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Two Different Types of Soft Near-Fields

Emin Aygün
Department of Mathematics,
Erciyes University, Kayseri, Turkey,
Email: eaygun@erciyes.edu.tr

Hüseyin Kamacı*
Department of Mathematics,
Yozgat Bozok University, Yozgat, Turkey,
Email: huseyin.kamaci@hotmail.com

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Abstract.: Molodtsov's soft set is a novel mathematical approach to address models containing uncertainty. The algebraic structures of this set approach are prominent issues. The main objective of this paper is to contribute algebraic structures in the soft set theory. Relatedly, the notions of soft intersection near-field and soft union near-field, and their fundamental properties are introduced. Also, some findings and results related to the emerging soft algebraic structures are included.

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1. INTRODUCTION

Principally the goal of soft set theory initiated by Molodtsov [15] is to provide a model with enough parameters to handle the uncertainty associated with the data, whereas on the other part it is capable of usefully characterizing the data. Since the emergence of this set theory, many studies on its operations, algebraic structures and applications have been published. Maji et al. [14] defined some fundamental operations on the soft sets. Immediately after their seminal work, the soft set theory started to progress rapidly. In [4, 5, 12, 18] the authors studied on the operations of soft sets in detail. Aktaş and Çağman [2] described and studied soft groups and Sezgin and Atagün [19] advanced the soft group theory by introducing normalistic soft groups. Moreover, the soft semiring [8], soft ring [1], soft int-ring [7], soft uni-ring [21], soft field [13], soft BCK/BCI algebras [9, 10], soft d-algebras [11], soft near-ring [17] and soft int near-ring [20] were developed and investigated in connection with the algebra properties of soft sets. In [6], the authors addressed the soft substructures of algebraic structures like rings, modules, and fields.

In this study, we focus on the soft algebraic structures in a systematic way. We discuss two different kinds of soft near-fields and obtain their basic properties.

2. PRELIMINARIES

In this part, we remind some fundamental concepts relevant to near-ring, near-field and soft sets.

Definition 2.1. *(*[16]*) A near-ring is a set $\mathcal{N}$ together with the binary operations "+" and "." such that*

(a): $(\mathcal{N}, +)$ *forms a group (need not be abelian).*
(b): $(\mathcal{N}, .)$ *forms a semi-group.*
(c): *For all* $\eta, \eta', \eta'' \in \mathcal{N}$, $(\eta + \eta')\eta'' = \eta\eta'' + \eta'\eta''$.

This near-ring $(\mathcal{N}, +, .)$ will be termed to right near-ring.

Definition 2.2. *(*[16]*) A near-ring $(\mathcal{N}, +, .)$ is said to be a near-field if the set $(\mathcal{N} - \{0_{\mathcal{N}}\}, .)$ is a group.*

Throughout this paper, X is an initial universe set and its power set is denoted by $\mathcal{P}(X)$. Also, P is a set of parameters.

Molodtsov [15] defined the soft set in the year 1999. In this paper, we focus on Molodtsov's soft set approach. For the following definitions, we may refer to the articles [4, 14, 15].

Definition 2.3. *A soft set Γ_P over the universe set X is a set described by*

$$\Gamma_P = \{(p, \Gamma(p)) : p \in P \text{ and } \Gamma(p) \in \mathcal{P}(X)\}$$

where $\Gamma : P \to \mathcal{P}(X)$ is a mapping. From now on we will denote this mapping Γ with domain P as Γ_P.

Definition 2.4. *Let Γ_P and Γ'_P be two soft sets on the common universal set X. Then, the intersection of these soft sets, symbolized by $\Gamma_P \sqcap \Gamma'_P = (\Gamma \sqcap \Gamma')_P$, is defined as $(\Gamma \sqcap \Gamma')_P(p) = \Gamma_P(p) \cap \Gamma'_P(p)$ for all $p \in P$.*

Definition 2.5. *Let Γ_P and Γ'_P be two soft sets on the common universal set X. Then, the union of these soft sets, symbolized by $\Gamma_P \sqcup \Gamma'_P = (\Gamma \sqcup \Gamma')_P$, is defined as $(\Gamma \sqcup \Gamma')_P(p) = \Gamma_P(p) \cup \Gamma'_P(p)$ for all $p \in P$.*

Example 2.6. *Let $X = \{x_1, x_2, x_3, x_4\}$ be a set of four found sources available for a manager in a Banking System and $P = \{p_1, p_2, p_3, p_4, p_5\}$ be a parameter set where p_i for $i = 1, 2, 3, 4, 5$ represent "demand deposit", "fund mobility", "term deposit", "liquidity" and "fund pricing" respectively. Then,*

$$\Gamma_P = \{(p_1, \{x_2, x_3\}), (p_2, \{x_4\}), (p_3, \emptyset), (p_4, \{x_1, x_3, x_4\}), (p_5, \{x_1, x_2, x_4\})\},$$
$$\Gamma'_P = \{(p_1, \{x_1, x_2, x_4\}), (p_2, \{x_3, x_4\}), (p_3, \{x_1, x_3, x_4\}), (p_4, X), (p_5, \{x_3\})\}$$

are two soft sets over X. The intersection and union of Γ_P and Γ'_P are obtained as follows:

$$(\Gamma \sqcap \Gamma')_P = \{(p_1, \{x_2\}), (p_2, \{x_4\}), (p_3, \emptyset), (p_4, \{x_1, x_3, x_4\}), (p_5, \emptyset)\},$$
$$(\Gamma \sqcup \Gamma')_P = \{(p_1, X), (p_2, \{x_3, x_4\}), (p_3, \{x_1, x_3, x_4\}), (p_4, X), (p_5, X)\}.$$

3. SOFT INTERSECTION NEAR-FIELD

In this part, we deal with the soft intersection near fields and the related results.

Definition 3.1. *Let $\mathcal{N}$ be a near-field and $\Gamma_{\mathcal{N}}$ be a soft set on the universal set X. $\Gamma_{\mathcal{N}}$ is said to be a soft intersection near-field over X if the following properties are satisfied, for all $p, q \in \mathcal{N}$*

(i): $\Gamma_{\mathcal{N}}(p+q) \supseteq \Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(q)$,

(ii): $\Gamma_{\mathcal{N}}(-p) = \Gamma_{\mathcal{N}}(p)$,

(iii): $\Gamma_{\mathcal{N}}(pq) \supseteq \Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(q)$,

(iv): $\Gamma_{\mathcal{N}}(p^{-1}) = \Gamma_{\mathcal{N}}(p)$, *where* $p \neq 0_{\mathcal{N}}$.

Example 3.2. *Let $(N,+)$ be a group, $M(N) = \{f : N \to N | f \text{ is bijective function}\}$ and $(M(N), +, \circ)$ be a near-field. Assume that $\Gamma_{M(N)}$ is a soft set on the universal set $X = \{x_1, x_2, x_3, x_4\}$, where $\Gamma_{M(N)} : M(N) \to \mathcal{P}(X)$ is a mapping given by*

$$\Gamma_{M(N)}(f(n)) = \begin{cases} X, & \text{if } f(n) = 0_N \\ \{x_1, x_2, x_4\}, & \text{if } f(n) = n \\ \{x_1, x_2\}, & \text{if } f(n) = m \ni m \neq 0_N, n \text{ for all } m \in N \end{cases}$$

(i): $\Gamma_{M(N)}((f+g)(n)) \supseteq \Gamma_{M(N)}(f(n)) \cap \Gamma_M(N)(g(n))$

- $f(n) = 0_N$ *and* $g(n) = n \Rightarrow (f+g)(n) = n$*; then* $\{x_1, x_2, x_4\} \supseteq X \cap \{x_1, x_2, x_4\}$
- $f(n) = 0_N$ *and* $g(n) = m \Rightarrow (f+g)(n) = m$*; then* $\{x_1, x_2\} \supseteq X \cap \{x_1, x_2\}$
- $f(n) = n$ *and* $g(n) = m \Rightarrow (f+g)(n) = n+m$ *(where* $n+m \neq n, 0_N$*); then* $\{x_1, x_2\} \supseteq \{x_1, x_2, x_4\} \cap \{x_1, x_2\}$.

Note that $n+m \neq n, 0_N$. *Because,*

(1) $n+m = n \Rightarrow -n+n+m = -n+n \Rightarrow m = 0_N$. *This is a contradiction.*

(2) $n+m = 0_N \Rightarrow -n+n+m = -n+0_N \Rightarrow m = -n$, *i.e.,* $m = n'$ *(cf. the following axiom (ii)). This is a contradiction.*

(ii): $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$

- $f(n) = 0_N$ *then it is obvious that* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$
- $f(n) = n \Rightarrow f(-n) = -n$*; so* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$
- $f(n) = m \Rightarrow f(-n) \neq n, 0_N$*; so* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$

(iii): $\Gamma_{M(N)}((f \circ g)(n)) \supseteq \Gamma_{M(N)}(f(n)) \cap \Gamma_{M(N)}(g(n))$

- $f(n) = 0_N$ *and* $g(n) = n \Rightarrow f(g(n)) = 0_N$*; then* $X \supseteq X \cap \{x_1, x_2, x_4\}$
- $f(n) = 0_N$ *and* $g(n) = m \Rightarrow f(g(n)) = 0_N$*; then* $X \supseteq X \cap \{x_1, x_2\}$
- $f(n) = n$ *and* $g(n) = 0_N \Rightarrow f(g(n)) = 0_N$*; then* $X \supseteq \{x_1, x_2, x_4\} \cap X$
- $f(n) = m$ *and* $g(n) = 0_N \Rightarrow f(g(n)) = m$*; then* $\{x_1, x_2\} \supseteq X \cap \{x_1, x_2\}$
- $f(n) = n$ *and* $g(n) = m \Rightarrow f(g(n)) = m$*; then* $\{x_1, x_2\} \supseteq \{x_1, x_2, x_4\} \cap \{x_1, x_2\}$
- $f(n) = m$ *and* $g(n) = n \Rightarrow f(g(n)) = m$*; then* $\{x_1, x_2\} \supseteq \{x_1, x_2, x_4\} \cap \{x_1, x_2\}$

(iv): $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(f^{-1}(n))$ *where* $f(n) \neq 0_N$

- $f(n) = n \Rightarrow f^{-1}(n) = f(n)$ *(since f is bijective function); then the equality is satisfied.*
- $f(n) = m \Rightarrow f^{-1}(n) \neq n, 0_N$ *(since f is bijective function); then the equality is satisfied.*

Therefore, we say that $\Gamma_{M(N)}$ *is a soft intersection near-field over* X.

Example 3.3. $M_2(\mathbb{Z}_2)$ *is a set of all* 2×2 *matrices with* $\mathbb{Z}_2$ *terms. Let*

$$\mathcal{M}=\{m_1=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, m_2=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, m_3=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, m_4=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\} \subseteq M_2(\mathbb{Z}_2)$$

with the following operations addition $(+)$ *and multiplication* $(*)$*:*

$+$	m_1	m_2	m_3	m_4
m_1	m_1	m_2	m_3	m_4
m_2	m_2	m_1	m_4	m_3
m_3	m_3	m_4	m_1	m_2
m_4	m_4	m_3	m_2	m_1

$*$	m_1	m_2	m_3	m_4
m_1	m_1	m_1	m_1	m_1
m_2	m_3	m_4	m_2	m_3
m_3	m_1	m_2	m_3	m_4
m_4	m_3	m_3	m_4	m_2

Then $(\mathcal{M},+)$ *is a group and* $(\mathcal{M}^\star,*)$ *is a group where* $\mathcal{M}^\star = \mathcal{M}-\{m_1\}$. *Also,* $(\mathcal{M},+,*)$ *is a (right) near-field (see* [3]*). Suppose that* $\Gamma_{\mathcal{M}}$ *is a soft set on the universal set* $X = \{x_1,x_2,x_3\}$, *where* $\Gamma_{\mathcal{M}}:\mathcal{M}\to\mathcal{P}(X)$ *is a mapping such that* $\Gamma_{\mathcal{M}}(m_1)=X$, $\Gamma_{\mathcal{M}}(m_2)=\{x_1,x_2\}$, $\Gamma_{\mathcal{M}}(m_3)=X$, *and* $\Gamma_{\mathcal{M}}(m_4)=\{x_1,x_2\}$. *Then, we can say that* $\Gamma_{\mathcal{M}}$ *is a soft intersection near-field over* X.

Proposition 3.4. *Let* $\Gamma_{\mathcal{N}}$ *be a soft intersection near-field on the universal set* X. *Then*

(i): $\Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \supseteq \Gamma_{\mathcal{N}}(p)$ *for all* $p \in \mathcal{N}$,

(ii): $\Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \supseteq \Gamma_{\mathcal{N}}(p)$ *for all* $p \in \mathcal{N}$ $(p \neq 0_{\mathcal{N}})$.

Proof. Suppose that $\Gamma_{\mathcal{N}}$ is a soft intersection near-field on the universal set X.
(i) For all $p \in \mathcal{N}$,

$$\Gamma_{\mathcal{N}}(0_{\mathcal{N}}) = \Gamma_{\mathcal{N}}(p-p) \supseteq \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(-p) = \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(p).$$

(ii) For all $p \in \mathcal{N}$ such that $p \neq 0_{\mathcal{N}}$,

$$\Gamma_{\mathcal{N}}(1_{\mathcal{N}}) = \Gamma_{\mathcal{N}}(pp^{-1}) \supseteq \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(p^{-1}) = \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(p).$$

These complete the proof.

Theorem 3.5. *Let* $\mathcal{N}$ *be a near-ring and* $\Gamma_{\mathcal{N}}$ *be a soft set on the universal set* X. $\Gamma_{\mathcal{N}}$ *is a soft intersection near-field over* X *iff for all* $p,q \in \mathcal{N}$

(i): $\Gamma_{\mathcal{N}}(p-q) \supseteq \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(q)$,

(ii): $\Gamma_{\mathcal{N}}(pq^{-1}) \supseteq \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(q)$, *where* $q \neq 0_{\mathcal{N}}$.

Proof. Assume that $\Gamma_{\mathcal{N}}$ is a soft intersection near-field over X. From Definition 3.1, for all $p,q \in \mathcal{N}$

$$\Gamma_{\mathcal{N}}(p-q) \supseteq \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(-q) = \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(q)$$

and

$$\Gamma_{\mathcal{N}}(pq^{-1}) \supseteq \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(q^{-1}) = \Gamma_{\mathcal{N}}(p)\cap\Gamma_{\mathcal{N}}(q) \text{ (where } q \neq 0_{\mathcal{N}}).$$

Conversely, suppose that the assertions (i) and (ii) are hold. If we take $p = 0_{\mathcal{N}}$ in the axiom (i) then we have

$$\Gamma_{\mathcal{N}}(0_{\mathcal{N}}-q) = \Gamma_{\mathcal{N}}(-q) \supseteq \Gamma_{\mathcal{N}}(0_{\mathcal{N}})\cap\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q)$$

by Proposition 3.4 (i) (i.e., $\Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \supseteq \Gamma_{\mathcal{N}}(q)$ for all $q \in \mathcal{N}$). Thus, we find $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(-(-q)) \supseteq \Gamma_{\mathcal{N}}(-q)$. So, it is obvious that for all $p,q \in \mathcal{N}$

$$\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(-q) \text{ and } \Gamma_{\mathcal{N}}(p+q) \supseteq \Gamma_{\mathcal{N}}(q)\cap\Gamma_{\mathcal{N}}(-q) = \Gamma_{\mathcal{N}}(q)\cap\Gamma_{\mathcal{N}}(q).$$

Likewise,if we take $p = 1_{\mathcal{N}}$ in the axiom (ii) then we have for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$

$$\Gamma_{\mathcal{N}}(1_{\mathcal{N}} q^{-1}) = \Gamma_{\mathcal{N}}(q^{-1}) \supseteq \Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \cap \Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q)$$

by Proposition 3.4 (ii) (i.e., $\Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \supseteq \Gamma_{\mathcal{N}}(q)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$). Hence, we can write $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}((q^{-1})^{-1}) = \Gamma_{\mathcal{N}}(q^{-1})$. Then, it is obvious that

$$\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q^{-1}) \text{ and } \Gamma_{\mathcal{N}}(pq) \supseteq \Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(q^{-1}) = \Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(q)$$

for all $p, q \in \mathcal{N}$ $(q \neq 0_{\mathcal{N}})$. Thus, the proof is completed.

Theorem 3.6. *Let $\Gamma_{\mathcal{N}}$ be a soft intersection near-field on the universal set X.*

- **(i):** *If $\Gamma_{\mathcal{N}}(pq^{-1}) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ for any $0_{\mathcal{N}} \neq p, q \in \mathcal{N}$ then $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.*
- **(ii):** *If $\Gamma_{\mathcal{N}}(p - q) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ for any $p, q \in \mathcal{N}$ then $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.*

Proof. (i) Suppose that $\Gamma_{\mathcal{N}}(pq^{-1}) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ for any $0_{\mathcal{N}} \neq p, q \in \mathcal{N}$. Then,

$$\begin{aligned} \Gamma_{\mathcal{N}}(p) &= \Gamma_{\mathcal{N}}((pq^{-1})q) \\ &\supseteq \Gamma_{\mathcal{N}}(pq^{-1}) \cap \Gamma_{\mathcal{N}}(q) \\ &= \Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \cap \Gamma_{\mathcal{N}}(q) \\ &= \Gamma_{\mathcal{N}}(q) \end{aligned}$$

and

$$\begin{aligned} \Gamma_{\mathcal{N}}(q) &= \Gamma_{\mathcal{N}}((qp^{-1})p) \\ &\supseteq \Gamma_{\mathcal{N}}(qp^{-1}) \cap \Gamma_{\mathcal{N}}(p) \\ &= \Gamma_{\mathcal{N}}((qp^{-1})^{-1}) \cap \Gamma_{\mathcal{N}}(p) \\ &= \Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \cap \Gamma_{\mathcal{N}}(p) \\ &= \Gamma_{\mathcal{N}}(p). \end{aligned}$$

Therefore, $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$ for any $0_{\mathcal{N}} \neq p, q \in \mathcal{N}$.
(ii) The proof is similar to that of (i). We omit it.

Corollary 3.7. *Let $\Gamma_{\mathcal{N}}$ be a soft intersection near-field on the universal set X. Then, the following statement hold.*

- **(i):** *If $\Gamma_{\mathcal{N}}(pq) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ for any $p, q \in \mathcal{N}$ then $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.*
- **(ii):** *If $\Gamma_{\mathcal{N}}(p - q) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ for any $p, q \in \mathcal{N}$ then $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.*

Theorem 3.8. *Let $\Gamma_{\mathcal{N}}$ be a soft intersection near-field on the universal set X.*

- **(i):** *$\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ for $0_{\mathcal{N}} \neq p \in \mathcal{N} \Leftrightarrow \Gamma_{\mathcal{N}}(pq) = \Gamma_{\mathcal{N}}(qp) = \Gamma_{\mathcal{N}}(q)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$.*
- **(ii):** *$\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ for $p \in \mathcal{N} \Leftrightarrow \Gamma_{\mathcal{N}}(p + q) = \Gamma_{\mathcal{N}}(q + p) = \Gamma_{\mathcal{N}}(q)$ for all $q \in \mathcal{N}$.*

Proof. (i) Let $\Gamma_{\mathcal{N}}$ be a soft intersection near-field on the universal set X.
$\Rightarrow$: Assume that $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ for $0_{\mathcal{N}} \neq p \in \mathcal{N}$. Then, by Proposition 3.4 (ii), we write $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \supseteq \Gamma_{\mathcal{N}}(q)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$. Since $\Gamma_{\mathcal{N}}$ is a soft intersection near-field over X, $\Gamma_{\mathcal{N}}(pq) \supseteq \Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$. Furthermore,

for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$

$$\begin{aligned}\Gamma_{\mathcal{N}}(q) &= \Gamma_{\mathcal{N}}((p^{-1}p)q)\\ &= \Gamma_{\mathcal{N}}(p^{-1}(pq))\\ &\supseteq \Gamma_{\mathcal{N}}(p^{-1}) \cap \Gamma_{\mathcal{N}}(pq)\\ &= \Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(pq)\\ &= \Gamma_{\mathcal{N}}(pq).\end{aligned}$$

It follows that $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(pq)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$. Then, we have that for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$

$$\Gamma_{\mathcal{N}}(1_{\mathcal{N}}) = \Gamma_{\mathcal{N}}(p) \supseteq \Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(pq).$$

Similarity, for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$

$$\begin{aligned}\Gamma_{\mathcal{N}}(qp) &= \Gamma_{\mathcal{N}}(qp(qq^{-1}))\\ &= \Gamma_{\mathcal{N}}(q(pq)q^{-1})\\ &\supseteq \Gamma_{\mathcal{N}}(q) \cap \Gamma_{\mathcal{N}}(pq) \cap \Gamma_{\mathcal{N}}(q)\\ &= \Gamma_{\mathcal{N}}(q) \cap \Gamma_{\mathcal{N}}(pq)\\ &= \Gamma_{\mathcal{N}}(q)\end{aligned}$$

since $\Gamma_{\mathcal{N}}(pq) = \Gamma_{\mathcal{N}}(q)$. Moreover, for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$

$$\begin{aligned}\Gamma_{\mathcal{N}}(q) &= \Gamma_{\mathcal{N}}(q(pp^{-1}))\\ &= \Gamma_{\mathcal{N}}((qp)p^{-1})\\ &\supseteq \Gamma_{\mathcal{N}}(qp) \cap \Gamma_{\mathcal{N}}(p)\\ &= \Gamma_{\mathcal{N}}(qp).\end{aligned}$$

since $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \supseteq \Gamma_{\mathcal{N}}(qp)$ by Proposition 3.4 (ii).Then, we have $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(qp)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$, and so $\Gamma_{\mathcal{N}}(pq) = \Gamma_{\mathcal{N}}(qp) = \Gamma_{\mathcal{N}}(q)$.
$\Leftarrow$: Assume that $\Gamma_{\mathcal{N}}(pq) = \Gamma_{\mathcal{N}}(qp) = \Gamma_{\mathcal{N}}(q)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$. If we choose $q = 1_{\mathcal{N}}$, then we have $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$.
(ii) It can be demonstrated in a similar way to the proof of (i).

Theorem 3.9. *If* $\Gamma_{\mathcal{N}}$ *and* $\Gamma'_{\mathcal{N}}$ *are two soft intersection near-fields on the universal set* X *then* $\Gamma_{\mathcal{N}} \sqcap \Gamma'_{\mathcal{N}}$ *is also soft intersection near-field over* X.

Proof. For all $p, q \in \mathcal{N}$,

$$\begin{aligned}(\Gamma \sqcap \Gamma')_{\mathcal{N}}(p-q) &= \Gamma_{\mathcal{N}}(p-q) \cap \Gamma'_{\mathcal{N}}(p-q)\\ &\supseteq (\Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(q)) \cap (\Gamma'_{\mathcal{N}}(p) \cap \Gamma'_{\mathcal{N}}(q))\\ &= (\Gamma_{\mathcal{N}}(p) \cap \Gamma'_{\mathcal{N}}(p)) \cap (\Gamma_{\mathcal{N}}(q) \cap \Gamma'_{\mathcal{N}}(q))\\ &= (\Gamma \sqcap \Gamma')_{\mathcal{N}}(p) \cap (\Gamma \sqcap \Gamma')_{\mathcal{N}}(q)\end{aligned}$$

and

$$\begin{aligned}(\Gamma \sqcap \Gamma^{'})_{\mathcal{N}}(pq^{-1}) &= \Gamma_{\mathcal{N}}(pq^{-1}) \cap \Gamma^{'}_{\mathcal{N}}(pq^{-1})\\ &\supseteq (\Gamma_{\mathcal{N}}(p) \cap \Gamma_{\mathcal{N}}(q)) \cap (\Gamma^{'}_{\mathcal{N}}(p) \cap \Gamma^{'}_{\mathcal{N}}(q))\\ &= (\Gamma_{\mathcal{N}}(p) \cap \Gamma^{'}_{\mathcal{N}}(p)) \cap (\Gamma_{\mathcal{N}}(q) \cap \Gamma^{'}_{\mathcal{N}}(q))\\ &= (\Gamma \sqcap \Gamma^{'})_{\mathcal{N}}(p) \cap (\Gamma \sqcap \Gamma^{'})_{\mathcal{N}}(q), \text{where} q \neq 0_{\mathcal{N}}.\end{aligned}$$

Thus, the proof is completed.

Example 3.10. *Let $(N,+)$ be any group, $M(N) = \{f : N \to N | f \text{ is bijective function}\}$ and $(M(N),+,\circ)$ be a near-field. We consider the soft set $\Gamma_{M(N)}$ given in Example 3.2. Also, we suppose that $\Gamma^{'}_{M(N)}$ is a soft set on the universal set $X = \{x_1,x_2,x_3,x_4\}$, where $\Gamma^{'}_{M(N)} : M(N) \to \mathcal{P}(X)$ is a mapping given by*

$$\Gamma^{'}_{M(N)}(f(n)) = \begin{cases} \{x_1,x_3,x_4\}, & \text{if } f(n) = 0_N \\ \{x_1,x_4\}, & \text{if } f(n) = n \\ \{x_1\}, & \text{if } f(n) = m \ni m \neq 0_N, n \text{ for all } m \in N \end{cases}$$

Then, we say that $\Gamma_{M(N)}$ and $\Gamma^{'}_{M(N)}$ are two soft intersection near-fields on X. Furthermore, we obtain the intersection of $\Gamma_{M(N)}$ and $\Gamma^{'}_{M(N)}$ as follows:

$$\begin{aligned}\Gamma^{''}_{M(N)}(f(n)) &= (\Gamma \sqcap \Gamma^{'})_{M(N)}(f(n))\\ &= \begin{cases} \{x_1,x_3,x_4\}, & \text{if } f(n) = 0_N \\ \{x_1,x_4\}, & \text{if } f(n) = n \\ \{x_1\}, & \text{if } f(n) = m \ni m \neq 0_N, n \text{ for all } m \in N \end{cases}\end{aligned}$$

Thus, we have that $\Gamma^{''}_{M(N)} = \Gamma_{M(N)} \sqcap \Gamma^{'}_{M(N)}$ is a soft intersection near-field over X.

4. SOFT UNION NEAR-FIELD

In this part, we address the soft union near-fields and investigate some of their properties.

Definition 4.1. *Let $\mathcal{N}$ be a near-field and $\Gamma_{\mathcal{N}}$ be a soft set on the universal set X. $\Gamma_{\mathcal{N}}$ is said to be a soft union near-field over X if the following properties are satisfied: For all $p, q \in \mathcal{N}$*

- **(i):** $\Gamma_{\mathcal{N}}(p+q) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)$,
- **(ii):** $\Gamma_{\mathcal{N}}(-p) = \Gamma_{\mathcal{N}}(p)$,
- **(iii):** $\Gamma_{\mathcal{N}}(pq) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)$,
- **(iv):** $\Gamma_{\mathcal{N}}(p^{-1}) = \Gamma_{\mathcal{N}}(p)$, *where* $p \neq 0_{\mathcal{N}}$.

Example 4.2. *Let $(N,+)$ be a group, $M(N) = \{f : N \to N | f \text{ is bijective function}\}$ and $(M(N),+,\circ)$ be a near-field. Assume that $\Gamma_{M(N)}$ is a soft set on the universal set $X = \{x_1,x_2,x_3,x_4\}$, where $\Gamma_{M(N)} : M(N) \to \mathcal{P}(X)$ is a mapping given by*

$$\Gamma_{M(N)}(f(n)) = \begin{cases} \{x_1\}, & \text{if } f(n) = 0_N \\ \{x_1,x_4\}, & \text{if } f(n) = n \\ \{x_1,x_2,x_4\}, & \text{if } f(n) = m \ni m \neq 0_N, n \text{ for all } m \in N \end{cases}$$

- **(i):** $\Gamma_{M(N)}((f+g)(n)) \subseteq \Gamma_{M(N)}(f(n)) \cup \Gamma_M(N)(g(n))$

- $f(n) = 0_N$ *and* $g(n) = n \Rightarrow (f + g)(n) = n$*; then* $\{x_1, x_4\} \subseteq \{x_1, x_4\} \cup \{x_1\}$
- $f(n) = 0_N$ *and* $g(n) = m \Rightarrow (f + g)(n) = m$*; then* $\{x_1, x_2, x_4\} \subseteq \{x_1\} \cup \{x_1, x_2, x_4\}$
- $f(n) = n$ *and* $g(n) = m \Rightarrow (f + g)(n) = n + m$ *(where* $n + m \neq n, 0_N$*); then* $\{x_1, x_2, x_4\} \subseteq \{x_1, x_2, x_4\} \cap \{x_1, x_4\}$.

(ii): $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$
- $f(n) = 0_N$ *then it is clear that* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$
- $f(n) = n \Rightarrow f(-n) = -n$*; so* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$
- $f(n) = m \Rightarrow f(-n) \neq n, 0_N$*; so* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(-f(n))$

(iii): $\Gamma_{M(N)}((f \circ g)(n)) \subseteq \Gamma_{M(N)}(f(n)) \cup \Gamma_{M(N)}(g(n))$
- $f(n) = 0_N$ *and* $g(n) = n \Rightarrow f(g(n)) = 0_N$*; then* $\{x_1\} \subseteq \{x_1\} \cup \{x_1, x_4\}$
- $f(n) = 0_N$ *and* $g(n) = m \Rightarrow f(g(n)) = 0_N$*; then* $\{x_1\} \subseteq \{x_1\} \cup \{x_1, x_4\}$
- $f(n) = n$ *and* $g(n) = 0_N \Rightarrow f(g(n)) = 0_N$*; then* $\{x_1\} \subseteq \{x_1, x_4\} \cup \{x_1\}$
- $f(n) = m$ *and* $g(n) = 0_N \Rightarrow f(g(n)) = m$*; then* $\{x_1, x_4\} \subseteq \{x_1\} \cup \{x_1, x_4\}$
- $f(n) = n$ *and* $g(n) = m \Rightarrow f(g(n)) = m$*; then* $\{x_1, x_4\} \subseteq \{x_1\} \cup \{x_1, x_2, x_4\}$
- $f(n) = m$ *and* $g(n) = n \Rightarrow f(g(n)) = m$*; then* $\{x_1, x_4\} \subseteq \{x_1\} \cup \{x_1, x_2, x_4\}$

(iv): $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(f^{-1}(n))$ *where* $f(n) \neq 0_N$
- $f(n) = n \Rightarrow f^{-1}(n) = f(n)$ *(since f is bijective function); then* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(f^{-1}(n))$.
- $f(n) = m \Rightarrow f^{-1}(n) \neq n, 0_N$ *(since f is bijective function); then* $\Gamma_{M(N)}(f(n)) = \Gamma_{M(N)}(f^{-1}(n))$.

Hence, $\Gamma_{M(N)}$ *is a soft union near-field over the universal set* X.

Example 4.3. *We consider the near-field* $(\mathcal{M}, +, *)$ *in Example 3.3. Suppose that* $\Gamma_{\mathcal{M}}$ *is a soft set on the universal set* $X = \{x_1, x_2, x_3\}$*, where* $\Gamma_{\mathcal{M}} : \mathcal{M} \to \mathcal{P}(X)$ *is a mapping such that* $\Gamma_{\mathcal{M}}(m_1) = \{x_2\}$, $\Gamma_{\mathcal{M}}(m_2) = X$, $\Gamma_{\mathcal{M}}(m_3) = \{x_2\}$*, and* $\Gamma_{\mathcal{M}}(m_4) = X$*. Then, we can say that* $\Gamma_{\mathcal{M}}$ *is a soft union near-field over* X.

Proposition 4.4. *Let* $\Gamma_{\mathcal{N}}$ *be a soft union near-field on the universal set* X*. Then*

(i): $\Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \subseteq \Gamma_{\mathcal{N}}(p)$ *for all* $p \in \mathcal{N}$,

(ii): $\Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \subseteq \Gamma_{\mathcal{N}}(p)$ *for all* $p \in \mathcal{N}$ $(p \neq 0_{\mathcal{N}})$.

Proof. Assume that $\Gamma_{\mathcal{N}}$ is a soft union near-field on the universal set X.
(i) For all $p \in \mathcal{N}$,

$$\Gamma_{\mathcal{N}}(0_{\mathcal{N}}) = \Gamma_{\mathcal{N}}(p - p) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(-p) = \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(p).$$

(ii) For all $p \in \mathcal{N}$ such that $p \neq 0_{\mathcal{N}}$,

$$\Gamma_{\mathcal{N}}(1_{\mathcal{N}}) = \Gamma_{\mathcal{N}}(pp^{-1}) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(p^{-1}) = \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(p).$$

These complete the proof.

Theorem 4.5. *Let* $\mathcal{N}$ *be a near-ring and* $\Gamma_{\mathcal{N}}$ *be a soft set on the universal set* X. $\Gamma_{\mathcal{N}}$ *is a soft union near-field over* X *iff for all* $p, q \in \mathcal{N}$

(i): $\Gamma_{\mathcal{N}}(p - q) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)$,

(ii): $\Gamma_{\mathcal{N}}(pq^{-1}) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)$, *where* $q \neq 0_{\mathcal{N}}$.

Proof. Suppose that $\Gamma_{\mathcal{N}}$ is a soft union near-field over X. From Definition 4.1, for all $p, q \in \mathcal{N}$

$$\Gamma_{\mathcal{N}}(p-q) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(-q) = \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)$$

and

$$\Gamma_{\mathcal{N}}(pq^{-1}) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q^{-1}) = \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q) \text{ (where } q \neq 0_{\mathcal{N}}\text{)}.$$

Conversely, assume that the assertions (i) and (ii) are hold. If we consider $p = 0_{\mathcal{N}}$ in the axiom (i) then we have

$$\Gamma_{\mathcal{N}}(0_{\mathcal{N}} - q) = \Gamma_{\mathcal{N}}(-q) \subseteq \Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \cup \Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q)$$

by Proposition 4.4 (i) (i.e., $\Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \subseteq \Gamma_{\mathcal{N}}(q)$ for all $q \in \mathcal{N}$). Thus, we find $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(-(-q)) \subseteq \Gamma_{\mathcal{N}}(-q)$. Therefore, we have following

$$\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(-q) \text{ and } \Gamma_{\mathcal{N}}(p+q) \subseteq \Gamma_{\mathcal{N}}(q) \cup \Gamma_{\mathcal{N}}(-q) = \Gamma_{\mathcal{N}}(q) \cup \Gamma_{\mathcal{N}}(q).$$

for all $p, q \in \mathcal{N}$.
Similarly, if we consider $p = 1_{\mathcal{N}}$ in the axiom (ii) then we have for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$

$$\Gamma_{\mathcal{N}}(1_{\mathcal{N}} q^{-1}) = \Gamma_{\mathcal{N}}(q^{-1}) \subseteq \Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \cup \Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q)$$

by Proposition 4.4 (ii) (i.e., $\Gamma_{\mathcal{N}}(1_{\mathcal{N}}) \subseteq \Gamma_{\mathcal{N}}(q)$ for all $0_{\mathcal{N}} \neq q \in \mathcal{N}$). Then, we obtain $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}((q^{-1})^{-1}) = \Gamma_{\mathcal{N}}(q^{-1})$. So, it is obvious

$$\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q^{-1}) \text{ and } \Gamma_{\mathcal{N}}(pq) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q^{-1}) = \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)$$

for all $p, q \in \mathcal{N}$ $(q \neq 0_{\mathcal{N}})$. Thus, the proof is completed.

Theorem 4.6. *Let* $\Gamma_{\mathcal{N}}$ *be a soft union near-field on the universal set* X.

(i): *If* $\Gamma_{\mathcal{N}}(p-q) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ *for any* $p, q \in \mathcal{N}$ *then* $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.
(ii): *If* $\Gamma_{\mathcal{N}}(pq^{-1}) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ *for any* $0_{\mathcal{N}} \neq p, q \in \mathcal{N}$ *then* $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.

Proof. (i) Suppose that $\Gamma_{\mathcal{N}}(p-q) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ for any $p, q \in \mathcal{N}$. Then,

$$\begin{aligned} \Gamma_{\mathcal{N}}(p) &= \Gamma_{\mathcal{N}}((p-q)+q) \\ &\subseteq \Gamma_{\mathcal{N}}(p-q) \cup \Gamma_{\mathcal{N}}(q) \\ &= \Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \cup \Gamma_{\mathcal{N}}(q) \\ &= \Gamma_{\mathcal{N}}(q) \end{aligned}$$

and

$$\begin{aligned} \Gamma_{\mathcal{N}}(q) &= \Gamma_{\mathcal{N}}((q-p)+p) \\ &\subseteq \Gamma_{\mathcal{N}}(q-p) \cup \Gamma_{\mathcal{N}}(p) \\ &= \Gamma_{\mathcal{N}}(-(q-p)) \cup \Gamma_{\mathcal{N}}(p) \\ &= \Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \cup \Gamma_{\mathcal{N}}(p) \\ &= \Gamma_{\mathcal{N}}(p). \end{aligned}$$

Hence, $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$ for any $p, q \in \mathcal{N}$.
(ii) The proof is similar to that of (i), therefore it is omitted.

Corollary 4.7. *Let* $\Gamma_{\mathcal{N}}$ *be a soft union near-field on the universal set* X.

(i): *If* $\Gamma_{\mathcal{N}}(p+q) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ *for any* $p, q \in \mathcal{N}$ *then* $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.

(ii): *If* $\Gamma_{\mathcal{N}}(pq) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ *for any* $p, q \in \mathcal{N}$ *then* $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(q)$.

Theorem 4.8. *Let* $\Gamma_{\mathcal{N}}$ *be a soft union near-field on the universal set* X.

(i): $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ *for* $p \in \mathcal{N} \Leftrightarrow \Gamma_{\mathcal{N}}(p+q) = \Gamma_{\mathcal{N}}(q+p) = \Gamma_{\mathcal{N}}(q)$ *for all* $q \in \mathcal{N}$.

(ii): $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(1_{\mathcal{N}})$ *for* $0_{\mathcal{N}} \neq p \in \mathcal{N} \Leftrightarrow \Gamma_{\mathcal{N}}(pq) = \Gamma_{\mathcal{N}}(qp) = \Gamma_{\mathcal{N}}(q)$ *for all* $0_{\mathcal{N}} \neq q \in \mathcal{N}$.

Proof. (i) Let $\Gamma_{\mathcal{N}}$ be a soft union near-field on the universal set X.
$\Rightarrow$: Suppose that $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$ for $p \in \mathcal{N}$. Then, by Proposition 4.4 (i), we write $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}}) \subseteq \Gamma_{\mathcal{N}}(q)$ for all $q \in \mathcal{N}$. Since $\Gamma_{\mathcal{N}}$ is a soft union near-field over X, $\Gamma_{\mathcal{N}}(pq) \subseteq \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q)$ for all $q \in \mathcal{N}$. Besides, for all $q \in \mathcal{N}$

$$\begin{aligned} \Gamma_{\mathcal{N}}(q) &= \Gamma_{\mathcal{N}}(((-p)+p)+q) \\ &= \Gamma_{\mathcal{N}}(-p+(p+q)) \\ &\subseteq \Gamma_{\mathcal{N}}(-p) \cup \Gamma_{\mathcal{N}}(p+q) \\ &= \Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(p+q) \\ &= \Gamma_{\mathcal{N}}(p+q). \end{aligned}$$

It follows that $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(p+q)\ \forall q \in \mathcal{N}$. Then, we achieve that

$$\Gamma_{\mathcal{N}}(0_{\mathcal{N}}) = \Gamma_{\mathcal{N}}(p) \subseteq \Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(p+q)\ \forall q \in \mathcal{N}.$$

Likewise, for all $q \in \mathcal{N}$

$$\begin{aligned} \Gamma_{\mathcal{N}}(q+p) &= \Gamma_{\mathcal{N}}(q+p+(q-q)) \\ &= \Gamma_{\mathcal{N}}(q+(p+q)-q) \\ &\subseteq \Gamma_{\mathcal{N}}(q) \cup \Gamma_{\mathcal{N}}(p+q) \cup \Gamma_{\mathcal{N}}(q) \\ &= \Gamma_{\mathcal{N}}(q) \cup \Gamma_{\mathcal{N}}(p+q) \\ &= \Gamma_{\mathcal{N}}(q) \end{aligned}$$

since $\Gamma_{\mathcal{N}}(p+q) = \Gamma_{\mathcal{N}}(q)$. Moreover, for all $q \in \mathcal{N}$

$$\begin{aligned} \Gamma_{\mathcal{N}}(q) &= \Gamma_{\mathcal{N}}(q+(p-p)) \\ &= \Gamma_{\mathcal{N}}((q+p)-q) \\ &\subseteq \Gamma_{\mathcal{N}}(q+p) \cup \Gamma_{\mathcal{N}}(p) \\ &= \Gamma_{\mathcal{N}}(q+p) \end{aligned}$$

since $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}}) = \Gamma_{\mathcal{N}}(q+p)$ by Proposition 4.4 (i).Then, we have $\Gamma_{\mathcal{N}}(q) = \Gamma_{\mathcal{N}}(q+p)\ \forall q \in \mathcal{N}$, and so $\Gamma_{\mathcal{N}}(p+q) = \Gamma_{\mathcal{N}}(q+p) = \Gamma_{\mathcal{N}}(q)\ \forall q \in \mathcal{N}$.
$\Leftarrow$: Suppose that $\Gamma_{\mathcal{N}}(p+q) = \Gamma_{\mathcal{N}}(q+p) = \Gamma_{\mathcal{N}}(q)$ for all $q \in \mathcal{N}$. If we choose $q = 0_{\mathcal{N}}$, then we have $\Gamma_{\mathcal{N}}(p) = \Gamma_{\mathcal{N}}(0_{\mathcal{N}})$.
(ii) It can be shown in a similar way to the proof of (i).

Theorem 4.9. *If* $\Gamma_{\mathcal{N}}$ *and* $\Gamma'_{\mathcal{N}}$ *are two soft union near-fields on the universal set* X *then* $\Gamma_{\mathcal{N}} \sqcup \Gamma'_{\mathcal{N}}$ *is also soft union near-field over* X.

Proof. For all $p, q \in \mathcal{N}$,

$$\begin{aligned}(\Gamma \sqcup \Gamma^{'})_{\mathcal{N}}(p-q) &= \Gamma_{\mathcal{N}}(p-q) \cup \Gamma^{'}_{\mathcal{N}}(p-q) \\ &\subseteq (\Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)) \cup (\Gamma^{'}_{\mathcal{N}}(p) \cup \Gamma^{'}_{\mathcal{N}}(q)) \\ &= (\Gamma_{\mathcal{N}}(p) \cup \Gamma^{'}_{\mathcal{N}}(p)) \cup (\Gamma_{\mathcal{N}}(q) \cup \Gamma^{'}_{\mathcal{N}}(q)) \\ &= (\Gamma \sqcup \Gamma^{'})_{\mathcal{N}}(p) \cup (\Gamma \sqcup \Gamma^{'})_{\mathcal{N}}(q)\end{aligned}$$

and

$$\begin{aligned}(\Gamma \sqcup \Gamma^{'})_{\mathcal{N}}(pq^{-1}) &= \Gamma_{\mathcal{N}}(pq^{-1}) \cup \Gamma^{'}_{\mathcal{N}}(pq^{-1}) \\ &\subseteq (\Gamma_{\mathcal{N}}(p) \cup \Gamma_{\mathcal{N}}(q)) \cup (\Gamma^{'}_{\mathcal{N}}(p) \cup \Gamma^{'}_{\mathcal{N}}(q)) \\ &= (\Gamma_{\mathcal{N}}(p) \cup \Gamma^{'}_{\mathcal{N}}(p)) \cup (\Gamma_{\mathcal{N}}(q) \cup \Gamma^{'}_{\mathcal{N}}(q)) \\ &= (\Gamma \sqcup \Gamma^{'})_{\mathcal{N}}(p) \cup (\Gamma \sqcup \Gamma^{'})_{\mathcal{N}}(q), \text{where} q \neq 0_{\mathcal{N}}.\end{aligned}$$

So, the proof is completed.

Example 4.10. *Let $(N,+)$ be any group, $M(N) = \{f : N \to N | f \text{ is bijective function}\}$ and $(M(N), +, \circ)$ be a near-field. Assume that $\Gamma_{M(N)}$ and $\Gamma^{'}_{M(N)}$ are two soft sets on the universal set $X = \{x_1, x_2, x_3, x_4\}$, where $\Gamma_{M(N)}, \Gamma^{'}_{M(N)} : M(N) \to \mathcal{P}(X)$ are the mappings given by*

$$\Gamma_{M(N)}(f(n)) = \begin{cases} \{x_1\}, & \text{if } f(n) = 0_N \\ \{x_1, x_4\}, & \text{if } f(n) = n \\ \{x_1, x_2, x_4\}, & \text{if } f(n) = m \ni m \neq 0_N, n \text{ for all } m \in N \end{cases}$$

$$\Gamma^{'}_{M(N)}(f(n)) = \begin{cases} \{x_2\}, & \text{if } f(n) = 0_N \\ \{x_2, x_4\}, & \text{if } f(n) = n \\ X, & \text{if } f(n) = m \ni m \neq 0_N, n \text{ for all } m \in N \end{cases}$$

Thus, we say that $\Gamma_{M(N)}$ and $\Gamma^{'}_{M(N)}$ are two soft union near-fields on X. Moreover, we find the union of $\Gamma_{M(N)}$ and $\Gamma^{'}_{M(N)}$ as follows:

$$\Gamma^{''}_{M(N)}(f(n)) = (\Gamma \sqcup \Gamma^{'})_{M(N)}(f(n))$$

$$= \begin{cases} \{x_1, x_2\}, & \text{if } f(n) = 0_N \\ \{x_1, x_2, x_4\}, & \text{if } f(n) = n \\ X, & \text{if } f(n) = m \ni m \neq 0_N, n \text{ for all } m \in N \end{cases}$$

Then, we obtain that $\Gamma^{''}_{M(N)} = \Gamma_{M(N)} \sqcup \Gamma^{'}_{M(N)}$ is a soft union near-field over X.

5. CONCLUSION

In this paper, we introduced two different kinds of soft near-fields, called soft intersection near-field and soft union near-field. By theoretical aspect we implemented some of the operations on soft sets to the proposed soft structures and then studied their characteristic. These contributed to the algebraic structures of soft sets. To extend this work, further research can be conducted improving the theoretical aspects of soft intersection near-field and soft union near-field. In the future, we will investigate the bipolar soft intersection near-ring, the bipolar soft union near-ring, and discuss some of their related properties.

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