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V_k- Super Vertex Out-Magic Labeling of Digraphs

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Abstract. Let $D(V, A)$ be a digraph of order p and size q. For an integer $k \geq 1$ and for $v \in V(D)$, let $w_k(v) = \sum_{e \in E_k(v)} f(e)$, where $E_k(v)$ is the set containing all arcs which are at distance at most k from v. The digraph D is said to be E_k*-regular* with regularity r if and only if $|E_k(e)| = r$ for some integer $r \geq 1$ and for all $e \in A(D)$. A V_k-super vertex out-magic labeling (V_k-SVOML) is an one-to-one onto function $f : V(D) \cup A(D) \to \{1, 2, \ldots, p+q\}$ such that $f(V(D)) = \{1, 2, \ldots, p\}$ and there exists a positive integer M such that $f(v) + w_k(v) = M$, $\forall\ v \in V(D)$. A digraph that admits a V_k-SVOML is called V_k-super vertex out-magic (V_k-SVOM). This paper contains several properties of V_k-SVOML in digraphs. We characterized the digraphs which are V_k-SVOM. Also, the magic constant for E_k-regular graphs has been obtained. Further, we characterized the unidirectional cycles and union of unidirectional cycles which are V_2-SVOM.

AMS (MOS) Subject Classification Codes: 05C78
Key Words: V_k-super vertex out-magic labeling, E_k-regular graphs, digraph labeling.

1. Introduction

Let $D(V, A)$ be a digraph of order p and size q. For a vertex $v \in V(D)$, the out-neighborhood of v is defined by $O(v) = \{u : (v, u) \in A(D)\}$. The out-degree of v is defined by $deg^+(v) = |O(v)|$. For basic definition and results we follow [3].

A graph *labeling* is an assignment of integers (usually positive or non-negative integers), which assigned to vertices /or edges /or both into a set of numbers. Lot of labelings have been defined and studied by many authors and an excellent survey of graph labeling can be found in [5].

The magic labeling in graphs was introduced by Sedlàček [11]. A *magic* labeling is an one-to-one onto function f from $E(G)$ onto $\{1, 2, \ldots, q\}$ such that for all $v \in V(G)$, $\sum_{u \in N(v)} f(uv) = M$ for some positive integer M.

In 2002, MacDougall et al. [8] introduced the notion of vertex magic total labeling (VMTL) in graphs. Let $G(V, E)$ be a graph with $|V(G)| = p$ and $|E(G)| = q$. A one-to-one map f from $E(G) \cup V(G)$ onto the integers $\{1, 2, \ldots, p + q\}$ is a VMTL if there is a constant M so that for every vertex $x \in V(G)$, $f(x) + \sum f(xy) = M$, where the sum is taken over all vertices y adjacent to x.

In 2004, MacDougall et al. [9] defined the super vertex-magic total labeling (SVMTL) in graphs. They call a VMTL is *super* if $f(V(G)) = \{1, 2, \ldots, p\}$. In this labeling, the smallest labels are assigned to the vertices.

Another labeling parameter called 'Super Edge magic total labeling' with different meaning has been defined and studied in [1, 6, 7, 10].

In 2008, Bloom et al. [2] extended the idea of magic labeling to digraphs.

The V-super vertex out-magic total labeling (V-SVOMTL) in digraph was introduced by Durga Devi et al. [4]. A V-SVOMTL is an one-to-one onto function $f : V(D) \cup A(D) \to \{1, 2, \ldots, p + q\}$ such that $f(V(D)) = \{1, 2, \ldots, p\}$ and for every $v \in V(D)$, $f(v) + \sum\limits_{u \in O(v)} f((u, v)) = M$ for some positive integer M.

This paper generalizes the definition of V-SVOMTL and defines a new labeling called V_k-super vertex out-magic labeling (V_k-SVOML). For an integer $k \geq 1$, let $E_k(u) = \{(u, v) : d(u, v) \leq k\}$ and $w_k(u) = \sum\limits_{e \in E_k(u)} f(e)$. Note that if (u, v) is an directed edge, then $d(u, v) = 1$. A V_k-SVOML is an one-to-one onto function $f : V(D) \cup A(D) \to \{1, 2, \ldots, p + q\}$ such that $f(V(D)) = \{1, 2, \ldots, p\}$ and there exists a positive integer M such that $f(v) + w_k(v) = M$, $\forall\, v \in V(D)$. A digraph that admits a V_k-SVOML is called V_k-super vertex out-magic (V_k-SVOM). If x_1 and y_1 are vertices of a digraph D then the distance from x_1 to y_1 in D, is the minimum length of a directed $x_1 - y_1$ path if y_1 is reachable from x_1, and otherwise it is taken as infinity.

Let k be an integer such that $1 \leq k \leq \text{diam}(D) + 1$. For $e = (u, v) \in A(D)$, we define $E_k(e) = \{w \in V(D) : 1 \leq d(u, w) \leq k\}$. The digraph D is said to be E_k*-regular* with regularity r if and only if $|E_k(e)| = r$ for some integer $r \geq 1$ and for all $e \in A(D)$. Consider the digraph D(see Figure 1). In D, $E_2(v_2) = \{e_2, e_3, e_8, e_9\}$, $E_2(v_7) = \{e_7, e_4\}$, $E_2(e_1) = \{v_2, v_3, v_8\}$ and $E_2(e_7) = \{v_4, v_5\}$.

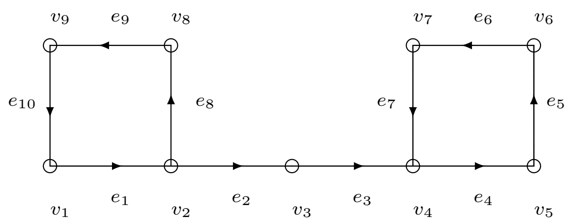

Figure 1: D

Observation 1.1. *Let D be a digraph of order $p(\geq 2)$. If $E_k(x_1) = E_k(x_2)$ for $x_1, x_2 \in V(D)$ ($x_1 \neq x_2$), then D is not V_k-SVOM.*

Proof. Let D be a digraph of order $p(\geq 2)$. Suppose $E_k(x_1) = E_k(x_2)$ for a pair of vertices x_1 and x_2 $(x_1 \neq x_2)$ of D. Then $f(x_1) + w_k(x_1) \neq f(x_2) + w_k(x_2)$ for any V_k-SVOML f of D (since f is one to one). In this case, D does not admit V_k-SVOML. □

A digraph D is said to be strongly connected if every pair of vertices are mutually reachable.

Remark 1.2. Let D be a strongly connected digraph. If $k \geq diam(D) + 1$, then $E_k(u) = A(D)$ for every $u \in V(D)$.

Proof. Let $u \in V(D)$. Then $E_k(u) \subseteq A(D)$. Let $(x, y) \in A(D)$. Since D is strongly connected, there exist directed $u-x$ path and $u-y$ path in D with length $\leq diam(D)$. Then there exist a directed $u - x - y$ path of length $\leq diam(D) + 1 \leq k$ and so $(x, y) \in E_k(u)$. Thus $A(D) \subseteq E_k(u)$. □

2. V_k-SVOML IN DIGRAPHS

This section will explore the basic properties of V_k-SVOML.

Theorem 2.1. *Let D be a digraph and f_1 be an one-to-one function from $A(D)$ onto $\{p + 1, p + 2, \ldots, p + q\}$. Then f_1 can be extended to a V_k-SVOML of D if and only if $\{w_k(v) : v \in V(D)\}$ is a set of p successive integers.*

Proof. Assume that $\{w_k(v) : v \in V(D)\}$ is a set of p successive integers. Let $t = min\{w_k(v) : v \in V(D)\}$. Define $f_2 : V(D) \cup A(D) \to \{1, 2, \ldots, p+q\}$ as $f_2((u, v)) = f_1((u, v))$ for $(u, v) \in A(D)$ and $f_2(v) = t+p-w_k(v)$ for $v \in V(D)$. Since $\{w_k(v)-t : v \in V(D)\}$ is a set of successive integers, $f_2(V(D)) = \{1, 2, \ldots, p\}$. Also $f_2(A(D)) = \{p + 1, p + 2, \ldots, p + q\}$. Hence f_2 is V_k-SVOML of D with magic constant $M = t + p$. Conversely, assume that f_1 can be extended to a V_k-SVOML f_2 of D. Let M be the magic constant. Since $f_1(v) + w_k(v) = M$ for every $v \in V(D)$, $\{w_k(v) : v \in V(D)\} = \{M - p, M - p + 1, \ldots, M - 1\}$ is a set of p successive integers. □

Lemma 2.2. *If a digraph $D(p, q)$ is V_k-SVOM and E_k-regular with regularity r, then the magic constant $M = \frac{p+1}{2} + rq + \frac{r}{p}\frac{q(q+1)}{2}$.*

Proof. Let f be a V_k-SVOML of D and M be the magic constant. Note that $M = f(x) + w_k(x)$ for all $x \in V(D)$. Summing over all $x \in V(D)$, we get $pM = \sum_{x \in V(D)} f(x) + \sum_{x\in V(D)} w_k(x) = \sum_{x\in V(D)} f(x) + \sum_{x\in V(D)} \sum_{e\in E_k(x)} f(e) = \sum_{x\in V(D)} f(x) + r \sum_{e\in A(D)} f(e)$ (since each edge is counted exactly r times in the sum $\sum_{x\in V(D)} \sum_{e\in E_k(x)} f(e)$).

Since $f(V(D)) = \{1, 2, \ldots, p\}$ and $f(A(D)) = \{p + 1, p + 2, \ldots, p + q\}$,
$pM = \frac{p(p+1)}{2} + r(pq) + r\frac{q(q+1)}{2}$ and so $M = \frac{p+1}{2} + rq + \frac{r}{p}\frac{q(q+1)}{2}$. □

Lemma 2.2 gives the magic constant for E_k-regular graphs which are V_k-SVOM for $k \geq 1$. In 2017, Durga Devi et al. [4] obtained the following result which gives the magic constant for all digraphs which admit V-SVOMTL.

Lemma 2.3. [4] *If a non-trivial digraph D is V-SVOMT, then the magic constant M is given by $M = q + \frac{p+1}{2} + \frac{q(q+1)}{2p}$.*

When $k = 1$, we have $r = |E_1(e)| = 1$ for all $e \in A(D)$. The above result is a corollary of Lemma 2.2 when $k = 1$.

Theorem 2.4. *For $k \geq 2$, trees are not V_k-SVOM.*

Proof. Suppose there is a tree that is V_k-SVOM. Let $V(D) = \{v_1, v_2, \ldots, v_p\}$. Since D is a tree, $q = p - 1$ and so at least one vertex of D has out-degree zero, let it be v_p. Then by Observation 1.1, v_p is the only vertex of D with out-degree zero. Since $w_k(v_p) = 0$, by Theorem 2.1, $\{w_k(v) : v \in V(D)\} = \{0, 1, 2, \ldots, p - 1\}$, which is impossible since $f(A) = \{p, p + 1, p + 2, p + 3, \ldots, 2p - 1\}$. □

Remark 2.5. From Theorem 2.4, we observe that a connected digraph is not V_k-SVOM ($k \geq 2$) when $q = p - 1$. Thus if a connected digraph is V_k-SVOM ($k \geq 2$), then $q \geq p$.

Corollary 2.6. *Let D be a connected E_k-regular digraph ($k \geq 2$) with regularity r. If D is V_k-SVOM, then $M \geq \frac{p+1}{2} + \frac{r(3p+1)}{2}$.*

Proof. When $q = p - 1$, D is a tree. By Theorem 2.4, D is not V_k-SVOM. Assume that $q \geq p$. Then by Lemma 2.2, $M \geq \frac{p+1}{2} + \frac{r(3p+1)}{2}$. □

Remark 2.7. From Corollary 2.6, we get $M = \frac{p+1}{2} + \frac{r(3p+1)}{2}$ when $p = q$. For example consider the following digraph $\overrightarrow{C_7}$.

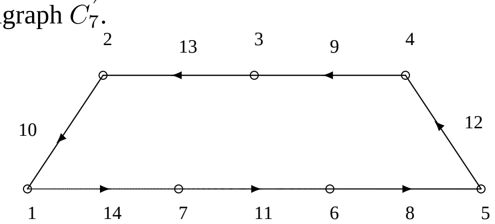

Figure 2: V_2-SVOML of $\overrightarrow{C_7}$

The unidirectional cycle $\overrightarrow{C_7}$ is E_2-regular with regularity $r = 2$ and V_2-SVOM with magic constant $M = \frac{p+1}{2} + \frac{r(3p+1)}{2} = 26$.

3. V_2-SVOML OF UNIDIRECTIONAL CYCLES AND UNION OF UNIDIRECTIONAL CYCLES

Durga Devi et al. [4] proved that all the unidirectional cycles admit V-SVOMTL. When $k = 2$, not all the unidirectional cycles admit V_2-SVOML. The next result characterize the unidirectional cycles which are V_2-SVOM.

Theorem 3.1. *Let $n (\geq 3)$ be an integer. Then the unidirectional cycle $\overrightarrow{C_n}$ is V_2-SVOM if and only if n is an odd integer.*

Proof. Suppose there exists a V_2-SVOML f of $\overrightarrow{C_n}$. Since $|E_2(e)| = r = 2$ for all $e \in A(\overrightarrow{C_n})$, by taking $k = 2$, $p = q = n$ and $r = 2$ in Lemma 2.2, we get $M = \frac{7n+3}{2}$. If n is an even integer, then M is not an integer, a contradiction. Thus n must be odd.

Conversely, assume that n is odd and $n \geq 3$. Let $V(\overrightarrow{C_n}) = \{a_i : 1 \leq i \leq n\}$ and $A(\overrightarrow{C_n}) = \{(a_i, a_{i \oplus_n 1}) : 1 \leq i \leq n\}$, where the operation $\oplus_n$ stands for addition modulo n. Define $f : V(D) \cup A(D) \to \{1, 2, \ldots, 2n\}$ as follows:

$f((a_i, a_{i+1})) = n + \frac{i+1}{2}$ when i is odd and $f((a_i, a_{i+1})) = \frac{3n+1}{2} + \frac{i}{2}$ when i is even. Then $\{w_2(a_n), w_2(a_1), \ldots, w_2(a_n - 1)\} = \{\frac{5n+3}{2}, \frac{5n+5}{2}, \ldots, \frac{7n+1}{2}\}$, is a set of n successive integers. Thus by Theorem 2.1, $\overrightarrow{C_n}$ is V_2-SVOM. □

Theorem 3.2. *Let $m \geq 1$ be an integer. Then $m\overrightarrow{C_n}$ is V_2-SVOM if and only if m and n are odd integers.*

Proof. Suppose there exists a V_2-SVOML f of $m\overrightarrow{C_n}$. Since $|E_2(e)| = r = 2$ for all $e \in A(m\overrightarrow{C_n})$, by taking $k = 2$, $p = q = mn$ and $r = 2$ in Lemma 2.2, we get $M = \frac{7mn+3}{2}$. Either m or n is even then M is not an integer, a contradiction. Thus both m and n are odd integers.

Conversely, assume that m and n are odd integers. Let $V(m\overrightarrow{C_n}) = V_1 \cup V_2 \cup \ldots \cup V_m$, where $V_i = \{v_i^1, v_i^2, \ldots, v_i^n\}$ for $i = 1, 2, \ldots, m$. Let $A(m\overrightarrow{C_n}) = A_1 \cup A_2 \cup \ldots \cup A_m$, where $A_i = \{e_i^1, e_i^2, \ldots, e_i^n\}$ with $e_i^j = (v_i^j, v_i^{j \oplus_n 1})$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.
Define $f : V(D) \cup A(D) \to \{1, 2, \ldots, 2mn\}$ as follows:
For $1 \leq i \leq \frac{m-1}{2}$,

$$f(v_i^j) = \begin{cases} (n-j)m + 1 - 2i & for\ j = 1, 2, \ldots n-2 \\ i & for\ j = n-1 \\ \frac{2mn-m+1}{2} + i & for\ j = n \end{cases}$$

$$f(e_i^j) = \begin{cases} \frac{(j-1)m}{2} + i + nm & for\ j = 1, 3, \ldots, n-2 \\ \frac{(n+j)m+1}{2} + i + nm & for\ j = 2, 4, \ldots, n-1 \\ \frac{(n+1)m}{2} + 1 - 2i + nm & for\ j = n. \end{cases}$$

For $\frac{m+1}{2} \leq i \leq m$,

$$f(v_i^j) = \begin{cases} (n+1-j)m + 1 - 2i & for\ j = 1, 2, \ldots, n-2 \\ i & for\ j = n-1 \\ \frac{(2n-3)m+1}{2} + i & for\ j = n \end{cases}$$

$$f(e_i^j) = \begin{cases} \frac{(j-1)m}{2} + i + nm & for\ j = 1, 3, \ldots, n-2 \\ \frac{(n+j-2)m}{2} + \frac{1}{2} + i + nm & for\ j = 2, 4, \ldots, n-1 \\ \frac{(n+3)m}{2} + 1 - 2i + nm & for\ j = n. \end{cases}$$

To prove $f(v) + w_2(v) = \frac{7mn+3}{2}$ for every vertex $v \in V(m\overrightarrow{C_n})$. Let $v \in V(m\overrightarrow{C_n})$.
Then $f(v_i^j) + w_2(v_i^j) = f(v_i^j) + f((v_i^j, v_i^{j+1})) + f((v_i^{j+1}, v_i^{j+2})) = f(v_i^j) + f(e_i^j) + f(e_i^{j+1})$.
Case 1: Suppose $1 \leq i \leq \frac{m-1}{2}$ and $j \geq 5$.
If j is even, then $f(v_i^j) + f(e_i^j) + f(e_i^{j+1}) = (n-j)m + 1 - 2i + \frac{(n+j)m+1}{2} + i + nm + \frac{jm}{2} + i + nm = nm - jm + 1 + nm + \frac{nm+jm+1}{2} + nm + \frac{jm}{2} = \frac{7mn}{2} + \frac{3}{2} = \frac{7mn+3}{2}$.
If j is odd, then $f(v_i^j) + f(e_i^j) + f(e_i^{j+1}) = (n-j)m + 1 - 2i + \frac{(j-1)m}{2} + i + nm + \frac{(n+j+1)m+1}{2} + i + nm = 3nm - jm + 1 + \frac{jm-m}{2} + \frac{nm+jm+m+1}{2} = 3nm + 1 + \frac{nm+1}{2} = \frac{7mn+3}{2}$.
Case 2: Suppose $\frac{m+1}{2} \leq i \leq m$ and $j \geq 5$.
If j is even, then $f(v_i^j) + f(e_i^j) + f(e_i^{j+1}) = (n+1-j)m + 1 - 2i + \frac{(n+j-2)m}{2} + \frac{1}{2} + i + nm + \frac{jm}{2} + i + nm = 3nm + m - jm + 1 + \frac{nm+jm-2m}{2} + \frac{1}{2} + \frac{jm}{2} =$

$3nm + m - jm + 1 + jm - m + \frac{nm}{2} + \frac{1}{2} = \frac{7mn+3}{2}$.
If j is odd, then $f(v_i^j) + f(e_i^j) + f(e_i^{j+1}) = (n + 1 - j)m + 1 - 2i + \frac{(j-1)m}{2} + i + nm + \frac{(n+j-1)m}{2} + \frac{1}{2} + i + nm = 3nm + m - jm + 1 + \frac{jm-m}{2} + \frac{nm+jm-m}{2} + \frac{1}{2} = 3nm + m - jm + 1 + \frac{nm}{2} + jm - m + \frac{1}{2} = \frac{7mn+3}{2}$.
Similarly, $f(v_i^j) + f(e_i^j) + f(e_i^{j+1}) = \frac{7mn+3}{2}$ for $1 \leq j \leq 4$ and $1 \leq i \leq m$. Hence f is a V_2-SVOML of $m\overrightarrow{C_n}$ with magic constant $M = \frac{7mn+3}{2}$. □

In the next result, we find the magic constant for unidirectional crown digraph which is not E_k-regular for all $k \geq 2$.

Theorem 3.3. *The unidirectional crown digraph $\overrightarrow{C_p^+}$ is V_2-SVOML if p is odd with magic constant $\frac{15p+3}{2}$.*

Proof. Let $V(\overrightarrow{C_p^+}) = \{a_i : 1 \leq i \leq p\} \cup \{b_i : 1 \leq i \leq p\}$ and $A(\overrightarrow{C_p^+}) = \{(a_i, a_{i \oplus_p 1}) : 1 \leq i \leq p\} \cup \{(b_i, a_i) : 1 \leq i \leq p\}$. Note that $\left|V(\overrightarrow{C_p^+})\right| = 2p$, $\left|A(\overrightarrow{C_p^+})\right| = 2p$.
Define $f : V(\overrightarrow{C_p^+}) \cup A(\overrightarrow{C_p^+}) \to \{1, 2, \ldots, 4p\}$ by
$f(b_i) = 2p - \frac{i-1}{2}$ when i is odd, $f(b_i) = \frac{3p-1}{2} - \frac{i-2}{2}$ when i is even. $f(a_i) = i, 1 \leq i \leq p$.
The arc labels are defined by $f((b_i, a_i)) = 2p + i, 1 \leq i \leq p$ and $f((a_i, a_{i \oplus_p 1})) = \frac{7p+2}{2} - \frac{i}{2}$ when i is odd, $f((a_i, a_{i \oplus_p 1})) = 4p - \frac{i-2}{2}$ when i is even.
To prove $f(b_i) + w_2(b_i) = \frac{15p+3}{2}$ for $1 \leq i \leq p$ and $b_i \in V(\overrightarrow{C_p^+})$.
Let $b_i \in V(\overrightarrow{C_p^+})$. Then $w_2(b_i) = f((b_i, a_i)) + f((a_i, a_{i \oplus_p 1}))$.
Case 1: Suppose i is odd, then $f(b_i) + f((b_i, a_i)) + f((a_i, a_{i \oplus_p 1}))$
$= 2p - \frac{i-1}{2} + 2p + i + + \frac{7p+2}{2} - \frac{i}{2} = 4p + \frac{7p+3}{2} = \frac{15p+3}{2}$.
Case 2: Suppose i is even, then $f(b_i) + f((b_i, a_i)) + f((a_i, a_{i \oplus_p 1}))$
$= \frac{3p-1}{2} - \frac{(i-2)}{2} + 2p + i + 4p - \frac{(i-2)}{2} = \frac{3p-1}{2} + 6p + 2 = \frac{15p+3}{2}$.
Similarly, we can prove that $f(a_i) + w_2(a_i) = \frac{15p+3}{2}$ for $1 \leq i \leq p$. □

Example 3.4. The above result has been illustrated through an example. Consider the following digraph $D_1 = \overrightarrow{C_5^+}$. Here $V(D_1) = \{a_1, a_2, \ldots, a_5\} \cup \{b_1, b_2, \ldots, b_5\}$ and $A(D_1) = \{(a_i, a_{i \oplus_5 1}) : 1 \leq i \leq 5\} \cup \{(b_i, a_i) : 1 \leq i \leq 5\}$.

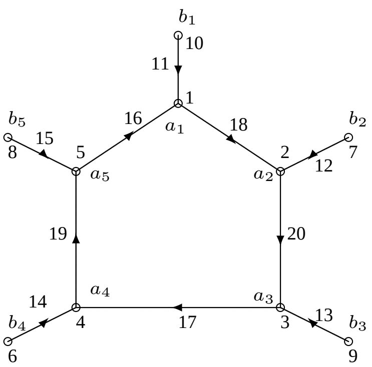

Figure 3: D_1

Here $p = 5$. The function f is given by $f : V(D_1) \cup A(D_1) \to \{1, 2, \ldots, 20\}$ by $f(b_i) = 2p - \frac{i-1}{2} = 10 - \frac{i-1}{2}$ when i is odd; $f(b_i) = \frac{3p-1}{2} - \frac{i-2}{2} = 7 - \frac{i-2}{2}$ when i is even; and $f(a_i) = i$ for $1 \leq i \leq 5$. The arc labels are given by $f((b_i, a_i)) = 2p + i = 10 + i, 1 \leq i \leq 5$; $f((a_i, a_{i \oplus_5 1})) = \frac{7p+2}{2} - \frac{i}{2} = \frac{37}{2} - \frac{i}{2}$ when i is odd; and $f((a_i, a_{i \oplus_5 1})) = 4p - \frac{i-1}{2} = 20 - \frac{i-2}{2}$ when i is even. From the above Figure, we can easily see that f is V_2-SVOML with magic constant $M = 39$.

CONCLUSION

In this paper, we introduced a new labeling in digraphs, namely V_k-SVOM. We obtain a necessary and sufficient condition for the existence of V_k-SVOML in digraphs and the magic constant for E_k-regular digraphs. Further, we characterized the unidirectional cycles and union of unidirectional cycles which are V_2-SVOM. In future we study V_k-SVOM$(k \geq 2)$ for directed circulant graph and the generalized de-Bruijn digraph.

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