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Analyzing the Dynamics of Cleaners Behavior in the Community for a Germ-Free Village Initiative

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Abstract. A mathematical model has been developed and analyzed along with a flowchart for cleaners in the community. The entire population is partitioned into Latent, candidate, passive, active, and non-cleaners. The analysis examines how the cleanliness present equilibrium achieves local and global asymptotic stability when the cleaning reproductive number exceeds one. It has been verified using the Routh-Hurwitz criteria that a positive, unique cleanliness present equilibrium has local asymptotic stability. By a graph-theoretic approach, it also has been proved that the cleanliness equilibrium exhibits global asymptotic stability. Parameter estimation has been carried out to determine the numerical value of the cleaning reproductive number. A sensitivity analysis is being conducted to identify the factors that discourage individuals from maintaining cleanliness and contribute to creating an infectious environment. Further, the dynamics of cleaner behavior in the community are analyzed, providing valuable insights for evidence-based policies and interventions promoting a clean and healthier environment.

AMS (MOS) Subject Classification Code: 92B05
Key Words: Mathematical Model; Cleanliness; Stability Analysis; Parameter Estimation; Sensitivity Analysis; Community Behavior.

1. Introduction

The significance of cleanliness and hygiene cannot be overstated in society. Everyone appreciates a clean home, although some individuals may be tidier than others, with certain individuals emphasizing hygiene more. Even those with busy schedules prioritize maintaining a clean and orderly environment. But why is cleanliness important? Does it serve as a genetic defense against disease?
Cleanliness can be described as devoid of dirt, germs, and undesirable substances [19]. It involves various methods and actions intended to uphold cleanliness, whether for oneself or as a group effort, fostering a hygienic setting. The significance of cleanliness dates back centuries, with historical evidence pointing towards implementing sanitation measures in ancient civilizations such as Mesopotamia, Egypt, and Indus Valley [11].
The link between cleanliness and disease prevention is well-documented throughout history. Epidemics, like the cholera epidemics that ravaged medieval Europe in the 19th century, highlighted how crucial sanitation is

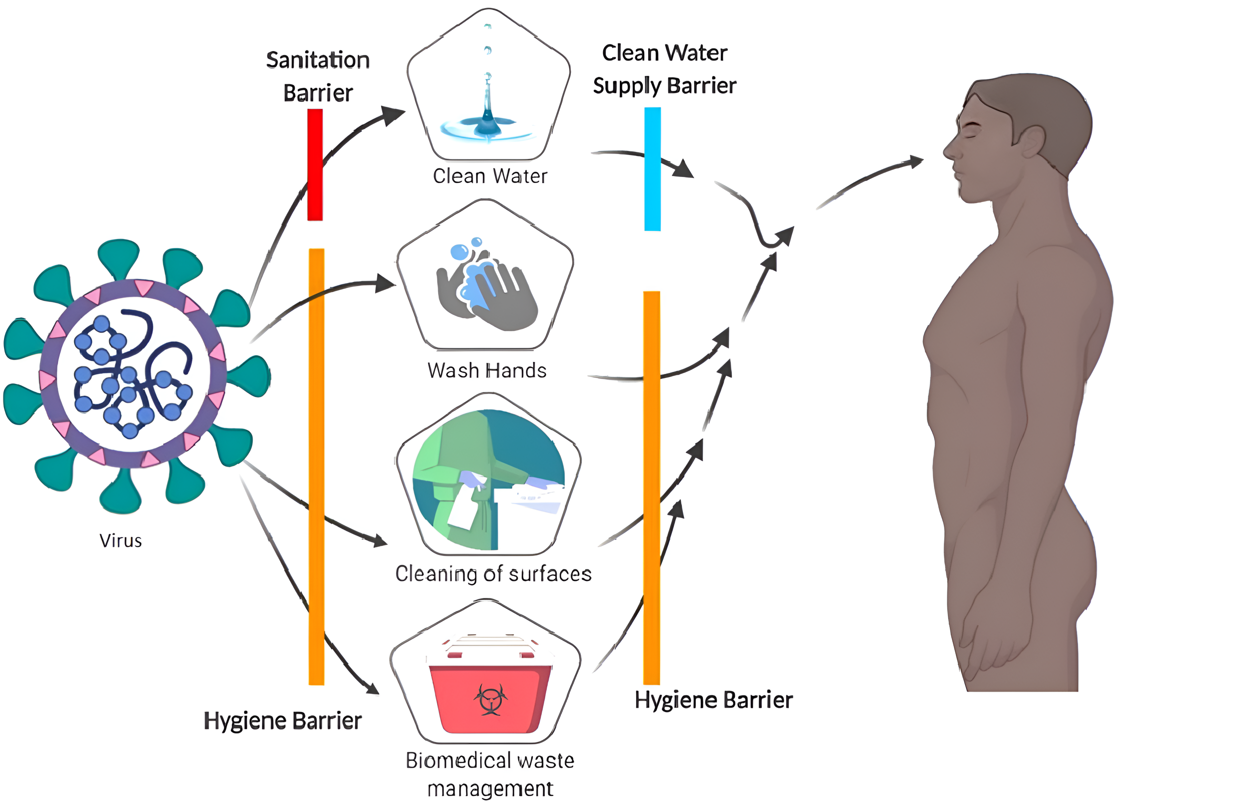


FIGURE 1. The Necessity of Cleanliness

to prevent contagious diseases. The germ theory of disease proposed by Louis Pasteur *et al.* [4] in the 19th century provided a scientific basis for understanding how pathogens spread through unsanitary conditions, further emphasizing the necessity of maintaining cleanliness to safeguard public health.

A quarter of the victims succumbed to conditions related to living or working in an unhealthy environment [2]. Unclean conditions, whether in the body, surroundings, or environment, can contribute to the spread of diseases and pose risks to our society. A disease is an abnormal state that interferes with an organism's structure or function and usually results from internal causes instead of external trauma. This term is commonly used to describe conditions that cause pain, dysfunction, social issues, or even death in individuals [33].

Vector-borne illnesses spread more widely when workers migrate to metropolitan areas, especially when they come from places where infections are common. The continual influx of migrants from underdeveloped areas to more developed regions has led to a substantial growth in urban populations, subsequently straining urban infrastructure. These factors collectively contribute to the onset of epidemics [21]. Additionally, vector-borne diseases can be transmitted through direct means such as organ transplants, blood transfusions, and parasite ingestion [10].

Figure 1 depicts the necessity of cleanliness to eliminate all infections. Decades have seen the use of mathematical modeling to depict natural phenomena such as [26, 8, 32, 17, 7, 3, 14]. Previous literature has explored various aspects of cleanliness and its impact on disease prevention [12, 35, 5]. Research has examined how well sanitation practices work to stop the spread of illnesses like respiratory ailments, diarrheal infections, and vector-borne infections. The variables affecting people's commitment to hygiene practices and the social norms that influence people's overall hygiene habits have been studied in behavioral research. Additionally, economic analyses have quantified the costs and benefits associated with investments in sanitation infrastructure and hygiene promotion programs [22, 16].

Mathematical modeling in population behavior uses equations and algorithms to describe how organisms or populations behave and interact, particularly in the context of health. The models outlined in [34, 30, 15, 20, 1, 25] help analyze interactions within biological systems, such as infection rates, recovery dynamics, and cleaning strategies. In 1962, Polya highlighted the importance of using equations to solve real-world problems, emphasizing how this approach can bridge mathematical concepts with practical situations. This perspective underscores the need for careful development of such connections [27]. A paradigm introducing three classes into the population, such as Susceptible (S), Infected (I), and Recovered (R), was presented by Kermack and McKendrick [18]. Graunt [13] was the first to systematically analyze causes of death in his book "Natural and Political Observations Made upon the Bills of Mortality." Daniel Bernoulli [6] pioneered early mathematical modeling of disease spread in 1766 by developing a model to assess the impact of smallpox vaccination, revealing its potential to increase life expectancy from 26 to 29 years. The association between cleanliness and religious practices was analyzed by Preston *et al.* [24] in their study of the relationship between religious beliefs and personal hygiene.

In this study, a novel mathematical model depicting the dynamics of cleaners is developed. The local and global stability of the model is analyzed under the condition of a cleanliness-present scenario. A graph-theoretic approach has been utilized to analyze the global stability at the cleanliness-present equilibrium point. Parameters are estimated to quantify cleaning reproduction and analyze the rate at which individuals are motivated toward cleaning activities. A sensitivity analysis is conducted to identify the factors that discourage individuals from maintaining cleanliness and contribute to creating an infectious environment. The behavior of the cleaners' community has been simulated to analyze their dynamics. This study adopts an interdisciplinary method to examine these interactions. It seeks to provide meaningful perspectives for crafting policies and interventions supported by evidence to advance a more sustainable and healthier environment.

2. Formulation of the Mathematical Model

The people who do not prioritize cleanliness are called "Latent Cleaners C_l." Those who prioritize cleanliness are called "Candidate Cleaners C_c." Individuals who clean irregularly are referred to as "Passive Cleaners C_p." Those who consistently attempt to clean everything are categorized as "Active Cleaners C_a," but some of them quit cleanliness and become ill and are marked as "Non-Cleaners C_n." Currently, several programs are being implemented to promote manual cleaning. A mathematical model has been developed to support these recognition programs. This model considers five compartments based on cleaners. Let $C(t)$ articulates the entire population size at time t, which is written as:

$$C(t) = C_l(t) + C_c(t) + C_p(t) + C_a(t) + C_n(t).$$

Assuming β as the recruitment rate into the latent class and α as the latent individuals' movement rate into the candidate class. The rate at which individuals move from the candidate cleaner class to the passive class is labeled ϵ. At the same time, a rate is denoted as γ for individuals transitioning from the candidate to the active cleaner class. Furthermore, individuals in the passive cleaner class have a rate of transitioning back to the latent class, represented by ρ. The transition rate from the passive to the candidate class is ν, and the transition rate from passive to active cleaners is η. Additionally, the transition rate for latent cleaners to the non-cleaners class resulting in illness is σ, and for candidate cleaners to the non-cleaners class resulting in illness is δ. Here, μ represents the departure rate from each respective class.

The model formulated with the various assumptions and parameters required for analyzing the dynamics of cleanliness is explained below with the help of a flow diagram. The notations and fitted parametric values used in the dynamical system of potential cleaners are given in the following Table 1.

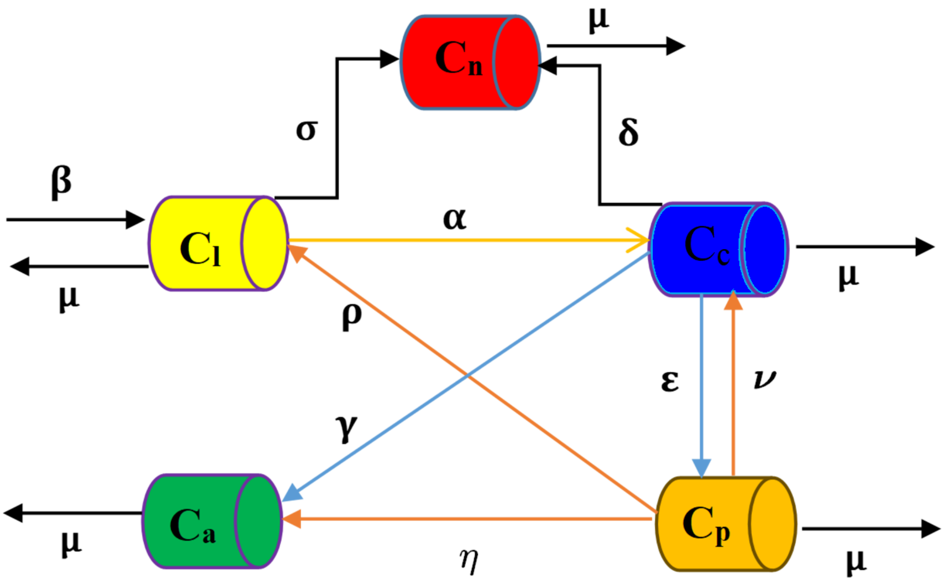


FIGURE 2. Flow Diagram for Model (2. 1)

The mathematical representation of the model is described by the subsequent set of nonlinear differential equations.

$$
\begin{aligned}
\frac{dC_l}{dt} &= \beta - \alpha C_l C_c - (\sigma + \mu)\, C_l + \rho C_p, \\
\frac{dC_c}{dt} &= \alpha C_l C_c - (\gamma + \epsilon + \delta + \mu)\, C_c + \nu C_p, \\
\frac{dC_p}{dt} &= \epsilon C_c - (\rho + \eta + \nu + \mu)\, C_p, \\
\frac{dC_a}{dt} &= \gamma C_c + \eta C_p - \mu C_a, \\
\frac{dC_n}{dt} &= \sigma C_l + \delta C_c - \mu C_n,
\end{aligned}
\tag{2. 1}
$$

with initial conditions

$$C_l \geq 0, \quad C_c \geq 0, \quad C_p \geq 0, \quad C_a \geq 0, \quad C_n \geq 0.$$

The cleaners population can be depicted using mathematical expression:

$$\frac{dC}{dt} = \beta - \mu\left(C_l + C_c + C_p + C_a + C_n\right).$$

The feasible region of the solution is

$$\mathcal{U} = \left\{C_l, C_c, C_p, C_a, C_n \in \mathbb{R}^5_+ : C = C_l + C_c + C_p + C_a + C_n \leq \frac{\beta}{\mu}\right\}.$$

TABLE 1. Estimated parameter values for the mild infected population

Notations	Values	Description	Source
β	1.47189	The rate of recruitment of cleaners	[23]
μ	0.0147189	The rate of mortality of cleaners	[23]
α	0.55	The rate at which latent individuals becomes candidate cleaners	fitted
σ	0.5	The rate at which candidate cleaner becomes non cleaner	fitted
γ	0.2	The rate at which candidate cleaners becomes active cleaners	fitted
ϵ	0.248	The rate at which candidate cleaners becomes passive cleaners	fitted
δ	0.6	The rate at which candidate cleaners becomes non cleaners	fitted
ν	0.1	The rate at which passive cleaners becomes candidate cleaners	fitted
ρ	0.1	The rate at which passive cleaners becomes latent cleaners	fitted
η	0.25	The rate at which passive cleaners becomes active cleaners	fitted

2.1. **Existence of Equilibria.** The system (2. 1) exhibits two points of equilibrium, which are:

- **Cleanliness-Free Equilibrium:** It refers to a state where cleanliness is unnecessary for a balanced or functional environment. It can be obtained by setting all the derivatives and the classes having some motivation for cleanliness equal to zero, as articulated below:

$$\mathcal{E}_0 = \left(\frac{\beta}{\mu+\sigma}, 0, 0, 0, \frac{\beta\sigma}{\mu(\mu+\sigma)}\right).$$

- **Cleanliness-Present Equilibrium:** It refers to a state where cleanliness practices become deeply ingrained and habitual within a community. It can be obtained by setting all derivatives equal to zero and then solving the equations, which is given as $\mathcal{E}_e^* = (C_l^*, C_c^*, C_p^*, C_a^*, C_n^*)$,
where

$$C_l^* = \frac{(\gamma+\delta+\epsilon+\mu)(\eta+\mu+\nu+\rho)-\epsilon\nu}{\alpha(\eta+\mu+\nu+\rho)},$$

$$C_c^* = \frac{\alpha\beta(\eta+\mu+\nu+\rho)-(\mu+\sigma)\left(\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta+\rho(\epsilon+\mu)+\epsilon\mu+\eta\mu+\mu^2+\mu\nu\right)}{\alpha\left(\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta+\epsilon\mu+\eta\mu+\mu^2+\mu\nu+\mu\rho\right)},$$

$$C_p^* = \frac{\alpha\beta\epsilon(\eta+\mu+\nu+\rho)-\epsilon(\mu+\sigma)\left(\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta+\rho(\epsilon+\mu)+\epsilon\mu+\eta\mu+\mu^2+\mu\nu\right)}{\alpha(\eta+\mu+\nu+\rho)\left(\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta+\epsilon\mu+\eta\mu+\mu^2+\mu\nu+\mu\rho\right)},$$

$$\begin{aligned}C_a^* = {} & [(\gamma(\eta+\mu+\nu+\rho)+\epsilon\eta)(\alpha\beta(\eta+\mu+\nu+\rho)-(\mu+\sigma)(\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta \\ & +\rho(\epsilon+\mu)+\epsilon\mu+\eta\mu+\mu^2+\mu\nu))\Big] \\ & \times\frac{1}{\alpha\mu(\eta+\mu+\nu+\rho)\left(\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta+\epsilon\mu+\eta\mu+\mu^2+\mu\nu+\mu\rho\right)},\end{aligned}$$

$$\begin{aligned}C_n^* = {} & \Big[\alpha\beta\delta(\eta+\mu+\nu+\rho)^2+\Big((\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta+\rho(\epsilon+\mu)+\epsilon\mu+\eta\mu+\mu^2+\mu\nu) \\ & \times\ \sigma(\gamma(\eta+\mu+\nu+\rho)+\epsilon(\eta+\mu)+\mu(\eta+\mu+\nu+\rho))-\delta\mu(\eta+\mu+\nu+\rho))] \\ & \times\frac{1}{\alpha\mu(\eta+\mu+\nu+\rho)\left(\gamma(\eta+\mu+\nu+\rho)+\delta(\eta+\mu+\nu+\rho)+\epsilon\eta+\epsilon\mu+\eta\mu+\mu^2+\mu\nu+\mu\rho\right)}.\end{aligned}$$

It is concluded that whenever $R_c > 1$, the model (2. 1) possesses a singular equilibrium point. The Cleanliness present equilibrium point with numerical values is $\mathcal{E}_e^* = (1.83519, 0.55156, 0.294343, 12.494, 84.8249)$.

Theorem 2.1. *The cleanliness-present point of equilibrium of the model (1) is locally asymptotically stable inside the region $\mathcal{U}$ if $C_c^* - C_l^* > 0$.*

Proof. To analyze the local stability of the cleanliness present equilibrium (CPE) of a dynamical system represented by the given Jacobian matrix, one can use the Routh-Hurwitz criteria [28]. This criterion will help determine the stability of the system's equilibrium points based on the eigenvalues of the Jacobian matrix evaluated at the

CPE point.

$$|J(\mathcal{E}_e^*) - \lambda I| = \begin{vmatrix} -\alpha C_c^* - \sigma - \mu - \lambda & -\alpha C_l^* & \rho & 0 & 0 \\ \alpha C_c^* & \alpha C_l^* - \gamma - \epsilon - \delta - \mu - \lambda & \nu & 0 & 0 \\ 0 & \epsilon & -\rho - \eta - \nu - \mu - \lambda & 0 & 0 \\ 0 & \gamma & \eta & -\mu - \lambda & 0 \\ \sigma & \delta & 0 & 0 & -\mu - \lambda \end{vmatrix}.$$

The characteristic equation of given Jacobian matrix is

$$(\mu + \lambda)^2 \mathcal{D}(\lambda) = 0,$$

such that

$$\mathcal{D}(\lambda) = \begin{vmatrix} -\alpha C_c^* - a - \lambda & -\alpha C_l^* & \rho \\ \alpha C_c^* & \alpha C_l^* - b - \lambda & \nu \\ 0 & \epsilon & c - \lambda \end{vmatrix},$$

where

$$a = \sigma + \mu, \quad b = \gamma + \epsilon + \delta + \mu, \quad c = \rho + \eta + \nu + \mu,$$

then $\mathcal{D}(\lambda)$ results into

$$\mathcal{D}(\lambda) = \lambda^3 + \mathcal{M}_1\lambda^2 + \mathcal{M}_2\lambda + \mathcal{M}_3,$$

where

$$\begin{aligned} \mathcal{M}_1 &= \alpha(C_c^* - C_l^*) + a + b + c > 0, \\ \mathcal{M}_2 &= \alpha(c + \mu)(C_c^* - C_l^*) + (\gamma + \epsilon + \delta)\alpha C_c^* + ab + ac + bc \\ &\quad +(\rho + \eta + \mu)(\gamma + \epsilon + \mu + \delta) + \nu(\epsilon + \mu + \delta) + \alpha(\delta C_c^* - \sigma C_l^*) > 0, \\ \mathcal{M}_3 &= (\mu + \sigma)\nu(\epsilon + \mu + \delta) + (\mu + \sigma)(\gamma + \epsilon + \mu + \delta)(\rho + \eta + \mu) + \mu c\alpha(C_c^* - C_l^*) \\ &\quad -\sigma c\alpha C_l^* + \gamma(\eta + \mu)\alpha C_c^* + (\epsilon + \delta)c\alpha C_c^* > 0, \end{aligned}$$

and

$$\begin{aligned} \mathcal{M}_1\mathcal{M}_2 - \mathcal{M}_3 &= a + b + c(\nu(\mu + \delta + \epsilon) + (\eta + \mu + \rho)(\mu + \delta + \gamma + \epsilon) + \delta\alpha C_c^* \\ &\quad +ac + bc + c\alpha(C_c^* - C_l^*)) + (\rho + \nu)\gamma\alpha C_c^* + \nu(\mu + \sigma)\gamma + \alpha(C_c^* - C_l^*) > 0, \end{aligned}$$

if

$$C_c^* - C_l^* > 0.$$

Therefore, by the Routh-Hurwitz criteria, the unique equilibrium point of the system (1) is locally asymptotically stable. □

Theorem 2.2. *The cleanliness-present point of equilibrium $\mathcal{E}_e^* = (C_l^*, C_c^*, C_p^*, C_a^*, C_n^*)$ of the model (1) is globally asymptotically stable inside the region $\mathcal{U}$.*

Proof. Suppose that

$$G_1 = C_l - {C_l}^* - {C_l}^* \ln \frac{C_l}{{C_l}^*},$$
$$G_2 = C_c - {C_c}^* - {C_c}^* \ln \frac{C_c}{{C_c}^*},$$
$$G_3 = C_p - {C_p}^* - {C_p}^* \ln \frac{C_p}{{C_p}^*},$$
$$G_4 = C_a - {C_a}^* - {C_a}^* \ln \frac{C_a}{{C_a}^*},$$
$$G_5 = C_n - {C_n}^* - {C_n}^* \ln \frac{C_n}{{C_n}^*}.$$

Taking derivative and utilizing inequality $1 - z + \ln z \leq 0$ for $z > 0$, yields

$$\begin{aligned}
\dot{G}_1 &= (1 - \frac{C_l^*}{C_l})\dot{C}_l, \\
\dot{G}_1 &= \left(1 - \frac{C_l^*}{C_l}\right)\left(\beta - \alpha C_l C_c - \sigma C_l - \mu C_l + \rho C_p\right), \\
\dot{G}_1 &= \left(1 - \frac{C_l^*}{C_l}\right)\left(\alpha C_l^* C_c^* + \sigma C_l^* + \mu C_l^* - \rho C_p^* - \alpha C_l C_c - \sigma C_l - \mu C_l + \rho C_p\right), \\
\dot{G}_1 &= \left(1 - \frac{C_l^*}{C_l}\right)\left(\alpha C_l^* C_c^* - \alpha C_l C_c\right) + \left(1 - \frac{C_l^*}{C_l}\right)\left(\sigma C_l^* - C_l \sigma\right) + \left(1 - \frac{C_l^*}{C_l}\right)\left(\mu C_l^* - \mu C_l\right) - \left(1 - \frac{C_l^*}{C_l}\right)\left(\rho C_p^* - \rho C_p\right), \\
\dot{G}_1 &= \alpha C_l^* C_c^* \left(1 - \frac{C_l^*}{C_l}\right)\left(1 - \frac{C_l C_c}{C_l^* C_c^*}\right) - (\sigma + \mu)\frac{\left(C_l^* - C_l\right)^2}{C_l} - \rho C_p^* \left(1 - \frac{C_l^*}{C_l}\right)\left(1 - \frac{C_p}{C_p^*}\right), \\
\dot{G}_1 &\leq \alpha C_l^* C_c^* \left(1 - \frac{C_l C_c}{C_l^* C_c^*} + \ln \frac{C_l C_c}{C_l^* C_c^*}\right) = a_{12} g_{12}.
\end{aligned}$$

Similarly

$$\begin{aligned}
&\dot{G}_2 \leq \alpha C_l^* C_c^* \left(\frac{C_l C_c}{C_l^* C_c^*} - \ln \frac{C_l C_c}{C_l^* C_c^*} - 1\right) + \nu C_p^* \left(1 + \ln \frac{C_c^* C_p}{C_c C_p^*} - \frac{C_c^*}{C_c}\frac{C_p}{C_p^*}\right) = a_{21} g_{21} + a_{23} g_{23}, \\
&\dot{G}_3 \leq \epsilon C_c^* \left(1 + \ln \frac{C_p^* C_c}{C_p C_c^*} - \frac{C_p^*}{C_p}\frac{C_c}{C_c^*}\right) = a_{34} g_{34}, \\
&\dot{G}_4 \leq \gamma C_c^* \left(1 + \ln \frac{C_a^* C_c}{C_a C_c^*} - \frac{C_a^*}{C_a}\frac{C_c}{C_c^*}\right) + \eta C_p^* \left(1 + \ln \frac{C_a^* C_p}{C_a C_p^*} - \frac{C_a^*}{C_a}\frac{C_p}{C_p^*}\right) = a_{43} g_{43} + a_{45} g_{45}, \\
&\dot{G}_5 \leq \sigma C_l^* \left(1 + \ln \frac{C_l}{C_l^*}\frac{C_n^*}{C_n} - \frac{C_l}{C_l^*}\frac{C_n^*}{C_n}\right) + \delta C_c^* \left(1 + \ln \frac{C_n^*}{C_n}\frac{C_c}{C_c^*} - \frac{C_n^*}{C_n}\frac{C_c}{C_c^*}\right) = a_{52} g_{52} + a_{54} g_{54}.
\end{aligned}$$

The associated weighted graph, as depicted in the figure, has five vertices and three cycles such that $G_{12} + G_{21} = 0$, $G_{34} + G_{43}$, and $G_{45} + G_{54} = 0$. Then, there exists c_i, $1 \leq i \leq 5$ such that

$$\mathcal{G} = \sum_{i=1}^{5} c_i G_i,$$

is Lyapunov function. By using Lemma (4.4), articulated in [31]

$$c_1 a_{12} = c_2 a_{21},$$
$$c_3 a_{34} = c_4 a_{43} + c_2 a_{23},$$
$$c_4 a_{45} = c_5 (a_{52} + a_{54}),$$

then the values

$$C_1 = C_2 = 1, \quad C_3 = \frac{2\nu}{(\rho + \eta + \nu + \mu)}, \quad C_4 = \frac{\nu\epsilon}{\gamma(\rho + \eta + \nu + \mu)},$$
$$C_5 = \frac{\eta\nu\epsilon^2 C_c^* (\sigma + \mu + \alpha C_c^*)}{\gamma(\rho + \eta + \nu + \mu)(\sigma(\beta\mu + \beta\nu + \beta\eta + \beta\rho + \epsilon\rho C_c^*) + \delta C_c^* (\sigma + \mu + \alpha C_c^*)(\mu + \nu + \eta + \rho))},$$

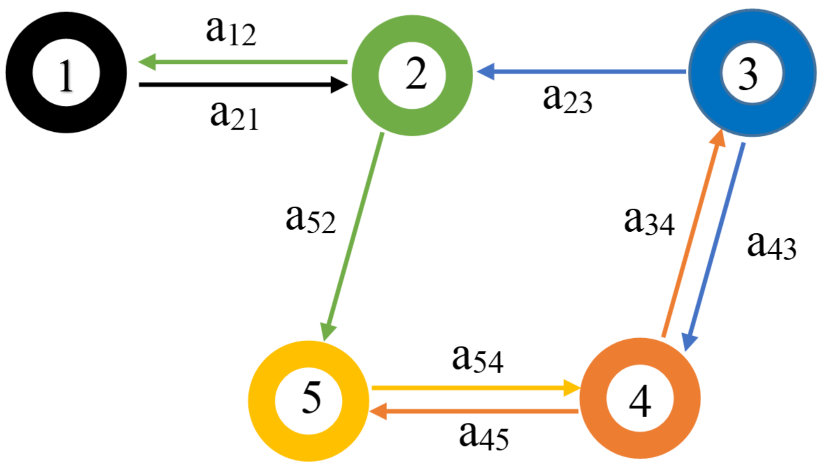


Figure 3. Weighted Graph

then it's evident that

$$\mathcal{G} = \sum_{i=1}^{5} c_i G_i \leq 0.$$

The LaSalle's principle of invariance [29] confirms that $\mathcal{E}_e^*$ is asymptotically stable globally in the region $\mathcal{U}$.

□

3. Cleaners Behavior in Community

This section calculates cleaning reproduction based on the parameters fitted to the model. The. The sensitivity of the parameters utilized is analyzed. The behavior of each cleaner class is also examined over time.

3.1. **Cleaning Reproduction Number.** The cleaning reproduction number R_c is a critical metric for quantifying the individuals motivated toward cleanliness. It represents the average number of secondary cleaners motivated by a single candidate cleaner in a population where everyone is a latent cleaner. It is calculated by next-generation matrix method [9] articulated as:

$$R_c = \frac{\alpha\beta(\mu+\nu+\eta+\rho)}{(\sigma+\mu)(\mu^2+\gamma\mu+\gamma\nu+\mu\delta+\mu\nu+\mu\epsilon+\delta\nu+\gamma\eta+\mu\eta+\gamma\rho+\mu\rho+\delta\eta+\delta\rho+\epsilon\eta+\epsilon\rho)}.$$

The estimated numerical value of the cleaning reproduction number is 1.55821.

3.2. **Sensitivity Analysis.** Sensitivity analysis is essential across engineering, economics, and environmental science. It helps determine how uncertainties in a model's output relate to input uncertainties, identifying which parameters most affect output variability. This analysis supports model validation, uncertainty quantification, and decision-making by prioritizing efforts to enhance predictive accuracy and robustness.

3.2.1. *Sensitivity Indices.* Sensitivity indices quantify how changes in model parameters affect model outcomes. The sensitivity indices are calculated by the formula $S_{R_c}^{p} = \frac{\partial R_c}{\partial p} \times \frac{p}{R_c}$ as utilized in [?]. The sensitivity indices are articulated in the Table 2 and visualized in Figure 4.

It can be observed that the parameters α and β have a direct relationship with the cleaning reproduction number, both having a sensitivity index of +1. β represents the individual recruitment rate into the latent class, but it is noteworthy that α possesses a high sensitivity index, indicating that more people are being motivated toward cleanliness. In contrast, σ and δ, which represent the transition rates from the latent and candidate classes to the non-cleaner class that becomes ill, respectively, have an inverse relationship with the cleaning reproduction number. This signifies that reducing the rates σ and δ is necessary to enhance cleanliness and develop a disease-free environment.

Table 2. Sensitivity Indices

Notations	Description	Indices	Sign
β	The rate of recruitment of cleaners	1	+ve
μ	The rate of mortality of cleaners	-0.0448531	-ve
α	The rate at which latent individuals become candidate cleaners	1	+ve
σ	The rate at which candidate cleaner becomes non-cleaner	-0.971404	-ve
γ	The rate at which candidate cleaners becomes active cleaners	-0.198147	-ve
ϵ	The rate at which candidate cleaners becomes passive cleaners	-0.191831	-ve
δ	The rate at which candidate cleaners becomes non cleaners	-0.59444	-ve
ν	The rate at which passive cleaners become candidate cleaners	0.0414941	+ve
ρ	The rate at which passive cleaners become latent cleaners	-0.011377	-ve
η	The rate at which passive cleaners become active cleaners	-0.0284425	-ve

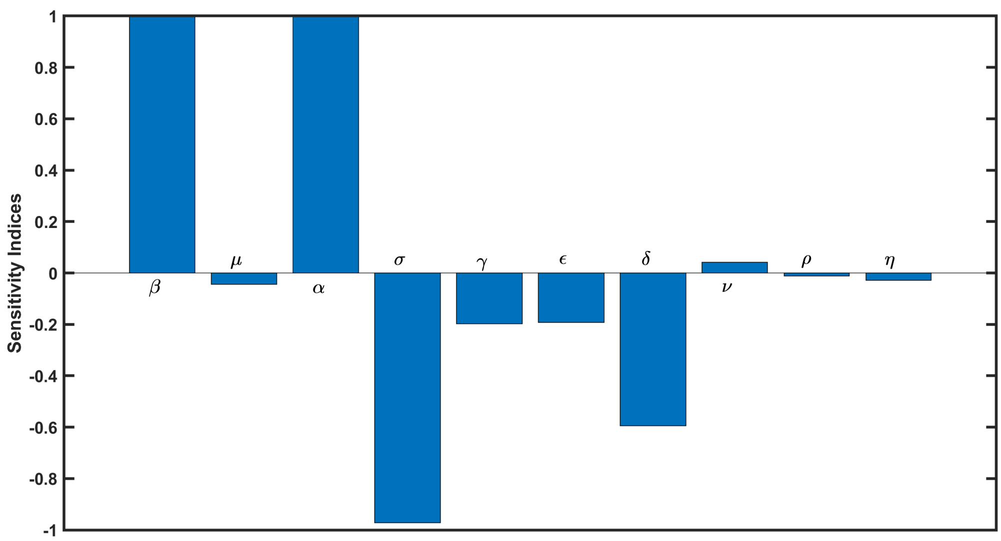
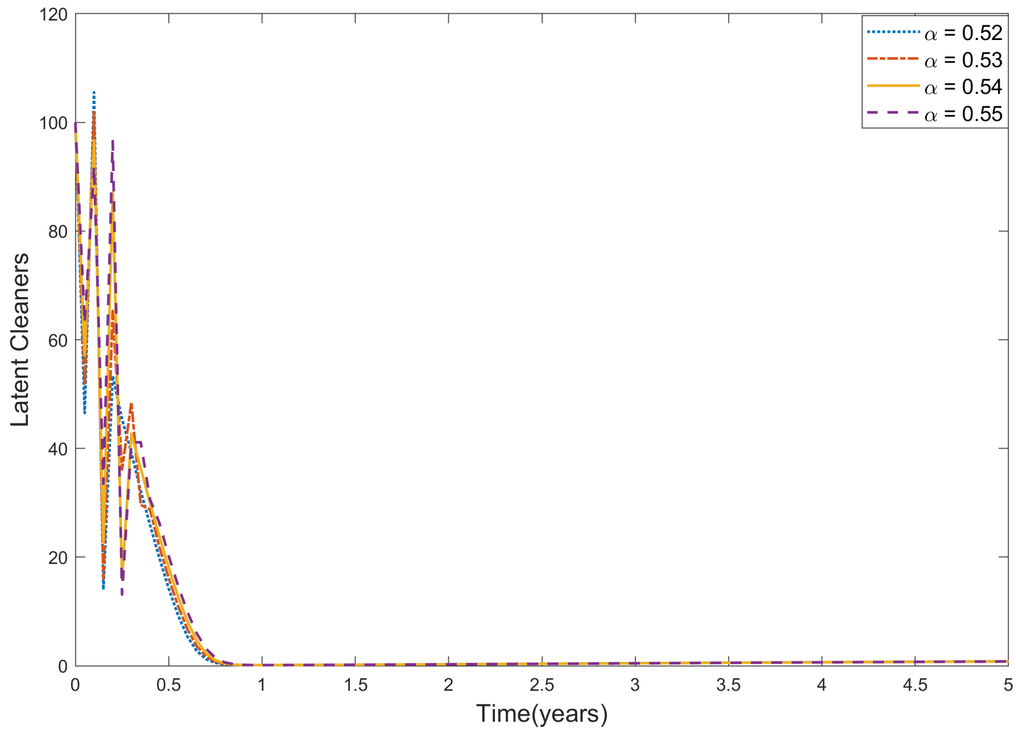
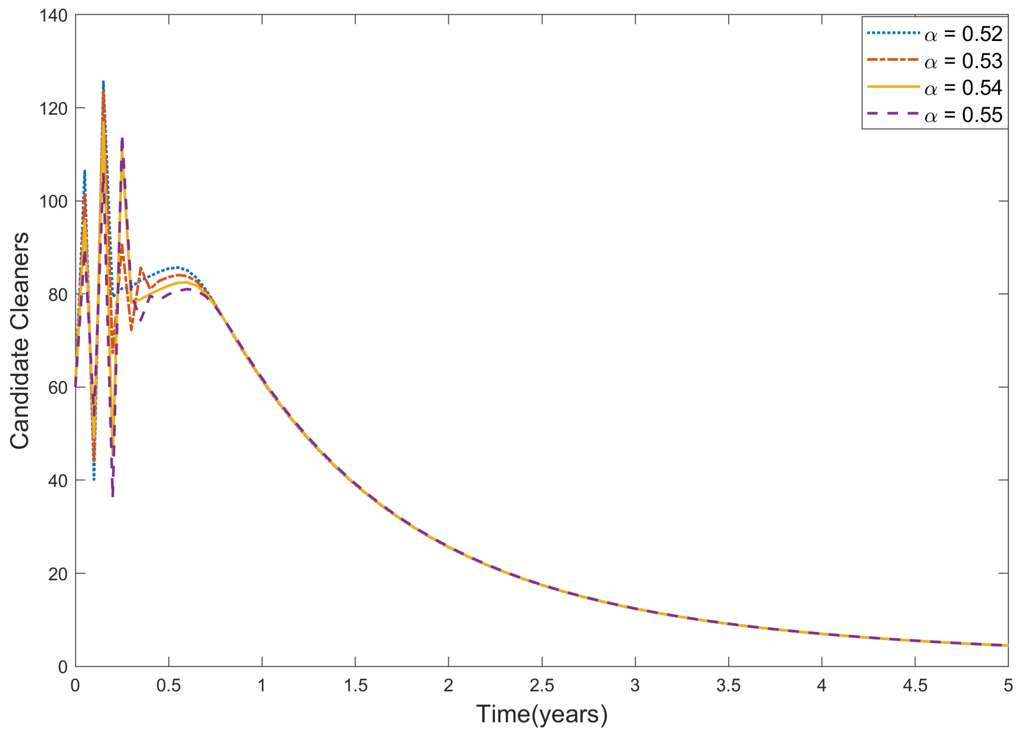


Figure 4. Plot of Sensitivity Indices

3.3. **Simulation of Cleaners Behavior.** The model's parameter α represents the rate at which people adopt cleaning behaviors, and it directly affects the dynamics of different cleaner categories, such as latent, candidate, passive, active, and non-cleaners. The following visuals demonstrate these behaviors.

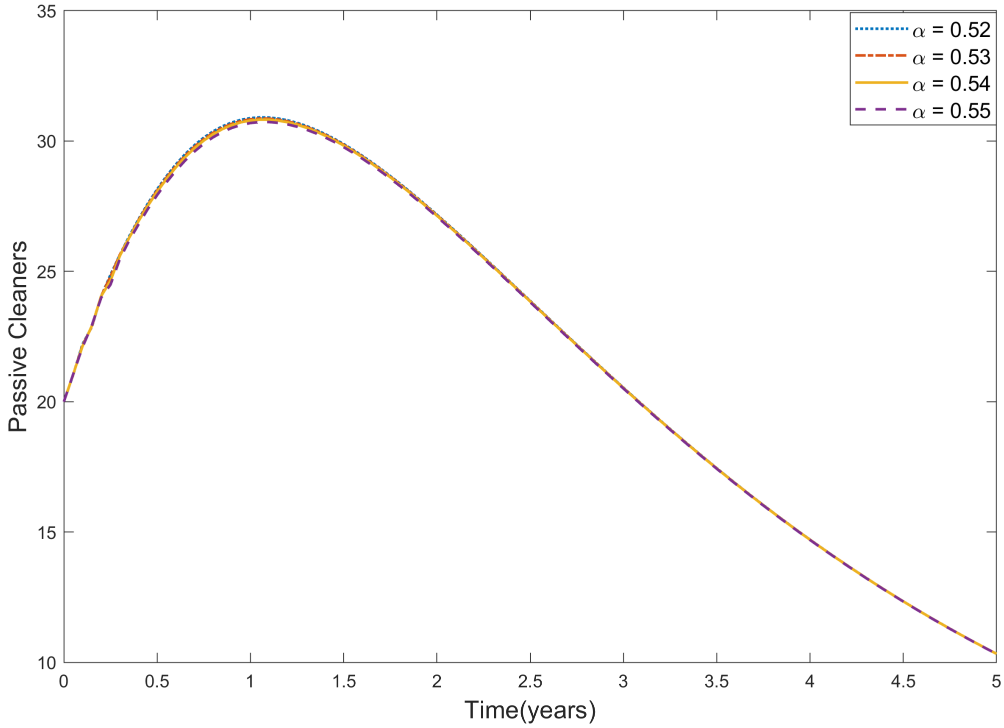


(A) Latent Cleaners

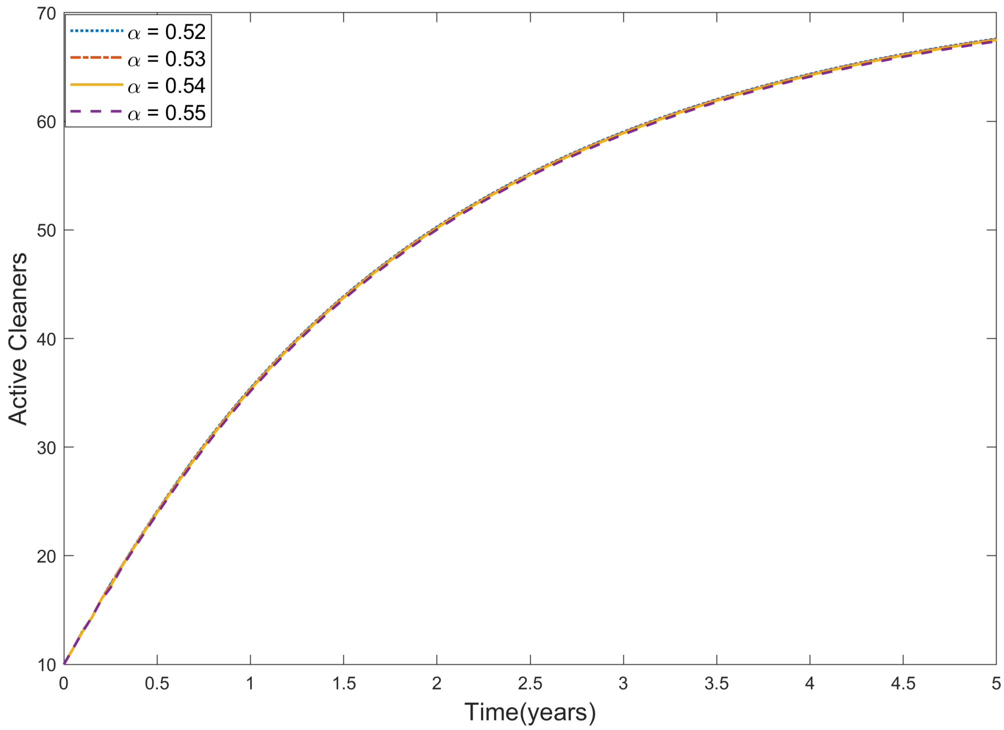


(B) Candidate Cleaners

FIGURE 5. Cleaners' behavior in the community

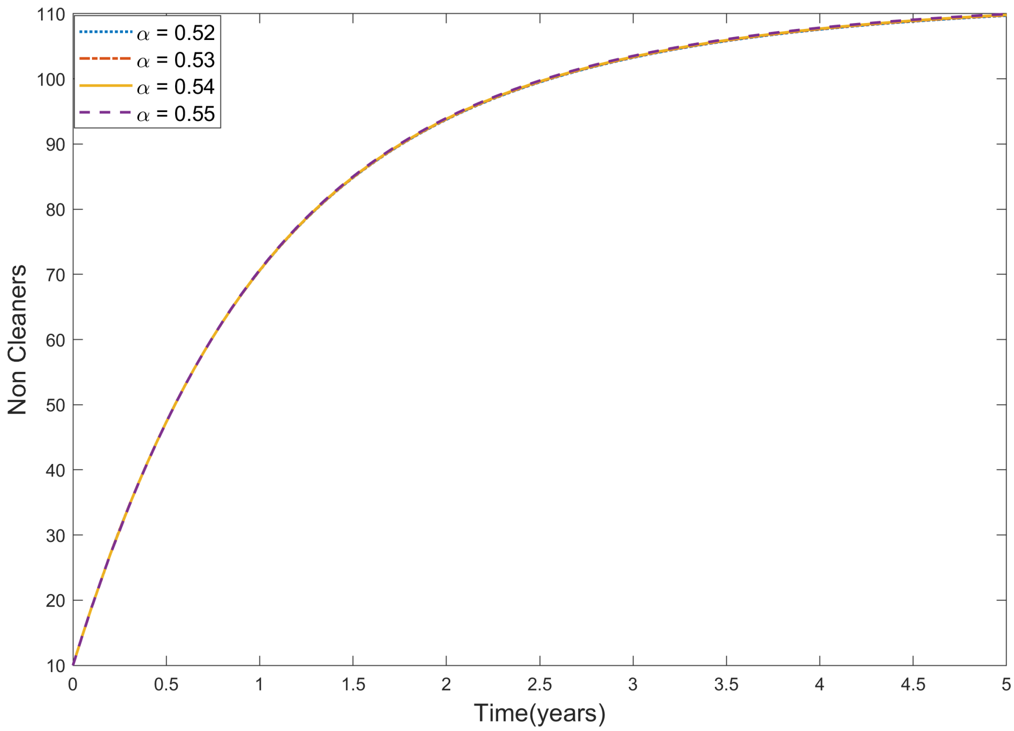


(c) Passive Cleaners

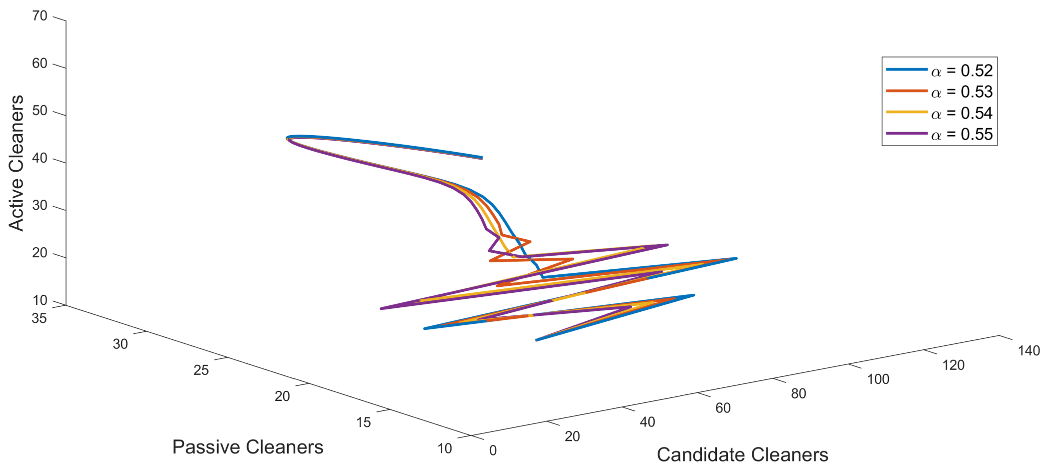


(d) Active Cleaners

Figure 5 (continued): Cleaners' behavior in the community

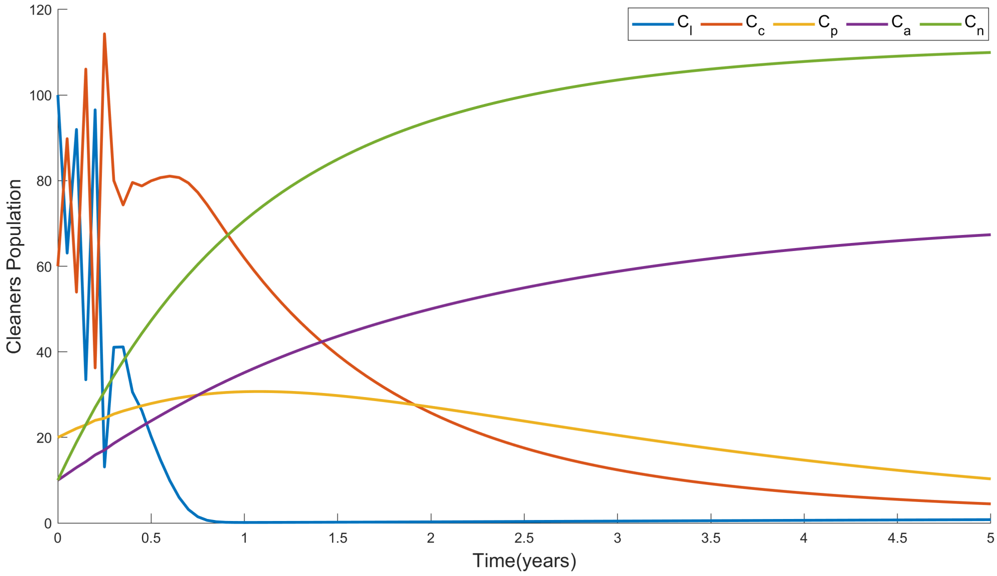


(E) Non-Cleaners



(F) Profile of candidate, passive and active Cleaners

Figure 5 (continued): Cleaners' behavior in the community



(G) Profile of entire cleaners Community

Figure 5 (continued): Cleaners' behavior in the community

In Figure 5a, the impact of α on the latent individuals is evident, with fluctuations observed within the first year. As time progresses, the influence of α diminishes, decreasing the latent population. Likewise, Figure 5b illustrates how α affects the behavior of candidate cleaners, showcasing fluctuations over a year followed by a gradual decrease in their numbers. This indicates that the rate of adoption of cleaning behaviors has a temporal influence on the population dynamics of candidates.

Meanwhile, Figures 5c and 5d demonstrate the contrasting trajectories of passive and active cleaners. Passive cleaners steadily increase over time, peaking at around one and a half years before declining gradually. On the other hand, active cleaners exhibit a more significant increase over time, suggesting a positive response to hygiene initiatives and awareness.

Interestingly, despite these positive trends, Figure 5e highlights the concerning rise in non-cleaners, individuals who remain unaffected by the rate α and show no motivation towards adopting cleaner practices. Figure 5f compares Candidate, Passive, and Active cleaners with the effect of motivation rate α. While Figure 5g visualizes the behavior of the entire cleaners population in the community. This underscores the ongoing challenge of motivating individuals towards hygiene and the importance of targeted interventions to address this population segment.

4. Conclusion

This study analyzes a Go-Clean and Go-Healthy approach by developing a novel mathematical model of five compartments: Latent cleaners, Candidate cleaners, Passive cleaners, Active cleaners, and Non-cleaners who get ill. The conditions under which the cleanliness present equilibrium achieves local and global asymptotic stability when the cleaning reproductive number exceeds one is analyzed. It has been verified using the Routh-Hurwitz criteria that a positive, unique cleanliness present equilibrium has local asymptotic stability. By a graph-theoretic approach, it also has been proved that the cleanliness equilibrium exhibits global asymptotic stability. Parameter estimation has been carried out to determine the numerical value of the cleaning reproductive number. A sensitivity analysis is being conducted to identify the factors that discourage individuals from maintaining cleanliness and contribute to creating an infectious environment. Further, the dynamics of cleaner behavior in the community are analyzed, providing valuable insights for evidence-based policies and interventions promoting a clean and healthier environment.

In summary, this study advances our knowledge of the dynamics of cleaner behavior in communities. It emphasizes the significance of focused interventions to support cleanliness, which reduces the risk of infectious diseases and improves public health results.

Future work focuses on refining the mathematical model with spatial and dynamic behavioral factors, validating and calibrating using real-world data, conducting simulations to evaluate intervention strategies, integrating social and psychological factors, translating findings into evidence-based policies, fostering interdisciplinary collaborations, and exploring technology for real-time data collection and intervention assessment, aiming to understand and promote cleaner behavior dynamics for improved community health.

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