

**Multi Q-Single Valued Neutrosophic Soft Expert Set and its Application in Decision Making**

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**Abstract.** Smarandache introduced the concept of neutrosophic set which is the generalization of fuzzy set, intuitionistic fuzzy set and a better mathematical tool to handle incomplete, inconsistency and vague information. In this work, we propose the concept of multi Q-single valued neutrosophic soft expert set and its basic operations such as, union, intersection, complement, AND and OR. Further, we construct an algorithm for decision-making method on multi Q-SVNSEs. Finally, an example is provided to show the practicality and effectiveness of the proposed algorithm which consist of indeterminate and inconsistent information. At the end, a comparison has been made through an example with the existing theory.

**AMS (MOS) Subject Classification Codes:** 0000,0003; 1049,430X

**Key Words:** Multi Q-fuzzy soft set, multi Q-single valued neutrosophic soft set, multi Q-single valued neutrosophic soft expert set, decision making.

## 1. INTRODUCTION

In our daily life most of problems in social sciences, management, engineering and medical sciences usually include data which are not surely crisp, accurate and deterministic in character due to variant uncertainties associated corresponding with these problems. Mostly such problems were solved by using fuzzy set introduced by Zadeh[60]. In Zadeh fuzzy set only membership function was considered and the non-membership function comes automatically. After fuzzy set Atanassov [15] proposed the concept of intuitionistic fuzzy set which is the extension of fuzzy set by introducing membership and non-membership functions and the hesitation function by subtracting the sum of membership and non-membership function from one. But the only limitation in this concept is that the hesitation degree can not be defined independently. To overcome this limitation, Smarandache [49, 50, 51] introduced the concept of neutrosophic set which is the generalization of fuzzy set and intuitionistic fuzzy set. Neutrosophic set is characterized by truth-membership, indeterminacy-membership and falsity-membership functions with the

condition that the sum of maximum of the three function is less or equal to three. After the introduction of neutrosophic sets many researcher proposed subclasses of neutrosophic sets and apply them to various areas [58, 59, 54, 55]. Liu et al. [30] and Khan et al. [27, 28] developed some power linguistic Heronian mean operators, power Bonferroni mean operator and power Muirhead mean operators for linguistic neutrosophic sets, interval neutrosophic sets and neutrosophic cubic sets and applied them to multiple attribute decision making and multiple attribute group decision making. Some other generalization of fuzzy sets were studied by many authors and give its applications in different field [47, 40, 48, 22, 26, 7].

The concept of soft set theory was first presented by Molodtsov [37] in 1999, which was a new concept of handling uncertainties and ambiguity. After the introduction of soft set theory Maji et al. [34, 35, 46, 33] presented the concept of fuzzy soft set and intuitionistic fuzzy soft set and proposed some basic operations and studied their related properties. Further, Fuzzy soft set theory, intuitionistic fuzzy soft set theory and fuzzy soft matrix theory were studied by Cagman et al. and Ahmad et al. [2, 18, 19, 20]. M. Riaz et al. [42, 43, 44] studied certain properties of bipolar fuzzy soft topology via  $Q$ -neighborhood, developed measurable soft mappings and fuzzy parametrized soft compact spaces with decision making. S. Roy et al [45] construct soft topology, soft base, soft subbase and discussed some related theorems. H. Kamaci et al [24] proposed difference operations for soft matrices and give its application in decision making. Yang et al. [57] combined soft set with interval valued fuzzy set and proposed the concept of interval valued soft set and gave some basic operations such as complement, AND and OR and prove De Morgan law for IVFSSs. Intuitionistic fuzzy soft set and interval valued intuitionistic fuzzy soft set were studied by Xu et al. and Jiang et al. [23, 56]. Maji [36] proposed the concept of neutrosophic soft set by combining soft set with neutrosophic set, proposed basic operational laws for it and studied their related properties. Alkhazaleh et al proposed the concept of possibility fuzzy soft set [9] and proposed operational rules and studied their basic properties. They also give applications of possibility soft set in decision-making. Alkhazaleh et al. and Zhu et al [10, 61] presented the concepts of fuzzy parametrized fuzzy soft set and fuzzy parametrized IVFSS and give its applications in decision-making. Broumi et al. [16] proposed neutrosophic parametrized soft set and give it application in decision-making. Alkhazaleh et al proposed the concept of soft multi-sets [11] which was the generalization of soft set proposed by Molodtsov, followed by the concepts of fuzzy soft multi-set [12], multi  $Q$ -fuzzy soft set [3], multi  $Q$ -intuitionistic fuzzy soft set [17], multi  $Q$ -single valued neutrosophic soft set [31]. Further, multi  $Q$ -fuzzy soft expert set was introduced by Adam et al. [6, 4, 5], proposed some operational laws such as, union, complement, intersection, AND and OR operation for it, and also give application in decision making. In recent years Mahmood et al. [32] proposed some generalized aggregation operators for cubic hesitant fuzzy sets and give its application in decision making. Voskoglou [52] give application of fuzzy number to human skills. Abbas et al. [1] proposed upper and lower contra-continuous fuzzy multi-functions. Othman [38] discussed the concept of fuzzy infra-semiopen set. Roa et al. [41] discussed general fixed point theorems in fuzzy metric spaces. Khan et al. [25] discussed Common Fixed Point Theorems for Converse Commuting and OWC Maps in Fuzzy Metric Spaces.

The aim of this article is to propose the concept of multi  $Q$ -single valued neutrosophic soft expert set [13] by extending the concept of expert set, defined some basic operational

laws such as complement, union, intersection, AND and OR along with illustrative example. We further give the applications in decision-making. Finally a comparison has been made with the existing theory of multi Q-fuzzy soft expert set through example.

## 2. PRELIMINARIES

In this section, some basic definitions about soft expert set, multi Q-single valued neutrosophic soft set.

Let  $U, E, X$  be respectively, universe of discourse set, set of parameters and set of experts. Let  $O$  be the set of opinions,  $Z = E \times X \times O$  and  $A \subseteq Z$ .

**Definition 1.** [13] A soft expert set over  $U$  is a pair  $(F, A)$ . Where  $F$  is a function given by  $F : A \rightarrow P(U)$ ,  $P(U)$  represent the power set of  $U$ .

**Definition 2.** [31] Let  $X$  be a universal set and  $Q \neq \emptyset$ . A Q-SVNS  $\tilde{N}_Q$  in  $X$  and  $Q$  is defined as,

$$\tilde{N}_Q = \{(\hat{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\hat{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(\hat{\theta}, \hat{u}), \xi_{\tilde{N}_Q}(\hat{\theta}, \hat{u}) : \hat{\theta} \in X, \hat{u} \in Q\}$$

Where  $\mu_{\tilde{N}_Q} : X \times Q \rightarrow [0, 1], \nu_{\tilde{N}_Q} : X \times Q \rightarrow [0, 1], \xi_{\tilde{N}_Q} : X \times Q \rightarrow [0, 1]$ , are respectively truth-membership, indeterminacy-membership and falsity membership functions for every  $\hat{\theta} \in X$  and  $\hat{u} \in Q$  and satisfy the condition  $0 \leq \mu_{\tilde{N}_Q}(\hat{\theta}, \hat{u}) + \nu_{\tilde{N}_Q}(\hat{\theta}, \hat{u}) + \xi_{\tilde{N}_Q}(\hat{\theta}, \hat{u}) \leq 3$ . The set of all Q-SVNS is denoted by  $M^K QSVN(X)$ .

**Definition 3.** [31] Let  $X$  be a universal set,  $Q \neq \emptyset$  and  $\tilde{N}_Q$  be a Q-SVNS. The complement of  $\tilde{N}_Q$  is denoted and defined as follows:

$$\tilde{N}_Q^c = \{(\hat{\theta}, \hat{u}), \xi_{\tilde{N}_Q}(\hat{\theta}, \hat{u}), 1 - \nu_{\tilde{N}_Q}(\hat{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\hat{\theta}, \hat{u}) : \hat{\theta} \in X, \hat{u} \in Q\}$$

**Definition 4.** [31] Let  $\tilde{A}_Q$  and  $\tilde{N}_Q$  be two Q-SVNS. Then the union and intersection is denoted and defined by

$$\tilde{A}_Q \cup \tilde{N}_Q = \left\{ \begin{array}{l} (\hat{\theta}, \hat{u}), \max(\mu_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\hat{\theta}, \hat{u})), \min(\nu_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(\hat{\theta}, \hat{u})), \\ \min(\xi_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \xi_{\tilde{N}_Q}(\hat{\theta}, \hat{u})) : \hat{\theta} \in X, \hat{u} \in Q \end{array} \right\}$$

$$\tilde{A}_Q \cap \tilde{N}_Q = \left\{ \begin{array}{l} (\hat{\theta}, \hat{u}), \min(\mu_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \mu_{\tilde{N}_Q}(\hat{\theta}, \hat{u})), \max(\nu_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \nu_{\tilde{N}_Q}(\hat{\theta}, \hat{u})), \\ \max(\xi_{\tilde{A}_Q}(\hat{\theta}, \hat{u}), \xi_{\tilde{N}_Q}(\hat{\theta}, \hat{u})) : \hat{\theta} \in X, \hat{u} \in Q \end{array} \right\}$$

**Definition 5.** [31] Let  $X$  be a non-empty set and  $Q$  be any non-empty set,  $l$  be any positive integer and  $I$  be a unit interval  $[0, 1]$ . A multi Q-SVNS  $\tilde{A}_Q$  in  $X$  and  $Q$  is a set of ordered sequences

$$\tilde{A}_Q = \{(\hat{\theta}, \hat{u}), \mu_j(\hat{\theta}, \hat{u}), \nu_j(\hat{\theta}, \hat{u}), \xi_j(\hat{\theta}, \hat{u}) : \hat{\theta} \in X, \hat{u} \in Q \text{ for all } j = 1, 2, \dots, l\}$$

Where  $\mu_j : X \times Q \rightarrow I^k, \nu_j : X \times Q \rightarrow I^k, \xi_j : X \times Q \rightarrow I^k$ , for all  $j = 1, 2, \dots, l$  and are respectively truth-membership, indeterminacy-membership and falsity membership functions for each  $\hat{\theta} \in X$  and  $\hat{u} \in Q$  and satisfy the condition  $0 \leq \mu_j(\hat{\theta}, \hat{u}) + \nu_j(\hat{\theta}, \hat{u}) + \xi_j(\hat{\theta}, \hat{u}) \leq 3$ , for all  $j = 1, 2, \dots, l$ . The functions  $\mu_j(\hat{\theta}, \hat{u}), \nu_j(\hat{\theta}, \hat{u}), \xi_j(\hat{\theta}, \hat{u})$  for

all  $j = 1, 2, \dots, l$  are called the truth-membership, indeterminacy-membership and falsity-membership functions respectively of the multi Q-SVNS  $\tilde{A}_Q$  and satisfy the condition  $0 \leq \mu_j(\hat{\theta}, \hat{u}) + \nu_j(\hat{\theta}, \hat{u}) + \xi_j(\hat{\theta}, \hat{u}) \leq 3$ , for all  $j = 1, 2, \dots, l$ .  $l$  is called the dimension of the Q-SVNS  $\tilde{A}_Q$ . The set of all Q-SVNS is denoted by  $Z^K QSVNS(X)$ .

### 3. MULTI Q-SINGLE VALUED NEUTROSOPHIC SOFT EXPERT SET

**Definition 6.** A multi Q-single valued neutrosophic soft expert set over  $U$  is a pair  $(F_Q, A)$ , where  $F_Q$  is a function given by

$$F_Q : A \rightarrow M^K QSVN(U)$$

such that  $M^K QSVN(U)$  denotes the set of all multi Q-single valued neutrosophic set over  $U$ .

Assume that  $F_Q : A \rightarrow M^K QSVN(U)$  is a function defined as  $F(x) = F(x)(q, u), \forall q \in Q, u \in U$ . For each  $x_i \in A, F(x_i) = F(x_i)(u, q)$ . Where  $F(x_i)$  represents the degree of truth-membership, degree of indeterminacy and falsity-membership of the elements of  $U$  in  $F(x_i)$ . Hence  $F(x_i)$  can be written as,

$$F(x_i)(u_i, q) = \{(u_i, q), F(x_i)(u_i, q)\} \text{ for } i = 1, 2, 3, \dots, m$$

Where,  $F(x_i)(u_i, q) = \langle \mu_j(u_i, q), \nu_j(u_i, q), \xi_j(u_i, q) \rangle$ , in which  $\mu_j(u_i, q), \nu_j(u_i, q)$  and  $\xi_j(u_i, q)$  respectively representing the truth-membership, indeterminacy-membership and falsity-membership degrees of each element  $u_i \in U$ .

Suppose that a professor wants to select three students for a research project from PhD and MS students, and wants to take the opinion of the two expert professors of these classes about these students. Let  $U = \{u_1, u_2, u_3\}$  be the set of students and  $Q = \{q_1, q_2\}$  be the set of students. Let the decision parameter under consideration is  $E = \{e_1 = \text{Hardworking}, e_2 = \text{Intelligence}\}$  and  $X = \{p, q\}$  be the set of expert professors of their classes. Suppose that

$$F_Q(e_1, p, 1) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right\}$$

$$F_Q(e_1, q, 1) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.3, 0.2, 0.4), (0.4, 0.1, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.2), (0.6, 0.1, 0.1), (0.7, 0.0, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.5), (0.3, 0.2, 0.3), (0.5, 0.1, 0.4) \rangle, \\ \langle (u_2, q_2), (0.9, 0.1, 0.1), (0.7, 0.2, 0.1), (0.5, 0.2, 0.1) \rangle, \\ \langle (u_3, q_1), (0.8, 0.1, 0.1), (0.4, 0.2, 0.1), (0.3, 0.2, 0.4) \rangle, \\ \langle (u_3, q_3), (0.6, 0.2, 0.1), (0.7, 0.2, 0.2), (0.8, 0.0, 0.1) \rangle \end{array} \right\}$$

$$F_Q(e_2, p, 1) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.2, 0.1, 0.4), (0.4, 0.2, 0.1), (0.5, 0.2, 0.3) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.1), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.3), (0.3, 0.2, 0.4), (0.4, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.1), (0.2, 0.1, 0.2), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_1), (0.4, 0.1, 0.4), (0.5, 0.2, 0.1), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_3, q_3), (0.2, 0.4, 0.3), (0.4, 0.3, 0.4), (0.5, 0.2, 0.1) \rangle \end{array} \right\}$$

$$F_Q(e_2, q, 1) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.3, 0.4), (0.2, 0.4, 0.4), (0.4, 0.2, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.3, 0.4), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.1), (0.6, 0.2, 0.3), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.3, 0.2), (0.4, 0.1, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.4, 0.2, 0.1), (0.6, 0.1, 0.2), (0.7, 0.1, 0.1) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.3), (0.5, 0.1, 0.1), (0.6, 0.2, 0.3) \rangle \end{array} \right\}$$

$$F_Q(e_1, p, 0) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right\}$$

$$F_Q(e_1, q, 0) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.3, 0.3, 0.4), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.2), (0.8, 0.0, 0.1), (0.9, 0.1, 0.1) \rangle \end{array} \right\}$$

$$F_Q(e_2, p, 0) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right\}$$

$$F_Q(e_2, q, 0) = \left\{ \begin{array}{l} \langle (u_1, q_1), (0.3, 0.2, 0.2), (0.4, 0.3, 0.2), (0.5, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.1), (0.5, 0.2, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_1), (0.6, 0.1, 0.3), (0.7, 0.1, 0.1), (0.8, 0.2, 0.1) \rangle, \\ \langle (u_2, q_2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3), (0.8, 0.0, 0.1) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.3, 0.2, 0.3), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle \end{array} \right\}$$

Then the multi Q-SVNSES  $(F_Q, E)$  consisting of the following group of approximations.

$$(F_Q, E) = \left( \left( F_Q(e_1, p, 1), \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right\} \right) \right), \\ \left( F_Q(e_1, p, 0), \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right\} \right) \right),$$

Each element of the multi Q-SVNSES implies the opinion of each expert professors based on each parameter about the students with their own classes.

Further, we shall define the definitions of agree, disagree, equal and subsets multi Q-SVNSESs.

**Definition 7.** Let  $(F_Q, A)$  and  $(G_Q, B)$  be two multi QSVNSESs. Then  $(F_Q, A)$  is said to be multi Q-SVNSE subset of  $(G_Q, B)$ , denoted by  $(F_Q, A) \subseteq (G_Q, B)$ , if  $A \subseteq B$  and  $F_Q(x) \subseteq G_Q(x)$ ,  $\forall x \in A$ .

**Definition 8.** Let  $(F_Q, A)$  and  $(G_Q, B)$  be two multi QSVNSESs. Then  $(F_Q, A)$  is said to be equal to  $(G_Q, B)$ , denoted by  $(F_Q, A) = (G_Q, B)$ , if  $(F_Q, A) \subseteq (G_Q, B)$  and  $(G_Q, B) \subseteq (F_Q, A)$ .

**Definition 9.** An agree-multi Q-SVNSES  $(F_Q, A)_1$  over  $U$  and  $Q$  is a multi Q-SVNSE subset of the multi Q-SVNSES  $(F_Q, A)$  defined as follows:

$$(F_Q, A)_1 = \{F_{Q_1}(\eta) : \eta \in E \times X \times \{1\}\}.$$

**Definition 10.** A disagree-multi Q-SVNSES  $(F_Q, A)_0$  over  $U$  and  $Q$  is a multi Q-SVNSE subset of the multi Q-SVNSES  $(F_Q, A)$  defined as follows:

$$(F_Q, A)_0 = \{F_{Q_1}(\eta) : \eta \in E \times X \times \{0\}\}.$$

**Example 1.** Consider Example (3). The agree-multi Q-SVNSES  $(F_Q, E)_1$  over  $U$  and  $Q$  is,

$$\left( F_Q(e_1, p, 1), \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right\} \right)$$

$$\left( \begin{array}{l} F_Q(e_1, q, 1), \\ F_Q(e_2, p, 1), \\ F_Q(e_2, q, 1), \end{array} \left\{ \begin{array}{l} \langle (u_1, q_1), (0.3, 0.2, 0.4), (0.4, 0.1, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.2), (0.6, 0.1, 0.1), (0.7, 0.0, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.5), (0.3, 0.2, 0.3), (0.5, 0.1, 0.4) \rangle, \\ \langle (u_2, q_2), (0.9, 0.1, 0.1), (0.7, 0.2, 0.1), (0.5, 0.2, 0.1) \rangle, \\ \langle (u_3, q_1), (0.8, 0.1, 0.1), (0.4, 0.2, 0.1), (0.3, 0.2, 0.4) \rangle, \\ \langle (u_3, q_3), (0.6, 0.2, 0.1), (0.7, 0.2, 0.2), (0.8, 0.0, 0.1) \rangle \end{array} \right. \right)$$

$$\left( \begin{array}{l} F_Q(e_2, p, 1), \\ F_Q(e_2, q, 1), \end{array} \left\{ \begin{array}{l} \langle (u_1, q_1), (0.2, 0.1, 0.4), (0.4, 0.2, 0.1), (0.5, 0.2, 0.3) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.1), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.3), (0.3, 0.2, 0.4), (0.4, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.1), (0.2, 0.1, 0.2), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_1), (0.4, 0.1, 0.4), (0.5, 0.2, 0.1), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_3, q_3), (0.2, 0.4, 0.3), (0.4, 0.3, 0.4), (0.5, 0.2, 0.1) \rangle \end{array} \right. \right)$$

$$\left( \begin{array}{l} F_Q(e_2, q, 1), \end{array} \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.3, 0.4), (0.2, 0.4, 0.4), (0.4, 0.2, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.3, 0.4), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.1), (0.6, 0.2, 0.3), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.3, 0.2), (0.4, 0.1, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.4, 0.2, 0.1), (0.6, 0.1, 0.2), (0.7, 0.1, 0.1) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.3), (0.5, 0.1, 0.1), (0.6, 0.2, 0.3) \rangle \end{array} \right. \right)$$

And the disagree-multi Q-SVNSES  $(F_Q, E)_0$  over  $U$  and  $Q$  is,

$$\left( \begin{array}{l} F_Q(e_1, p, 0), \\ F_Q(e_1, q, 0), \\ F_Q(e_2, p, 0), \\ F_Q(e_2, q, 0), \end{array} \left\{ \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right. \right)$$

$$\left( \begin{array}{l} F_Q(e_1, q, 0), \\ F_Q(e_2, p, 0), \\ F_Q(e_2, q, 0), \end{array} \left\{ \begin{array}{l} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.3, 0.3, 0.4), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.2), (0.8, 0.0, 0.1), (0.9, 0.1, 0.1) \rangle \end{array} \right. \right)$$

$$\left( \begin{array}{l} F_Q(e_2, p, 0), \\ F_Q(e_2, q, 0), \end{array} \left\{ \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right. \right)$$

$$\left( \begin{array}{l} F_Q(e_2, q, 0), \end{array} \left\{ \begin{array}{l} \langle (u_1, q_1), (0.3, 0.2, 0.2), (0.4, 0.3, 0.2), (0.5, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.1), (0.5, 0.2, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_1), (0.6, 0.1, 0.3), (0.7, 0.1, 0.1), (0.8, 0.2, 0.1) \rangle, \\ \langle (u_2, q_2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3), (0.8, 0.0, 0.1) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.3, 0.2, 0.3), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle \end{array} \right. \right)$$

Further we shall define the definition of complement of multi Q-SVNSES and it related proposition.

**Definition 11.** *The complement of multi Q-SVNSES  $(F_Q, A)$  is denoted by  $(F, A)^c$  and is defined by  $(F_Q, A)^c = (F_Q^c; \lceil A)$ , where  $F_Q^c : \lceil A \rightarrow M^k QF(U)$  is a function given by  $F_Q^c(x) = c(F_Q(\lceil x))$ ,  $\forall x \in \lceil A$  such that  $c$  is the multi Q-SVN complement.*

**Proposition 3.1.** *Let  $(F_Q, A)$  be a multi Q-SVNSES. Then*

$$\begin{aligned} (1) \quad ((F_Q, A)^c)^c &= (F_Q, A) \\ (2) \quad (F_Q, A)_1^c &= (F_Q, A)_0^c \\ (3) \quad (F_Q, A)_0^c &= (F_Q, A)_1^c \end{aligned}$$

*Proof.* The proof of this proposition is easy by using the properties of multi Q-SVNS.  $\square$

#### 4. UNION AND INTERSECTION OF MULTI Q-SVNSESs

In this section we propose the definitions of union and intersection of multi Q-SVNSES, give some examples and discussed their related properties.

**Definition 12.** *Let  $(F_Q, A)$  and  $(G_Q, B)$  be two multi Q-SVNSESs over  $U$  and  $Q$ . Then their union is a multi Q-SVNSES denoted by  $(M_Q, C) = (F_Q, A)(G_Q, B)$ , where  $C = A \cup B$  and  $\forall x \in C$ , the multi Q-SVNSES  $(M_Q, C)$  is defined as follows:*

$$M_Q(x) = \begin{cases} F_Q(x) & \text{if } x \in A - B; \\ G_Q(x) & \text{if } x \in B - A; \\ F_Q(x)G_Q(x) & \text{if } x \in A \cap B. \end{cases}$$

Let us suppose that  $U = \{u_1, u_2, u_3, u_4\}$ ,  $E = \{e_1, e_2, e_3\}$  are respectively, the universe of discourse and the set of parameters. Let  $Q = \{q_1, q_2\}$  be a non-empty set and  $X = \{p, q\}$  be the expert set. Then  $(F_Q, A)$  and  $(G_Q, B)$  are two multi Q-SVNSESs over  $U$  and  $Q$  such that

$$\begin{aligned} A &= \{(e_1, p, 1), (e_2, p, 0), (e_1, q, 1), (e_2, q, 1)\}; \\ B &= \{(e_1, p, 1), (e_1, q, 1), (e_2, q, 1)\} \end{aligned}$$

$$(F_Q, A) = \left\{ \begin{array}{l} (e_1, p, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right), \\ (e_1, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.3, 0.2, 0.4), (0.4, 0.1, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.2), (0.6, 0.1, 0.1), (0.7, 0.0, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.5), (0.3, 0.2, 0.3), (0.5, 0.1, 0.4) \rangle, \\ \langle (u_2, q_2), (0.9, 0.1, 0.1), (0.7, 0.2, 0.1), (0.5, 0.2, 0.1) \rangle, \\ \langle (u_3, q_1), (0.8, 0.1, 0.1), (0.4, 0.2, 0.1), (0.3, 0.2, 0.4) \rangle, \\ \langle (u_3, q_3), (0.6, 0.2, 0.1), (0.7, 0.2, 0.2), (0.8, 0.0, 0.1) \rangle \end{array} \right), \\ (e_2, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.1, 0.4), (0.4, 0.2, 0.1), (0.5, 0.2, 0.3) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.1), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.3), (0.3, 0.2, 0.4), (0.4, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.1), (0.2, 0.1, 0.2), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_1), (0.4, 0.1, 0.4), (0.5, 0.2, 0.1), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_3, q_3), (0.2, 0.4, 0.3), (0.4, 0.3, 0.4), (0.5, 0.2, 0.1) \rangle \end{array} \right), \\ (e_2, p, 0), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right) \end{array} \right\},$$

and

$$(G_Q, B) = \left\{ (e_1, p, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right) \right\}$$

$$(e_1, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.3, 0.3, 0.4), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.2), (0.8, 0.0, 0.1), (0.9, 0.0, 0.1) \rangle \end{array} \right)$$

$$(e_2, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right)$$

Then the union of  $(F_Q, A)$  and  $(G_Q, B)$  is  $(M_Q, C) = (F_Q, A) (G_Q, B)$ , such that

$$(M_Q, C) = \left\{ \begin{array}{l} (e_1, p, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.2), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.1), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right), \\ (e_1, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.5, 0.1, 0.2), (0.6, 0.1, 0.1), (0.7, 0.0, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.9, 0.1, 0.1), (0.7, 0.2, 0.1), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_3, q_1), (0.8, 0.1, 0.1), (0.4, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.1), (0.8, 0.0, 0.1), (0.9, 0.0, 0.1) \rangle \end{array} \right), \\ (e_2, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.1, 0.4), (0.4, 0.2, 0.1), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.5, 0.2, 0.1), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.1), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.4, 0.1, 0.3), (0.5, 0.2, 0.1), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle \end{array} \right), \\ (e_2, p, 0), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right) \end{array} \right\},$$

**Definition 13.** Let  $(F_Q, A)$  and  $(G_Q, B)$  be two multi  $Q$ -SVNSEs over  $U$  and  $Q$ . Then their intersection is a multi  $Q$ -SVNSE denoted by  $(M_Q, C) = (F_Q, A) (G_Q, B)$ , where  $C = A \cup B$  and  $\forall x \in C$ , the multi  $Q$ -SVNSE  $(M_Q, C)$  is defined as follows:

$$M_Q(x) = \begin{cases} F_Q(x) & \text{if } x \in A - B; \\ G_Q(x) & \text{if } x \in B - A; \\ (F_Q, A) (G_Q, B) & \text{if } x \in A \cap B. \end{cases}$$

**Example 2.** From Example(4), we have  $(M_Q, C) = (F_Q, A) (G_Q, B)$ , where

$$(H_Q, C) = \left\{ \begin{array}{l} (e_1, p, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.5), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.3), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.3, 0.5) \rangle, \\ \langle (u_3, q_2), (0.1, 0.4, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right), \\ (e_1, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.4, 0.2, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.5), (0.3, 0.3, 0.4), (0.5, 0.1, 0.4) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.2, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.2, 0.1), (0.3, 0.2, 0.4) \rangle, \\ \langle (u_3, q_3), (0.6, 0.2, 0.2), (0.7, 0.2, 0.2), (0.8, 0.0, 0.1) \rangle \end{array} \right), \\ (e_2, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.2, 0.3) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.1, 0.4, 0.3), (0.3, 0.2, 0.4), (0.4, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.3), (0.2, 0.2, 0.2), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.2, 0.4, 0.3), (0.4, 0.3, 0.4), (0.5, 0.2, 0.3) \rangle \end{array} \right), \\ (e_2, p, 0), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right) \end{array} \right\},$$

Now from the definitions of union and intersection of multi Q-SVNSESs we have the following proposition.

**Proposition 4.1.** Let  $(F_Q, A)$ ,  $(H_Q, B)$  and  $(N_Q, C)$  are any three multi Q-SVNSESs over  $U$  and  $Q$ . Then we have,

$$\begin{aligned} (1) \quad (F_Q, A) \cup (F_Q, A) &= (F_Q, A) \\ (2) \quad (F_Q, A) \cap (F_Q, A) &= (F_Q, A) \end{aligned}$$

$$(3) \quad (F_Q, A) \cup ((H_Q, B) \cup (N_Q, C)) = ((F_Q, A) \cup (H_Q, B)) \cup (N_Q, C);$$

$$(4) \quad (F_Q, A) \cap ((H_Q, B) \cap (N_Q, C)) = ((F_Q, A) \cap (H_Q, B)) \cap (N_Q, C);$$

$$(5) \quad (F_Q, A) \cap ((H_Q, B) \cup (N_Q, C)) = ((F_Q, A) \cap (H_Q, B)) \cup ((F_Q, A) \cap (N_Q, C));$$

$$(6) \quad (F_Q, A) \cup ((H_Q, B) \cap (N_Q, C)) = ((F_Q, A) \cup (H_Q, B)) \cap ((F_Q, A) \cup (N_Q, C)).$$

*Proof.* The proof of this proposition is easy, by using the properties of multi Q-SVNSESs.  $\square$

## 5. AND AND OR OPERATIONS OF MULTI Q-SVNSESs

In this section we propose the definitions of AND and OR operations for multi Q-SVNSESs and give some examples.

**Definition 14.** Let  $(F_Q, A)$  and  $(G_Q, B)$  be two multi  $Q$ -SVNSESs over  $U$  and  $Q$ . Then the "AND" operation for  $(F_Q, A)$  and  $(G_Q, B)$  is denoted by  $(F_Q, A) \wedge (G_Q, B)$  and is defined as  $(M, A \times B) = (F_Q, A) \wedge (G_Q, B)$  such that  $M_Q(\rho, \varphi) = (F_Q(\rho) \cap G_Q(\varphi)) ; \forall (\rho, \varphi) \in A \times B$ .

. Suppose Example (4) Let

$$A = \{(e_1, p, 1), (e_2, p, 0)\}, \text{ and } B = \{(e_1, p, 1), (e_1, q, 1)\}$$

$$(F_Q, A) = \left\{ \begin{array}{l} (e_1, p, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.4, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right) \\ (e_2, p, 0), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right) \end{array} \right\}$$

and

$$(G_Q, B) = \left\{ \begin{array}{l} (e_1, p, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.3), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right) \\ (e_1, q, 1), \left( \begin{array}{l} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.3, 0.3, 0.4), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.2), (0.8, 0.0, 0.1), (0.9, 0.0, 0.1) \rangle \end{array} \right) \end{array} \right\}$$

Then  $(F_Q, A) \text{ AND } (G_Q, B) =$

$$\left\{ \begin{array}{l} ((e_1, p, 1), (e_1, p, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.5), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.3), (0.1, 0.4, 0.4), (0.6, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.2, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right), \\ ((e_1, p, 1), (e_1, q, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.2, 0.4, 0.4), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.5), (0.4, 0.2, 0.3), (0.6, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.1, 0.4, 0.4), (0.5, 0.2, 0.2) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.2), (0.3, 0.1, 0.1), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.1, 0.2, 0.5), (0.3, 0.2, 0.4) \rangle \end{array} \right), \\ ((e_1, p, 0), (e_1, p, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.4), (0.3, 0.3, 0.2), (0.4, 0.3, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.3, 0.4), (0.3, 0.1, 0.3) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.3), (0.5, 0.1, 0.1), (0.7, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.2, 0.3, 0.4), (0.3, 0.4, 0.2), (0.4, 0.1, 0.5) \rangle, \\ \langle (u_3, q_3), (0.1, 0.4, 0.4), (0.3, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right), \\ ((e_1, p, 0), (e_1, q, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_1, q_2), (0.1, 0.2, 0.4), (0.2, 0.2, 0.3), (0.4, 0.2, 0.1) \rangle, \\ \langle (u_2, q_1), (0.2, 0.3, 0.2), (0.3, 0.3, 0.4), (0.5, 0.2, 0.3) \rangle, \\ \langle (u_2, q_2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_1), (0.2, 0.4, 0.3), (0.3, 0.2, 0.2), (0.4, 0.2, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.2, 0.3) \rangle \end{array} \right) \end{array} \right\},$$

**Definition 15.** Let  $(F_Q, A)$  and  $(G_Q, B)$  be two multi Q-SVNSESSs over  $U$  and  $Q$ . Then the "OR" operation for  $(F_Q, A)$  and  $(G_Q, B)$  is denoted by  $(F_Q, A) \vee (G_Q, B)$  and is defined as  $(M, A \times B) = (F_Q, A) \vee (G_Q, B)$  such that  $M_Q(\rho, \varphi) = (F_Q(\rho) \cup G_Q(\varphi)); \forall (\rho, \varphi) \in A \times B$ .

**Example 3.** Suppose Example (5)  $(F_Q, A) OR (G_Q, B) =$

$$\left\{ \begin{array}{l} ((e_1, p, 1), (e_1, p, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.1, 0.2, 0.3), (0.3, 0.3, 0.2), (0.4, 0.3, 0.1) \rangle, \\ \langle (u_1, q_2), (0.3, 0.1, 0.2), (0.4, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.1), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.1, 0.2) \rangle, \\ \langle (u_3, q_3), (0.2, 0.3, 0.4), (0.3, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right), \\ \\ ((e_1, p, 1), (e_1, q, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.2, 0.1), (0.3, 0.3, 0.4), (0.6, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.6, 0.1, 0.2), (0.8, 0.1, 0.1), (0.6, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.7, 0.2, 0.2), (0.6, 0.1, 0.1), (0.5, 0.3, 0.2) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.4), (0.8, 0.0, 0.1), (0.9, 0.0, 0.1) \rangle \end{array} \right), \\ \\ ((e_2, p, 0), (e_1, p, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.2, 0.2, 0.4), (0.4, 0.2, 0.2), (0.5, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.2, 0.1, 0.2), (0.6, 0.1, 0.1), (0.7, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.4, 0.1, 0.1), (0.6, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.2), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.4, 0.1, 0.1), (0.5, 0.2, 0.3), (0.6, 0.1, 0.3) \rangle \end{array} \right), \\ \\ ((e_2, p, 0), (e_1, q, 1)), \left( \begin{array}{l} \langle (u_1, q_1), (0.6, 0.1, 0.1), (0.7, 0.2, 0.2), (0.8, 0.1, 0.1) \rangle, \\ \langle (u_1, q_2), (0.4, 0.1, 0.2), (0.5, 0.1, 0.3), (0.6, 0.1, 0.1) \rangle, \\ \langle (u_2, q_1), (0.3, 0.2, 0.1), (0.5, 0.1, 0.1), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_2, q_2), (0.4, 0.1, 0.3), (0.6, 0.1, 0.2), (0.7, 0.1, 0.2) \rangle, \\ \langle (u_3, q_1), (0.3, 0.2, 0.2), (0.4, 0.1, 0.1), (0.6, 0.1, 0.3) \rangle, \\ \langle (u_3, q_3), (0.7, 0.1, 0.1), (0.8, 0.0, 0.1), (0.9, 0.0, 0.1) \rangle \end{array} \right) \end{array} \right\}$$

**Definition 16.** Let  $N_Q$  be a multi  $Q$ -single valued neutrosophic soft expert element. Then the score function of  $N_Q$  is defined as follows:

$$S_c(N_Q) = \frac{1}{k_j} \frac{2 + \mu_j(u_i, q) - \nu_j(u_i, q) - \xi_j(u_i, q)}{3}$$

## 6. AN APPLICATION OF MULTI Q-SVNSESS IN DECISION MAKING

In this section, we present an application of multi Q-SVNSES theory in a decision making problem by using the score values of each element in agree multi Q-SVNSESs and disagree multi Q-SVNSESs.

**Example 4.** Let us suppose that a university wants to fill the vacancy of assistant professor in the department of mathematics to be chosen by the expert committee. Suppose that the candidates passed the entry test is three. Let the  $U = \{u_1, u_2, u_3\}$  be the set of candidates with two types of qualifications  $Q = \{q_1 = \text{master of science}, q_2 = \text{PhD}\}$  and the set of parameters consider by the hiring committee is  $E = \{e_1 = \text{experience}, e_2 =$

teaching skills}. Let  $X = \{p, q\}$  be the two expert of the committee. After a lot of discussion the committee formulate the following multi  $Q$ -SVNSES.

$$(M_Q, D) = \left\{ (e_1, p, 1), \begin{pmatrix} (u_1, q_1), (0.3, 0.1, 0.2), (0.3, 0.2, 0.2), (0.4, 0.1, 0.1), \\ (u_1, q_2), (0.2, 0.1, 0.1), (0.2, 0.2, 0.1), (0.6, 0.2, 0.3), \\ (u_2, q_1), (0.2, 0.3, 0.4), (0.4, 0.2, 0.4), (0.4, 0.3, 0.4), \\ (u_2, q_2), (0.1, 0.2, 0.3), (0.4, 0.3, 0.3), (0.5, 0.3, 0.3), \\ (u_3, q_1), (0.2, 0.4, 0.4), (0.3, 0.3, 0.3), (0.4, 0.2, 0.3), \\ (u_3, q_2), (0.2, 0.1, 0.3), (0.2, 0.2, 0.2), (0.5, 0.2, 0.2) \end{pmatrix}, \right.$$

$$(e_1, p, 1), \begin{pmatrix} (u_1, q_1), (0.2, 0.1, 0.2), (0.3, 0.2, 0.3), (0.5, 0.1, 0.1), \\ (u_1, q_2), (0.1, 0.2, 0.3), (0.2, 0.1, 0.2), (0.3, 0.2, 0.1), \\ (u_2, q_1), (0.2, 0.4, 0.4), (0.3, 0.2, 0.3), (0.5, 0.2, 0.2), \\ (u_2, q_2), (0.2, 0.2, 0.3), (0.3, 0.1, 0.2), (0.5, 0.2, 0.1), \\ (u_3, q_1), (0.1, 0.2, 0.4), (0.2, 0.1, 0.3), (0.4, 0.2, 0.2), \\ (u_3, q_2), (0.2, 0.2, 0.2), (0.3, 0.2, 0.4), (0.4, 0.2, 0.3) \end{pmatrix},$$

$$(e_2, p, 1), \begin{pmatrix} (u_1, q_1), (0.1, 0.3, 0.4), (0.3, 0.2, 0.2), (0.5, 0.1, 0.4), \\ (u_1, q_2), (0.2, 0.3, 0.3), (0.3, 0.1, 0.3), (0.4, 0.2, 0.2), \\ (u_2, q_1), (0.2, 0.2, 0.4), (0.3, 0.2, 0.4), (0.5, 0.2, 0.3), \\ (u_2, q_2), (0.1, 0.2, 0.4), (0.3, 0.2, 0.3), (0.5, 0.2, 0.2), \\ (u_3, q_1), (0.2, 0.1, 0.4), (0.3, 0.2, 0.3), (0.4, 0.1, 0.2), \\ (u_3, q_2), (0.3, 0.1, 0.2), (0.3, 0.1, 0.1), (0.4, 0.1, 0.2) \end{pmatrix},$$

$$(e_2, q, 1), \begin{pmatrix} (u_1, q_1), (0.2, 0.2, 0.4), (0.3, 0.1, 0.2), (0.4, 0.1, 0.3), \\ (u_1, q_2), (0.1, 0.1, 0.5), (0.4, 0.1, 0.2), (0.5, 0.2, 0.3), \\ (u_2, q_1), (0.3, 0.1, 0.4), (0.4, 0.1, 0.3), (0.4, 0.1, 0.2), \\ (u_2, q_2), (0.2, 0.1, 0.3), (0.3, 0.2, 0.2), (0.5, 0.1, 0.3), \\ (u_3, q_1), (0.1, 0.1, 0.2), (0.2, 0.2, 0.1), (0.3, 0.1, 0.2), \\ (u_3, q_2), (0.1, 0.2, 0.4), (0.2, 0.2, 0.3), (0.3, 0.2, 0.3) \end{pmatrix},$$

$$(e_1, p, 0), \begin{pmatrix} (u_1, q_1), (0.1, 0.2, 0.5), (0.3, 0.1, 0.3), (0.5, 0.1, 0.2), \\ (u_1, q_2), (0.2, 0.3, 0.3), (0.3, 0.2, 0.3), (0.4, 0.1, 0.2), \\ (u_2, q_1), (0.1, 0.1, 0.4), (0.2, 0.2, 0.3), (0.4, 0.2, 0.2), \\ (u_2, q_2), (0.1, 0.1, 0.3), (0.2, 0.1, 0.3), (0.3, 0.2, 0.3), \\ (u_3, q_1), (0.2, 0.2, 0.2), (0.3, 0.2, 0.2), (0.5, 0.1, 0.1), \\ (u_3, q_2), (0.1, 0.3, 0.2), (0.2, 0.1, 0.2), (0.3, 0.1, 0.3) \end{pmatrix},$$

$$(e_1, q, 0), \begin{pmatrix} (u_1, q_1), (0.2, 0.1, 0.1), (0.3, 0.1, 0.1), (0.5, 0.1, 0.1), \\ (u_1, q_2), (0.2, 0.3, 0.4), (0.3, 0.1, 0.2), (0.4, 0.1, 0.3), \\ (u_2, q_1), (0.1, 0.2, 0.3), (0.2, 0.3, 0.4), (0.3, 0.4, 0.4), \\ (u_2, q_2), (0.2, 0.1, 0.3), (0.3, 0.2, 0.1), (0.4, 0.2, 0.3), \\ (u_3, q_1), (0.3, 0.1, 0.1), (0.3, 0.2, 0.1), (0.4, 0.3, 0.3), \\ (u_3, q_2), (0.2, 0.2, 0.2), (0.3, 0.3, 0.4), (0.5, 0.2, 0.2) \end{pmatrix}$$

$$\left. \begin{aligned}
 (e_2, p, 0), & \left( \begin{array}{l}
 (u_1, q_1), (0.3, 0.1, 0.1), (0.3, 0.2, 0.3), (0.4, 0.2, 0.1), \\
 (u_1, q_2), (0.2, 0.3, 0.4), (0.3, 0.2, 0.3), (0.5, 0.2, 0.2), \\
 (u_2, q_1), (0.1, 0.2, 0.4), (0.2, 0.3, 0.3), (0.4, 0.2, 0.2), \\
 (u_2, q_2), (0.2, 0.2, 0.2), (0.3, 0.2, 0.1), (0.5, 0.2, 0.3), \\
 (u_3, q_1), (0.2, 0.1, 0.2), (0.2, 0.1, 0.3), (0.6, 0.2, 0.2), \\
 (u_3, q_2), (0.1, 0.1, 0.2), (0.4, 0.2, 0.2), (0.5, 0.2, 0.2)
 \end{array} \right) \\
 (e_2, q, 0), & \left( \begin{array}{l}
 (u_1, q_1), (0.1, 0.3, 0.5), (0.2, 0.2, 0.3), (0.3, 0.3, 0.4), \\
 (u_1, q_2), (0.2, 0.2, 0.6), (0.2, 0.1, 0.2), (0.5, 0.1, 0.2), \\
 (u_2, q_1), (0.3, 0.1, 0.1), (0.3, 0.1, 0.2), (0.4, 0.1, 0.2), \\
 (u_2, q_2), (0.2, 0.2, 0.4), (0.4, 0.2, 0.3), (0.4, 0.2, 0.2), \\
 (u_3, q_1), (0.2, 0.1, 0.3), (0.3, 0.1, 0.2), (0.5, 0.2, 0.3), \\
 (u_3, q_2), (0.1, 0.4, 0.3), (0.3, 0.4, 0.3), (0.6, 0.2, 0.3)
 \end{array} \right)
 \end{aligned} \right\}$$

The following algorithm may be followed by the university committee to fill the vacancy of assistant professor.

- (1) Input the multi Q-SVNSES  $(M_Q, D)$ .
- (2) Find agree-multi Q-SVNSES and disagree-multi Q-SVNSES.
- (3) Find the score values using Definition (16) of each agree-multi Q-SVN soft expert element and disagree-multi Q-SVN soft expert element.
- (4) Find  $Y_j = \tilde{S}(u, q)_{ij}$  for agree-multi Q-SVN soft expert elements.
- (5) Find  $Z_j = \tilde{S}(u, q)_{ij}$  for disagree-multi Q-SVN soft expert elements.
- (6) Find  $K_j = Y_j - Z_j$ .
- (7) Find  $k$ , for which  $K_k = \max K_j$ , where  $K_k$  is the optimal choice object. If  $k$  has more than one value, then any one could be selected by the university committee according to their option.

Table 1 and Table 2 respectively represent the agree-multi Q-SVNSES, disagree-multi Q-SVNSES and the score values of each multi Q-SVN soft expert elements.

Table 1 presentation of agree-multi Q-SVNSES

$U \times Q$	$(u_1, q_1)$	$(u_1, q_2)$	$(u_2, q_1)$	$(u_2, q_2)$	$(u_3, q_1)$	$(u_3, q_2)$
$(e_1, p, 1)$	0.678	0.667	0.556	0.589	0.556	0.633
$(e_1, q, 1)$	0.667	0.611	0.589	0.656	0.589	0.600
$(e_2, p, 1)$	0.589	0.611	0.589	0.600	0.622	0.689
$(e_2, q, 1)$	0.622	0.622	0.656	0.644	0.633	0.556
$Y_j = \tilde{S}(u, q)_{ij}$	$y_1 = 2.556$	$y_2 = 2.511$	$y_3 = 2.389$	$y_4 = 2.489$	$y_5 = 2.400$	$y_6 = 2.478$

Table 2 presentation of disagree-multi Q-SVNSES

$U \times Q$	$(u_1, q_1)$	$(u_1, q_2)$	$(u_2, q_1)$	$(u_2, q_2)$	$(u_3, q_1)$	$(u_3, q_2)$
$(e_1, p, 0)$	0.611	0.611	0.589	0.589	0.667	0.600
$(e_1, q, 0)$	0.711	0.611	0.511	0.633	0.656	0.611
$(e_2, p, 0)$	0.667	0.600	0.567	0.644	0.656	0.656
$(e_2, q, 0)$	0.511	0.611	0.689	0.611	0.644	0.567
$Z_j = \tilde{S}(u, q)_{ij}$	$z_1 = 2.500$	$z_2 = 2.433$	$z_3 = 2.356$	$z_4 = 2.478$	$z_5 = 2.622$	$z_6 = 2.433$

From Table 1 and Table 2 we able to find the value of  $K_j$  as shown in Table 3.

Table. 3  $K_j = Y_j - Z_j$

$j$	$U \times Q$	$Y_j$	$Z_j$	$K_j$
1	$(u_1, q_1)$	2.556	2.500	0.056
2	$(u_1, q_2)$	2.511	2.433	0.078
3	$(u_2, q_1)$	2.389	2.356	0.033
4	$(u_2, q_2)$	2.489	2.478	0.011
5	$(u_3, q_1)$	2.400	2.622	-0.222
6	$(u_3, q_2)$	2.478	2.433	0.044 height

So from Table 3 the maximum value of  $K_j = 0.078$ , so  $k = 0.078$ , hence the university committee will choose the candidate  $u_1$  with *PhD* degree for the job.

## 7. COMPARISON AND DISCUSSION

In this we propose the concept of multi Q-SVNSES which is the generalization of Q-fuzzy soft expert set defined by F. Adam et al.[6]. The concept of multi Q-fuzzy soft expert set can not explain the universal set  $U$  in detail while the concept of multi Q-SVNSES defined in this article can explain the universal set  $U$  in more detail with three membership functions namely, truth-membership, indeterminacy-membership and falsity-membership. To illustrate the advantages of our proposed method with that of the method defined by F.Adam et al. [6] let us consider the above Example 4 by neglecting the falsity-membership and indeterminacy-membership functions. That is  $\nu_j(u_i, q) = 0$  and  $\xi_j(u_i, q) = 0$ . Then the multi Q-fuzzy soft expert set can define the above information as follows:

$$(M_Q, D) = \left\{ (e_1, p, 1), \left( \begin{array}{l} ((u_1, q_1), (0.3, 0.3, 0.4)), ((u_1, q_2), (0.2, 0.2, 0.6)), \\ ((u_2, q_1), (0.2, 0.4, 0.4)), ((u_2, q_2), (0.1, 0.4, 0.5)), \\ ((u_3, q_1), (0.2, 0.3, 0.4)), ((u_3, q_2), 0.2, 0.2, 0.5) \end{array} \right), \dots \right\}$$

Now using the algorithm defined by F. Adam et al. [6] we have,

Table 4 and Table 5 respectively represent the agree-multi Q-fuzzy soft expert set, disagree-multi Q-fuzzy soft expert sets and the mean of each multi Q-fuzzy soft expert sets.

Table 4 presentation of agree-multi Q-fuzzy soft expert set

$U \times Q$	$(u_1, q_1)$	$(u_1, q_2)$	$(u_2, q_1)$	$(u_2, q_2)$	$(u_3, q_1)$	$(u_3, q_2)$
$(e_1, p, 1)$	0.333	0.333	0.333	0.333	0.333	0.300
$(e_1, q, 1)$	0.333	0.200	0.333	0.333	0.233	0.300
$(e_1, p, 1)$	0.300	0.300	0.333	0.300	0.300	0.333
$(e_2, q, 1)$	0.300	0.333	0.367	0.333	0.200	0.200
$C_j =_{ij} (u, q)_{ij}$	$c_1 = 1.267$	$c_2 = 1.167$	$c_3 = 1.367$	$c_4 = 1.300$	$c_5 = 1.067$	$c_6 = 1.133$ height

Table 5 presentation of disagree-multi Q-fuzzy soft expert set

$U \times Q$	$(u_1, q_1)$	$(u_1, q_2)$	$(u_2, q_1)$	$(u_2, q_2)$	$(u_3, q_1)$	$(u_3, q_2)$
$(e_1, p, 0)$	0.300	0.300	0.233	0.200	0.333	0.2
$(e_1, p, 0)$	0.333	0.300	0.200	0.300	0.333	0.333
$(e_1, p, 0)$	0.333	0.333	0.233	0.333	0.333	0.333
$(e_2, q, 0)$	0.200	0.300	0.333	0.333	0.333	0.333
$K_j =_{ij} (u, q)_{ij}$	$k_1 = 1.167$	$k_2 = 1.233$	$k_3 = 1.000$	$k_4 = 1.167$	$k_5 = 1.333$	$k_6 = 1.200$ height

Table. 3  $S_j = C_j - K_j$

$j$	$U \times Q$	$C_j$	$K_j$	$S_j$
1	$(u_1, q_1)$	1.267	1.167	0.100
2	$(u_1, q_2)$	1.167	1.233	-0.067
3	$(u_2, q_1)$	1.367	1.000	0.367
4	$(u_2, q_2)$	1.300	1.167	0.133
5	$(u_3, q_1)$	1.067	1.333	-0.267
6	$(u_3, q_2)$	1.133	1.200	-0.067 height

So from Table 3 we can see that the maximum value  $r = 0.367$ . So the university committee will choose the candidate  $u_2$  with master of science degree.

As we can see that the result obtained through using the method defined by F. Adam et al. [6] and the result obtained by using the method in this article is totally different. The reason is that the multi Q-fuzzy soft expert set can handle a limited information while the concept of multi Q-SVNSES proposed in this article can handle incomplete, inconsistent and indeterminate data. Which makes the concept and method proposed in this article more accurate and realistic than that defined by F. Adam et al.[6].

## 8. CONCLUSION

In this article we reviewed the basic concept of Q-SVNSESs, proposed some operational laws such as, complement, intersection, union, OR and AND and discussed some basic properties related to these operations. Further, we defined an algorithm and applied to solve a decision making problem of selecting assistant professor. Finally a comparison had been made through example to show the advantages and practicality of the proposed concept and decision making algorithm with the existing concept of multi Q-fuzzy soft expert set. This new concept of multi Q-SVNSES will provide a remarkable addition to the existing theories for handling inconsistent and indeterminate information. This new concept of multi Q-SVNSES will stimulant more progress in research and relevant applications.

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