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# Non-Response in Sample Surveys - A New Approach

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**Abstract.** In performing sample surveys, the investigator often faces the problem of non-response and the possibility of bias as a result of this phenomenon. This article discusses the issue of estimating the population mean in the presence of non-response under simple random sampling scheme using two auxiliary variables. These missing observations may occur on auxiliary, study or both of the variables. A weighted ratio-cum-exponential factor-type estimator is suggested and its properties have been studied under large sample approximation. The mean square error of suggested estimator is minimum as compared to selected estimators in the study. The theoretical results are verified through a simulation study.

**AMS (MOS) Subject Classification Codes: 62D05**
**Key Words:** Auxiliary; bias; factor-type; investigator; mean square error.

## 1. Introduction

The precision of the estimators is improved when, in sample surveys, information on the auxiliary variables is incorporated only at the estimation stage, designing stage or both stages provided that study and auxiliary variables are correlated. Gain in efficiency is obtained when information on suitable supplementary variables is properly utilized. When the study and supplementary variables are positively correlated, then ratio type of estimation is used Cochran [6]. When negative correlation exists between study and auxiliary variables, then Murthy [18] and Robson [23] proposed product type of estimation. Under various sampling schemes Dayyeh et al. [7], Ahmad [1], Azeem and Hanif [4], Chaudhary and Kumar [5], Gupta and Shabbir [9], Gupta and Shabbir [10], Azaz et al. [2], Kadilar and Cingi [12], Khan and Khan [13], Khare and Sinha [14], Kumar and Viswanathaiah [15], Muneer et al. [17], Noor-ul-Amin [20], Ozturk [21], Shabbir and Gupta [25], Srivastava and Jhajj [28], Tripathy [29] and Verma and Singh [30] have proposed a large number of estimators of population parameters using knowledge on auxiliary variable(s).

Under two phase sampling, Shukla [26] studied factor-type estimator and expression for minimum mean square error (MSE) is derived for a fixed cost. Estimating population mean under simple random sampling scheme, Shukla et al. [27] proposed three classes of estimators utilizing knowledge on two auxiliary variables and derived expressions for minimum MSE. Under two-phase sampling, Audu and Adewara [3] developed a modified factor-type estimator and obtained the expression of minimum mean square error, first and second-phase sample sizes for a (a) fixed cost and (b) fixed variance. For the first time, the seriousness of missing observations was dealt by Hansen and Hurwitz [11] in estimating population mean. An alternative approach to Hansen and Hurwitz [11] approach was developed by Grover and Kaur [8]. The selected respondents are divided into three mutually exclusive sub-groups i.e.

(i) responded to both study and auxiliary variables, (sub-sample $s_1$)

(ii) responded to auxiliary variable, only (sub-sample $s_2$)

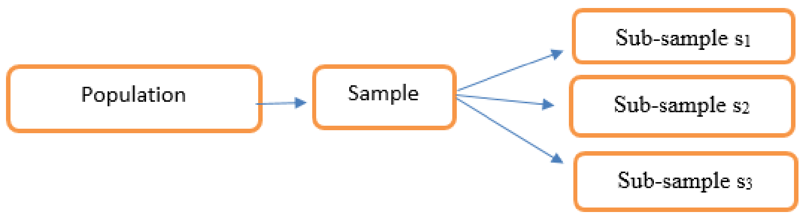
(iii) responded to study variable, only (sub-sample $s_3$).

In the present article, a new method has been proposed where selected respondents are divided into four mutually exclusive sub-groups, $s_1$, $s_2$, and $s_3$ as defined in (i), (ii), and (iii) above and $s_4$ defined as: did not respond to any of the variables in the study. In this method, a questionnaire is initially mailed to the selected sample and those who did not answer to any variable are directly contacted with the assumption that all respondents provide information through direct contact method.

## 2. Proposed sampling plan and Notations

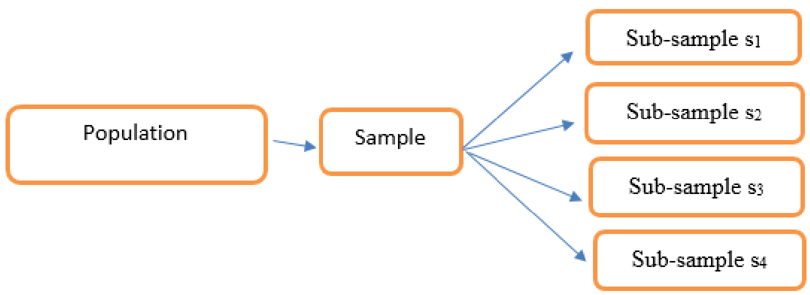
Let there be N units $(U_1, U_2, \ldots, U_N)$ in the population. Let $Y$ and $X_1, X_2$ be the characteristics on study and auxiliary variables respectively. For $i^{th}$ unit of population, let the triplet $(y_i, x_{1i}, x_{2i}); i = 1, 2, 3 \ldots N$ designates the values of study and auxiliary variables respectively. From this population a sample "$s$" of $n$ units is drawn using simple random sampling without replacement (SRSWOR). The population mean $\bar{Y}$ is estimated utilizing known means $\bar{X}_1$ and $\bar{X}_2$ of auxiliary variables. A questionnaire is mailed to whole of the n units selected in sample "$s$". Let in the sub-sample "$s_1$", for $(n - p - q - r)$ units full response occurs on all three variables, in sub-sample "$s_2$", p units responded on both auxiliary variables $X_1, X_2$ but no-reply on study variable $Y$, in sub-sample "$s_3$", $q$ units responded on study variable but did not respond on both auxiliary variables and for $r$ units in "$s_4$", information is absent on all variables. As in sub-sample "$s_4$", information is not available on all three variables, information is obtained by using a face-to-face interview method.

**Figure 1: Grover and Kaur (2014) Method**




$s = s_1 \cup s_2 \cup s_3,\quad s_j \cap s_k = \emptyset$ for $j, k = 1, 2, 3$ and $j \neq k$

**Figure 2: Suggested Method**




$s = s_1 \cup s_2 \cup s_3 \cup s_4,\quad s_j \cap s_k = \emptyset$ for $j, k = 1, 2, 3, 4$ and $j \neq k$

For different sub-samples, the sample means of study and auxiliary variables are;

$$\bar{x}_l = \frac{\sum_{i\epsilon s_1}^{p} x_{li}}{(n-p-q-r)},\ \bar{y} = \frac{\sum_{i\epsilon s_1}^{p} y_i}{(n-p-q-r)},\ \bar{x}'_l = \frac{\sum_{i\epsilon s_2}^{p} x'_{li}}{p},\ \bar{y}'' = \frac{\sum_{i\epsilon s_3}^{q} y''_i}{q},\ \bar{x}'''_l = \frac{\sum_{i\epsilon s_4}^{r} x'''_{li}}{r},$$
$$\bar{y}''' = \frac{\sum_{i\epsilon s_4}^{r} y'''_i}{r}, \qquad \text{for} \quad l = 1, 2. \tag{2. 1}$$
$$\bar{x}_{lA} = \frac{(n-p-q-r)\,\bar{x}_l + p\bar{x}'_l + r\bar{x}'''_l}{(n-q)} \quad \text{(pooled mean of } X_l \text{ based on } S_1 \cap S_2 \cap S_4\text{)},$$
$$\bar{y}_A = \frac{(n-p-q-r)\,\bar{y} + q\,\bar{y}'' + r\,\bar{y}'''}{(n-p)} \quad \text{(pooled mean of } Y \text{ based on } S_1 \cap S_3 \cap S_4\text{)}.$$

$$f_0 = \frac{1}{(n-p-q-r)} - \frac{1}{N},\ f_1 = \frac{1}{(n-p)} - \frac{1}{N},\ f_2 = \frac{1}{(n-q)} - \frac{1}{N},\ f_3 = \frac{1}{(n-r)} - \frac{1}{N}, \tag{2. 2}$$

$$\bar{Y} = N^{-1}\left(\sum_{i=1}^{N} Y_i\right),\ \bar{X}_l = N^{-1}\left(\sum_{i=1}^{N} X_{li}\right),\ S_0^2 = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^2}{N-1},\ S_l^2 = \frac{\sum_{i=1}^{N}(X_{li} - \bar{X}_l)^2}{(N-1)},$$
$$S_{0l} = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})(X_{li} - \bar{X}_l)}{(N-1)},\ S_{12} = \frac{\sum_{i=1}^{N}(X_{1i} - \bar{X}_1)(X_{2i} - \bar{X}_2)}{(N-1)},\ C_0 = \frac{S_y}{\bar{Y}},\ C_l = \frac{S_l}{\bar{X}_l}, \tag{2. 3}$$
$$\rho_{0l} = \frac{S_{0l}}{S_0 S_l},\ \rho_{12} = \frac{S_{12}}{S_1\, S_2}.$$

## 3. Some Available estimators

Selected estimators of population mean and their corresponding MSE's are given as follows.

(i) The mean per unit estimator of population mean is:

$$t_0 = \bar{y}_A \tag{3.1}$$

The mean square error of $t_0$ is:

$$MSE(t_0) \cong \bar{Y}^2 f_1 C_0^2 \tag{3.2}$$

(ii) Sanaullah et al. [24] suggested exponential estimator-I

$$t_1 = \bar{y}_A exp\left[\alpha\left(\frac{\bar{X}_2 - \bar{x}_{2A}}{\bar{X}_2 + \bar{x}_{2A}}\right) - (1-\alpha)\left(\frac{\bar{X}_1 - \bar{x}_{1A}}{\bar{X}_1 + \bar{x}_{1A}}\right)\right] \tag{3.3}$$

where $\alpha$ is the optimization constant. The expression of mean square error is:

$$MSE(t_1) \approx \bar{Y}^2 \left[\begin{array}{c} f_1\left\{C_0^2 + (1-\alpha) H_{01} C_1^2 - \alpha\ H_{02} C_2^2\right\} \\ +f_2\left\{\frac{(1-\alpha)^2}{4} C_1^2 + \frac{\alpha^2}{4} C_2^2 - \frac{\alpha(1-\alpha)}{2} H_{12} C_2^2\right\} \end{array}\right] \tag{3.4}$$

(iii) Vishwakarma and Kumar [31] exponential chain ratio-product type estimator:

$$t_2 = \bar{y}_A\left[k \exp\left\{\frac{\left(\frac{\bar{x}_{1A}}{\bar{x}_{2A}}\right)\bar{X}_2 - \bar{X}_1}{\left(\frac{\bar{x}_{1A}}{\bar{x}_{2A}}\right)\bar{X}_2 + \bar{X}_1}\right\} + (1-k)\exp\left\{\frac{\bar{X}_1 - \left(\frac{\bar{x}_{1A}}{\bar{x}_{2A}}\right)\bar{X}_2}{\bar{X}_1 + \left(\frac{\bar{x}_{1A}}{\bar{x}_{2A}}\right)\bar{X}_2}\right\}\right] \tag{3.5}$$

where $k$ is the optimization constant. The expression of mean square error is given by:

$$MSE(t_2) \approx \bar{Y}^2 \left[\begin{array}{c} f_1\left\{C_0^2 + (2k-1)\left(H_{01}C_1^2 - H_{02}C_2^2\right)\right\} \\ +f_2\frac{(2k-1)^2}{4}\left(C_1^2 + C_2^2 - 2H_{12}C_2^2\right) \end{array}\right] \tag{3.6}$$

(iv) Khan and Khan [13] proposed the following estimator

$$t_3 = \left\{\left[\overline{y}_A + \alpha_1\left(\overline{x}_{1A} - \overline{X}_1\right)\right] * \left[\alpha_2 \exp\left(\frac{\left(\frac{\overline{x}_1 A}{\overline{x}_{2A}}\right)\overline{X}_2 - \overline{X}_1}{\left(\frac{\overline{x}_{1A}}{\overline{x}_{2A}}\right)\overline{X}_2 + \overline{X}_1}\right) + (1-\alpha_2)\exp\left(\frac{\overline{X}_1 - \left(\frac{\overline{x}_{1A}}{\overline{x}_{2A}}\right)\overline{X}_2}{\overline{X}_1 + \left(\frac{\overline{x}_{1A}}{\overline{x}_{2A}}\right)\overline{X}_2}\right)\right]\right\}. \tag{3.7}$$

The mean square error of $t_3$ is given by:

$$MSE(t_3) \approx \overline{Y}^2 \left\{\begin{array}{c} f_1\left[C_0^2 + H_{01}C_1^2\left(2\alpha_1 R_1 + (2\alpha_2 - 1)\right) - (2\alpha_2 - 1) H_{02} C_2^2\right] \\ +f_2 C_1^2\left[\alpha_1^2 R_1^2 + \frac{(2\alpha_2-1)^2}{4} + \alpha_1(2\alpha_2-1) R_1\right] + \\ f_2 C_2^2\left[\frac{(2\alpha_2-1)^2}{4} - \frac{(2\alpha_2-1)^2}{2} H_{12} - \alpha_1(2\alpha_2-1) R_1 H_{12}\right] \end{array}\right\} \tag{3.8}$$

(v) Riaz et al. [22] proposed a regression-cum-exponential estimator

$$t_4 = \left[a_1 \bar{y}_A + a_2 \ \left(\bar{X}_2 - \bar{x}_{2A}\right)\right] exp\left[\left(\frac{\bar{X}_1 - \bar{x}_{1A}}{\bar{X}_1 + \bar{x}_{1A}}\right)\right] \tag{3.9}$$

where $\alpha_1$ and $\alpha_2$ are real positive constants.

$$MSE(t_4) \approx \bar{Y}^2 \left[ \begin{array}{c} (a_1 - 1)^2 + f_1 \left\{\alpha_1^2 C_0^2 - a_1^2 H_{01} C_1^2 - 2a_1 a_2 \ R_2 H_{02} C_2^2\right\} \\ +f_2 \left\{\left(\frac{a_1}{2}\right)^2 C_1^2 + (a_2 R_2)^2 C_2^2 + a_1 a_2 \ R_2 H_{12} C_2^2\right\} \end{array} \right] \tag{3.10}$$

$$t_5 = [\alpha_1 \bar{y}_A + \alpha_2(\bar{X}_2 - \bar{x}_{2A})] \exp[-\frac{(\bar{X}_1 - \bar{x}_{1A})}{(\bar{X}_1 - \bar{x}_{1A})}] \tag{3.11}$$

$$MSE(t_5) \approx \bar{Y}^2 \left[ \begin{array}{c} (a_1 - 1)^2 + f_1 \left\{\alpha_1^2 C_0^2 + a_1^2 H_{01} C_1^2 - 2a_1 a_2 \ R_2 H_{02} C_2^2\right\} \\ +f_2 \left\{\left(\frac{a_1}{2}\right)^2 C_1^2 + (a_2 R_2)^2 C_2^2 - a_1 a_2 \ R_2 H_{12} C_2^2\right\} \end{array} \right] \tag{3.12}$$

where $R_2 = \frac{\bar{X}_2}{\bar{Y}}$.

(vi) Muneer et al. [17] Estimator

$$t_6 = \left\{\left[k_3 \overline{y}_A + k_4 \ \left(\overline{x}_{1A} - \overline{X}_1\right)\right] * \left[w\left(2 - \exp\left(\frac{\overline{x}_{2A} - \overline{X}_2}{\overline{x}_{2A} + \overline{X}_2}\right)\right) + (1-w)\exp\left(\frac{\overline{X}_2 - \overline{x}_{2A}}{\overline{X}_2 + \overline{x}_{2A}}\right)\right]\right\} \tag{3.13}$$

$$MSE(t_6) \approx \overline{Y}^2 \left\{ \begin{array}{c} (k_3 - 1)^2 + f_1\left[k_3^2 C_0^2 - 2k_3 k_4 R_1 H_{01} C_1^2 \ - k_3^2 H_{02} C_2^2\}\right] \\ +f_2\left[k_4^2 R_1^2 C_1^2 + \frac{k_3^2}{4} C_2^2 + k_3 k_4 R_1 H_{12} C_2^2\right] \end{array} \right\}. \tag{3.14}$$

## 4. Proposed estimator

The suggested weighted ratio-cum-exponential factor-type estimator of finite population mean is based upon, $s_1 \cup s_2 \cup s_3 \cup s_4$, which is as follows:

$$t_p = \overline{y}\left\{w\left[\frac{N_1}{D_1} \exp\left(\frac{N_2 - \ D_2}{N_2 + \ D_2}\right)\right] + (1 - w)\left[\frac{N_2}{D_2} \exp\left(\frac{N_1 - \ D_1}{N_1 + \ D_1}\right)\right]\right\}. \tag{4.1}$$

where $\quad 0 \leq w \leq 1$ and

$$N_l = (A_l + C_l)\overline{X}_l + f B_l \ \overline{x}_{lA}, D_l = (A_l + f B_l)\overline{X}_l + C_l \ \overline{x}_{lA}, A_l = (k_l - 1)(k_l - 2),$$
$$B_l = (k_l - 1)(k_l - 4), C_l = (k_l - 2)(k_l - 3)(k_l - 4), f = \frac{n}{N},$$
$$\alpha_l \ = \frac{f \ B_l}{A_l \ + \ f \ B_l \ + \ C_l}, \beta_l \ = \frac{C_l}{A_l \ + \ f \ B_l \ + \ C_l}, \theta_l = \alpha_l - \beta_l, \tag{4.2}$$

where $l$=1, 2.

Using (4.2) in (4.1), we get:

$$t_p = \bar{y}_A \left[ \begin{array}{c} w\left(\left\{\frac{(A_1+C_1)\bar{X}_1+fB_1\bar{x}_{1A}}{(A_1+fB_1)\bar{X}_1+C_1\bar{x}_{1A}}\right\} * \right. \\ \left. \exp\left\{\frac{\{(A_2+C_2)\bar{X}_2+fB_2\bar{x}_{2A}\}-\{(A_2+fB_2)\bar{X}_2+C_2\bar{x}_{2A}\}}{\{(A_2+C_2)\bar{X}_2+fB_2\bar{x}_{2A}\}+\{(A_2+fB_2)\bar{X}_2+C_2\bar{x}_{2A}\}}\right\}\right) \\ +(1-w)\left(\left\{\frac{(A_2+C_2)\bar{X}_2+fB_2\bar{x}_{2A}}{(A_2+fB_2)\bar{X}_2+C_2\bar{x}_{2A}}\right\} * \right. \\ \left. \exp\left\{\frac{\{(A_1+C_1)\bar{X}_1+fB_1\bar{x}_{1A}\}-\{(A_1+fB_1)\bar{X}_1+C_1\bar{x}_{1A}\}}{\{(A_1+C_1)\bar{X}_1+fB_1\bar{x}_{1A}\}+\{(A_1+fB_1)\bar{X}_1+C_1\bar{x}_{1A}\}}\right\}\right) \end{array} \right]. \tag{4.3}$$

## 5. Bias and MSE of Suggested Estimator

To obtain expressions of bias and MSE of proposed estimator up to first degree of approximation, we define the following error terms:

$$\begin{aligned} & e_0 = \frac{\bar{y}}{\bar{Y}} - 1,\ e_l = \frac{\bar{x}_l}{\bar{X}_l} - 1,\ e'_l = \frac{\bar{x}'_l}{\bar{X}_l} - 1, e''_0 = \frac{\bar{y}''}{\bar{Y}} - 1, e'''_0 = \frac{\bar{y}'''}{\bar{Y}} - 1, e'''_l = \frac{\bar{x}'''_l}{\bar{X}_l} - 1. \\ & E(e_0) = E(e_l) = E(e'_l) = E(e''_0) = E(e'''_0) = E(e'''_l) = 0, E(e_0^2) = f_0C_0^2, E(e_l^2) = f_0C_l^2, \\ & E(e'^2_l) = (r^{-1} - N^{-1})C_l^2, E(e_0e_l) = f_0\rho_{0l}C_0C_l, E(e_1e_2) = f_0\rho_{12}C_1C_2, \\ & E(e'_1\ e'_2) = (r^{-1} - N^{-1})\rho_{12}C_1, E(e'''_0 e'''_l) = (r^{-1} - N^{-1})\rho_{0l}C_0C_l, \\ & E(e'''_1\ e'''_2) = (r^{-1} - N^{-1})\rho_{12}C_1C_2, E(e_0e'_l) = E(e_0e''_0) = E(e_0e'''_0) = \\ & E(e_0\ e'''_l) = E(e'_le''_0) = E(e'_le'''_0) = E(e''_0e'''_0) = E(e''_0e'''_l) = 0, \\ & H_{0l} = \rho_{0l}\ \frac{C_0}{C_l}, H_{12} = \rho_{12}\ \frac{C_1}{C_2}. \end{aligned} \tag{5.1}$$

For bias and MSE of the suggested estimator, replacing $\bar{y}_A$ and $\bar{x}_{lA}$ by corresponding expressions from (2.1) and then using notations from (4.2), we get:

$$t_p = \bar{Y}\left\{(1+\Omega_0)\left[\begin{array}{c} w\left(\left(\frac{1+\alpha_1\ \Omega_1}{1+\beta_1\Omega_1}\right)\exp\left(\frac{\langle 1+\alpha_2\ \Omega_2\rangle-\langle 1+\beta_2\ \Omega_2\rangle}{\langle 1+\alpha_2\ \Omega_2\rangle+\langle 1+\beta_2\ \Omega_2\rangle}\right)\right) \\ +(1-w)\left(\left(\frac{1+\alpha_2\ \Omega_2}{1+\beta_2\Omega_2}\right)\exp\left(\frac{\langle 1+\alpha_1\ \Omega_1\rangle-\langle 1+\beta_1\ \Omega_1\rangle}{\langle 1+\alpha_1\ \Omega_1\rangle+\langle 1+\beta_1\ \Omega_1\rangle}\right)\right)\end{array}\right]\right\}, \tag{5.2}$$

where $\Omega_0 = \left\{\left(\frac{n-p-q-r}{n-p}\right)\ e_0 + \left(\frac{q}{n-p}\right)e''_0 + \left(\frac{r}{n-p}\right)e'''_0\right\}$,
$\Omega_1 = \left\{\left(\frac{n-p-q-r}{n-q}\right)e_1 + \frac{p}{n-q}\ e'_1 + \frac{r}{n-q}e'''_1\right\}$ and $\Omega_2 = \left\{\left(\frac{n-p-q-r}{n-q}\right)e_2 + \frac{p}{n-q}\ e'_2 + \frac{r}{n-q}e'''_2\right\}$.

$$t_p - \bar{Y} = \bar{Y}\left(\Omega_0 + b_1\Omega_1 + b_2\Omega_2 + b_1\Omega_0\Omega_1 + b_2\Omega_0\Omega_2 + b_3\Omega_1\Omega_2 + b_4\Omega_1^2 + b_5\Omega_2^2\right), \tag{5.3}$$

where

$$\begin{aligned} b_1 &= \frac{(1+w)}{2}\theta_1, b_2 = \frac{(2-\ w)}{2}\theta_2, b_3 = \frac{\theta_1\theta_2}{2}, \\ b_4 &= (1-w)\ \left\{\frac{\theta_1^2}{8} - \frac{\theta_1}{4}(\alpha_1\ \ + \beta_1\ )\right\} - w\theta_1\beta_1, \\ b_5 &= \left[w\ \left\{\frac{\theta_2^2}{8} - \frac{\theta_2}{4}(\alpha_2 + \beta_2\ )\right\} - (1-w)\,\theta_2\beta_2\right]. \end{aligned} \tag{5.4}$$

Taking expectation of (5.3), the bias of $t_p$ becomes:

$$Bias(t_p) \cong \bar{Y}\left[f_1\left(b_1H_{01}C_1^2 + b_2H_{02}C_2^2\right) + f_2\left(b_3H_{12}C_2^2 + b_4C_1^2 + b_5C_2^2\right)\right], \tag{5.5}$$

Squaring (5.3), taking expectation, the MSE of $t_p$ is:

$$MSE\left(t_p\right) \cong \overline{Y}^2 \left[ \begin{array}{c} f_1\left(C_0^2+2b_1H_{01}C_1^2+2b_2H_{02}C_2^2\right)+ \\ f_2\left(b_1^2C_1^2+b_2^2C_2^2+2b_1b_2H_{12}C_2^2\right) \end{array} \right]. \tag{5.6}$$

For minimum MSE, differentiate (5.6) with respect to $b_1$ and $b_2$ respectively and equating to zero, $b_{1m}$ and $b_{2m}$ are:

$$b_{1m} = \frac{f_1\left(H_{02}H_{12}C_2^2-H_{01}C_1^2\right)}{f_2\left(C_1^2-H_{12}^2C_2^2\right)}, \quad b_{2m} = \frac{f_1C_1^2\left(H_{01}H_{12}-H_{02}\right)}{f_2\left(C_1^2-H_{12}^2C_2^2\right)}. \tag{5.7}$$

Rearranging (5.7), $\theta_{1m}$ and $\theta_{2m}$ are given by:

$$\theta_{1m} = \frac{2f_1\left(H_{02}H_{12}C_2^2-H_{01}C_1^2\right)}{(1+w)\,f_2\left(C_1^2-H_{12}^2C_2^2\right)}, \quad \theta_{2m} = \frac{2f_1C_1^2\left(H_{01}H_{12}-H_{02}\right)}{(2-w)\,f_2\left(C_1^2-H_{12}^2C_2^2\right)}. \tag{5.8}$$

In terms of characterization scalar $k_l$, (5.8) can be rearranged in the form:

$$\left[ \begin{array}{l} (1+\theta_{1m})\,k_1^3+(f\theta_{1\mathrm{m}}-f-8\theta_{1\mathrm{m}}-9)\,k_1^2+(23\theta_{1\mathrm{m}} \\ -5f\theta_{1\mathrm{m}}+5f+26)\,k_1+(4f\theta_{1\mathrm{m}}-4f-22\theta_{1\mathrm{m}}-24) \end{array} \right]=0,$$

$$\left[ \begin{array}{l} (1+\theta_{2\mathrm{m}})\,k_2^3+(f\theta_{2\mathrm{m}}-f-8\theta_{2\mathrm{m}}-9)\,k_2^2+(23\theta_{2\mathrm{m}} \\ -5f\theta_{2\mathrm{m}}+5f+26)\,k_2+(4f\theta_{2\mathrm{m}}-4f-22\theta_{2\mathrm{m}}-24) \end{array} \right]=0. \tag{5.9}$$

Solving (5.9), gives multiple choices of $k_1$ and $k_2$ combinations, which result multiple choices of minimum mean square error.

## 6. Comparison

The suggested estimator is compared to the selected estimators on the basis of MSE and conditions are obtained under which the proposed estimator outperforms the selected estimators.

(i) From (5.6) and (3.2), MSE ($t_p$) < MSE ($t_0$), if
$\omega_1\, C_1^2+\,\omega_2\, C_1^2+\,\omega_3\, C_2^2\,+\,\omega_4\, C_2^2\,<0$, where $\omega_1 = 2f_1b_1H_{01}$ , $\omega_2 = f_2b_1^2, \omega_3 = 2f_1H_{02}$ and $\omega_4 = f_2\left(b_2^2+2b_1b_2H_{12}\right)$.

(ii) From (5.6) and (3.4), MSE ($t_p$) < MSE ($t_2$), if
$\lambda_1\, C_1^2+\,\lambda_2\, C_1^2+\,\lambda_3\, C_2^2\,+\,\lambda_4\, C_2^2\,<0$,
where $\lambda_1 = f_1\left\{2b_1-(1-\alpha)\right\}H_{01}, \lambda_2 = f_2\left(b_1^2-\frac{(1-\alpha)^2}{4}\right)$, $\lambda_3 = f_1\left(2b_2+\alpha\right)\, H_{02}$,
and $\lambda_4 = f_2\left\{\left(b_2^2-\frac{\alpha^2}{4}\right)+\left(2b_1b_2+\frac{\alpha(1-\alpha)}{2}\right)H_{12}\right\}$.

(iii) From (5.6) and (3.6), MSE ($t_p$) < MSE ($t_2$), if
$\pi_1\, C_1^2\,+\,\pi_2\, C_1^2+\,\pi_3\, C_2^2\,+\,\pi_4\, C_2^2\,<0$,
where $\pi_1 = f_1\left\{2b_1-(2k-1)\right\}H_{01}$, $\pi_2 = f_2\left\{b_1^2-\frac{(2k-1)^2}{4}\right\}$,
$\pi_3 = f_1\left\{2b_2+(2k-1)\right\}H_{02}$, and $\pi_4 = f_2\left\{\left(b_2^2-\frac{(2k-1)^2}{4}\right)+\left(2b_1b_2+\frac{(2k-1)^2}{2}\right)H_{12}\right\}$.

(iv) From (5.6) and (3.8), MSE ($t_p$) < MSE ($t_3$), if
$\phi_1\, C_1^2+\,\phi_2\, C_1^2+\,\phi_3\, C_2^2\,+\,\phi_4\, C_2^2<0$, $\phi_1 = f_1\left\{2b_1-2\alpha_1R_1-(2\alpha_2-1)\right\}H_{01}$,

$\phi_2 = f_2\left\{b_1^2 - \alpha_1^2 R_1^2 - \frac{(2\alpha_2-1)^2}{4} - \alpha_1(2\alpha_2-1)R_1\right\}, \phi_3 = f_1\{2b_2 + (2\alpha_2 - 1)\}H_{02},$
$\phi_4 = f_2\left\{\left(b_2^2 - \frac{(2\alpha_2-1)^2}{4}\right) + \left(2b_1b_2 + \frac{(2\alpha_2-1)^2}{2} + \alpha_1(2\alpha_2-1)R_1\right)H_{12}\right\}$

(v) From (5.6) and (3.10), MSE ($t_p$) < MSE ($t_4$), if
$\left\{\left(\psi_1C_0^2+\psi_2C_1^2+\psi_3C_1^2+\psi_4C_2^2+\psi_5C_2^2\right) - (\alpha_1-1)^2\right\} < 0,$
where $\psi_1 = f_1\left(1-\alpha_1^2\right)$, $\psi_2 = f_1\left(2b_1+\alpha_1^2\right)H_{01}$, $\psi_3 = f_2\left\{b_1^2 - \left(\frac{\alpha_1}{2}\right)^2\right\}$,
$\psi_4 = 2f_1(b_2+\alpha_1\alpha_2R_2)H_{02}$ and $\psi_5 = f_2\left\{\left(b_2^2-\alpha_2^2R_2^2\right) + (2b_1b_2 - \alpha_1\alpha_2R_2)H_{12}\right\}$.

(vi) From (5.6) and (3.12), $MSE(t_p) < MSE(t_5)$, if
$\{(\eta_1C_0^2 + \eta_2C_1^2 + \eta_3C_1^2 + \eta_4C_2^2 + \eta_5C_2^2) - (\alpha_1-1)^2\} < 0$, where $\eta_1 = f_1(1-\alpha_1^2)$,
$\eta_2 = f_1(2b_1 - \alpha_1^2)H_{01}, \eta_3 = f_2\{b_1^2 - (\frac{\alpha_1}{2})^2\}$,
$\eta_4 = 2f_1(b_2 + \alpha_1\alpha_2R_2)H_{02}$ and $\eta_5 = f_2\{(b_2^2 - \alpha_2^2R_2^2) + (2b_1b_2 + \alpha_1\alpha_2R_2)H_{12}\}$.

(vii) From (5.6) and (3.14), MSE ($t_p$) < MSE ($t_6$), if
$\left\{\left(\tau_1C_0^2+\tau_2C_1^2+\tau_3C_1^2+\tau_4C_2^2+\tau_5C_2^2\right) - (k_3-1)^2\right\} < 0,$
where $\tau_1 = f_1\left(1-k_3^2\right)$, $\tau_2 = 2f_1(b_1+k_3k_4R_1)H_{01}$, $\tau_3 = f_2\left(b_1^2-k_4^2R_1^2\right)$,
$\tau_4 = f_1\left(2b_2+k_3^2\right)H_{02}$ and $\tau_5 = f_2\left\{\left(b_2^2-\frac{k_3^2}{4}\right) + (2b_1b_2-k_3k_4R_1)H_{12}\right\}$.

## 7. Simulation Results and Discussion

To make the results more encouraging, simulation study has been conducted. From the following fixed populations, 10,000 samples were generated using (SRSWOR) scheme.

**Population 1.** Source: Murthy [19]
$Y$ = wheat cultivated area in 1964, $X_1$ = wheat cultivated area in 1963, $X_2$ = Cultivated land in 1961.
$N = 34, \bar{Y} = 199.44, \bar{X}_1 = 208.89, \bar{X}_2 = 747.59, C_0^2 = 0.5673, C_1^2 = 0.5191, C_2^2 = 0.35274,$ $\rho_{0l} = 0.9801, \rho_{02} = 0.9043, \rho_{12} = 0.9097.$

**Population 2.** Source: Kutner et al. [16]
$Y$ = No. of females employed, $X_1$ = No. of active physicians, $X_2$ = No. of females who are educated. $N = 61, \bar{Y} = 7.46, \bar{X}_1 = 5.31, \bar{X}_2 = 179, C_0^2 = 0.7103, C_1^2 = 0.7587, C_2^2 = 0.2515, \rho_{01} = 0.7373, \rho_{02} = -0.207, \rho_{12} = -0.0033.$

**Population 3.** Source: Kutner et al. [16]
$Y$ =Total cost of claims by subscriber, $X_1$ = subscriber's age (year), $X_2$ = Treatment duration in days.
$N = 788, \bar{Y} = 2799.955, \bar{X}_1 = 58.72, \bar{X}_2 = 164.03, C_0^2 = 5.70931, C_1^2 = 0.013231, C_2^2 = 0.543397, \rho_{01} = -0.059, \rho_{02} = 0.176, \rho_{12} = 0.136.$

Under the proposed sampling plan and with different missingness rates in each variable, two different sample sizes (30% and 20% of the corresponding population sizes) were taken. For optimal mean square error (MSE), $\theta_{1m}$ and $\theta_{2m}$ are obtained in (5.8). Rearranging (5.8), two polynomials of degree 3 in $k_1$ and $k_2$ are obtained in (5.9). Solving equation (5.9), we get multiple choices for minimum MSE of the suggested estimator, which depend on the number of combinations of real roots of $k_1$ and $k_2$. (see Table 1, 2 and 3, in Appendix). In population-1, at serial no. 5, 9 and 10 both $k_1$ and $k_2$ have three real roots so there are nine possibilities for minimum MSE in each case. For rest of the quadruplets $(n, p, q, r)$

observed, there is only one real and two imaginary roots for $k_1$ and all roots of $k_2$ are real so the estimator attains three minimum values of MSE in each case. For population 2, at serial no.8, $k_1$ has only one real and two complex roots and all roots of $k_2$ are real so there are three possible values of minimum MSE and for remaining combinations of quadruplet, all roots of $k_1$ and $k_2$ are real so there are nine possibilities for minimum MSE in each case. In population 3, $k_1$ has only one real and two complex roots for all quadruplets $(n, p, q, r)$ examined, while all roots of $k_2$ are real so in each case, there are three possible values of minimum MSE. The percent relative efficiency (PRE) is computed by using the following expression:

$$PRE = Var(t_0)/Var(t_i) * 100,$$

for $i = 0, 1, 2, 3, 4, 5, 6.$ and results are displayed in Tables 4, 5 and 6 in appendix.

## 8. Conclusion

A pooled ratio-cum-exponential factor-type estimator is proposed for estimating the population mean under non-response incorporating knowledge on two supplementary variables, and comparison of suggested estimator with the chosen estimators is made on the basis of percent relative efficiency. From Table 1, 2 and 3, (Appendix), number of real roots of $k_1$ and $k_2$ reveal that the selected estimator provides a wide range for optimal MSE, which depends on the number of combinations of real roots of the characterizing constant $k_l$ $(l = 1, 2)$. The compiled values of PRE in Table 4 (Appendix) reveal that the suggested estimator outperforms all the estimators chosen, for high positive correlations between variables (Population-1) under both large and small sample sizes and different missingness rates. Similarly results of PRE in Table 5 and 6 (Appendix) reveal that the suggested estimator outperforms under low correlation structures (Population-2 and 3) for both large and small sample sizes and different missingness rates. Thus, due to the dominant nature of the proposed estimator, it may be recommended for practical use.

## Appendix

Table 1. Real Roots of $k_1$ & $k_2$

| S.No | Population-1 | | | | | |
|---|---|---|---|---|---|---|
| | SS* | MR** | | | Real Roots | |
| | n | p | q | r | $k_1$ | $k_2$ |
| 1 | 14 | 3 | 3 | 1 | 2.417 | 1.89 |
| | | | | | ——— | 3.8826 |
| | | | | | ——— | 4.5206 |
| 2 | 14 | 3 | 2 | 1 | 2.459 | 1.3471 |
| | | | | | ——— | 3.6326 |
| | | | | | ——— | 4.0048 |
| 3 | 14 | 3 | 1 | 1 | 2.9905 | 1.605 |
| | | | | | ——— | 3.9521 |
| | | | | | ——— | 4.696 |
| 4 | 14 | 2 | 3 | 1 | 2.0236 | 1.1489 |
| | | | | | ——— | 4.014 |
| | | | | | ——— | 4.0679 |
| 5 | 14 | 1 | 3 | 1 | 1.252 | 1.0404 |
| | | | | | 2.0326 | 3.9324 |
| | | | | | 20.358 | 5.9947 |
| 6 | 10 | 3 | 3 | 1 | 2.0538 | 1.8819 |
| | | | | | ——— | 3.3938 |
| | | | | | ——— | 4.2241 |
| 7 | 10 | 3 | 2 | 1 | 3.1645 | 1.7774 |
| | | | | | ——— | 3.4736 |
| | | | | | ——— | 4.2457 |
| 8 | 10 | 3 | 1 | 1 | 2.5912 | 1.6309 |
| | | | | | ——— | 2.9118 |
| | | | | | ——— | 7.1187 |
| 9 | 10 | 2 | 3 | 1 | 1.0024 | 1.9801 |
| | | | | | 2.6624 | 3.2576 |
| | | | | | 46.3146 | 4.8124 |
| 10 | 10 | 1 | 3 | 1 | 2.2029 | 1.7764 |
| | | | | | 2.7004 | 3.9985 |
| | | | | | 12.7291 | 5.0065 |

*SS= Sample size
*MR=Missingness rate

TABLE 2. Real Roots of $k_1$ & $k_2$

| S.No | Population-2 | | | | | |
|---|---|---|---|---|---|---|
| | SS | MR | | | Real Roots | |
| | n | p | q | r | $k_1$ | $k_2$ |
| 1 | 24 | 4 | 4 | 2 | 1.5935 | 1.5416 |
| | | | | | 2.9321 | 3.0708 |
| | | | | | 16.8056 | 6.6751 |
| 2 | 24 | 4 | 3 | 2 | 1.8923 | 1.2408 |
| | | | | | 2.8071 | 2.9008 |
| | | | | | 30.215 | 15.0743 |
| 3 | 24 | 4 | 2 | 2 | -11.911 | 1.5687 |
| | | | | | 0.7704 | 3.1773 |
| | | | | | 2.8995 | 6.9621 |
| 4 | 24 | 3 | 4 | 2 | 1.0579 | 1.4447 |
| | | | | | 2.998 | 2.9012 |
| | | | | | 13.0561 | 9.8998 |
| 5 | 24 | 2 | 4 | 2 | 1.0468 | 1.8716 |
| | | | | | 1.9049 | 3.1094 |
| | | | | | 12.0641 | 5.4489 |
| 6 | 12 | 4 | 4 | 2 | -4.7028 | 1.6773 |
| | | | | | 1.2117 | 2.7932 |
| | | | | | 2.5801 | 6.3889 |
| 7 | 12 | 4 | 3 | 2 | -6.3356 | 1.699 |
| | | | | | 0.7811 | 2.0912 |
| | | | | | 2.5002 | 5.5432 |
| 8 | 12 | 4 | 2 | 2 | 2.4333 | 1.0143 |
| | | | | | ——— | 2.5674 |
| | | | | | ——— | 7.4151 |
| 9 | 12 | 3 | 4 | 2 | 1.323 | 1.7439 |
| | | | | | 2.328 | 2.2366 |
| | | | | | 14.5108 | 5.1487 |
| 10 | 12 | 2 | 4 | 2 | 1.4884 | 1.7556 |
| | | | | | 2.2208 | 2.5112 |
| | | | | | 9.2163 | 5.1003 |

TABLE 3. Real Roots of $k_1$ & $k_2$

| S.No | Population-3 | | | | | |
|---|---|---|---|---|---|---|
| | SS | MR | | | Real Roots | |
| | n | p | q | r | $k_1$ | $k_2$ |
| 1 | 315 | 49 | 49 | 25 | 2.9408 | -56.8404 |
| | | | | | ——— | 1.961 |
| | | | | | ——— | 2.8347 |
| 2 | 315 | 49 | 48 | 25 | 2.21644 | -25.2419 |
| | | | | | ——— | 1.9853 |
| | | | | | ——— | 2.8453 |
| 3 | 315 | 49 | 47 | 25 | 2.039 | -6.2396 |
| | | | | | ——— | 0.8991 |
| | | | | | ——— | 2.8629 |
| 4 | 315 | 48 | 49 | 25 | 2.2952 | 1.0756 |
| | | | | | ——— | 3.8895 |
| | | | | | ——— | 50.8061 |
| 5 | 315 | 47 | 49 | 25 | 2.0341 | -24.3362 |
| | | | | | ——— | 0.3227 |
| | | | | | ——— | 2.1712 |
| 6 | 158 | 49 | 49 | 25 | 1.9035 | 1.6588 |
| | | | | | ——— | 2.1808 |
| | | | | | ——— | 5.0833 |
| 7 | 158 | 49 | 48 | 25 | 2.0926 | 1.102 |
| | | | | | ——— | 2.5007 |
| | | | | | ——— | 19.0513 |
| 8 | 158 | 49 | 47 | 25 | 2.0806 | 1.958 |
| | | | | | ——— | 2.3201 |
| | | | | | ——— | 15.0032 |
| 9 | 158 | 48 | 49 | 25 | 2.0914 | 1.4461 |
| | | | | | ——— | 2.5774 |
| | | | | | ——— | 60.3952 |
| 10 | 158 | 47 | 49 | 25 | 2.0918 | 1.3554 |
| | | | | | ——— | 2.7354 |
| | | | | | ——— | 9.6115 |

TABLE 4. PRE of the selected estimators with respect to mean per unit estimator for Population-1

| S.No | SS | MR | | | Percentage Relative Efficiency | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n | p | q | r | $t_o$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_p$ |
| 1 | 12 | 3 | 3 | 1 | 100 | 90.7 | 98.5 | 99.8 | 106.7 | 102.7 | 103.2 | 155.6 |
| 2 | 12 | 3 | 2 | 1 | 100 | 105.5 | 103.7 | 117.6 | 119.3 | 100.7 | 116.5 | 168.2 |
| 3 | 12 | 3 | 1 | 1 | 100 | 118.1 | 133.3 | 107.3 | 129.9 | 133.4 | 107.3 | 202.3 |
| 4 | 12 | 2 | 3 | 1 | 100 | 117.5 | 130.5 | 107.7 | 130.6 | 127.3 | 125.8 | 230.2 |
| 5 | 12 | 1 | 3 | 1 | 100 | 135.4 | 113.4 | 145.6 | 141.1 | 106.4 | 137.4 | 336.8 |
| 6 | 10 | 3 | 3 | 1 | 100 | 73.6 | 93.8 | 93.2 | 90.8 | 81.1 | 91.4 | 119.3 |
| 7 | 10 | 3 | 2 | 1 | 100 | 82.1 | 97.5 | 105.8 | 96.3 | 97.8 | 98.5 | 130.2 |
| 8 | 10 | 3 | 1 | 1 | 100 | 103 | 101.7 | 117 | 124.1 | 100.6 | 123.5 | 178.1 |
| 9 | 10 | 2 | 3 | 1 | 100 | 96.1 | 93.8 | 115.5 | 110.3 | 104.2 | 113.2 | 173.8 |
| 10 | 10 | 1 | 3 | 1 | 100 | 127.2 | 110.5 | 131.6 | 128.2 | 94 | 122.1 | 336.1 |

TABLE 5. PRE of the selected estimators with respect to mean per unit estimator for Population 2

| S.No | SS | MR | | | Percentage Relative Efficiency | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n | p | q | r | $t_o$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_p$ |
| 1 | 18 | 4 | 4 | 2 | 100 | 103.6 | 105.3 | 102.6 | 107.4 | 62.8 | 102.7 | 230.3 |
| 2 | 18 | 4 | 3 | 2 | 100 | 103.1 | 107.2 | 106.3 | 113.5 | 63.5 | 109.7 | 204.7 |
| 3 | 18 | 4 | 2 | 2 | 100 | 99.9 | 108.7 | 119.5 | 129.7 | 71.8 | 123.8 | 176.9 |
| 4 | 18 | 3 | 4 | 2 | 100 | 105.2 | 110.3 | 111.5 | 113.8 | 61.5 | 109 | 226.7 |
| 5 | 18 | 2 | 4 | 2 | 100 | 103.7 | 109.3 | 124.8 | 128.6 | 66.4 | 125.6 | 205.5 |
| 6 | 12 | 4 | 4 | 2 | 100 | 62.9 | 76.3 | 82.8 | 85.7 | 76 | 93.9 | 104.8 |
| 7 | 12 | 4 | 3 | 2 | 100 | 72.2 | 94 | 88.5 | 106.6 | 69.4 | 97.6 | 110.2 |
| 8 | 12 | 4 | 2 | 2 | 100 | 97.9 | 101.1 | 88.2 | 102.8 | 62.8 | 98.9 | 203.3 |
| 9 | 12 | 3 | 4 | 2 | 100 | 80.3 | 98.1 | 99 | 103.8 | 64.8 | 102.1 | 129.3 |
| 10 | 12 | 2 | 4 | 2 | 100 | 104.6 | 108.3 | 104.2 | 100.2 | 60.4 | 96.4 | 293.2 |

TABLE 6. PRE of the selected estimators with respect to mean per unit estimator for Population 3

| S.No | SS | MR | | | Percentage Relative Efficiency | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $n$ | $p$ | $q$ | $r$ | $t_o$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ | $t_6$ | $t_p$ |
| 1 | 236 | 49 | 49 | 25 | 100 | 100.6 | 101.4 | 100.4 | 91.7 | 92.9 | 92 | 164.2 |
| 2 | 236 | 49 | 48 | 25 | 100 | 100.9 | 102.9 | 101.2 | 113.5 | 115.4 | 114.2 | 174.9 |
| 3 | 236 | 49 | 47 | 25 | 100 | 100.7 | 101.8 | 101 | 113.3 | 114.8 | 113.9 | 175.3 |
| 4 | 236 | 48 | 49 | 25 | 100 | 100.8 | 101.8 | 101 | 113.6 | 114.3 | 112.8 | 175.8 |
| 5 | 236 | 47 | 49 | 25 | 100 | 100.6 | 101.6 | 100.6 | 91.1 | 92.4 | 91.7 | 168.4 |
| 6 | 158 | 49 | 49 | 25 | 100 | 100.1 | 100.8 | 99.8 | 90.9 | 92 | 91.2 | 132.9 |
| 7 | 158 | 49 | 48 | 25 | 100 | 100 | 100.7 | 99.7 | 92 | 93.2 | 92.5 | 129 |
| 8 | 158 | 49 | 47 | 25 | 100 | 99.5 | 100.6 | 99.3 | 119.3 | 120.8 | 119.5 | 139.4 |
| 9 | 158 | 48 | 49 | 25 | 100 | 99.9 | 100.9 | 99.7 | 92 | 93.1 | 92.4 | 129.3 |
| 10 | 158 | 47 | 49 | 25 | 100 | 99.6 | 100.3 | 99.2 | 119.4 | 120.6 | 119.1 | 140.1 |

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