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On Separately Irresolute and Pre semi open Multiplication Mapping of Topological Spaces defined on Loops

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Abstract. This paper is devoted to explore semi topological loops with respect to irresoluteness. It is also investigated that how separately irresolute and pre semi open multiplication mapping of topological spaces are defined on Inverse property loops. We mainly show that, semi closure of semi discrete sub-loop of a semi Irr-topological loop is also a sub-loop; a sub-loop of semi Irr-topological loop has semi isolated point if and only if it is semi discrete; Image under the composition of inverse mapping and left translation of a semi compact set in a semi Irr-topological IP-loop is semi compact. In fact, we investigate the properties of Semi Irr-topological loops by enfeebling the conditions of continuity and openness.

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1. Introduction

Relationship between topological spaces and algebraic structures always remain very captivating. Mostly it requires the continuity of algebraic operations. Here, we join them through the weaker form of openness and continuity. When N. Levine introduced semi open set for the first time in 1963, many of the mathematicians examined several results by using semi continuity and semi open set. A number of new results are being obtained when open sets are replaced by semi open sets, continuity is replaced by irresoluteness. A subset M of a space X is called semi open set if there is an open set O in X satisfying $M \subseteq Cl(Int(M))$. Set consisting of all the semi open sets in X is denoted by $SO(X)$

and set of all the semi open sets containing t in X is denoted by $SO(X,t)$. He proved that every union of the semi open sets always remain semi open, but their finite intersection may not be semi open [8]. Later, it was examined by Hildebrand and Crossley that, the intersection of a semi open and an open set is always semi open [3]. Semi closed set is the compliment of semi open set. Every closed (open) set is semi closed (semi open). If $M_1 \subseteq X$ and $M_2 \subseteq Y$ are semi open, then $M_1 \times M_2$ is semi open in $X \times Y$. A set $M_2 \subseteq X$ is semi open neighborhood of $t \in X$, if there is a $M_1 \in SO(X)$ such that $t \in M_1 \subseteq M_2$. A point $t \in X$ is said to be a semi interior point of M, if there is a semi open set M' such that $t \in M' \subseteq M$. Set consisting of all the semi interior points of M is denoted by $sInt(M)$. For any semi open set $M_t, t \in sCl(M')$ if and only if $M_t \cap M' \neq \phi$. A mapping $f : X \to Y$ is said to be

- Irresolute, if for each $M_2 \in SO(Y)$ the set $f^{-1}(M_2) \in SO(X)$. Equivalently, f is irresolute if and only if for all $t \in X$, there is $M_1 \in SO(X,t)$ for every $M_2 \in SO(Y, f(t))$, such that $f(M_1) \subseteq M_2$ or $t \in M_1 \subseteq f^{-1}(M_2)$ [1];
- Pre semi open, if for every $M \in SO(X)$, $f(M) \in SO(Y)$ [7];
- Semi homeomorphism, if f is irresolute, pre semi open and bijective [7].

A subset M of a space X is semi compact (semi-Lindelof), if there is a finite (countable) subcover for any semi open cover of $M \in X$ [4, 17]. A space (L,τ) is s-regular if for every $t \in L$ and every closed set $Q \subseteq L$ there exists semi open neighbourhoods M containing t such that $t \notin Q$ and N containing Q such that $M \cap N = \phi$ [9]. A point t of subset N of topological space (L,τ) is said to be semi isolated if there is a $M \in SO(L)$ such that $M \cap N = t$. If every point of N is semi isolated point, then N is said to be semi discrete [11]. If topological space (L,τ) cannot be expressed as a union of two nonempty disjoint semi open sets in L, then L is called semi connected. Semi component of topological space (L,τ) are the maximal semi connected subsets of L. $S.C.(t)$ is semi component containing t, for $t \in L$ [5]. A space (L,τ) is semi homogeneous if there is a semi homeomorphism f of L onto itself such that for every $r, s \in L$ implies $f(r) = s$. Let $M \in SO(L,e)$ in a semi Irr-topological loop $(L,*,\tau)$. A subset S of L is said to be M-semi disjoint if for every disjoint $r, s \in S$ implies $r \notin s * M$ [1]. A groupoid $(L,*)$ is said to be a loop, if the following conditions are satisfied;

- L contains an identity element.
- For every $s \in L$, the mapping $l_s : L \to L$ and $r_s : L \to L$ are bijective, where $l_s(t) = s * t$ and $r_s(t) = t * s$ for all $t \in L$ [15].

An inverse property loop (IP-loop) L is a loop having two sided inverse t^{-1} such that $(r * t) * t^{-1} = r = t^{-1} * (t * r)$ for all $r, t \in L$. Left and right translations are defined on a loop $(L,*)$ as follows:

- Left translation $l_{t_1} : L \to L$ is given by $l_{t_1}(t_2) = t_1 * t_2$;
- Right translation $r_{t_1} : L \to L$ is given as $r_{t_1}(t_2) = t_2 * t_1$;

where $t_1, t_2 \in L$. Here the condition of the irresoluteness of left and right translations is equivalent to separately irresoluteness of multiplication mapping [18]. We used the standard notions and terminologies as in [6].

In this article, we derive some significant results on semi topological loops with respect to irresoluteness and bring into light some properties of semi Irr-topological inverse property loops. We provide comparison which actuate us to bring out this notion. In fact we explore, extend and generalize the idea of M.S. Bosan [1], T. Oner [14, 12, 13] and S. Lin [2].

2. Some significant results on semi topological loops with respect to irresoluteness

Definition 2.1. *A triplet loop $(L, *, \tau)$ is semi Irr-topological loop, if the following conditions are satisfied:*

- *$(L, *)$ is a loop.*
- *(L, τ) is a topological space.*
- *Multiplication mapping is separately irresolute on $(L, *, \tau)$.*
- *Multiplication mapping is separately pre semi open on $(L, *, \tau)$.*

Remark 2.2. *In a semi Irr-topological loop, right and left translations are semi homeomorphism.*

Lemma 2.3. *For a semi Irr-topological loop X. If X_0 is an open set in X and $f : X \to Y$ is pre semi open. Then the restricted function $f_{|X_0} : X_0 \to Y$ is also pre semi open.*

Proof. Let $U \in SO(X)$ and $X_0 \in O(X)$. So $U \cap X_0$ is a semi open set in X. Therefore, $f(U \cap X_0)$ is semi open in Y. Also $U \cap X_0$ is semi open in X_0 [10] and $f_{|X_0}(U \cap X_0)$ is semi open in Y. Hence $f_{|X_0}$ is pre semi open. In the following theorem it will be proved that in a semi Irr-topological loop, every open sub-loop is also semi Irr-topological loop.

Theorem 2.4. *Every open sub-loop M of a semi Irr-topological loop $(L, *, \tau)$ is also a semi Irr-topological loop, which is infact a semi Irr-topological sub-loop of L.*

Proof. Since $(L, *, \tau)$ is a semi Irr-topological loop and M is its open sub-loop, it is to be shown that l_x and r_x are irresolute and pre semi open in M. Since $(M, *)$ being a loop is open in L and (M, τ_M) is a topological space, by Lemma 2.3 l_x, r_x are pre semi open in M. Also l_x, r_x are irresolute in M [1]. Hence, $(M, *, \tau_M)$ is Semi Irr-topological sub-loop.

The theorem given below is about the property of semi closure of a sub-loop.

Theorem 2.5. *Let $(L, *, \tau)$ be a semi Irr-topological loop and M be a sub-loop of L. Then $sCl(M)$ is a sub-loop of L.*

Proof. It is obvious that $e \in sCl(M)$ since M is a sub-loop. Moreover, in Theorem 4.5 [14], it was shown that semi-homeomorphism property of $l_t, r_t : L \to L$ implies $sCl(M) * sCl(M) \subseteq sCl(M)$. Hence $l_t(sCl(M)) \subseteq sCl(M)$ and $r_t\ (sCl(M)) \subseteq sCl(M)$ for any $t \in sCl(M)$. Therefore, for every $t \in sCl(M)$, we have bijective mappings $l_t : sCl(M) \to sCl(M)$ and $r_t : sCl(M) \to sCl(M)$ means that $sCl(M)$ is a sub-loop of L.

A property of semi open sub-loop is discussed in the next theorem.

Theorem 2.6. *In a semi Irr-topological loop every semi open sub-loop is semi closed.*

Proof. Suppose that M is semi open sub-loop of semi Irr-topological loop L. Every semi open neighbourhood of p meets M if and only if $p \in sCl(M)$. Since $p * M$ is semi open neighbourhood of p, it meets M. Thus, there is a $q \in M$ such that $q = p * r$, where $r \in M$. But then $p = q/r \in M$. Hence, $M = sCl(M)$.

Theorem 2.7. *For a semi Irr-topological loop $(L, *, \tau)$, if $M \subseteq L$ is semi open and $n \in L$ then $n * M$ and $M * n$ are semi open.*

Proof. As l_n and r_n are pre semi open in semi Irr-topological loop. So, for a semi open set M of L, $l_n(M) = n * M$ and $r_n(M) = M * n$ are semi open.

Theorem 2.8. *Let L be a semi Irr-topological loop. If $N \subseteq L$ and $M \in SO(L)$, then $M.N$ and $N.M$ are semi open.*

Proof. As $M \in SO(L)$, so for $n \in N$, $n * M$ and $M * n$ are semi open. So, by Theorem 2.7 $M * N = \cup_{n \in N} M * n$ and $N * M = \cup_{n \in N} n * M$ are semi open.

Theorem 2.9. *Let $(L, *, \tau)$ be a semi Irr-topological loop and M, N be subsets of L. Then,*

(1) *$M * N$ and $N * M$ are semi compact, if N is finite and M is semi compact.*
(2) *$M * N$ and $N * M$ are semi-Lindelof, if N is countable and M is semi-Lindelof.*

Proof. (1) Since given is that M is semi compact, and l_n, r_n are irresolute for all $n \in N$. So, $l_n(M) = n * M$ and $r_n(M) = M * n$ are semi compact [4]. Furthermore, $M * N = \cup_{n \in N} M * n$ and $N * M = \cup_{n \in N} n * M$ are semi compact.
(2) Similar as above [17].

Theorem 2.10. *Semi Irr-topological loop is semi homogeneous space.*

Proof. Let $m, n \in L$. Since left translation is semi homeomorphism and there must exists $t \in L$ such that $l_t(m) = n$. This shows that semi Irr-topological loop is semi homogeneous.

Theorem 2.11. *Let M be a sub-loop of a semi Irr-topological loop L containing a non-void semi open set N then M is also semi open in L.*

Proof. As N is semi open, and l_m is pre semi open in L. So, for every $m \in M, l_m(N) = m * N$ is semi open in L. Hence, $M = \cup_{m \in M} m * N$ is semi open in L.

Theorem 2.12. *In a semi Irr-topological loop L, semi interior of a non-empty sub-loop M is non-empty if and only if it is semi open.*

Proof. Let M is semi open non-void sub-loop of L, so $sInt(M) = M \neq \phi$. Conversely, suppose that semi interior of M is non-empty. Let $t \in sInt(M)$ then there is $N \in SO(L)$ such that $t \in N \subseteq M$. For every $m \in M, l_m(N) = m * N \subseteq m * M = M$. As N is semi open so $m * N$, and we have $M = \cup\{m * N : m \in M\}$ is semi open.

Theorem 2.13. *In a semi Irr-topological loop L, if M is semi open then $S = \cup_{n=1}^{\infty} M^n$ is semi open.*

Proof. As M is semi open, so by Theorem 2.8 $M * M = M^2 \in SO(L)$ and $M^2 * M = M^3 \in SO(L)$. Similarly, $M^4,\ M^5, \ldots$ are semi open sets in L. Thus, $S = \cup_{n=1}^{\infty} M^n$ is semi open.

Theorem 2.14. *Let L' be a sub-loop of semi Irr-topological loop L. Then L' has a semi isolated point if and only if it is semi discrete.*

Proof. Let a sub-loop L' has a semi isolated point t. So, there is $S \in SO(L)$ such that $L' \cap S = \{t\}$. Then for every $r \in L', r = l_r(l_{(t)_L^{-1}}(t)) = l_r(l_{(t)_L^{-1}}(L' \cap S)) = l_r(l_{(t)_L^{-1}}(L') \cap l_{(t)_L^{-1}}(S)) = l_r(l_{(t)_L^{-1}}(L')) \cap l_r(l_{(t)_L^{-1}}(S)) = L' \cap r * ((t)_L^{-1} * S)$. Here $r * ((t)_L^{-1} * S)$ is semi open. Hence, L' is semi discrete. Conversely, if L' is semi discrete then its every point is semi isolated.

Theorem 2.15. *Consider a semi connected semi Irr-topological loop L. Let L' is a sub-loop of L containing a semi interior point t, then $L' = L$. In particular L coincides its semi open sub-loop.*

Proof. $L' \in SO(L)$ because it contains a semi interior point [14]. Moreover L' is semi closed by Theorem 2.6 Therefore, $L = L'$ because L is semi connected.

Theorem 2.16. *Let $(L, *, \tau)$ be a semi connected semi Irr-topological loop and $L' \in SO(L, e_L)$ be symmetric. Then L' generates L.*

Proof. Let L^* be sub-loop of L generated by L'. Then L^* consists of $t_1 * t_2 * \ldots * t_n$, all the products of finite number of sequences of the elements of L'. As $e_L \in sInt(L^*)$, so L^* is semi open. By Theorem 2.15 $L^* = L$. Hence, L is generated by L'.

Theorem 2.17. *If the homeomorphism $f : (L, *, \tau_L) \to (M, *', \tau_M)$ of semi Irr-topological loops is irresolute at e_L then it is irresolute on L.*

Proof. Let n be an arbitrary element of L and $H \in SO(M, m)$, where $m = f(n)$. Since l_m is irresolute in M. So, there is $I \in SO(M, e_M)$ such that $l_m\ (I) = m * I \subseteq H$. Moreover, $f(W) \subseteq I$ for $W \in SO(L, e_L)$ because as f is iresolute at e_L. Also $n * W \in SO(L, n)$ due to pre semi openness of $l_n : L \to L$. We have $f(n*W) = f(n)*f(W) = m*f(W) \subseteq m * I \subseteq H$. Thus, f is irresolute at any arbitrary point $n \in L$. Hence, f is irresolute on L.

Theorem 2.18. *Let $(L, *, \tau)$ be a semi Irr-topological loop and $S.C.(e)$ be open semi connected component of e_L. Then $S.C.(e)$ is sub-loop, if for $t \in S.C.(e), r_{(t)_L^{-1}}$ and $r_{(t)_R^{-1}}$ are open.*

Proof. Let $t \in S.C.(e)$, then $(S.C.(e)).(t)_L^{-1} = r_{(t)_L^{-1}}(S.C.(e))$ is semi connected and contain e [16]. Since $r_{(t)_L^{-1}}$ is open, irresolute and $S.C.(e)$ is open. Hence, $(S.C.(e)).(t)_L^{-1} \subseteq S.C.(e)$ because $S.C.(e)$ is maximal semi connected set. As $(t)_L^{-1} \in (S.C.(e))_L^{-1}$, therefore, $(S.C.(e))(S.C.(e))_L^{-1} \subseteq S.C.(e)$. Similarly, $(S.C.(e))(S.C.(e))_R^{-1} \subseteq S.C.(e)$. Hence, $S.C.(e)$ is sub-loop.

In the next theorem we will define some properties of the collection of semi open neighbourhoods of identity element.

Theorem 2.19. *Let $(L, *, \tau)$ be a semi Irr-topological loop and the collection of all semi open neighbourhoods containing e_L is β_e, then $\forall M \in \beta_e, \exists N \in \beta_e$ such that $m * N, N * m \subseteq M, \forall m \in M$.*

Proof. As given $(L, *, \tau)$ is semi Irr-irresolute topological loop, so $\forall M \in \beta_e$ containing $m, \exists N \in \beta_e$ such that $l_m(N) = m*N \subseteq M$. Similarly, $r_m(N) = N*m \subseteq M, \forall m \in M$.

Theorem 2.20. *Let $(L, *, \tau)$ be a semi Irr-topological loop and collection of all the semi open neighbourhoods containing e_L be β_e. Then,*

(1) $\forall M \in \beta_e$ *and* $t \in L$, $\exists N \in \beta_e$ *such that* $(t * N) * (t)_R^{-1} \subseteq M$.
(2) $\forall M \in \beta_e$ *and* $t \in L$, $\exists N \in \beta_e$ *such that* $(t)_L^{-1} * (N * t) \subseteq M$.

Proof. (1) As l_t and $r_{(t)_R^{-1}}$ are semi homeomorphisms, $r_{(t)_R^{-1}} \circ l_t$ is also semi homeomorphism. Then, for all $M \in \beta_e$ and all $t \in L$, there exists $N \in \beta_e$ such that $(t * N) * (t)_R^{-1} \subseteq M$.
(2) Similar.

Theorem 2.21. *If* $(L, *, \tau)$ *is a semi Irr-topological loop and* $t \in L$*, then for each* $S \in SO(L, t)$*, there is* $M, N \in SO(L, e_L)$ *such that* $t * M \subseteq S$ *and* $N * t \subseteq S$.

Proof. Since r_t and l_t are irresolute at e_L and $r_t(e_L) = t, l_t(e_L) = t$, for every $S \in SO(L, t)$, there exists $M, N \in SO(L, e_L)$ such that $l_t(M) = t * M \subseteq S$ and $r_t(N) = N * t \subseteq S$.

3. Some properties of semi Irr-topological inverse property loops

Definition 3.1. *A triplet loop* $(L, *, \tau)$ *is said to be a semi Irr-topological IP-loop, if the following conditions are satisfied:*

- $(L, *)$ *is an IP-loop.*
- (L, τ) *is a topological space.*
- *Multiplication mapping is separately irresolute on* $(L, *, \tau)$.

Theorem 3.2. *Let* $(L, *)$ *be an IP loop and* (L, τ) *is a topological space. If* l_t *and* r_t *are pre semi open in* L *then* $(L, *, \tau)$ *is semi Irr-topological IP-loop.*

Proof. Let l_t be pre semi open for all $t \in L$. For any $M \subseteq L$ we have $l_t^{-1}(M) = l_{t^{-1}}(M)$. Then l_t is irresolute. Similarly, r_t is irresolute. Therefore, $(L, *, \tau)$ is semi Irr-topological loop.

Corollary 3.3. *In a semi Irr-topological IP-loop multiplication mapping is separately pre semi open.*

Theorem 3.4. *For a semi Irr-topological IP-loop* L*. If* $N^{-1} \in SO(L, e)$ *then* $N \subseteq sCl(N) \subseteq N^2$.

Proof. Assume that $r \in sCl(N)$. As $r * N^{-1} \in SO(L, r)$ so it meets N. Furthermore, there exists $s \in N$ such that $s = r * t^{-1}$ for $t \in N$. Then $r = s * t \in N * N = N^2$. Hence $sCl(N) \subseteq N^2$. Therefore, $N \subseteq sCl(N) \subseteq N^2$.

Theorem 3.5. *A semi Irr-topological IP-loop* $(L, *, \tau)$ *is s-regular at* e*, if* μ_e *is base at* e *and every* $N \in \mu_e$ *there exists symmetric* $M \in SO(L, e)$ *such that* $M * M \subseteq N$.

Proof. Let $r \in sCl(M)$. Then $r * M$ is semi open neighbourhood of r. Clearly, $r * M \cap M \neq \phi$. So, there exists $s, t \in M$ such that $t = r * s$, $r = t * s^{-1} \in M * M^{-1} = M * M \subseteq N$. Thus, $sCl(M) \subseteq N$. It gives L is s-regular at e.

Theorem 3.6. *For* $A, B \in SO(L, e)$ *in semi Irr-topological IP-loop* $(L, *, \tau)$ *such that* $B^4 \subseteq A$ *and* $B^{-1} = B$*. If* $M \subseteq L$ *is A-semi disjoint then* $\{m * B : m \in M\}$ *is semi discrete in* L.

Proof. To prove semi discreteness of $\{m * B : m \in M\}$, it is enough to prove that for every $r \in L$, $r * B$ intersects at most one of $\{m * B : m \in M\}$. Assume contrarily that for some $r \in L$, there exists $s, t \in M$ such that $r * B \cap t * B \neq \phi$ and $r * B \cap s * B \neq \phi$, then $r^{-1} * t \in B^2$ and $r \in s * B^2$. Here $t = r * (r^{-1} * t) \in s * B^4 \subseteq s * A$. Hence, $t \in s * A$, a contradiction to the fact that M is A-semi disjoint. So $\{m * B : m \in M\}$ is semi discrete in L.

Theorem 3.7. *If M is semi compact then $r * M^{-1}$ is semi compact in semi Irr-topological IP-loop $(L, *, \tau)$.*

Proof. Consider semi open cover $\{S_i : i \in I\}$ of $r * M^{-1}$. So $r * M^{-1} \subseteq \cup_{i \in I} S_i$, $M^{-1} \subseteq r^{-1} * \cup_{i \in I} S_i = \cup_{i \in I} r^{-1} * S_i$. Hence, $M \subseteq \cup_{i \in I} S_i^{-1} * r$. By semi compactness of M, there exists a finite $I_0 \subseteq I$ such that $M \subseteq \cup_{i \in I_0} S_i^{-1} * r, M * r^{-1} \subseteq \cup_{i \in I_0} S_i^{-1}$. Thus, $r * M^{-1} \subseteq \cup_{i \in I_0} S_i$. Therefore, $r * M^{-1}$ has semi open finite subcover in L.

4. Conclusion

This paper is based on the characterization of the properties of semi topological loops with respect to irresoluteness. We investigated that in what manner separately irresolute and pre semi open multiplication mapping of topological spaces are defined on loops. We also bring into light some properties of semi Irr-topological IP-loops. We provide comparison between two algebraic structures with the weaker form of continuity and openness. As, this paper is generalization of [1, 14, 12, 13], so it dominates all the previous results which are examined in this field. Factually, these results have a useful contribution in the field of Topological Algebra.

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