{G}_1}(x,t),\widetilde{\nu}_{\mathcal{G}_2}(x,t)\}\succ} : (x,t) \in X \times T^{\cup}\}$.

**c):** $\mathcal{G}_1^{(T')} \widetilde{\cup}\, \mathcal{G}_2^{(T'')} = \{\frac{x^{(t)}}{\prec\max\{\widetilde{\mu}_{\mathcal{G}_1}(x,t),\widetilde{\mu}_{\mathcal{G}_2}(x,t)\},\min\{\widetilde{\nu}_{\mathcal{G}_1}(x,t),\widetilde{\nu}_{\mathcal{G}_2}(x,t)\}\succ} : (x,t) \in X \times T^{\cup}\}$.

where

$$\widetilde{\mu}_{\mathcal{G}_1}(x,t) = \begin{cases} \mu_{\mathcal{G}_1}(x,t), & \text{if } t \in T' \\ 0, & \text{if } t \in T''\backslash T' \end{cases}, \qquad \widetilde{\mu}_{\mathcal{G}_2}(x,t) = \begin{cases} \mu_{\mathcal{G}_2}(x,t), & \text{if } t \in T'' \\ 0, & \text{if } t \in T'\backslash T'' \end{cases},$$

$$\widetilde{\nu}_{\mathcal{G}_1}(x,t) = \begin{cases} \nu_{\mathcal{G}_1}(x,t), & \text{if } t \in T' \\ 1, & \text{if } t \in T''\backslash T' \end{cases}, \qquad \widetilde{\nu}_{\mathcal{G}_2}(x,t) = \begin{cases} \nu_{\mathcal{G}_2}(x,t), & \text{if } t \in T'' \\ 1, & \text{if } t \in T'\backslash T'' \end{cases}.$$

It is important to note that Propositions 3.6, 3.7, and 3.8 follow directly from the fact that a temporal intuitionistic fuzzy set (TIFS) defined over a universe $X$ and a temporal scale $T$ can be viewed as an intuitionistic fuzzy set (IFS) over the Cartesian product $X \times T$. **This understanding of temporal IFSs has been recognized in the literature since the late 20th century.** Therefore, the significance of these propositions in the context of our work lies not in the introduction of new mathematical concepts but in their application within the framework of generalized temporal intuitionistic fuzzy sets (GTIFS). **The novelty of our approach lies in extending this established theoretical foundation to integrate GTIFS with soft set theory, thereby offering a new perspective on decision-making processes that involve temporal and fuzzy uncertainty.**

3.6. **Proposition.** Let $\mathcal{G}_1^{(T)}$, $\mathcal{G}_2^{(T)}$ and $\mathcal{G}_3^{(T)}$ be three $gtif$-sets on $X$. Then, we have the following:

**i):** $\widetilde{c}(\widetilde{c}(\mathcal{G}_1^{(T)})) = \mathcal{G}_1(T)$.

**ii):** If $\mathcal{G}_1^{(T)} \widetilde{\subseteq} \mathcal{G}_2^{(T)}$ and $\mathcal{G}_2^{(T)} \widetilde{\subseteq} \mathcal{G}_3^{(T)}$ then $\mathcal{G}_1^{(T)} \widetilde{\subseteq} \mathcal{G}_3^{(T)}$.

*Proof.* The proofs are clear and, therefore, omitted. □

3.7. **Proposition.** Let $\mathcal{G}_1^{(T')}$, $\mathcal{G}_2^{(T'')}$ be two $gtif$-sets on $X$. Then, we have the following.

**i):** $\widetilde{c}(\mathcal{G}_1^{(T')} \widetilde{\cup}\, \mathcal{G}_2^{(T'')}) = \widetilde{c}(\mathcal{G}_1^{(T')}) \widetilde{\cap}\, \widetilde{c}(\mathcal{G}_2^{(T'')})$.

**ii):** $\widetilde{c}(\mathcal{G}_1^{(T')} \widetilde{\cap}\, \mathcal{G}_2^{(T'')}) = \widetilde{c}(\mathcal{G}_1^{(T')}) \widetilde{\cup}\, \widetilde{c}(\mathcal{G}_2^{(T'')})$.

*Proof.* (i) By Definition 3.5 (a) and (c), we write that

$$\widetilde{c}(\mathcal{G}_1^{(T')} \widetilde{\cup}\, \mathcal{G}_2^{(T'')}) = \{\frac{x^{(t)}}{\prec min\{\widetilde{\nu}_{\mathcal{G}_1}(x,t),\widetilde{\nu}_{\mathcal{G}_2}(x,t), max\{\widetilde{\mu}_{\mathcal{G}_1}(x,t),\widetilde{\mu}_{\mathcal{G}_2}(x,t)\}\}\succ} : (x,t) \in X \times T^{\cup}\}.$$

Also, we know that

$$\widetilde{c}(\mathcal{G}_1^{(T')}) = \{\frac{x^{(t)}}{\widetilde{\nu}_{\mathcal{G}_1}(x,t),\widetilde{\mu}_{\mathcal{G}_1}(x,t)} : (x,t) \in X \times T'\}, \widetilde{c}(\mathcal{G}_2^{(T'')}) = \{\frac{x^{(t)}}{\widetilde{\nu}_{\mathcal{G}_2}(x,t),\widetilde{\mu}_{\mathcal{G}_2}(x,t)} : (x,t) \in X \times T''\}.$$

From Definition 3.5 (b), we obtain that

$$\widetilde{c}(\mathcal{G}_1^{(T')}) \widetilde{\cap} \widetilde{c}(\mathcal{G}_2^{(T'')}) = \{\frac{x^{(t)}}{\prec min\{\widetilde{\nu}_{\mathcal{G}_1}(x,t),\widetilde{\nu}_{\mathcal{G}_2}(x,t), max\{\widetilde{\mu}_{\mathcal{G}_1}(x,t),\widetilde{\mu}_{\mathcal{G}_2}(x,t)\}\}\succ} : (x,t) \in X \times T^{\cup}\}.$$

So, the proof is completed.
(ii) Similarly, it can be proved. □

3.8. **Proposition.** Let $\mathcal{G}_1^{(T')}$, $\mathcal{G}_2^{(T'')}$ and $\mathcal{G}_3^{(T''')}$ be three $gtif$-sets on $X$. For each $\diamond \in \{\widetilde{\cup}, \widetilde{\cap}\}$, we have the following.

**i):** $\mathcal{G}_1^{(T')} \diamond \mathcal{G}_1^{(T')} = \mathcal{G}_1^{(T')}$.
**ii):** $\mathcal{G}_1^{(T')} \diamond \mathcal{G}_2^{(T'')} = \mathcal{G}_2^{(T'')} \diamond \mathcal{G}_1^{(T')}$
**iii):** $(\mathcal{G}_1^{(T')} \diamond \mathcal{G}_2^{(T'')}) \diamond \mathcal{G}_3^{(T''')} = \mathcal{G}_1^{(T')} \diamond (\mathcal{G}_2^{(T'')} \diamond \mathcal{G}_3^{(T''')})$.

*Proof.* The proofs can be easily seen from Definition 3.5. □

## 4. Introduction to Generalized Temporal Intuitionistic Fuzzy Soft Sets

4.1. **Definition.** Let $E$ be a set of parameters, $A \subseteq E$. The generalized temporal intuitionistic fuzzy soft set ($gtif$-soft set) $\mathfrak{S}_A^{(T)}$ over $X$ is a collection of pairs as follows:

$$\mathfrak{S}_A^{(T)} = \{(e, \mathfrak{S}^{(T)}(e)) : e \in A,\ \mathfrak{S}^{(T)}(e) \in GTIFS(X^{(T)})\},$$

where $\mathfrak{S}^{(T)} : A \to GTIFS(X^{(T)})$ is a mapping. The collection of all $gtif$-soft sets on $X$ (with $\kappa$) is denoted by $GTIFSS(X^{(T)})(GTIFSS_\kappa(X^{(T)}))$.

4.2. **Example.** Assume that an investment company wants to invest some money in the best option from different types of finance companies. For this purpose, let us assume that these companies plan to analyze their data for three months, separately for each month (e.g., April, May, and June). Let $X = \{x_1, x_2, x_3\}$ be a set of finance companies, $E = \{e_1 = environmental\ \ risks, e_2 = political\ risks\}$ be a set of parameters and $T = \{t_1, t_2, t_3\}$ be a set of time moments. Also, we take $A = E$. Then, the investment company evaluates the data of the finance companies in the specified months and obtains the following mapping: $\mathfrak{S}^{(T)} : A \to \mathcal{GTIFS}(X^{(T)})$ such that

$$\mathfrak{S}^{(T)}(e_1) = \{\frac{x_1^{(t_1)}}{\prec 0.7,0.6\succ}, \frac{x_2^{(t_1)}}{\prec 0.8,0.4\succ}, \frac{x_3^{(t_1)}}{\prec 0.5,0.4\succ}, \frac{x_1^{(t_2)}}{\prec 0.9,0.6\succ}, \frac{x_2^{(t_2)}}{\prec 0.8,0.7\succ}, \frac{x_3^{(t_2)}}{\prec 0.7,0.7\succ}, \frac{x_1^{(t_3)}}{\prec 0.4,0.9\succ}, \frac{x_2^{(t_3)}}{\prec 0.5,0.7\succ}, \frac{x_3^{(t_3)}}{\prec 0.6,0.6\succ}\},$$

$$\mathfrak{S}^{(T)}(e_2) = \{\frac{x_1^{(t_1)}}{\prec 0.2,0.8\succ}, \frac{x_2^{(t_1)}}{\prec 0.6,0.5\succ}, \frac{x_3^{(t_1)}}{\prec 0.5,0.5\succ}, \frac{x_1^{(t_2)}}{\prec 0.7,0.4\succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.7\succ}, \frac{x_3^{(t_2)}}{\prec 0.5,0.3\succ}, \frac{x_1^{(t_3)}}{\prec 0.5,0.7\succ}, \frac{x_2^{(t_3)}}{\prec 0.9,0.6\succ}, \frac{x_3^{(t_3)}}{\prec 0.7,0.2\succ}\}.$$

Thus, the generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_A^{(T)}$ can be written as:

$$\mathfrak{S}_A^{(T)} = \{(e_1, \{\frac{x_1^{(t_1)}}{\prec 0.7,0.6\succ}, \frac{x_2^{(t_1)}}{\prec 0.8,0.4\succ}, \frac{x_3^{(t_1)}}{\prec 0.5,0.4\succ}, \frac{x_1^{(t_2)}}{\prec 0.9,0.6\succ}, \frac{x_2^{(t_2)}}{\prec 0.8,0.7\succ}, \frac{x_3^{(t_2)}}{\prec 0.7,0.7\succ}, \frac{x_1^{(t_3)}}{\prec 0.4,0.9\succ}, \frac{x_2^{(t_3)}}{\prec 0.5,0.7\succ}, \frac{x_3^{(t_3)}}{\prec 0.6,0.6\succ}\}),$$
$$(e_2, \{\frac{x_1^{(t_1)}}{\prec 0.2,0.8\succ}, \frac{x_2^{(t_1)}}{\prec 0.6,0.5\succ}, \frac{x_3^{(t_1)}}{\prec 0.5,0.5\succ}, \frac{x_1^{(t_2)}}{\prec 0.7,0.4\succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.7\succ}, \frac{x_3^{(t_2)}}{\prec 0.5,0.3\succ}, \frac{x_1^{(t_3)}}{\prec 0.5,0.7\succ}, \frac{x_2^{(t_3)}}{\prec 0.9,0.6\succ}, \frac{x_3^{(t_3)}}{\prec 0.7,0.2\succ}\})\}.$$

Considering the $gtif$-soft set $\mathfrak{S}_A^{(T)}$, we can interpret that for the parameter "environment risks", the generalized intuitionistic fuzzy value of a finance company $x_1$ at the time-moment $t_1 \in T$ is $\prec 0.7, 0.6 \succ$. In addition to this, the generalized intuitionistic fuzzy value of a finance company $x_1$ at the time-moment $t_2 \in T$ is $\prec 0.6, 0.6 \succ$. Others can be interpreted similarly.
Moreover, the generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_A^{(T)}$ can be illustrated in Figure 1.

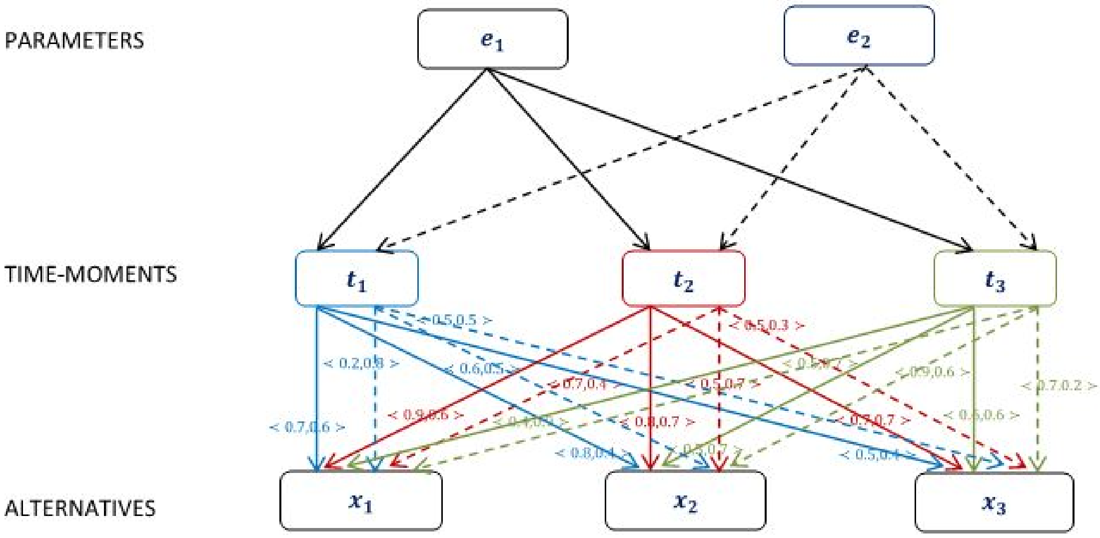


FIGURE 1. The figuration of $gtif$-soft set $\mathfrak{S}_A^{(T)}$ in Example 4.2.

4.3. **Definition.** Let $A \subseteq E$, $T = \{t_1, t_2, \ldots, t_k\}$ be a set of time moments, and $w_{t_1}, w_{t_2}, \ldots, w_{t_k}$ be the weightage of time moments $t_1, t_2, \ldots, t_k$ respectively. Also, let $\mathfrak{S}_A^{(T)}$ be a $gtif$-soft set over $X$. Then the generalized non-temporal intuitionistic fuzzy soft set ($gntif$- soft set) $\mathfrak{S}_A$ concerning $\mathfrak{S}_A^{(T)}$ over X is a collection of pairs $\mathfrak{S}_A = \{(e, \mathfrak{S}(e)) : e \in A, \mathfrak{S}(e) \in GIFS(X)\}$, where $\mathfrak{S} : A \to GIFS(X)$ is a mapping defined by $\mathfrak{S}(e) = \{\frac{x}{\prec \mu_{\mathfrak{S}(e)}(x), \nu_{\mathfrak{S}(e)(x)} \succ} : x \in X\}$ such that

$$\mu_{\mathfrak{S}(e)}(x) = \sum_{s=1}^{k} \frac{(w_{t_s})[\mu_{\mathfrak{S}^{(T)}(e)}(x, t_s)]}{kd}, \tag{4. 1}$$

$$\nu_{\mathfrak{S}(e)}(x) = \sum_{s=1}^{k} \frac{(w_{t_s})[\nu_{\mathfrak{S}^{(T)}(e)}(x, t_s)]}{kd} \tag{4. 2}$$

where $d = max\{w_{t_s} : i = 1, 2, \ldots, k\}$.

4.4. **Example.** Consider the $gtif$-soft set $\mathfrak{S}_A^{(T)}$ given in Example 4.2. If the weightage of time $t_1, t_2$ and $t_3$ are $0.2, 0.5$, and $0.8$ respectively, then the generalized non-temporal intuitionistic fuzzy soft set ($gntif$-soft set) $\mathfrak{S}_A$ concerning $\mathfrak{S}_A^{(T)}$ over $X$ is a collection of pairs as follows:

$$\mathfrak{S}_A = \{(e_1, \{\frac{x_1}{\prec 0.3791667, 0.475 \succ}, \frac{x_2}{\prec 0.4, 0.4125 \succ}, \frac{x_3}{\prec 0.3875, 0.3791667 \succ}\}),$$
$$(e_2, \{\frac{x_1}{\prec 0.3291667, 0.3833333 \succ}, \frac{x_2}{\prec 0.4541667, 0.3875 \succ}, \frac{x_3}{\prec 0.3791667, 0.1708333 \succ}\})\}.$$

The constitution of the $gntif$-soft set $\mathfrak{S}_A$ concerning the $gtif$-soft set $\mathfrak{S}_A^{(T)}$ over $X$ can be illustrated as in Figure 2.

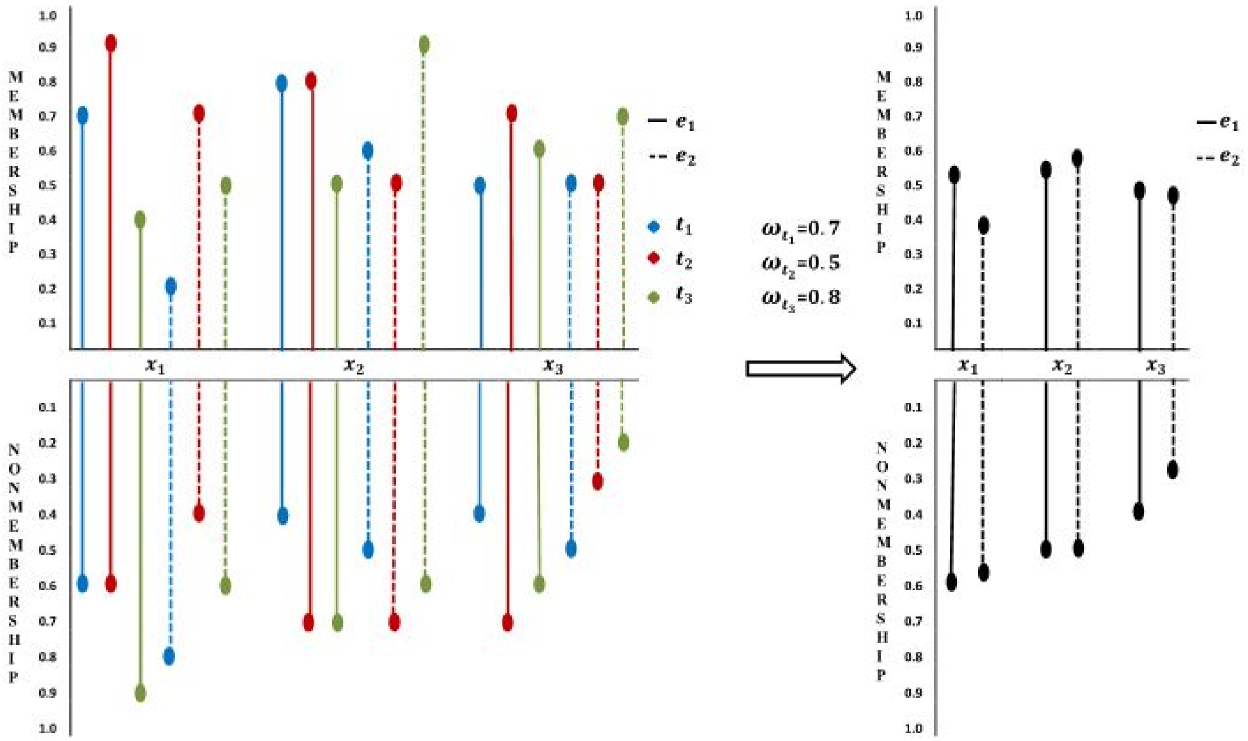


FIGURE 2. The figuration of $gntif$-soft set $\mathfrak{S}_A$ concerning $gtif$-soft set $\mathfrak{S}_A^{(T)}$.

To point out that our calculation in the decision-making process is possible by Scilab codes, we define the generalized temporal (non-temporal) intuitionistic fuzzy soft matrix as in Definition 4.5 (Definition 4.7) as follows. Further, we will not discuss the other properties of generalized temporal (non-temporal) intuitionistic fuzzy soft matrices throughout this paper.

4.5. **Definition.** Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universal set and $E$ be a set of parameters, $A = \{e_1, e_2, \ldots, e_m\} \subseteq E$. Also, let $T = \{t_1, t_2, \ldots, t_k\}$ be a set of time moments, and $\mathfrak{S}_A^{(T)}$ be the $gtif$-soft set over $X$. Then the generalized temporal intuitionistic fuzzy soft matrix ($gtif$-soft matrix) for $\mathfrak{S}_A^{(T)}$ is $[\mathfrak{S}_{ij}^T]_{mk\times 2n}$, where

$$\mathfrak{S}_{ij}^T = \begin{cases} \mu_{\mathfrak{S}^{(T)}(e_l)}(x_p, t_q) & \text{if } j \text{ is odd} \\ \nu_{\mathfrak{S}^{(T)}(e_l)}(x_p, t_q) & \text{if } j \text{ is even} \end{cases} \tag{4. 3}$$

such that $p = \begin{cases} \frac{j+1}{2} & \text{if } j \text{ is odd} \\ \frac{j}{2} & \text{if } j \text{ is even} \end{cases}$ and $q = \begin{cases} i \pmod{k} & \text{if } i \pmod{k} \neq 0 \\ \frac{i}{l} & \text{if } i \pmod{k} = 0 \end{cases}$ where $l$ is the smallest positive integer providing $l \geq \dfrac{i}{k}$.

4.6. **Example.** The generalized temporal intuitionistic fuzzy soft matrix, as $\mathfrak{S}_A^{(T)}$ given in Example 4.2 is

$$[\mathfrak{S}_{ij}^T]_{6\times 6} = \begin{bmatrix} 0.7 & 0.6 & 0.8 & 0.4 & 0.5 & 0.4 \\ 0.9 & 0.6 & 0.8 & 0.7 & 0.7 & 0.7 \\ 0.4 & 0.9 & 0.5 & 0.7 & 0.6 & 0.6 \\ 0.2 & 0.8 & 0.6 & 0.5 & 0.5 & 0.5 \\ 0.7 & 0.4 & 0.5 & 0.7 & 0.5 & 0.3 \\ 0.5 & 0.7 & 0.9 & 0.6 & 0.7 & 0.2 \end{bmatrix}$$

4.7. **Definition.** Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universal set and $E$ be a set of parameters, $A = \{e_1, e_2, \ldots, e_m\} \subseteq E$. Let $\mathfrak{S}_A$ be the generalized non-temporal intuitionistic fuzzy soft set concerning $\mathfrak{S}_A^{(T)}$. Then the generalized non-temporal intuitionistic fuzzy soft matrix ($gntif$-soft matrix) for $\mathfrak{S}_A$ is $[\mathfrak{S}_{ij}]_{m\times 2n}$, where

$$\mathfrak{S}_{ij} = \begin{cases} \mu_{\mathfrak{S}(e_i)}(x_p) \text{ if } j \text{ is odd} \\ \nu_{\mathfrak{S}(e_i)}(x_q) \text{ if } j \text{ is even} \end{cases} \tag{4. 4}$$

such that $p = \frac{j+1}{2}$ and $q = \frac{j}{2}$.

4.8. **Example.** The generalized non-temporal intuitionistic fuzzy soft matrix, as $\mathfrak{S}_A$ given in Example 4.4 is

$$[\mathfrak{S}_{ij}]_{2\times 6} = \begin{bmatrix} 0.3791667 & 0.475 & 0.4 & 0.4125 & 0.3875 & 0.3791667 \\ 0.3291667 & 0.3833333 & 0.4541667 & 0.3875 & 0.3791667 & 0.1708333 \end{bmatrix}$$

**Note:** The $gntif$-soft matrix $[\mathfrak{S}_{ij}]_{m\times 2n}$ can be formed by using $gtif$-soft matrix $[\mathfrak{S}_{ij}^T]_{mk\times 2n}$ corresponding to the $gtif$-soft set $\mathfrak{S}_A^{(T)}$. Each component of $gntif$-soft matrix $[\mathfrak{S}_{ij}]_{m\times 2n}$ is calculated as follows:

$$\mathfrak{S}_{ij} = \frac{\sum\limits_{s=(i-1)k+1}^{ik} w_{t_s} \mathfrak{S}_{sj}^T}{kd} \tag{4. 5}$$

where $d = max\{w_{t_s} : s = 1, 2, ..., k\}$.

4.9. **Definition.** Let $\dot{\mathfrak{S}}_A^{(T)}$ and $\ddot{\mathfrak{S}}_B^{(T)}$ be two $gtif$-soft set over $X$.

- **a):** $\dot{\mathfrak{S}}_A^{(T)}$ is called a $gtif$-soft subset of $\ddot{\mathfrak{S}}_B^{(T)}$, denoted by $\dot{\mathfrak{S}}_A^T \sqsubseteq \ddot{\mathfrak{S}}_B^{(T)}$, if $B \subseteq A$, and $\dot{\mathfrak{S}}^{(T)}(e)$ is $gtif$-subset $\ddot{\mathfrak{S}}^{(T)}(e)$ (i.e., $\dot{\mathfrak{S}}^{(T)}(e) \widetilde{\subseteq} \ddot{\mathfrak{S}}^{(T)}(e)$) for $e \in B$.
- **b):** $\dot{\mathfrak{S}}_A^{(T)}$ and $\ddot{\mathfrak{S}}_B^{(T)}$ are called a $gtif$-soft equal sets, denoted by $\dot{\mathfrak{S}}_A^{(T)} = \ddot{\mathfrak{S}}_B^{(T)}$, if $\dot{\mathfrak{S}}_A^{(T)} \sqsubseteq \ddot{\mathfrak{S}}_B^{(T)}$ and $\ddot{\mathfrak{S}}_B^{(T)} \sqsubseteq \dot{\mathfrak{S}}_A^{(T)}$.

4.10. **Example.** Consider the $gtif$-soft set $\mathfrak{S}_A^{(T)}$ given in Example 4.2. For $B = \{e_2\}$, we define a new $gtif$-soft set over $X$ as follows:

$$\dot{\mathfrak{S}}_B^{(T)} = \{(e_2, \{\frac{x_1^{(t_1)}}{\prec 0.2, 0.8 \succ}, \frac{x_2^{(t_1)}}{\prec 0.4, 0.7 \succ}, \frac{x_3^{(t_1)}}{\prec 0.4, 0.5 \succ}, \frac{x_1^{(t_2)}}{\prec 0.2, 0.7 \succ}, \frac{x_2^{(t_2)}}{\prec 0.5, 0.9 \succ}, \frac{x_3^{(t_2)}}{\prec 0.2, 0.4 \succ}, \frac{x_1^{(t_3)}}{\prec 0.1, 0.8 \succ}, \frac{x_2^{(t_3)}}{\prec 0.4, 0.9 \succ}, \frac{x_3^{(t_3)}}{\prec 0.2, 0.7 \succ}\})\}.$$

Then, we have $\dot{\mathfrak{S}}_B^{(T)} \sqsubseteq \mathfrak{S}_A^{(T)}$.

4.11. **Definition.** The relative complement of $gtif$-soft set $\mathfrak{S}_A^{(T)}$ over $X$ is symbolized by $\overline{\mathfrak{S}_A^{(T)}}$ and is defined by a mapping $\overline{\mathfrak{S}^{(T)}} : A \to \mathcal{GTIFS}(X^{(T)})$ such that $\overline{\mathfrak{S}^{(T)}}(e) = \widetilde{c}(\mathfrak{S}^{(T)}(e))$ for all $e \in A$.

4.12. **Example.** Let us assume $gtif$-soft set $\mathfrak{S}_A^T$ given in Example 4.2. Then, the relative complement of $\mathfrak{S}_A^T$ is $\overline{\mathfrak{S}_A^T} = \{(e_1, \{\frac{x_1^{(t_1)}}{\prec 0.6,0.7\succ}, \frac{x_2^{(t_1)}}{\prec 0.4,0.8\succ}, \frac{x_3^{(t_1)}}{\prec 0.4,0.5\succ}, \frac{x_1^{(t_2)}}{\prec 0.6,0.9\succ}, \frac{x_2^{(t_2)}}{\prec 0.7,0.8\succ}, \frac{x_3^{(t_2)}}{\prec 0.7,0.7\succ}, \frac{x_1^{(t_3)}}{\prec 0.9,0.4\succ}, \frac{x_2^{(t_3)}}{\prec 0.7,0.5\succ}, \frac{x_3^{(t_3)}}{\prec 0.6,0.6\succ}\}),$
$(e_2, \{\frac{x_1^{(t_1)}}{\prec 0.8,0.2\succ}, \frac{x_2^{(t_1)}}{\prec 0.5,0.6\succ}, \frac{x_3^{(t_1)}}{\prec 0.5,0.5\succ}, \frac{x_1^{(t_2)}}{\prec 0.4,0.7\succ}, \frac{x_2^{(t_2)}}{\prec 0.7,0.5\succ}, \frac{x_3^{(t_2)}}{\prec 0.3,0.5\succ}, \frac{x_1^{(t_3)}}{\prec 0.7,0.5\succ}, \frac{x_2^{(t_3)}}{\prec 0.6,0.9\succ}, \frac{x_3^{(t_3)}}{\prec 0.2,0.7\succ}\})\}$

4.13. **Definition.** Let $\dot{\mathfrak{S}}_A^{(T')}$ and $\ddot{\mathfrak{S}}_B^{(T'')}$ be two $gtif$-soft sets on $X$. Then, the extended union of these $gtif$-soft sets, symbolized by $\dddot{\mathfrak{S}}_C^{(T^{\cup})} = \dot{\mathfrak{S}}_A^{(T')} \sqcup \ddot{\mathfrak{S}}_B^{(T'')}$, is defined by

$$\dddot{\mathfrak{S}}^{(T^{\cup})}(e) = \begin{cases} \dot{\mathfrak{S}}^{(T')}(e), & \text{if } e \in A - B \\ \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in B - A \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cup}\, \ddot{\mathfrak{S}}^{(T'')}(e) & \text{if } e \in A \cap B \end{cases}$$

for all $e \in C = A \cup B$.

4.14. **Definition.** Let $\dot{\mathfrak{S}}_A^{(T')}$ and $\ddot{\mathfrak{S}}_B^{(T'')}$ be two $gtif$-soft sets on $X$. Then, the restricted union of these $gtif$-soft sets, symbolized by $\dddot{\mathfrak{S}}_C^{(T^{\cup})} = \dot{\mathfrak{S}}_A^{(T')} \Cup \ddot{\mathfrak{S}}_B^{(T'')}$, is defined by

$$\dddot{\mathfrak{S}}^{(T^{\cup})}(e) = \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cup}\, \ddot{\mathfrak{S}}^{(T'')}(e)$$

for all $e \in C = A \cap B \neq \emptyset$.

4.15. **Definition.** Let $\dot{\mathfrak{S}}_A^{(T')}$ and $\ddot{\mathfrak{S}}_B^{(T'')}$ be two $gtif$-soft sets on $X$. Then, the extended intersection of these $gtif$-soft sets, symbolized by $\dddot{\mathfrak{S}}_C^{(T^{\cup})} = \dot{\mathfrak{S}}_A^{(T')} \sqcap \ddot{\mathfrak{S}}_B^{(T'')}$, is defined by

$$\dddot{\mathfrak{S}}^{(T^{\cup})}(e) = \begin{cases} \dot{\mathfrak{S}}^{(T')}(e), & \text{if } e \in A - B \\ \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in B - A \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \ddot{\mathfrak{S}}^{(T'')}(e) & \text{if } e \in A \cap B \end{cases}$$

for all $e \in C = A \cup B$.

4.16. **Definition.** Let $\dot{\mathfrak{S}}_A^{(T')}$ and $\ddot{\mathfrak{S}}_B^{(T'')}$ be two $gtif$-soft sets on $X$. Then, the restricted intersection of these $gtif$-soft sets, symbolized by $\dddot{\mathfrak{S}}_C^{(T^{\cup})} = \dot{\mathfrak{S}}_A^{(T')} \Cap \ddot{\mathfrak{S}}_B^{(T'')}$, is defined by

$$\dddot{\mathfrak{S}}^{(T^{\cup})}(e) = \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \ddot{\mathfrak{S}}^{(T'')}(e)$$

for all $e \in C = A \cap B \neq \emptyset$.

4.17. **Example.** Imagine a bustling financial district where investment firms are meticulously scrutinizing potential ventures across different sectors. Let $X = \{x_1, x_2\}$ represent the array of investment opportunities, while $E = \{e_1, e_2, e_3\}$ symbolizes the parameters under evaluation, such as environmental and political risks. Now, picture this scenario unfolding over a dynamic three-month period, encompassing April, May, and June, denoted by $T' = \{t_1, t_2, t_3\}$. Firstly, let's examine the findings for $A = \{e_1\}$, where the investment firm's analysis yields the following insights: $\dot{\mathfrak{S}}_A^{(T')} = \{(e_1, \{\frac{x_1^{(t_1)}}{\prec 0.5,0.4\succ}, \frac{x_2^{(t_1)}}{\prec 0.6,0.5\succ}, \frac{x_1^{(t_2)}}{\prec 0.8,0.3\succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.3\succ}, \frac{x_1^{(t_3)}}{\prec 0.2,0.4\succ}, \frac{x_2^{(t_3)}}{\prec 0.7,0.5\succ}\})\}$

$\dot{\mathfrak{S}}_A^{(T')}$ reveals the nuanced evaluations of finance companies across the months, capturing the fluctuating landscape of opportunities and risks. Moving on to a broader assessment with $B = \{e_1, e_2\}$ and $T'' = \{t_1, t_2\}$, we uncover deeper layers of analysis:

$\ddot{\mathfrak{S}}_B^{(T'')} = \{(e_1, \{\frac{x_1^{(t_1)}}{\prec 0.3,0.7\succ}, \frac{x_2^{(t_1)}}{\prec 0.4,0.3\succ}, \frac{x_1^{(t_2)}}{\prec 0.7,0.4\succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.3\succ}\}), (e_2, \{\frac{x_1^{(t_1)}}{\prec 0.2,0.9\succ}, \frac{x_2^{(t_1)}}{\prec 0.5,0.5\succ}, \frac{x_1^{(t_2)}}{\prec 0.3,0.4\succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.7\succ}\})\}$

$\ddot{\mathfrak{S}}_B^{(T'')}$ unveils a richer tapestry of data, considering not just environmental, but also political risks across the selected months. Bringing these perspectives together, we embark on an exploration of combined strategies: The amalgamation of $\dot{\mathfrak{S}}_A^{(T')} \sqcup \ddot{\mathfrak{S}}_B^{(T'')}$ reflects a comprehensive view of investment options, integrating insights from both environmental and political risk factors. Furthermore, the focused examination through $\dot{\mathfrak{S}}_A^{(T')} \Cap \ddot{\mathfrak{S}}_B^{(T'')}$ offers a distilled perspective, highlighting areas of alignment between the two sets of evaluations.

$\dot{\mathfrak{S}}_A^{(T')} \sqcup \ddot{\mathfrak{S}}_B^{(T'')} =$
$\{(e_1, \{\frac{x_1^{(t_1)}}{\prec 0.5,0.4 \succ}, \frac{x_2^{(t_1)}}{\prec 0.6,0.3 \succ}, \frac{x_1^{(t_2)}}{\prec 0.8,0.3 \succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.3 \succ}, \frac{x_1^{(t_3)}}{\prec 0.2,0.4 \succ}, \frac{x_2^{(t_3)}}{\prec 0.7,0.5 \succ}\}), (e_2, \{\frac{x_1^{(t_1)}}{\prec 0.2,0.9 \succ}, \frac{x_2^{(t_1)}}{\prec 0.5,0.5 \succ}, \frac{x_1^{(t_2)}}{\prec 0.3,0.4 \succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.7 \succ}\})\}$
$\dot{\mathfrak{S}}_A^{(T')} \Cap \ddot{\mathfrak{S}}_B^{(T'')} = \{(e_1, \{\frac{x_1^{(t_1)}}{\prec 0.5,0.7 \succ}, \frac{x_2^{(t_1)}}{\prec 0.4,0.5 \succ}, \frac{x_1^{(t_2)}}{\prec 0.7,0.4 \succ}, \frac{x_2^{(t_2)}}{\prec 0.5,0.3 \succ}, \frac{x_1^{(t_3)}}{\prec 0.2,0.4 \succ}, \frac{x_2^{(t_3)}}{\prec 0.7,0.5 \succ}\})\}$

In this dynamic financial landscape, where decisions are as fluid as market conditions, the fusion of temporal evaluations offers a robust foundation for strategic investment decisions.

**Proposition 4.18.** *Let $\dot{\mathfrak{S}}_A^{(T')}$, $\ddot{\mathfrak{S}}_B^{(T'')}$ and $\dddot{\mathfrak{S}}_C^{(T''')}$ be three $gtif$-soft sets on $X$. For each $\diamond \in \{\sqcap, \sqcup, \Cap, \Cup\}$, we have the following properties.*

**i):** $\dot{\mathfrak{S}}_A^{(T')} \diamond \dot{\mathfrak{S}}_A^{(T')} = \dot{\mathfrak{S}}_A^{(T')}$.
**ii):** $\dot{\mathfrak{S}}_A^{(T')} \diamond \ddot{\mathfrak{S}}_B^{(T'')} = \ddot{\mathfrak{S}}_B^{(T'')} \diamond \dot{\mathfrak{S}}_A^{(T')}$.
**iii):** $(\dot{\mathfrak{S}}_A^{(T')} \diamond \ddot{\mathfrak{S}}_B^{(T'')}) \diamond \dddot{\mathfrak{S}}_C^{(T''')} = \dot{\mathfrak{S}}_A^{(T')} \diamond (\ddot{\mathfrak{S}}_B^{(T'')} \diamond \dddot{\mathfrak{S}}_C^{(T''')})$.

*Proof.* The proofs of (i) and (ii) are straightforward, so they are passed.
Let's prove the assertion (iii) for the operation of the extended intersection.
We consider the left side of equality, i.e., $(\dot{\mathfrak{S}}_A^{(T')} \sqcap \ddot{\mathfrak{S}}_B^{(T'')}) \sqcap \dddot{\mathfrak{S}}_C^{(T''')}$.
Assume that $(\dot{\mathfrak{S}}_A^{(T')} \sqcap \ddot{\mathfrak{S}}_B^{(T'')}) = \mathfrak{S}_D^{(T^*)}$, where $D = A \cup B$ and for all $e \in D$

$$\mathfrak{S}^{(T^*)}(e) = \begin{cases} \dot{\mathfrak{S}}^{(T')}(e), & \text{if } e \in A - B \\ \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in B - A \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in A \cap B \end{cases}$$

And assume that $\mathfrak{S}_D^{(T^*)} \sqcap \dddot{\mathfrak{S}}_C^{(T''')} = \mathfrak{S}_F^{(T^{**})}$, where $F = D \cup C$ and for all $e \in F$

$$\begin{aligned} \mathfrak{S}^{(T^{**})}(e) &= \begin{cases} \mathfrak{S}^{(T^*)}(e), & \text{if } e \in D - C \\ \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in C - D \\ \mathfrak{S}^{(T^*)}(e) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in D \cap C \end{cases} \\ &= \begin{cases} \dot{\mathfrak{S}}^{(T')}(e), & \text{if } e \in (A - B) - C \\ \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in (B - A) - C \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \ddot{\mathfrak{S}}^{(T'')}, & \text{if } e \in (A \cap B) - C \\ \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } \varepsilon_j \in C - (A \cup B) \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in (A - B) \cap C \\ \ddot{\mathfrak{S}}^{(T'')}(e) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in (B - A) \cap C \\ (\dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \ddot{\mathfrak{S}}^{(T'')}(e)) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in A \cap B \cap C \end{cases} \end{aligned} \qquad (4.\ 6)$$

consider the other side of equality, i.e., $\dot{\mathfrak{S}}_A^{(T')} \diamond (\ddot{\mathfrak{S}}_B^{(T'')} \diamond \dddot{\mathfrak{S}}_C^{(T''')})$.
Suppose that $\ddot{\mathfrak{S}}_B^{(T'')} \diamond \dddot{\mathfrak{S}}_C^{(T''')} = \mathfrak{S}_H^{(T^{\star})}$, where $H = B \cup C$ and for all $e \in H$

$$\mathfrak{S}^{(T^\star)}(e) = \begin{cases} \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in B - C \\ \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in C - B \\ \ddot{\mathfrak{S}}^{(T'')}(e) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in B \cap C \end{cases}$$

And suppose that $\dot{\mathfrak{S}}_A^{(T')} \sqcap \mathfrak{S}_H^{(T^\star)} = \mathfrak{S}_I^{(T^{\star\star})}$, where $I = A \cup H$ and for all $e \in I$

$$\begin{aligned} \mathfrak{S}^{(T^{\star\star})}(e) &= \begin{cases} \dot{\mathfrak{S}}^{(T')}(e), & \text{if } e \in A - H \\ \mathfrak{S}^{(T^\star)}(e), & \text{if } e \in H - A \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \mathfrak{S}^{(T^\star)}(e), & \text{if } e \in A \cap H \end{cases} \\ &= \begin{cases} \dot{\mathfrak{S}}^{(T')}(e), & \text{if } e \in A - (B \cup C) \\ \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in (B - C) - A \\ \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in (C - B) - A \\ \ddot{\mathfrak{S}}^{(T'')}(e) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in (B \cap C) - A \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \ddot{\mathfrak{S}}^{(T'')}(e), & \text{if } e \in A \cap (B - C) \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e), & \text{if } e \in A \cap (C - B) \\ \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cap}\, (\ddot{\mathfrak{S}}^{(T'')}(e) \,\widetilde{\cap}\, \dddot{\mathfrak{S}}^{(T''')}(e)), & \text{if } e \in A \cap B \cap C \end{cases} \end{aligned} \tag{4. 7}$$

Considering Proposition 3.8 (iii), we have $(\dot{\mathfrak{S}}_A^{(T')} \sqcap \ddot{\mathfrak{S}}_B^{(T'')}) \sqcap \dddot{\mathfrak{S}}_C^{(T''')} = \dot{\mathfrak{S}}_A^{(T')} \sqcap (\ddot{\mathfrak{S}}_B^{(T'')} \sqcap \dddot{\mathfrak{S}}_C^{(T''')})$ by ( 4. 6 ) and ( 4. 7 ).
For the operations of extended union, restricted intersection, and restricted union can be proved similarly. □

**Proposition 4.19.** *Let $\dot{\mathfrak{S}}_A^{(T')}$ and $\ddot{\mathfrak{S}}_B^{(T'')}$ be two $gtif$-soft sets on $X$.*

**i):** $\overline{(\dot{\mathfrak{S}}_A^{(T')} \Cup \ddot{\mathfrak{S}}_B^{(T'')})} = \overline{\dot{\mathfrak{S}}_A^{(T')}} \Cap \overline{\ddot{\mathfrak{S}}_B^{(T'')}}$ *and* $\overline{(\dot{\mathfrak{S}}_A^{(T')} \Cap \ddot{\mathfrak{S}}_B^{(T'')})} = \overline{\dot{\mathfrak{S}}_A^{(T')}} \Cup \overline{\ddot{\mathfrak{S}}_B^{(T'')}}$.

**ii):** $\overline{(\dot{\mathfrak{S}}_A^{(T')} \sqcup \ddot{\mathfrak{S}}_B^{(T'')})} = \overline{\dot{\mathfrak{S}}_A^{(T')}} \sqcap \overline{\ddot{\mathfrak{S}}_B^{(T'')}}$ *and* $\overline{(\dot{\mathfrak{S}}_A^{(T')} \sqcap \ddot{\mathfrak{S}}_B^{(T'')})} = \overline{\dot{\mathfrak{S}}_A^{(T')}} \sqcup \overline{\ddot{\mathfrak{S}}_B^{(T'')}}$.

*Proof.* (i) Let, $\dot{\mathfrak{S}}_A^{(T')} \Cup \ddot{\mathfrak{S}}_B^{(T'')} = \dddot{\mathfrak{S}}_C^{(T''')}$ where $T''' = T' \cup T''$ and $\dddot{\mathfrak{S}}^{(T''')}(e) = \dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cup}\, \ddot{\mathfrak{S}}^{(T'')}(e)$ for all $e \in C = A \cap B \neq \emptyset$. By Definition 4.11, we write that $\overline{\dot{\mathfrak{S}}_A^{(T')} \Cup \ddot{\mathfrak{S}}_B^{(T'')}} = \overline{\dddot{\mathfrak{S}}_C^{(T''')}}$ where

$$\widetilde{c}(\dddot{\mathfrak{S}}^{(T''')}(e)) = \widetilde{c}(\dot{\mathfrak{S}}^{(T')}(e) \,\widetilde{\cup}\, \ddot{\mathfrak{S}}^{(T'')}(e)) = \widetilde{c}(\dot{\mathfrak{S}}^{(T')}(e)) \,\widetilde{\cap}\, \widetilde{c}(\ddot{\mathfrak{S}}^{(T'')}(e)) \tag{4. 8}$$

for all $e \in C$.
On the other hand, from Definition 4.16, we can write $\overline{\dot{\mathfrak{S}}_A^{(T')}} \Cap \overline{\ddot{\mathfrak{S}}_B^{(T'')}} = \ddddot{\mathfrak{S}}_D^{(T'''')}$ where $T'''' = T' \cup T''$ and

$$\ddddot{\mathfrak{S}}^{(T'''')}(e) = \widetilde{c}(\dot{\mathfrak{S}}^{(T')}(e)) \,\widetilde{\cap}\, \widetilde{c}(\ddot{\mathfrak{S}}^{(T'')}(e)) \tag{4. 9}$$

for all $e \in D = A \cap B \neq \emptyset$.
Thus, we have $\overline{(\dot{\mathfrak{S}}_A^{(T')} \Cup \ddot{\mathfrak{S}}_B^{(T'')})} = \overline{\dot{\mathfrak{S}}_A^{(T')}} \Cap \overline{\ddot{\mathfrak{S}}_B^{(T'')}}$ by ( 4. 8 ) and ( 4. 9 ).
The remainder of (i) can be proved similarly.
(ii) It is similar to the proof of (i). □

## 5. Novel Decision-Making Approaches

This section proposes two algorithms for single and multiple decision-makers within the generalized temporal intuitionistic fuzzy soft set environment.

**Algorithm 1: Generalized Temporal Intuitionistic Fuzzy Soft Set-Based Decision-Making.**

***Purpose***. To rank alternatives based on their performance over time, considering both membership and non-membership degrees within a generalized temporal intuitionistic fuzzy soft set framework. The flow chart of Algorithm 1 is depicted in Figure 3.

***Input***.

- **Universal Set**: $X = \{x_1, x_2, \ldots, x_n\}$
- **Parameter Set**: $A = \{e_1, e_2, \ldots, e_m\} \subseteq E$
- **Time Moments Set**: $T = \{t_1, t_2, \ldots, t_k\}$
- **Weights of Time Moments**: $w_{t_1}, w_{t_2}, \ldots, w_{t_k}$

**Steps.**

**Step** 1. **Generation of Generalized Temporal Intuitionistic Fuzzy Soft Set**:
- Generate the Generalized Temporal Intuitionistic Fuzzy Soft Set $\mathfrak{S}_A^{(T)}$ over the universal set $X$.

**Step** 2. **Construction of Generalized Non-Temporal Intuitionistic Fuzzy Soft Matrix**:
- Construct the Generalized Non-Temporal Intuitionistic Fuzzy Soft Matrix $[\mathfrak{S}_{ij}]_{m\times 2n}$ for the Generalized Temporal Intuitionistic Fuzzy Soft Set $\mathfrak{S}_A^{(T)}$ created in Step 1 by using Equation ( 4. 5 ).

**Step** 3. **Membership Comparison Matrix**:
- Calculate the membership comparison matrix $[\mathfrak{S}_{ij}^{\mu}]_{n\times n}$:
  - **For** $i \neq j$: $\mathfrak{S}_{ij}^{\mu}$ is the count of rows in $[\mathfrak{S}_{ij}]_{m\times 2n}$ where the value in the $(2i-1)^{\text{th}}$ column exceeds that in the $(2j-1)^{\text{th}}$ column.
  - **For** $i = j$: $\mathfrak{S}_{ij}^{\mu} = 0$.

**Step** 4. **Non-Membership Comparison Matrix**:
- Compute the non-membership comparison matrix $[\mathfrak{S}_{ij}^{\nu}]_{n\times n}$:
  - **For** $i \neq j$: $\mathfrak{S}_{ij}^{\nu}$ is the count of rows in $[\mathfrak{S}_{ij}]_{m\times 2n}$ where the value in the $(2i)^{\text{th}}$ column is less than that in the $(2j)^{\text{th}}$ column.
  - **For** $i = j$: $\mathfrak{S}_{ij}^{\nu} = 0$.

**Step** 5. **Row and Column Sum for Membership**:
- Compute the row sums $r_i^{\mu}$ and column sums $c_j^{\mu}$ of the membership comparison matrix:
  - $r_i^{\mu} = \sum_{j=1}^{n} \mathfrak{S}_{ij}^{\mu}$ for each $i = 1, 2, \ldots, n$.
  - $c_j^{\mu} = \sum_{i=1}^{n} \mathfrak{S}_{ij}^{\mu}$ for each $j = 1, 2, \ldots, n$.

**Step** 6. **Row and Column Sum for Non-Membership**:
- Calculate the row sums $r_i^{\nu}$ and column sums $c_j^{\nu}$ of the non-membership comparison matrix:
  - $r_i^{\nu} = \sum_{j=1}^{n} \mathfrak{S}_{ij}^{\nu}$ for each $i = 1, 2, \ldots, n$.
  - $c_j^{\nu} = \sum_{i=1}^{n} \mathfrak{S}_{ij}^{\nu}$ for each $j = 1, 2, \ldots, n$.

**Step** 7. **Determine the Final Information Scores and Rank of Alternatives**:
- Compute the final information scores using $\Delta_k = r_k^{\mu} - c_k^{\mu} + r_k^{\nu} - c_k^{\nu}$.

**Step** 8. **Obtain the Rank of Alternatives**:
- Rank the alternatives within the universe set $X$ based on the computed values of $\Delta_k$.

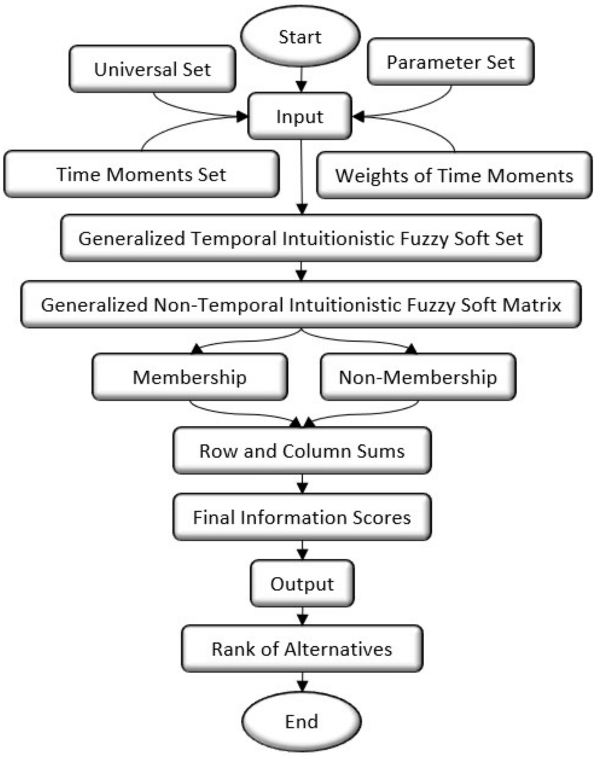


FIGURE 3. Flow Chart of Decision-Making Approaches: Algorithm 1

Let us give a fictitious numerical example to illustrate the calculations in the steps of the algorithm in detail.

5.1. **Example.** Imagine you're in charge of selecting the best alternatives for a project. You have four options, labeled $x_1$, $x_2$, $x_3$, and $x_4$. Each option has its own set of parameters, denoted by $e_1$, $e_2$, $e_3$, and $e_4$. However, you're only interested in evaluating parameters $e_1$, $e_2$, and $e_4$ for your decision-making process. Now, let's add another layer of complexity: time. You'll analyze your options over two specific periods, the first and second quarters of the year, labeled as $t_1$ and $t_2$. These periods have different weights based on their significance: $t_1$ holds a weight of 0.7, indicating higher importance than $t_2$, which carries a weight of 0.5. To find the ranking of alternatives based on their overall performance, we can utilize Algorithm 1 as follows.

**Step 1.** Let us generate the generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_A^{(T)}$ over $X$ as follows:

$$\mathfrak{S}_A^{(T)} =$$
$$\{(e_1, \{\frac{x_1^{(t_1)}}{\prec 0.42, 0.75 \succ}, \frac{x_2^{(t_1)}}{\prec 0.34, 0.54 \succ}, \frac{x_3^{(t_1)}}{\prec 0.72, 0.41 \succ}, \frac{x_4^{(t_1)}}{\prec 0.67, 0.76 \succ}, \frac{x_1^{(t_2)}}{\prec 0.63, 0.63 \succ}, \frac{x_2^{(t_2)}}{\prec 0.51, 0.87 \succ}, \frac{x_3^{(t_2)}}{\prec 0.78, 0.34 \succ}, \frac{x_4^{(t_2)}}{\prec 0.64, 0.55 \succ}\}),$$
$$(e_2, \{\frac{x_1^{(t_1)}}{\prec 0.51, 0.87 \succ}, \frac{x_2^{(t_1)}}{\prec 0.63, 0.63 \succ}, \frac{x_3^{(t_1)}}{\prec 0.61, 0.47 \succ}, \frac{x_4^{(t_1)}}{\prec 0.55, 0.55 \succ}, \frac{x_1^{(t_2)}}{\prec 0.34, 0.54 \succ}, \frac{x_2^{(t_2)}}{\prec 0.42, 0.75 \succ}, \frac{x_3^{(t_2)}}{\prec 0.72, 0.71 \succ}, \frac{x_4^{(t_2)}}{\prec 0.64, 0.63 \succ}\}),$$
$$(e_3, \{\frac{x_1^{(t_1)}}{\prec 0.62, 0.59 \succ}, \frac{x_2^{(t_1)}}{\prec 0.44, 0.93 \succ}, \frac{x_3^{(t_1)}}{\prec 0.65, 0.45 \succ}, \frac{x_4^{(t_1)}}{\prec 0.76, 0.52 \succ}, \frac{x_1^{(t_2)}}{\prec 0.44, 0.93 \succ}, \frac{x_2^{(t_2)}}{\prec 0.62, 0.59 \succ}, \frac{x_3^{(t_2)}}{\prec 0.74, 0.74 \succ}, \frac{x_4^{(t_2)}}{\prec 0.77, 0.81 \succ}\})\}.$$

**Step 2.** By using Equation ( 4. 5 ), the generalized non-temporal intuitionistic fuzzy soft matrix $[\mathfrak{S}_{ij}]_{3\times 8}$ is obtained as

$$[\mathfrak{S}_{ij}]_{3\times 8} = \begin{bmatrix} 0.435 & 0.6 & 0.3521 & 0.5807 & 0.6385 & 0.3264 & 0.5635 & 0.5764 \\ 0.3764 & 0.6278 & 0.465 & 0.5828 & 0.5621 & 0.4885 & 0.5035 & 0.5 \\ 0.4671 & 0.6271 & 0.4414 & 0.6757 & 0.5892 & 0.4892 & 0.655 & 0.5492 \end{bmatrix}$$

**Step 3-4.** For the generalized non-temporal intuitionistic fuzzy soft matrix $[\mathfrak{S}_{ij}]_{3\times 8}$, the membership comparison matrix $[\mathfrak{S}^{\mu}_{ij}]_{4\times 4}$ and the non-membership comparison matrix $[\mathfrak{S}^{\nu}_{ij}]_{4\times 4}$ are given as follows:

$$[\mathfrak{S}^{\mu}_{ij}]_{4\times 4} = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 3 & 3 & 0 & 2 \\ 3 & 3 & 1 & 0 \end{bmatrix} \text{ and } [\mathfrak{S}^{\nu}_{ij}]_{4\times 4} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 3 & 0 & 3 \\ 3 & 3 & 0 & 0 \end{bmatrix}.$$

**Step 5-8.** The row and column sums for the membership and non-membership comparison matrices, along with the final information scores $\Delta_k$ for $k = 1, 2, 3, 4$, are obtained as shown in Table 1.

TABLE 1. The table of $\Delta_k$

| $k$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $r^{\mu}_k$ | 2 | 1 | 8 | 7 |
| $c^{\mu}_k$ | 7 | 8 | 1 | 2 |
| $r^{\nu}_k$ | 1 | 2 | 9 | 6 |
| $c^{\nu}_k$ | 8 | 7 | 0 | 3 |
| $\Delta_k$ | -12 | -12 | 16 | 8 |

Based on the values of $\Delta_k$ given in Table 1, the ranking of the preference order of alternatives is obtained as $x_3 \succ x_4 \succ x_1 = x_2$.
This algorithm can be adapted to solve many real-world problems in various fields involving uncertainty, and ambiguous and unknown data. Many decision-making problems in real life consist of multiple alternatives, parameters, or time moments. In handling such decision-making problems, manual calculations are almost impossible, therefore the Scilab codes given in the Appendix can be used.

5.2. **Example.** Let's consider an individual intrigued by the prospect of investing in foreign currencies, denoted by $X = \{x_1 = \text{Turkish lira},\ x_2 = \text{euro},\ x_3 = \text{Chinese Yuan},\ x_4 = \text{dollar},\ x_5 = \text{Japanese Yen},\ x_6 = \text{sterlin},\ x_7 = \text{Hong Kong Dollar},\ x_8 = \text{Swiss Frank}\}$. This individual has meticulously studied these foreign currencies over twelve months, represented by $T = \{t_1, ..., t_{12}\}$. The objective is to gather insights into the optimal currency for investment. Additionally, an assessment of various risks, including environmental risks, political instability, financial fluctuations, and exchange rate volatility, denoted by $E = \{e_1 = \text{environmental risks}, e_2 = \text{political risks}, e_3 = \text{financial fluctuations}, e_4 = \text{exchange rate fluctuations}\}$ has been conducted during this period. Consequently, a generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}^{(T)}_E$ (referenced in Table 4, Appendix) has been formulated over $X$.
Suppose that weights of time moments $t_1, t_2 \ldots, t_{12}$ are $w_{t_1} = 0.1, w_{t_2} = 0.3, w_{t_3} = 0.4, w_{t_4} = 0.6, w_{t_5} = 0.1, w_{t_6} = 0.8, w_{t_7} = 0.7, w_{t_8} = 0.4, w_{t_9} = 0.2, w_{t_{10}} = 0.4, w_{t_{11}} = 0.8$, and $w_{t_{12}} = 0.3$ respectively, then we obtain the final information scores $\Delta_k$ for $k = 1, 2, \ldots, 8$ as Table 2 by Algorithm 1.

TABLE 2. The table of $\Delta_k$

| $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $r_k^\mu$ | 16 | 3 | 16 | 14 | 5 | 25 | 6 | 27 |
| $c_k^\mu$ | 12 | 25 | 12 | 14 | 23 | 3 | 22 | 1 |
| $r_k^\nu$ | 1 | 23 | 12 | 16 | 7 | 18 | 12 | 23 |
| $c_k^\nu$ | 27 | 5 | 16 | 12 | 21 | 10 | 16 | 5 |
| $\Delta_k$ | -22 | -4 | 0 | 4 | -32 | 30 | -20 | 44 |

The ranking of foreign currencies based on the values of $\Delta_k$ mentioned in Table 2 is $x_8 \succ x_6 \succ x_4 \succ x_3 \succ x_2 \succ x_7 \succ x_1 \succ x_5$. Therefore, the suggested investment is in the currency of Swiss Franc. Note that Example 5.2 can be solved using the Scilab code provided in Table 8 (Appendix).

**Algorithm 2:**

Let $X = \{x_1, x_2, \dots x_p\}$ be a universal set, $A_1, A_1, ..., A_n \subseteq E$. Also, let $T1, T2, \dots Tn$ be a different set of time moments considered by $n \geq 2$ several decision-makers. Let $T = \bigcup_{q=1}^{n} Tq = \{t_1, t_2, \dots, t_k\}$ be the set of time moments and $w_{t_1}, w_{t_2}, \dots, w_{t_k}$ be the weights of time moments, $t_1, t_2, \dots, t_k$ respectively.

Step 1. First step: Find the generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_{A_1}^{(T1)}, \mathfrak{S}_{A_2}^{(T2)}, \dots \mathfrak{S}_{A_n}^{(Tn)}$ over $X$ concerning $n$ decision makers.

Step 2. Second step: Find the product $\mathfrak{S}_C^{(T)} = \mathfrak{S}_{A_1}^{(T1)} * \mathfrak{S}_{A_2}^{(T2)} * \dots * \mathfrak{S}_{A_n}^{(Tn)}$, where $'*'$ may be any one of the extended union, restricted union, extended intersection, or restricted intersection based on the real situation of the problem.

Step 3. Third step: Repeat Step 2 to Step 8 of Algorithm 1 to find the best alternatives.

5.3. **Example.** In a bustling scene unfolding in India, picture a non-public healthcare organization seizing the reins to tackle the pressing challenge of environmental pollution in North India. With pollution levels soaring, especially in bustling urban hubs, the imperative to assess and rank the most polluted cities looms large. This mission isn't just about numbers; it's about steering public health interventions, directing resources, and crafting policies aimed at curbing the harmful impact of pollution on local communities. By pinpointing and prioritizing these pollution hotspots, the organization ignites targeted interventions, ignites awareness campaigns, and champions sustainable environmental practices. In essence, this endeavor marks a pivotal stride toward preserving public health and nurturing a greener, healthier tomorrow for generations to come.

A group of cities, denoted by $X = \{x_1 = \text{Kanpur}, x_2 = \text{Faridabad}, x_3 = \text{Varanasi}, x_4 = \text{Gaya}, x_5 = \text{Patna}, x_6 = \text{Delhi}, x_7 = \text{Lucknow}, x_8 = \text{Agra}\}$, is selected for the experiment. We consider a set of criteria contributing to pollution, denoted as $E = \{$ $e_1$=Vehicle Exhaust Fumes, $e_2$=Fossil Fuel-Based Power Plants, $e_3$=Exhaust from Industrial Plants and Factories, $e_4$=Construction and Agricultural Activities, $e_5$=Natural Causes, $e_6$=Household Activities$\}$ for evaluation. Three decision-makers have been strategically appointed to meticulously evaluate the criteria across distinct sets of time moments. These time moments are delineated as follows:

$T1 = \{t_1^1 = \text{January}, t_2^1 = \text{March}, t_3^1 = \text{June}, t_4^1 = \text{August}, t_5^1 = \text{September}, t_6^1 = \text{November}, t_7^1 = \text{December}\}$

$T2 = \{t_1^2 = \text{January}, t_2^2 = \text{February}, t_3^2 = \text{March}, t_4^2 = \text{June}, t_5^2 = \text{August}, t_6^2 = \text{September}, t_7^2 = \text{October}, t_8^2 = \text{December}\}$

$T3 = \{t_1^3 = \text{January}, t_2^3 = \text{March}, t_3^3 = \text{May}, t_4^3 = \text{June}, t_5^3 = \text{September}, t_6^3 = \text{November}, t_7^3 = \text{December}\}$

The decision-makers furnish their findings through a set of comprehensive frameworks known as generalized temporal intuitionistic fuzzy soft sets $\mathfrak{S}_{A_1}^{(T1)}$, $\mathfrak{S}_{A_2}^{(T2)}$, and $\mathfrak{S}_{A_3}^{(T3)}$. These sets, detailed in Tables 5, 6, and 7 provided in the

Appendix, encapsulate the intricate dynamics of the evaluation criteria across the set $X$. Notably, the criteria $A_1$, $A_2$, and $A_3$ remain consistent, aligning with the overarching set $E$. This holistic approach ensures a thorough analysis of pollution trends, facilitating informed decision-making to address environmental challenges with precision.

To pinpoint the city most profoundly affected by pollution within the framework of generalized temporal intuitionistic fuzzy soft sets $\mathfrak{S}_{A_1}^{(T1)}$, $\mathfrak{S}_{A_2}^{(T2)}$, and $\mathfrak{S}_{A_3}^{(T3)}$ over the set $X$, a pivotal adjustment is made in the Algorithm 2. Here, the traditional product operator $*$ is replaced by the extended intersection operator $\sqcap$, aligning with the real-world intricacies of the problem at hand. This adaptation acknowledges the multi-faceted nature of pollution dynamics over time, ensuring a more accurate representation of its impact across different temporal phases.

Notably, the time set $T$ encompasses the union of $T1$, $T2$, and $T3$, consisting of distinct time moments ranging from January to December. It is clear that $T = T1 \cup T2 \cup T3 = \{$ $t_1$=January, $t_2$=February, $t_3$=March, $t_4$=May, $t_5$=June, $t_6$=August, $t_7$=September, $t_8$=October, $t_9$=November, $t_{10}$=December$\}$. Assuming weighted time moments $t_1, t_2, \ldots, t_{10}$ with corresponding weights $w_{t_1} = 0.6, w_{t_2} = 0.8, w_{t_3} = 0.5, w_{t_4} = 0.6, w_{t_5} = 0.7, w_{t_6} = 0.9, w_{t_7} = 0.7, w_{t_8} = 0.3, w_{t_9} = 0.4$, and $w_{t_{10}} = 0.9$, the culmination of these factors yields the final information scores $\Delta_k$ for $k = 1, 2, \ldots, 8$ as depicted in Table 3 through the application of Algorithm 2. This meticulous approach offers deeper insights into the severity of pollution across different cities, empowering stakeholders with actionable intelligence to drive targeted interventions and policy decisions.

TABLE 3. The table of $\Delta_k$

| $k$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $r_k^{\mu}$ | 0 | 6 | 13 | 25 | 13 | 31 | 37 | 37 |
| $c_k^{\mu}$ | 42 | 36 | 23 | 17 | 23 | 11 | 5 | 5 |
| $r_k^{\nu}$ | 0 | 32 | 19 | 42 | 13 | 26 | 7 | 29 |
| $c_k^{\nu}$ | 42 | 10 | 23 | 0 | 29 | 16 | 35 | 13 |
| $\Delta_k$ | -84 | -8 | -14 | 50 | -26 | 30 | 4 | 48 |

We get the rank of polluted cities as $x_4 \succ x_8 \succ x_6 \succ x_7 \succ x_2 \succ x_3 \succ x_5 \succ x_1$ by the values of $\Delta_k$ mentioned in Table 3. Therefore, we identify Gaya as the most polluted city in North India for the concerned period of evaluation.

## 6. CONCLUSION

In this paper, we introduced the concepts of generalized temporal intuitionistic fuzzy sets and generalized temporal intuitionistic fuzzy soft sets, exploring their foundational principles and tailored classical set operations. We developed novel algorithms to model time-related uncertainties in decision-making, accommodating both single decision-makers and multi-observer inputs.

Our contributions include expanding the theory of intuitionistic fuzzy sets by incorporating temporal dynamics, resulting in generalized temporal intuitionistic fuzzy sets. This novel framework, integrated with soft set theory, offers a fresh perspective on uncertainty modeling. We introduced two multi-criteria decision-making methods specifically designed for generalized temporal intuitionistic fuzzy soft sets, demonstrating their practical applicability. By providing Scilab codes and applying our methodologies to complex real-life examples, we showcased their versatility and effectiveness.

However, our study has limitations. The computational complexity of the proposed algorithms may pose challenges for large-scale problems. Additionally, the reliance on expert judgment for determining membership and non-membership values may introduce subjectivity.

Future research should focus on developing optimization algorithms to reduce the computational burden, investigating automated techniques for membership and non-membership value assignment to minimize subjectivity, and expanding the application of generalized temporal intuitionistic fuzzy soft sets to diverse real-world problems, including those in economics, healthcare, and environmental science.

In conclusion, our study provides a comprehensive framework for integrating temporal dynamics into intuitionistic fuzzy sets and soft sets, offering novel tools for decision-making in uncertain and dynamic environments. By addressing limitations and pursuing future research directions, we aim to advance this field and contribute to the development of more robust and flexible decision-making methodologies.

**Author Contributions:** Hüseyin Kamacı contributed to the conceptualization and methodology of the study. Subramanian Petchimuthu was responsible for data analysis and interpretation. Fathima Banu M participated in the investigation and literature review. Şerif Özlü provided supervision, validation, and overall project administration. All authors contributed to the writing, reviewing, and editing of the manuscript and approved the final version for publication.

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## APPENDIX

TABLE 4. The table of generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_E^{(T)}$

| Para-meter | Time-moment | Alternatives | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
| | $t_1$ | $\prec 0.51, 0.73 \succ$ | $\prec 0.44, 0.54 \succ$ | $\prec 0.55, 0.46 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.55, 0.87 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.88 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_3$ | $\prec 0.31, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_4$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_6$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.79, 0.74 \succ$ |
| $e_1$ | $t_7$ | $\prec 0.45, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.88, 0.45 \succ$ | $\prec 0.45, 0.87 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_8$ | $\prec 0.81, 0.83 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.79, 0.49 \succ$ |
| | $t_9$ | $\prec 0.91, 0.79 \succ$ | $\prec 0.72, 0.74 \succ$ | $\prec 0.75, 0.96 \succ$ | $\prec 0.76, 0.45 \succ$ | $\prec 0.95, 0.77 \succ$ | $\prec 0.77, 0.75 \succ$ | $\prec 0.99, 0.78 \succ$ | $\prec 0.78, 0.49 \succ$ |
| | $t_{10}$ | $\prec 0.49, 0.79 \succ$ | $\prec 0.24, 0.24 \succ$ | $\prec 0.25, 0.26 \succ$ | $\prec 0.46, 0.65 \succ$ | $\prec 0.25, 0.27 \succ$ | $\prec 0.67, 0.25 \succ$ | $\prec 0.29, 0.78 \succ$ | $\prec 0.88, 0.44 \succ$ |
| | $t_{11}$ | $\prec 0.90, 0.73 \succ$ | $\prec 0.28, 0.24 \succ$ | $\prec 0.65, 0.26 \succ$ | $\prec 0.26, 0.65 \succ$ | $\prec 0.27, 0.97 \succ$ | $\prec 0.67, 0.25 \succ$ | $\prec 0.24, 0.27 \succ$ | $\prec 0.94, 0.44 \succ$ |
| | $t_{12}$ | $\prec 0.41, 0.93 \succ$ | $\prec 0.22, 0.54 \succ$ | $\prec 0.65, 0.26 \succ$ | $\prec 0.27, 0.45 \succ$ | $\prec 0.25, 0.87 \succ$ | $\prec 0.29, 0.95 \succ$ | $\prec 0.99, 0.28 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_1$ | $\prec 0.51, 0.73 \succ$ | $\prec 0.52, 0.64 \succ$ | $\prec 0.57, 0.56 \succ$ | $\prec 0.76, 0.45 \succ$ | $\prec 0.55, 0.97 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.58, 0.58 \succ$ | $\prec 0.58, 0.64 \succ$ |
| | $t_2$ | $\prec 0.74, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.67, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.85 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_3$ | $\prec 0.49, 0.93 \succ$ | $\prec 0.82, 0.74 \succ$ | $\prec 0.85, 0.96 \succ$ | $\prec 0.76, 0.45 \succ$ | $\prec 0.95, 0.87 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.98, 0.44 \succ$ |
| | $t_4$ | $\prec 0.87, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.86 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.98, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.78 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5$ | $\prec 0.28, 0.73 \succ$ | $\prec 0.32, 0.84 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.38, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.38, 0.38 \succ$ | $\prec 0.78, 0.84 \succ$ |
| | $t_6$ | $\prec 0.77, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.96 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.88, 0.74 \succ$ |
| $e_2$ | $t_7$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.47, 0.46 \succ$ | $\prec 0.46, 0.75 \succ$ | $\prec 0.75, 0.47 \succ$ | $\prec 0.48, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_8$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.52, 0.84 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.67 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.47 \succ$ |
| | $t_9$ | $\prec 0.71, 0.77 \succ$ | $\prec 0.76, 0.74 \succ$ | $\prec 0.75, 0.66 \succ$ | $\prec 0.76, 0.45 \succ$ | $\prec 0.65, 0.77 \succ$ | $\prec 0.77, 0.75 \succ$ | $\prec 0.99, 0.78 \succ$ | $\prec 0.78, 0.64 \succ$ |
| | $t_{10}$ | $\prec 0.49, 0.73 \succ$ | $\prec 0.64, 0.24 \succ$ | $\prec 0.25, 0.26 \succ$ | $\prec 0.46, 0.75 \succ$ | $\prec 0.26, 0.27 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.29, 0.78 \succ$ | $\prec 0.88, 0.44 \succ$ |
| | $t_{11}$ | $\prec 0.47, 0.73 \succ$ | $\prec 0.28, 0.24 \succ$ | $\prec 0.65, 0.86 \succ$ | $\prec 0.26, 0.65 \succ$ | $\prec 0.27, 0.87 \succ$ | $\prec 0.67, 0.25 \succ$ | $\prec 0.28, 0.27 \succ$ | $\prec 0.74, 0.44 \succ$ |
| | $t_{12}$ | $\prec 0.81, 0.73 \succ$ | $\prec 0.22, 0.84 \succ$ | $\prec 0.65, 0.26 \succ$ | $\prec 0.28, 0.45 \succ$ | $\prec 0.25, 0.87 \succ$ | $\prec 0.29, 0.95 \succ$ | $\prec 0.29, 0.28 \succ$ | $\prec 0.88, 0.43 \succ$ |
| | $t_1$ | $\prec 0.46, 0.63 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.56, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.56, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.68, 0.64 \succ$ |
| | $t_2$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.46 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.45, 0.67 \succ$ | $\prec 0.67, 0.45 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.74, 0.44 \succ$ |
| | $t_3$ | $\prec 0.44, 0.76 \succ$ | $\prec 0.82, 0.44 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.84, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.47, 0.85 \succ$ | $\prec 0.89, 0.48 \succ$ | $\prec 0.79, 0.44 \succ$ |
| | $t_4$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.45, 0.94 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.45, 0.97 \succ$ | $\prec 0.97, 0.45 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.74, 0.44 \succ$ |
| | $t_5$ | $\prec 0.24, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.55 \succ$ | $\prec 0.55, 0.37 \succ$ | $\prec 0.37, 0.55 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_6$ | $\prec 0.75, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.55, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.56, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.68, 0.74 \succ$ |
| $e_3$ | $t_7$ | $\prec 0.61, 0.75 \succ$ | $\prec 0.62, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.45, 0.67 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.46, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_8$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.75, 0.66 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.57, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.75, 0.46 \succ$ |
| | $t_9$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.72, 0.74 \succ$ | $\prec 0.65, 0.76 \succ$ | $\prec 0.76, 0.45 \succ$ | $\prec 0.75, 0.67 \succ$ | $\prec 0.77, 0.75 \succ$ | $\prec 0.89, 0.78 \succ$ | $\prec 0.78, 0.64 \succ$ |
| | $t_{10}$ | $\prec 0.87, 0.73 \succ$ | $\prec 0.24, 0.24 \succ$ | $\prec 0.25, 0.76 \succ$ | $\prec 0.46, 0.65 \succ$ | $\prec 0.75, 0.27 \succ$ | $\prec 0.67, 0.25 \succ$ | $\prec 0.29, 0.78 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_{11}$ | $\prec 0.47, 0.73 \succ$ | $\prec 0.28, 0.24 \succ$ | $\prec 0.67, 0.26 \succ$ | $\prec 0.26, 0.67 \succ$ | $\prec 0.27, 0.27 \succ$ | $\prec 0.67, 0.25 \succ$ | $\prec 0.24, 0.27 \succ$ | $\prec 0.74, 0.74 \succ$ |
| | $t_{12}$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.27, 0.54 \succ$ | $\prec 0.65, 0.26 \succ$ | $\prec 0.27, 0.45 \succ$ | $\prec 0.75, 0.87 \succ$ | $\prec 0.29, 0.97 \succ$ | $\prec 0.29, 0.28 \succ$ | $\prec 0.78, 0.47 \succ$ |
| | $t_1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.76 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.57, 0.58 \succ$ | $\prec 0.75, 0.74 \succ$ |
| | $t_2$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.66, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.66 \succ$ | $\prec 0.76, 0.46 \succ$ |
| | $t_3$ | $\prec 0.61, 0.76 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.86, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.76, 0.46 \succ$ |
| | $t_4$ | $\prec 0.68, 0.73 \succ$ | $\prec 0.97, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.75 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.79, 0.98 \succ$ | $\prec 0.78, 0.47 \succ$ |
| | $t_5$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.66 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.36, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.36, 0.38 \succ$ | $\prec 0.58, 0.45 \succ$ |
| | $t_6$ | $\prec 0.81, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.76 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.67 \succ$ | $\prec 0.87, 0.55 \succ$ | $\prec 0.59, 0.88 \succ$ | $\prec 0.58, 0.74 \succ$ |
| $e_4$ | $t_7$ | $\prec 0.81, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.86 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.87, 0.45 \succ$ | $\prec 0.49, 0.88 \succ$ | $\prec 0.77, 0.44 \succ$ |
| | $t_8$ | $\prec 0.46, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.65, 0.56 \succ$ | $\prec 0.56, 0.65 \succ$ | $\prec 0.65, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_9$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.72, 0.74 \succ$ | $\prec 0.75, 0.76 \succ$ | $\prec 0.76, 0.65 \succ$ | $\prec 0.65, 0.77 \succ$ | $\prec 0.77, 0.75 \succ$ | $\prec 0.79, 0.78 \succ$ | $\prec 0.68, 0.64 \succ$ |
| | $t_{10}$ | $\prec 0.49, 0.73 \succ$ | $\prec 0.64, 0.24 \succ$ | $\prec 0.25, 0.26 \succ$ | $\prec 0.46, 0.75 \succ$ | $\prec 0.25, 0.27 \succ$ | $\prec 0.77, 0.25 \succ$ | $\prec 0.29, 0.77 \succ$ | $\prec 0.88, 0.44 \succ$ |
| | $t_{11}$ | $\prec 0.49, 0.73 \succ$ | $\prec 0.28, 0.27 \succ$ | $\prec 0.65, 0.26 \succ$ | $\prec 0.27, 0.65 \succ$ | $\prec 0.29, 0.27 \succ$ | $\prec 0.67, 0.25 \succ$ | $\prec 0.24, 0.97 \succ$ | $\prec 0.79, 0.44 \succ$ |
| | $t_{12}$ | $\prec 0.91, 0.73 \succ$ | $\prec 0.22, 0.54 \succ$ | $\prec 0.65, 0.26 \succ$ | $\prec 0.29, 0.95 \succ$ | $\prec 0.25, 0.87 \succ$ | $\prec 0.29, 0.95 \succ$ | $\prec 0.29, 0.98 \succ$ | $\prec 0.55, 0.44 \succ$ |

TABLE 5. The table of generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_{A_1}^{(T1)}$

| Para-meter | Time-moment | Alternatives $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | $t_1^1$ | $\prec 0.41, 0.53 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_2^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.55 \succ$ |
| | $t_3^1$ | $\prec 0.51, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.55, 0.44 \succ$ |
| | $t_4^1$ | $\prec 0.48, 0.55 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5^1$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.33 \succ$ |
| | $t_6^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^1$ | $\prec 0.41, 0.55 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.22 \succ$ |
| $e_2$ | $t_1^1$ | $\prec 0.55, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.33 \succ$ |
| | $t_2^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_3^1$ | $\prec 0.44, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.66, 0.44 \succ$ |
| | $t_4^1$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5^1$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.78 \succ$ |
| | $t_6^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^1$ | $\prec 0.46, 0.87 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| $e_3$ | $t_1^1$ | $\prec 0.32, 0.76 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_2^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_3^1$ | $\prec 0.66, 0.77 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.66, 0.44 \succ$ |
| | $t_4^1$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5^1$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_6^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^1$ | $\prec 0.41, 0.54 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| $e_4$ | $t_1^1$ | $\prec 0.56, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_2^1$ | $\prec 0.71, 0.76 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.87 \succ$ |
| | $t_3^1$ | $\prec 0.32, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.87 \succ$ |
| | $t_4^1$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.59, 0.44 \succ$ |
| | $t_5^1$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.98, 0.44 \succ$ |
| | $t_6^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.87 \succ$ |
| | $t_7^1$ | $\prec 0.88, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| $e_5$ | $t_1^1$ | $\prec 0.41, 0.99 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.43, 0.44 \succ$ |
| | $t_2^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.57, 0.44 \succ$ |
| | $t_3^1$ | $\prec 0.54, 0.87 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.68 \succ$ |
| | $t_4^1$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5^1$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.77 \succ$ |
| | $t_6^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.77, 0.74 \succ$ |
| | $t_7^1$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| $e_6$ | $t_1^1$ | $\prec 0.42, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.54 \succ$ |
| | $t_2^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.65, 0.44 \succ$ |
| | $t_3^1$ | $\prec 0.76, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_4^1$ | $\prec 0.65, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.67, 0.44 \succ$ |
| | $t_5^1$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_6^1$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^1$ | $\prec 0.32, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.56, 0.76 \succ$ |

TABLE 6. The table of generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_{A_2}^{(T2)}$

| Para-meter | Time-moment | Alternatives $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|---|
| | $t_1^2$ | $\prec 0.55, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.99, 0.77 \succ$ |
| | $t_2^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_3^2$ | $\prec 0.55, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.43 \succ$ |
| | $t_4^2$ | $\prec 0.54, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.67, 0.44 \succ$ |
| $e_1$ | $t_5^2$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.65, 0.44 \succ$ |
| | $t_6^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.78 \succ$ |
| | $t_7^2$ | $\prec 0.46, 0.59 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.79, 0.44 \succ$ |
| | $t_8^2$ | $\prec 0.88, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.68 \succ$ |
| | $t_1^2$ | $\prec 0.98, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.64, 0.65 \succ$ |
| | $t_2^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.32, 0.44 \succ$ |
| | $t_3^2$ | $\prec 0.46, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.56 \succ$ |
| | $t_4^2$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.90, 0.44 \succ$ |
| $e_2$ | $t_5^2$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.98 \succ$ |
| | $t_6^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^2$ | $\prec 0.96, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_8^2$ | $\prec 0.94, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.68, 0.68 \succ$ |
| | $t_1^2$ | $\prec 0.89, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.86, 0.65 \succ$ |
| | $t_2^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.14, 0.84 \succ$ |
| | $t_3^2$ | $\prec 0.56, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.87, 0.44 \succ$ |
| | $t_4^2$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.89 \succ$ |
| $e_3$ | $t_5^2$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.24, 0.96 \succ$ |
| | $t_6^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^2$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_8^2$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_1^2$ | $\prec 0.57, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_2^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_3^2$ | $\prec 0.49, 0.93 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.58, 0.44 \succ$ |
| | $t_4^2$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.88, 0.44 \succ$ |
| $e_4$ | $t_5^2$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.98, 0.44 \succ$ |
| | $t_6^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^2$ | $\prec 0.91, 0.33 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.98, 0.94 \succ$ |
| | $t_8^2$ | $\prec 0.91, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.79, 0.44 \succ$ |
| | $t_1^2$ | $\prec 0.49, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.94 \succ$ |
| | $t_2^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.98, 0.44 \succ$ |
| | $t_3^2$ | $\prec 0.49, 0.93 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_4^2$ | $\prec 0.48, 0.79 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.79, 0.44 \succ$ |
| $e_5$ | $t_5^2$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.84 \succ$ |
| | $t_6^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.88, 0.74 \succ$ |
| | $t_7^2$ | $\prec 0.81, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.84 \succ$ |
| | $t_8^2$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.88, 0.44 \succ$ |
| | $t_1^2$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.64 \succ$ |
| | $t_2^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.68, 0.44 \succ$ |
| | $t_3^2$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.64 \succ$ |
| | $t_4^2$ | $\prec 0.48, 0.63 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| $e_6$ | $t_5^2$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.68, 0.64 \succ$ |
| | $t_6^2$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.58, 0.74 \succ$ |
| | $t_7^2$ | $\prec 0.51, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.58, 0.44 \succ$ |
| | $t_8^2$ | $\prec 0.41, 0.53 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.54 \succ$ |

TABLE 7. The table of generalized temporal intuitionistic fuzzy soft set $\mathfrak{S}_{A_3}^{(T3)}$

| Parameter | Time-moment | Alternatives $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
|---|---|---|---|---|---|---|---|---|---|
| $e_1$ | $t_1^3$ | $\prec 0.45, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.75, 0.44 \succ$ |
| | $t_2^3$ | $\prec 0.71, 0.75 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.75, 0.44 \succ$ |
| | $t_3^3$ | $\prec 0.45, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.54 \succ$ |
| | $t_4^3$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.68, 0.44 \succ$ |
| | $t_5^3$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_6^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^3$ | $\prec 0.46, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.68, 0.44 \succ$ |
| $e_2$ | $t_1^3$ | $\prec 0.41, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.64 \succ$ |
| | $t_2^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.68, 0.44 \succ$ |
| | $t_3^3$ | $\prec 0.61, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.68, 0.44 \succ$ |
| | $t_4^3$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5^3$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.45 \succ$ |
| | $t_6^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.54 \succ$ |
| | $t_7^3$ | $\prec 0.51, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.54 \succ$ |
| $e_3$ | $t_1^3$ | $\prec 0.41, 0.53 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.75, 0.44 \succ$ |
| | $t_2^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.38, 0.44 \succ$ |
| | $t_3^3$ | $\prec 0.31, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.34 \succ$ |
| | $t_4^3$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.73, 0.44 \succ$ |
| | $t_5^3$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.38, 0.44 \succ$ |
| | $t_6^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.73 \succ$ |
| | $t_7^3$ | $\prec 0.31, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.44 \succ$ |
| $e_4$ | $t_1^3$ | $\prec 0.45, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.28, 0.44 \succ$ |
| | $t_2^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.28, 0.24 \succ$ |
| | $t_3^3$ | $\prec 0.31, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.38, 0.44 \succ$ |
| | $t_4^3$ | $\prec 0.43, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.43 \succ$ |
| | $t_5^3$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.58, 0.54 \succ$ |
| | $t_6^3$ | $\prec 0.51, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.75 \succ$ |
| | $t_7^3$ | $\prec 0.11, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.66, 0.44 \succ$ |
| $e_5$ | $t_1^3$ | $\prec 0.22, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.55, 0.44 \succ$ |
| | $t_2^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.56 \succ$ |
| | $t_3^3$ | $\prec 0.66, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.66 \succ$ |
| | $t_4^3$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_5^3$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.66, 0.44 \succ$ |
| | $t_6^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.74 \succ$ |
| | $t_7^3$ | $\prec 0.66, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.66 \succ$ |
| $e_6$ | $t_1^3$ | $\prec 0.41, 0.66 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.45 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.66, 0.44 \succ$ |
| | $t_2^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.62, 0.64 \succ$ | $\prec 0.65, 0.66 \succ$ | $\prec 0.66, 0.45 \succ$ | $\prec 0.65, 0.67 \succ$ | $\prec 0.67, 0.65 \succ$ | $\prec 0.69, 0.68 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_3^3$ | $\prec 0.55, 0.73 \succ$ | $\prec 0.82, 0.84 \succ$ | $\prec 0.85, 0.86 \succ$ | $\prec 0.86, 0.45 \succ$ | $\prec 0.85, 0.87 \succ$ | $\prec 0.87, 0.85 \succ$ | $\prec 0.89, 0.88 \succ$ | $\prec 0.78, 0.55 \succ$ |
| | $t_4^3$ | $\prec 0.48, 0.73 \succ$ | $\prec 0.92, 0.94 \succ$ | $\prec 0.95, 0.96 \succ$ | $\prec 0.96, 0.45 \succ$ | $\prec 0.95, 0.97 \succ$ | $\prec 0.97, 0.95 \succ$ | $\prec 0.99, 0.98 \succ$ | $\prec 0.67, 0.44 \succ$ |
| | $t_5^3$ | $\prec 0.21, 0.73 \succ$ | $\prec 0.32, 0.34 \succ$ | $\prec 0.35, 0.36 \succ$ | $\prec 0.36, 0.45 \succ$ | $\prec 0.35, 0.37 \succ$ | $\prec 0.37, 0.35 \succ$ | $\prec 0.39, 0.38 \succ$ | $\prec 0.78, 0.44 \succ$ |
| | $t_6^3$ | $\prec 0.71, 0.73 \succ$ | $\prec 0.52, 0.54 \succ$ | $\prec 0.55, 0.56 \succ$ | $\prec 0.56, 0.75 \succ$ | $\prec 0.55, 0.57 \succ$ | $\prec 0.57, 0.55 \succ$ | $\prec 0.59, 0.58 \succ$ | $\prec 0.78, 0.89 \succ$ |
| | $t_7^3$ | $\prec 0.75, 0.73 \succ$ | $\prec 0.42, 0.44 \succ$ | $\prec 0.45, 0.46 \succ$ | $\prec 0.46, 0.45 \succ$ | $\prec 0.45, 0.47 \succ$ | $\prec 0.47, 0.45 \succ$ | $\prec 0.49, 0.48 \succ$ | $\prec 0.78, 0.89 \succ$ |

TABLE 8. Scilab Codes for Generalized Non-Temporal Intuitionistic Fuzzy Soft Set Algorithms

| Algorithm 1 (Single expert)/ Generalized non-temporal intuitionistic fuzzy soft set | Algorithm 2 (Multi expert) | Extended intersection of gtif-soft sets (Multi set of time moments with common parameter) |
|---|---|---|

**Algorithm 1 (Single expert)/ Generalized non-temporal intuitionistic fuzzy soft set**

**Algorithm 1:**

```
function G=singleexpert()
B=Mataver()
A1=zeros(size(B,'r'),size(B,'c')/2);
B1=zeros(size(B,'r'),size(B,'c')/2);
A2=zeros(size(B,'c')/2,size(B,'c')/2);
B2=zeros(size(B,'c')/2,size(B,'c')/2);
k=0; l=0; for j=1:size(B,'c')
if modulo(j,2)==1; k=k+1;
else l=l+1; end
for i=1:size(B,'r')
if modulo(j,2)==1; A1(i,k)=B(i,j);
else B1(i,l)=B(i,j); end  end  end
for i=1:size(B,'c')/2
for j=1:size(B,'c')/2; u=0; v=0;
for k=1:size(B,'r')
if A1(k,i)>A1(k,j); u=u+1; end
if B1(k,i)<B1(k,j); v=v+1; end end
A2(i,j)=u; B2(i,j)=v; end end
for i=1:size(B,'c')/2; c=0; d=0; e=0; f=0;
for j = 1:size(B,'c')/2
c=c+A2(i,j); d=d+A2(j,i); e=e+B2(i,j);
f=f+B2(j,i); end
G(i,1)=(c-d)+(e-f); end
endfunction
function B=Mataver()
[x,y]= size(A);
Tc= size(T, "c")
B=zeros(x/Tc,y); count=0
for l=1:Tc; count=count+1
c(count)=T(1,l); end
b=max(c);
for i = 1 :x/Tc
for j = 1 : y; a=0;
for k = 1 : Tc
a = a+T(1,k)*A((i-1)*Tc+k,j); end
a=a/(Tc*b);
B(i,j) = a; end end
endfunction
```

**Generalized non-temporal intuitionistic fuzzy soft set:**

```
function B=GNTIFSS()
[x,y]= size(A);
Tc= size(T, "c")
B=zeros(x/Tc,y);
count=0
for l=1:Tc
count=count+1
c(count)=T(1,l); end
b=max(c);
for i = 1 :x/Tc
for j = 1 : y
a=0;
for k = 1 : Tc
a = a+T(1,k)*A((i-1)*Tc+k,j); end
d=a/(Tc*b);
B(i,j) = d; end end
endfunction
```

**Algorithm 2 (Multi expert)**

```
function G=multiexpert()
B=Mataver()
A1=zeros(size(B,'r'),size(B,'c')/2);
B1=zeros(size(B,'r'),size(B,'c')/2);
A2=zeros(size(B,'c')/2,size(B,'c')/2);
B2=zeros(size(B,'c')/2,size(B,'c')/2);
k=0; l=0; for j=1:size(B,'c')
if modulo(j,2)==1; k=k+1;
else l=l+1; end
for i=1:size(B,'r')
if modulo(j,2)==1; A1(i,k)=B(i,j);
else B1(i,l)=B(i,j); end  end end
for i=1:size(B,'c')/2
for j=1:size(B,'c')/2
u=0; v=0; for k=1:size(B,'r')
if A1(k,i)>A1(k,j); u=u+1; end
if B1(k,i)<B1(k,j); v=v+1; end end
A2(i,j)=u; B2(i,j)=v; end end
for i=1:size(B,'c')/2
c=0; d=0; e=0; f=0; for j = 1:size(B,'c')/2
c=c+A2(i,j); d=d+A2(j,i); e=e+B2(i,j);
f=f+B2(j,i); end
G(i,1)=(c-d)+(e-f); end
endfunction
function B=Mataver()
D=Extintersection()
[x,y]= size(D); Tc= size(T, "c");
B=zeros(x/Tc,y); count=0
for l=1:Tc; count=count+1; c(count)=T(1,l);
end
b=max(c);
for i = 1 :x/Tc
for j = 1 : y; a=0;
for k = 1 : Tc;
a = a+T(1,k)*D((i-1)*Tc+k,j); end
a=a/(Tc*b); B(i,j) = a; end end
endfunction
```

**Extended intersection of gtif-soft sets (Multi set of time moments with common parameter)**

```
function D=Extintersection()
Fac= size(Fa, "c")
Fbc= size(Fb, "c")
Fcc= size(Fc, "c")
[Ax,l]= size(A)
[Bx,l]= size(B)
[Cx,l]= size(C)
E = Ax/Fac
Faub = union(Fa,Fb)
Fauc = union(Fa,Fc)
T = union(Faub,Fauc)
Tc= size(T, "c")
Faib = intersect(Fa,Fb)
Faic = intersect(Fa,Fc)
V = intersect(Faib,Faic)
D=zeros(E*Tc,l)
for k = 1 : E
for i = 1 : 12
u=size(intersect(i,T), "c")
if u==0 then
else
w = size(intersect(i,V), "c")
[nb, locFa] = members(i, Fa, "last")
[nb, locFb] = members(i, Fb, "last")
[nb, locFc] = members(i, Fc, "last")
[nb, locT] = members(i, T, "last")
Ra= (locFa+(k-1)*Fac)
Rb= (locFb+(k-1)*Fbc)
Rc= (locFc+(k-1)*Fcc)
Rt= (locT+(k-1)*Tc)
for j = 1 : l
if w == 0 then
if modulo(j,2) == 0 then
D(Rt,j)= 1
else
D(Rt,j)= 0
end
else
if modulo(j,2) == 0 then
D(Rt,j)= max(A(Ra,j),B(Rb,j),C(Rc,j))
else
D(Rt,j)= min(A(Ra,j),B(Rb,j),C(Rc,j))
end end end end end end
endfunction
```

### Procedure to use Scilab codes:

**(a) To solve Example 5.2**:

(i) Input the matrix $\mathbf{A}$ as the $gtif$-soft matrix of $gtif$-soft set $\mathfrak{S}_A^{(T)}$.

(ii) Input the row matrix $T = \begin{bmatrix} 0.1 & 0.3 & 0.4 & 0.6 & 0.1 & 0.8 & 0.7 & 0.4 & 0.2 & 0.4 & 0.8 & 0.3 \end{bmatrix}_{1\times 12}$ as the weights of time moments $t_1, t_2, \ldots, t_{12}$.

**(b) To solve Example 5.3:**

(i) Input the matrices $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$ as the $gtif$-soft matrices of $gtif$-soft sets $\mathfrak{S}_A^{(T1)}, \mathfrak{S}_A^{(T2)}$, and $\mathfrak{S}_A^{(T3)}$ respectively.

(ii) Input the row matrices $\mathbf{Fa} = \begin{bmatrix} 1 & 3 & 6 & 8 & 9 & 11 & 12 \end{bmatrix}_{\mathbf{1\times 7}}$, $\mathbf{Fb} = \begin{bmatrix} 1 & 2 & 3 & 6 & 8 & 9 & 10 & 12 \end{bmatrix}_{\mathbf{1\times 8}}$, $\mathbf{Fc} = \begin{bmatrix} 1 & 3 & 5 & 6 & 9 & 11 & 12 \end{bmatrix}_{\mathbf{1\times 7}}$
as the time moments $T1, T2, T3$ respectively, after assigning the numbers 1 to 12 for the months from January to December.

(iii) Input the row matrix $\mathbf{T} = \begin{bmatrix} 0.6 & 0.8 & 0.5 & 0.6 & 0.7 & 0.9 & 0.7 & 0.3 & 0.4 & 0.9 \end{bmatrix}_{\mathbf{1\times 10}}$ as the weights of time moments $t_1, t_2, \ldots, t_{10}$ because, $T = T1 \cup T2 \cup T3 = \{t_1, t_2, \ldots, t_{10}\}$.