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Similarity Measures for New Hybrid Models: mF Sets and mF Soft Sets

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Abstract. Similarity measure for fuzzy systems plays a very substantial role in handling problems that contain uncertain information, but unable to deal the vagueness and uncertainty of the problems having multipolar information. In this research article, we define certain distances between two m-polar fuzzy sets (mF sets) and m-polar fuzzy soft sets (mF soft sets). We also propose a new similarity measure (SM) for mF sets and mF soft sets based on the distances. We demonstrate with an application that the proposed SM for mF sets is capable of recognizing different patterns. Moreover, we apply the concept of SM of mF soft sets to medical diagnosis. Finally, we summarize our proposed method as an algorithm in each application.

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1. Introduction

The scholastic mathematical models and tools are unable to handle the complexity of the information containing uncertainties. Molodtsov [43] pointed out the difficulties of these models. In order to tackle these problems, he [43] introduced a novel idea of soft set theory. Maji *et al.* [37] extended the idea of soft sets to fuzzy soft sets. Feng *et al.* [20] in 2010 gave deeper insights into the decision-making based on fuzzy soft sets. Zhang [60] proposed the idea of bipolar fuzzy sets and relations as a computational framework for cognitive modeling and multi-agent decision analysis. Further, Chen *et al.* [17] introduced the concept of mF sets in 2014, as a generalization of bipolar fuzzy sets. They [17] showed that 2-polar fuzzy sets and bipolar fuzzy sets are cryptomorphic mathematical notions. The idea behind this is, that the “multipolar information” (not just bipolar information which correspond to two-valued logic) exists because data for a real world problems are sometimes from n agents $(n \geq 2)$. There are many other examples such as truth degrees of a logic formula which are based on n logic implication operators $(n \geq 2)$. For examples, the exact degree of telecommunication safety of mankind is a point in $[0, 1]^n$ $(n \approx 7\times 10^9)$ because different person has been monitored different times. There are many other examples such as truth degree of two logic formula which are based on n logic implication operators $(n \geq 2)$, ordering results of magazines, ordering results of university and inclusion degrees (accuracy measures, rough measures, approximation qualities, fuzziness measures and decision preformation evaluations) of rough set. We can obtain concisely one from the corresponding one in [17]. Akram [2] introduced many new concepts including $m-$polar fuzzy graphs, $m-$polar fuzzy line graphs, $m-$polar fuzzy labeling graphs and certain metrics in $m-$polar fuzzy graphs. Akram *et al.* [3, 7, 8] proposed multi-attribute decision-making methods based on mF rough and mF soft rough information. For other related models and decision-making techniques, readers are referred to ([4], [5], [6], [12]).

Chen ([18], [19], [16]) studied the concept of SM of vague sets and elements. SMs proposed by Chen fails to hold in some cases. In order to overcome this issue, Hong and Kim [24] introduced some modified measures. The uncertainty measures of soft sets and fuzzy soft sets were introduced by Majumdar and Samanta ([40],[42]). Some set theoretic operations based on SMs of soft sets were presented by Kharal [29]. Li and Cheng [32] proposed the idea of new SMs between intuitionistic fuzzy sets. They also presented numerical examples to illustrate the application of these measures. Szmidt and Kacprzyk ([53], [54], [55]) also proposed the distance measures for intuitionistic fuzzy sets. Jiang *et al.* [25] initiated the concept of SM based on the distances, under intuitionistic fuzzy soft set and interval valued fuzzy soft set environment. Majumdar and Samanta [41] also studied the idea of uncertainty measure of intuitionistic fuzzy soft sets. Several researchers published their work on SMs (see [15], [33], [34], [47], [51], [52], [57]). Similarity measures have a variety of applications in many research fields including medical diagnosis, pattern recognition, coding theory, game theory and region extraction. Similarity measure for fuzzy sets and fuzzy soft sets plays a very substantial role in handling problems that contain uncertain information. We have used standard definitions and terminologies in this paper. For other notations, terminologies and applications not mentioned in the paper, the readers are referred to ([10]-[59]).

In many practical problems, multipolar information exists which cannot be represented well using the existing models. In many real life problems, it is often necessary to compare two sets of information. We usually interested to know whether two images or patterns are similar or approximately similar or at least to what degree they are similar. To handle this kind of multipolar information, we have extended the idea of these SMs to discuss the multipolar information. In this research article, we propose a new SM for mF sets and mF soft sets based on the distances. Moreover, we apply the concept of SM of mF soft sets to medical diagnosis. The organization of this research article is as follows.

In section 2, we review some basic concept. In section 3, we introduce our new concepts related to SM. In section 4, we present an application in pattern recognition problem. In section 5, we introduce the SM for mF soft sets. In section 6, we show an application related to medical diagnosis. In section 7, we present the conclusion and future directions.

2. Preliminaries

In this section, we review some basic and fundamental definitions related to our proposed concept.

Definition 2.1. [43] A pair (F, A) is called a *soft set* over U, where F is a mapping given by

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\epsilon \in A$. $F(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set (F, A).

Definition 2.2. [17] An *mF set* on a universe U is a function $R = (p_1 \circ R(r), p_2 \circ R(r), \cdots, p_m \circ R(r)) : U \to [0,1]^m$, where the $i - th$ projection mapping is defined as $p_i \circ R : [0,1]^m \to [0,1]$. $\mathbf{0} = (0, 0, \cdots, 0)$ is the smallest element in $[0,1]^m$ and $\mathbf{1} = (1, 1, \cdots, 1)$ is the largest element in $[0,1]^m$.

Definition 2.3. [8] Let U be a universe, T a set of parameters and $N \subseteq T$. Define $\zeta : N \to mF^S$. Then (ζ, N) is called an *mF soft set* over a universe U, which is defined by,

$$(\zeta, N) = \Big\{(x, p_i \circ N_\epsilon(x)) : x \in U \text{ and } \epsilon \in N\Big\}.$$

Definition 2.4. [8] $SM(A, B) = \frac{1}{1+DM(A,B)}$, where SM is the similarity measure, DM is the distance measure of two fuzzy sets and A, B are the examined fuzzy sets.

3. Similarity Measure for mF Sets

In this section, we introduce our novel concepts including distances between two mF sets and SM for mF sets.

Definition 3.1. Let $\mathcal{N}$ and $\mathcal{S}$ be two mF sets on $\mathbb{U} = \{u_1, u_2, u_3, \ldots, u_n\}$. Then the distance between $\mathcal{N}$ and $\mathcal{S}$ is defined as:

(1) Hamming distance:

$$d_H(\mathcal{N}, \mathcal{S}) = \frac{1}{m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big|p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{S}(u_j)\Big|\right\}$$

(2) Normalized Hamming distance:

$$d_{NH}(\mathcal{N}, \mathcal{S}) = \frac{1}{mn}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big|p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{S}(u_j)\Big|\right\}$$

(3) Euclidean distance:

$$d_E(\mathcal{N}, \mathcal{S}) = \sqrt{\frac{1}{m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{S}(u_j)\Big)^2\right\}}$$

(4) Normalized Euclidean distance:

$$d_{NE}(\mathcal{N}, \mathcal{S}) = \sqrt{\frac{1}{mn}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big(p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{S}(u_j)\Big)^2\right\}}$$

Theorem 3.1. The distances between $\mathcal{N}$ and $\mathcal{S}$ satisfy the following inequalities.

(i). $d_H(\mathcal{N}, \mathcal{S}) \leq n$,

(ii). $d_{NH}(\mathcal{N},\mathcal{S}) \leq 1$,
(iii). $d_E(\mathcal{N},\mathcal{S}) \leq \sqrt{n}$,
(iv). $d_{NE}(\mathcal{N},\mathcal{S}) \leq 1$.

Theorem 3.2. The distance functions d_H, d_{NH}, d_E, and d_{NE}, defined from $m(\mathbb{U}) \to R^+$, are metric.

Proof. Let $\mathcal{N}$, $\mathcal{S}$ and $\mathcal{R}$ be three mF sets over $\mathbb{U}$, then

(i). $d_H(\mathcal{N},\mathcal{S}) \geq 0$.
(ii). Suppose $d_H(\mathcal{N},\mathcal{S}) = 0$.

$$\Leftrightarrow \frac{1}{m}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\Big|p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{S}(u_j)\Big|\right\} = 0,$$

$$\Leftrightarrow \Big|p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{S}(u_j)\Big| = 0,$$

$$\Leftrightarrow p_i \circ \mathcal{N}(u_j) = p_i \circ \mathcal{S}(u_j), \text{ for all } 1 \leq i \leq m,\ 1 \leq j \leq n,$$

$$\Leftrightarrow \mathcal{N} = \mathcal{S}.$$

(iii). $d_H(\mathcal{N},\mathcal{S}) = d_H(\mathcal{S},\mathcal{N})$.
(iv). For any three mF sets $\mathcal{N}, \mathcal{S}$ and $\mathcal{R}$,

$$\Big|p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{S}(u_j)\Big|$$

$$=\Big|p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{R}(u_j) + p_i \circ \mathcal{R}(u_j) - p_i \circ \mathcal{S}(u_j)\Big|, \text{ for all } i,\ j.$$

$$\leq\Big|p_i \circ \mathcal{N}(u_j) - p_i \circ \mathcal{R}(u_j)\Big| + \Big|p_i \circ \mathcal{R}(u_j) - p_i \circ \mathcal{S}(u_j)\Big|, \text{ for all } i,\ j.$$

Thus $d_H(\mathcal{N},\mathcal{S}) \leq d_H(\mathcal{N},\mathcal{R}) + d_H(\mathcal{R},\mathcal{S})$.

□

Definition 3.2. The *SM* of two mF sets $\mathcal{N}$ and $\mathcal{S}$ is defined as

$$\mathbb{S}(\mathcal{N},\mathcal{S}) = \frac{1}{1 + d(\mathcal{N},\mathcal{S})}$$

where $d(\mathcal{N},\mathcal{S})$ is any of the above distances defined in Definition 3.1.

Definition 3.3. The *SM* of two mF sets $\mathcal{N}$ and $\mathcal{S}$ is defined as

$$\mathbb{S}'(\mathcal{N},\mathcal{S}) = \exp^{-\beta d(\mathcal{N},\mathcal{S})}$$

where $\beta > 0$ is called the steepness measure.

Definition 3.4. The two mF sets $\mathcal{N}$ and $\mathcal{S}$ are β similar if and only if $\mathbb{S}(\mathcal{N},\mathcal{S}) \geq \beta$, i.e.,

$$\mathcal{N} \approx^{\beta} \mathcal{S} \Leftrightarrow \mathbb{S}(\mathcal{N},\mathcal{S}) \geq \beta,\ \beta \in (0,1).$$

$\mathcal{N}$ and $\mathcal{S}$ are *significantly similar* if $\mathbb{S}(\mathcal{N},\mathcal{S}) \geq \frac{1}{2}$.

Theorem 3.3. The SM of two mF sets $\mathcal{N}$ and $\mathcal{S}$ satisfies the following.

(i). $0 \leq \mathbb{S}(\mathcal{N}, \mathcal{S}) \leq 1$,
(ii). $\mathbb{S}(\mathcal{N}, \mathcal{S}) = \mathbb{S}(\mathcal{S}, \mathcal{N})$,
(iii). $\mathbb{S}(\mathcal{N}, \mathcal{S}) = 1 \Leftrightarrow \mathcal{N} = \mathcal{S}$.

4. Application in Pattern Recognition Problem

In this section, we use mF information to solve pattern recognition problem by applying the concept of distance based SM.

Algorithm:

1. Assume that there are n patterns which are represented by mF sets R_j, $j = 1, 2, 3, \ldots, n$, in the feature space $\mathbb{U} = \{u_1, u_2, u_3, \ldots, u_k\}$.
2. Consider an mF set $\mathcal{N}$, which is another sample to be recognized.
3. Calculate the SM $\mathbb{S}(R_j, \mathcal{N})$ between R_j and $\mathcal{N}$.
4. The sample R_j is similar to $\mathcal{N}$, if $\mathbb{S}(R_j, \mathcal{N}) \geq \frac{1}{2},\ j = 1, 2, 3, \ldots, n.$
5. Our result is R_k, if $\mathbb{S}(R_k, \mathcal{N})$ is greater than $\mathbb{S}(R_j, \mathcal{N}),\ j,\ k = 1, 2, 3, \ldots, n.$

Now we present an application of pattern recognition problem in order to classify hybrid rocks.
Suppose that there are four types of rock fields denoted by R_1, R_2, R_3 and R_4. Let $\mathbb{U} = \{u_1 = \text{Texture}, u_2 = \text{Fracture}, u_3 = \text{Grain Size}, u_4 = \text{Crystalline Structure}\}$ be the feature space of rock fields. The feature "Texture" of the rock refers to the arrangement, shape and distribution of minerals in the rock. The minerals within the rock may have uneven, conchoidal or hackly "Fracture." Rocks may have no visible grain, medium or very coarse grain size. "Crystalline Structure" is another feature to classify what type of rock it is. The rock may have angular, medium or rounded crystalline structure. Table 1 represents the four types of rock fields by 3-polar fuzzy sets in the feature space $\mathbb{U}$.

.	u_1	u_2	u_3	u_4
R_1	(0.55, 0.27, 0.73)	(0.33, 0.83, 0.24)	(0.13, 0.76, 0.65)	(0.78, 0.46, 0.22)
R_2	(0.76, 0.54, 0.34)	(0.56, 0.44, 0.21)	(0.79, 0.10, 0.33)	(0.89, 0.36, 0.11)
R_3	(0.11, 0.91, 0.25)	(0.85, 0.15, 0.35)	(0.20, 0.80, 0.22)	(0.90, 0.05, 0.15)
R_4	(0.78, 0.36, 0.45)	(0.34, 0.26, 0.83)	(0.93, 0.20, 0.10)	(0.10, 0.71, 0.09)

TABLE 1. 3-polar fuzzy set for rock fields

Let $\mathcal{N}$ be an unknown hybrid rock, which is to be recognized.
$\mathcal{N} = \Big\{(u_1,\ 0.10, 0.92, 0.22), (u_2,\ 0.85, 0.16, 0.35), (u_3,\ 0.22, 0.81, 0.20),$
$(u_4,\ 0.90, 0.06, 0.17)\Big\}.$

The Euclidean distance between R_j and $\mathcal{N}$ is calculated as:

$$d_E(R_1, \mathcal{N}) = 0.8178,$$
$$d_E(R_2, \mathcal{N}) = 0.7563,$$
$$d_E(R_3, \mathcal{N}) = 0.0294,$$
$$d_E(R_4, \mathcal{N}) = 1.0463.$$

The SM of R_j and $\mathcal{N}$ is calculated as:

$$\mathbb{S}(R_1, \mathcal{N}) = 0.5501,$$
$$\mathbb{S}(R_2, \mathcal{N}) = 0.5694,$$
$$\mathbb{S}(R_3, \mathcal{N}) = 0.9714,$$
$$\mathbb{S}(R_4, \mathcal{N}) = 0.4887.$$

Since $\mathbb{S}(R_3, \mathcal{N})$ is highest, so R_3 and $\mathcal{N}$ have same pattern. Thus hybrid rock $\mathcal{N}$ belongs to the rock field R_3.

5. SIMILARITY MEASURE FOR mF SOFT SETS

In this section, we introduce the concept of SM for mF soft sets and investigate its properties.

Definition 5.1. "Let $\mathbb{U}$ be a universe, $\mathcal{T}$ a set of parameters and $\mathcal{N} \subseteq \mathcal{T}$. Define $\psi : \mathcal{N} \to mF^{\mathbb{U}}$, where $mF^{\mathbb{U}}$ is the collection of all mF subsets of $\mathbb{U}$. Then $(\Psi, \mathcal{N})$ is called an mF *soft set* (shortly, mF soft set) over a universe $\mathbb{U}$, which is defined by,

$$\Psi_{\mathcal{N}} = (\Psi, \mathcal{N}) = \Big\{ \big(t, \psi_N(t)\big) : t \in \mathcal{T},\ \psi_{\mathcal{N}}(t) \in mF^{\mathbb{U}} \Big\},$$

and $\psi_{\mathcal{N}}(t)$ is an mF set, denoted by,

$$\psi_{\mathcal{N}}(t) = \Big\{ \big(u, p_i \circ \mathcal{N}(u) : u \in \mathbb{U}\big) \Big\}.$$

"

Example 5.1. Let $\mathbb{U} = \{s_1, s_2, s_3, s_4, s_5\}$ be the set of five couches, and let $\mathcal{T} = \{t_1, t_2, t_3, t_4\}$ be the set of parameters, where the parameter,

't_1' stands for the Fabric of Couch,
't_2' stands for the Style of Couch,
't_3' stands for the Frame of Couch,
't_4' stands for the Price of Couch.

We give further characteristics of these parameters.

- The "Fabric of Couch" may be leather, polyester and velvet.
- The "Style of Couch" may be modern, contemporary and sectional.
- The "Frame of Couch" may be of hardwood, particle board and metal.
- The "Price of Couch" may be very costly, costly and cheap for the buyer.

Suppose that a family wants to purchase a couch of $\mathbb{U}$. They consider three parameters t_1, t_2, t_4 for the selection of a couch. Let $\mathcal{N} = \{t_1, t_2, t_4\}$ be subset of $\mathcal{T}$. Then we can formulate all possible information on these couches as a 3-polar fuzzy soft set $(\Psi, \mathcal{N})$.

$$(\Psi, \mathcal{N}) = \begin{Bmatrix} \psi(t_1) & = \Big\{(c_1,\ 7/10, 1/2, 3/10), (c_2,\ 1/5, 1/2, 3/10), (c_3,\ 3/5, 2/5, 7/10), \\ & (c_4,\ 1/2, 3/5, 3/10), (c_5,\ 7/10, 3/5, 2/5)\Big\}, \\ \psi(t_2) & = \Big\{(c_1,\ 9/10, 3/5, 7/10), (c_2,\ 3/5, 1/2, 2/5), (c_3,\ 1/2, 3/5, 7/10), \\ & (c_4,\ 7/10, 7/10, 3/5), (c_5,\ 2/5, 3/10, 3/5)\Big\}, \\ \psi(t_4) & = \Big\{(c_1,\ 4/5, 3/5, 2/5), (c_2,\ 3/5, 1/2, 3/5), (c_3,\ 2/5, 3/10, 1/2), \\ & (c_4,\ 3/5, 2/5, 1/2), (c_5,\ 7/10, 3/5, 7/10)\Big\}. \end{Bmatrix}$$

Thus $(\Psi, \mathcal{N})$ is a 3-polar fuzzy soft set in which we have chosen the Fabric, Style and Price of the Couch as desired parameters for the selection. For example, if we consider the parameter "Fabric of Couch", $(c_2, 0.2, 0.5, 0.3)$ shows that according to the family couch c_2 has 20% leather, 50% polyester and 30% velvet fabric.

Definition 5.2. Let $\mathbb{U} = \{u_1, u_2, \ldots, u_n\}$ be a universe, $\mathcal{T} = \{t_1, t_2, \ldots, t_q\}$ a set of parameters, $\mathcal{N}, \mathcal{S} \subseteq \mathcal{T}$ and $\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}$ two mF soft sets on $\mathbb{U}$ with their mF approximate functions

$$\psi_{\mathcal{N}}(t_j) = \Big\{\big(u, p_i \circ \mathcal{N}(u)\big) : u \in \mathbb{U}\Big\},$$

$$\omega_{\mathcal{S}}(t_j) = \Big\{\big(u, p_i \circ \mathcal{S}(u)\big) : u \in \mathbb{U}\Big\},$$

respectively. Then the distance between $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ is defined as:

(1) Hamming distance:

$$d_H(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = \frac{1}{mq}\left\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}\Big|p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big|\right\}$$

(2) Normalized Hamming distance:

$$d_{NH}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = \frac{1}{mqn}\left\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}\Big|p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big|\right\}$$

(3) Euclidean distance:

$$d_E(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = \sqrt{\frac{1}{mq}\left\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}\Big(p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big)^2\right\}}$$

(4) Normalized Euclidean distance:

$$d_{NE}(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})=\sqrt{\frac{1}{mqn}\left\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}\Big(p_i\circ\mathcal{N}(t_j)(u_k)-p_i\circ\mathcal{S}(t_j)(u_k)\Big)^2\right\}}$$

Example 5.2. Let $\mathbb{U}=\{u_1,u_2,u_3,u_4,u_5\}$ be the feature space, $\mathcal{T}=\{t_1,t_2,t_3,t_4,t_5\}$ a set of parameters and $\mathcal{N}=\mathcal{S}=\{t_2,t_3,t_4\}\subseteq\mathcal{T}$. We can define the 3-polar fuzzy soft sets $(\Psi,\mathcal{N})$ and $(\Omega,\mathcal{S})$ over $\mathbb{U}$ as follows:

$$(\Psi,\mathcal{N})=$$

$$\left\{\begin{array}{ll}\psi(t_2) & =\Big\{(u_1,\ 7/10,1/2,3/10),(u_2,\ 3/5,4/5,1/2),(u_3,\ 3/5,2/5,7/10),\\ & (u_4,\ 1/2,3/5,3/10),(u_5,\ 7/10,3/5,2/5)\Big\},\\ \psi(t_3) & =\Big\{(u_1,\ 9/10,3/5,7/10),(u_2,\ 3/5,1/2,3/5),(u_3,\ 1/2,3/5,7/10),\\ & (u_4,\ 7/10,4/5,3/5),(u_5,\ 2/5,3/10,3/5)\Big\},\\ \psi(t_4) & =\Big\{(u_1,\ 4/5,3/5,2/5),(u_2,\ 3/5,1/2,1/2),(u_3,\ 2/5,3/10,1/2),\\ & (u_4,\ 3/5,3/5,1/2),(u_5,\ 7/10,3/5,7/10)\Big\}.\end{array}\right\}$$

$$(\Omega,\mathcal{S})=$$

$$\left\{\begin{array}{ll}\omega(t_2) & =\Big\{(u_1,\ 4/5,4/5,1/2),(u_2,\ 3/5,7/10,7/10),(u_3,\ 9/10,3/10,1/2),\\ & (u_4,\ 3/5,3/5,1/2),(u_5,\ 3/5,4/5,2/5)\Big\},\\ \omega(t_3) & =\Big\{(u_1,\ 4/5,3/5,1/2),(u_2,\ 9/10,2/5,1/2),(u_3,\ 1/2,7/10,4/5),\\ & (u_4,\ 4/5,9/10,3/5),(u_5,\ 7/10,1/5,3/5)\Big\},\\ \omega(t_4) & =\Big\{(u_1,\ 9/10,7/10,1/2),(u_2,\ 3/5,1/2,1/2),(u_3,\ 1/2,3/5,1/2),\\ & (u_4,\ 3/5,1/2,0.7),(u_5,\ 1/2,7/10,7/10)\Big\}.\end{array}\right\}$$

Then, by using Definition 5.2, we can calculate the distance between $\Psi_{\mathcal{N}}=(\Psi,\mathcal{N})$ and $\Omega_{\mathcal{S}}=(\Omega,\mathcal{S})$ as:

(i). $d_H(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})=0.555$,
(ii). $d_{NH}(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})=0.1111$,
(iii). $d_E(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})=0.3266$,
(iv). $d_{NE}(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})=0.1461$.

Theorem 5.3. The distances between $(\Psi,\mathcal{N})$ and $(\Omega,\mathcal{S})$ satisfy the following inequalities.

(i). $d_H(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})\leq n$,
(ii). $d_{NH}(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})\leq 1$,
(iii). $d_E(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})\leq\sqrt{n}$,
(iv). $d_{NE}(\Psi_{\mathcal{N}},\Omega_{\mathcal{S}})\leq 1$.

Theorem 5.4. The distance functions d_H, d_{NH}, d_E, and d_{NE}, defined from $mF^{\mathbb{U}} \to R^+$, are metric.

Proof. Let $\Psi_{\mathcal{N}} = (\Psi, \mathcal{N})$, $\Omega_{\mathcal{S}} = (\Omega, \mathcal{S})$ and $\Lambda_{\mathcal{R}} = (\Lambda, \mathcal{R})$ be three mF soft sets over $\mathbb{U}$, then

(1) $d_H(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) \geq 0$.
(2) Suppose $d_H(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = 0$.

$$\Leftrightarrow \frac{1}{qm}\left\{\sum_{i=1}^{m}\sum_{j=1}^{q}\sum_{k=1}^{n}\Big|p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big|\right\} = 0, \text{ for all } i, j, k,$$

$$\Leftrightarrow \Big|p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big| = 0,$$

$$\Leftrightarrow p_i \circ \mathcal{N}(t_j)(u_k) = p_i \circ \mathcal{S}(t_j)(u_k), \text{ for all } 1 \leq i \leq m,\ 1 \leq j \leq q \text{ and } 1 \leq k \leq n,$$

$$\Leftrightarrow \Psi_{\mathcal{N}} = \Omega_{\mathcal{S}}.$$

(3) $d_H(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = d_H(\Omega_{\mathcal{S}}, \Psi_{\mathcal{N}})$.
(4) For any three mF soft sets Ψ_N, Ω_S and Λ_R,

$$\Big|p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big|$$

$$=\Big|p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{R}(t_j)(u_k) + p_i \circ \mathcal{R}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big|, \text{ for all } i,\ j,\ k.$$

$$\leq\Big|p_i \circ \mathcal{N}(t_j)(u_k) - p_i \circ \mathcal{R}(t_j)(u_k)\Big| + \Big|p_i \circ \mathcal{R}(t_j)(u_k) - p_i \circ \mathcal{S}(t_j)(u_k)\Big|, \text{ for all } i,\ j,\ k.$$

Thus $d_H(\Psi_{\mathcal{N}}, \Lambda_{\mathcal{R}}) \leq d_H(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) + d_H(\Omega_{\mathcal{S}}, \Lambda_{\mathcal{R}})$.

□

Definition 5.3. The *SM* of $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ is defined as

$$\mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = \frac{1}{1 + d(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}})}$$

where $d(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}})$ is any of the above distances defined in Definition 5.2.

Definition 5.4. The *SM* of $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ is also defined as

$$\mathbb{S}'(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = \exp^{-\beta d(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}})}$$

where $\beta > 0$ is called the steepness measure.

Definition 5.5. The two mF soft sets $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ are β similar if and only if $\mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) \geq \beta$, i.e.,

$$\Psi_{\mathcal{N}} \approx^{\beta} \Omega_{\mathcal{S}} \Leftrightarrow \mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) \geq \beta,\ \beta \in (0, 1).$$

$\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ are *significantly similar* if $\mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) \geq \frac{1}{2}$.

Example 5.5. Consider the two mF soft sets $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ as in Example 5.2. The SM of $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ by using Euclidean distance is calculated as,

$$\mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = 0.7538 \geq \frac{1}{2}.$$

Hence $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ are significantly similar.

Theorem 5.6. The SM of $\Psi_{\mathcal{N}}$ and $\Omega_{\mathcal{S}}$ over $\mathbb{U}$ satisfies the following.

(i). $0 \leq \mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) \leq 1$.
(ii). $\mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = \mathbb{S}(\Omega_{\mathcal{S}}, \Psi_{\mathcal{N}})$.
(iii). $\mathbb{S}(\Psi_{\mathcal{N}}, \Omega_{\mathcal{S}}) = 1 \Leftrightarrow \Psi_{\mathcal{N}} = \Omega_{\mathcal{S}}$.

6. Applications in Medical Diagnosis

In this section, we apply the concept of SM for soft sets in medical diagnosis.

1. We present an application to show that the distance based similarity of two mF soft sets can be used to decide whether a patient has anemia or not.

   Consider the universal set $\mathbb{U} = \{u_1 = \text{anemia}, u_2 = \text{not anemia}\}$ consisting of only two elements and let $\mathcal{T} = \{t_1, t_2, t_3\}$ be the set of parameters where the parameter,

   't_1' denotes the General Fatigue,
   't_2' denotes the Heart Symptoms,
   't_3' denotes the Strange Cravings,

   We give further characteristics of these parameters.

   • The symptom "General Fatigue" can cause headache, dizziness, poor concentration and irritability.

   • The "Heart Symptoms" of the patient may include shortness of breath, chest pain, low blood pressure and arrhythmia.

   • The patient may have "Strange Cravings" to eat items that are not food such as clay, dirt, ice and starch.

   Then we can formulate all attainable information on these symptoms under discussion as a 4-polar fuzzy soft set $(\omega, \mathcal{N})$ and this 4-polar fuzzy soft set can be constructed with the help of Anemia Specialist.

$$\Omega = (\omega, \mathcal{N}) =$$
$$\begin{Bmatrix} \omega(t_1) & = \big\{(u_1,\ 3/5, 7/10, 3/5, 7/10), (u_2,\ 1/2, 3/5, 3/5, 2/5)\big\}, \\ \omega(t_2) & = \big\{(u_1,\ 7/10, 4/5, 3/5, 4/5), (u_2,\ 3/5, 1/2, 7/10, 3/5)\big\}, \\ \omega(t_3) & = \big\{(u_1,\ 3/5, 7/10, 7/10, 3/5), (u_2,\ 1/2, 2/5, 1/2, 3/5)\big\}. \end{Bmatrix}$$

Now, we construct a 4-polar fuzzy soft set $(\psi, \mathcal{S})$, based on the medical reports of the patient.

$$\Psi = (\psi, \mathcal{S}) =$$
$$\begin{Bmatrix} \psi(t_1) & = \big\{(u_1,\ 3/5, 3/5, 3/5, 1/2), (u_2,\ 1/2, 1/2, 3/5, 3/10)\big\}, \\ \psi(t_2) & = \big\{(u_1,\ 4/5, 9/10, 7/10, 4/5), (u_2,\ 1/2, 2/5, 3/5, 1/2)\big\}, \\ \psi(t_3) & = \big\{(u_1,\ 7/10, 3/5, 3/5, 7/10), (u_2,\ 1/2, 2/5, 3/10, 1/2)\big\}. \end{Bmatrix}$$

Calculate the Hamming distance between $(\omega, \mathcal{N})$ and $(\psi, \mathcal{S})$, we have

$$d_H(\Omega_{\mathcal{N}}, \Psi_{\mathcal{S}}) = 0.1583$$

The SM between $(\zeta, \mathcal{N})$ and $(\psi, \mathcal{S})$ is

$$\mathbb{S}(\Omega_{\mathcal{N}}, \Psi_{\mathcal{S}}) = \frac{1}{1.1583} = 0.8633 > \frac{1}{2}$$

Since $\mathbb{S}(\Omega_{\mathcal{N}}, \Psi_{\mathcal{S}}) > \frac{1}{2}$, it is clear that the two 4-polar fuzzy soft sets are significantly similar. Thus we conclude that the patient has the disease anemia.

Algorithm:

1. Construct a 4-polar fuzzy soft set $\Omega_{\mathcal{N}}$ with the help of Anemia Specialist.
2. Construct a 4-polar fuzzy soft set $\Psi_{\mathcal{S}}$ based on the medical reports of ill person.
3. Calculate the Hamming distance between $\Omega_{\mathcal{N}}$ and $\Psi_{\mathcal{S}}$, using the formula

$$d_H(\Omega_{\mathcal{N}}, \Psi_{\mathcal{S}}) = \frac{1}{mq} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{q} \Big| p_i \circ \mathcal{N}(t_k)(u_j) - p_i \circ \mathcal{S}(t_k)(u_j) \Big|.$$

4. Calculate the SM of $\Omega_{\mathcal{N}}$ and $\Psi_{\mathcal{S}}$.
5. Evaluate result by using the similarity.

2. Now, we give another application of SM in medical diagnosis to determine which patient is running a dengue fever.

Suppose that there are four patients in a hospital with symptoms, High Fever, Severe Pain and Bleeding. Let $\mathbb{U} = \{s = \text{severe}, m = \text{mild}, n = \text{normal}\}$ be the universal set and $\mathcal{T} = \{t_1, t_2, t_3\}$ be the set of parameters, which are symptoms of dengue fever. The parameter

't_1' stands for High Fever,
't_2' stands for Severe Pain,
't_3' stands for Bleeding.

We give further characteristics of these parameters.

- The patient suffering from "High Fever" may also have headache, irritability and loss of appetite .
- The patient may have "Severe Pain" in muscles, joints or pain behind the eyes.
- The symptom "Bleeding" may include the bleeding of nose, gums and bleeding under the skin.

We construct the 3-polar fuzzy soft set for dengue fever with the help of doctor.

$(\Psi, \mathcal{T})$	t_1	t_2	t_3
severe	(9/10, 4/5, 4/5)	(7/10, 4/5, 7/10)	(4/5, 7/10, 4/5)
mild	(3/5, 2/5, 1/2)	(1/2, 3/5, 1/2)	(1/2, 2/5, 2/5)
normal	(2/5, 2/5, 1/5)	(2/5, 3/10, 1/5)	(1/5, 3/10, 0)

TABLE 2. 3-polar fuzzy soft set $(\Psi, \mathcal{T})$

Now, we will construct the 3-polar fuzzy soft set based on the medical reports of these four patients.

$(\Omega^1, \mathcal{N})$	t_1	t_2	t_3
severe	(1/2, 2/5, 1/2)	(2/5, 3/10, 3/5)	(3/10, 2/5, 3/10)
mild	(2/5, 1/5, 1/5)	(2/5, 1/2, 1/5)	(2/5, 3/10, 2/5)
normal	(2/5, 3/10, 1/5)	(3/10, 1/5, 1/10)	(1/5, 3/10, 0)

TABLE 3. 3-polar fuzzy soft set for patient $\mathcal{P}_1$

$(\Omega^2, \mathcal{N})$	t_1	t_2	t_3
severe	(7/10, 3/5, 7/10)	(3/5, 1/2, 7/10)	(7/10, 3/5, 1/2)
mild	(1/2, 3/5, 3/5)	(2/5, 1/2, 1/2)	(2/5, 3/10, 2/5)
normal	(3/10, 1/5, 1/5)	(1/5, 1/10, 1/10)	(1/10, 1/5, 0)

TABLE 4. 3-polar fuzzy soft set for patient $\mathcal{P}_2$

$(\Omega^3, \mathcal{N})$	t_1	t_2	t_3
severe	(8/10, 7/10, 4/5)	(4/5, 9/10, 7/10)	(7/10, 7/10, 7/10)
mild	(7/10, 3/5, 3/5)	(3/5, 3/5, 3/5)	(1/2, 3/10, 2/5)
normal	(1/2, 2/5, 1/2)	(3/10, 1/2, 3/10)	(3/10, 3/10, 1/10)

TABLE 5. 3-polar fuzzy soft set for patient $\mathcal{P}_3$

$(\Omega^4, \mathcal{N})$	t_1	t_2	t_3
severe	(3/5, 1/2, 3/5)	(7/10, 1/2, 2/5)	(3/5, 1/2, 1/2)
mild	(1/2, 3/10, 2/5)	(3/10, 1/2, 3/10)	(3/10, 1/5, 3/10)
normal	(1/5, 1/5, 1/5)	(1/10, 1/5, 1/5)	(1/10, 1/5, 1/10)

TABLE 6. 3-polar fuzzy soft set for patient $\mathcal{P}_4$

The distances between $(\Psi, \mathcal{T})$ and $(\Psi^j, \mathcal{T})$ obtained from the medical reports of four patients is calculated in the following table.

.	d^1_∞	d^2_∞	d^3_∞
$\mathcal{P}_1$	(2/5, 2/5, 3/10)	(3/10, 1/2, 3/10)	(1/2, 3/10, 1/2)
$\mathcal{P}_2$	(1/5, 1/5, 1/10)	(1/5, 3/10, 1/10)	(1/10, 1/10, 3/10)
$\mathcal{P}_3$	(1/10, 1/5, 3/10)	(1/10, 1/5, 1/10)	(1/10, 1/10, 1/10)
$\mathcal{P}_4$	(3/10, 3/10, 1/5)	(3/10, 3/10, 3/10)	(1/5, 1/5, 3/10)

TABLE 7. Distance between 3-polar fuzzy soft sets

From Table 8, it is clear that the patient $\mathcal{P}_3$ is suffering from dengue fever.

Algorithm

1. Construct an mF soft set $\Psi_{\mathcal{T}} = (\psi, \mathcal{T})$ for dengue fever with the help of doctor.
2. Construct mF soft sets $\Omega^r_{\mathcal{N}} = (\omega^r, \mathcal{N})$ based on the medical reports of patients $\mathcal{P}_r$.
3. Calculate the distance between $(\psi, \mathcal{T})$ and $(\omega^r, \mathcal{N})$ using the formula

$$d_\infty(\Psi_{\mathcal{T}}, \Omega^r_{\mathcal{N}}) = \left(d^1_\infty(\Psi_{\mathcal{T}}, \Omega^r_{\mathcal{N}}), d^2_\infty(\Psi_{\mathcal{T}}, \Omega^r_{\mathcal{N}}), \ldots, d^m_\infty(\Psi_{\mathcal{T}}, \Omega^r_{\mathcal{N}})\right)$$

where,

$$d^i_\infty(\Psi_{\mathcal{T}}, \Omega^r_{\mathcal{T}}) = \sup |p_i \circ \mathcal{T}(t_j)(u_k) - p_i \circ \mathcal{N}(t_j)(u_k)|$$

.	$\mathbb{S}^1$	$\mathbb{S}^2$	$\mathbb{S}^3$	$\mathbb{S} = \inf\{\mathbb{S}^1, \mathbb{S}^2, \mathbb{S}^3\}$
$\mathcal{P}_1$	(0.71, 0.71, 0.7)	(0.77, 0.67, 0.77)	(0.67, 0.77, 0.67)	(0.67, 0.67, 0.67)
$\mathcal{P}_2$	(0.83, 0.83, 0.91)	(0.83, 0.77, 0.91)	(0.91, 0.91, 0.77)	(0.83, 0.77, 0.77)
$\mathcal{P}_3$	(0.91, 0.83, 0.77)	(0.91, 0.83, 0.91)	(0.91, 0.91, 0.91)	(0.91, 0.83, 0.77)
$\mathcal{P}_4$	(0.77, 0.77, 0.83)	(0.77, 0.77, 0.77)	(0.83, 0.83, 0.77)	(0.77, 0.77, 0.77)

TABLE 8. Similarity measure of 3-polar fuzzy soft sets

4. Calculate the SM $\mathbb{S}^j = (\mathbb{S}^j_1, \mathbb{S}^j_2, \ldots, \mathbb{S}^j_m)$ of $\Psi_{\mathcal{T}}$ and $\Omega^r_{\mathcal{N}}$.
5. Put $\mathbb{S} = \inf \mathbb{S}^j$.
6. The patient $\mathcal{P}_r$ is suffering from dengue fever if $\mathbb{S}(\Psi_{\mathcal{T}}, \Omega^r_{\mathcal{N}})$ is maximum for each $i \in m$.

7. CONCLUSION

In many practical problems, multipolar information exists which cannot be represented well using the existing models. An mF model is used to handle uncertain data having multipolar information and it has an increasing number of applications in numerous fields, including, robotics, industrial automation, and optimization. Distance based SMs have a great deal of importance in solving many practical problems containing uncertainty. In this research article, we have studied the concept of distance based on SM of mF sets and mF soft sets. We have applied the concept of SM of mF sets to pattern recognition problem. Moreover, we have applied the concept of SM of mF soft sets to medical diagnosis. Finally, we have developed algorithms of our proposed methods. In future, our proposed methods may be extended to new directions including: (1) SM for mF soft rough sets; (2) mF rough soft sets.
Conflict of interest: The authors declare that they have no conflict of interest regarding the publication of the research article.

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