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Completion of Complex Valued Dislocated Metric Space

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Abstract.: In this paper, we prove an important property of metric space which is the existence and uniqueness of completion. Firstly we gave completion of a complex-valued dislocated metric space and then prove its uniqueness.

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1. Introduction and Mathematical preliminaries

A well-known property of metric spaces is the existence and uniqueness of a metric space's completion. In recent years, researchers have looked into the completion of other types of metric spaces. Ge and Lin [19] investigated the presence of partial metric space completion, and Dung [10] responded to Ge and Lin's denseness property question with an example of partial metric space completion. Any strong b-metric space has a completion, according to An et al. [2]. Andrikopoulos [3] looked into the completion of quasi pseudo metric spaces.

Dahliatul and Supeno [20] investigated and proved the existence and uniqueness of a complex-valued metric space. Kumari et al. [22] suggested a procedure for completing a dislocated metric space. Beg et al. [8] explored the completion of complex-valued strong b-metric space in their recent paper. Some recent work about fixed point is disscussed in [4],[6], [13], [14], [15], [16] and [9]. A new extension of the double controlled metric-type spaces, called double controlled metric-like spaces is discussed in[25], by considering that the self-distance may not be zero. On the other hand, if the value of the metric is zero, then it has to be a self-distance. A fixed point theorem in complete metric-like spaces for a Lipschitz map with bound is provided in [21]. This paper aims to show that complex-valued dislocated metric space is complete. The definition of complex-valued dislocated metric space was introduced by Ege et al. [18].

Definition 1.1. [5]***(Dislocated metric space.)*** *A dislocated metric space is a pair (S, ϑ), where S is a set and ϑ is a dislocated metric on S, that is, a function defined on $S \times S$ such that for all $\varrho, \varsigma, \sigma \in S$ we have:*

M1: $\vartheta(\varrho, \varsigma) \geq 0$
M2: $\vartheta(\varrho, \varsigma) = 0 \Rightarrow \varrho = \varsigma$
M3: $\vartheta(\varrho, \varsigma) = \vartheta(\varsigma, \varrho)$
M4: $\vartheta(\varrho, \varsigma) \leq \vartheta(\varrho, \sigma) + \vartheta(\sigma, \varsigma)$ *for all* $\varrho, \varsigma, \sigma \in S$

Definition 1.2. $z_1 \precsim z_2$ *if and only if* $Re(z_1) \leq Re(z_2)$ *and* $Im(z_1) \leq Im(z_2)$

Definition 1.3. [18]***(Complex valued d-metric space.)*** *Let S be a nonempty set. Suppose that for all $\varrho, \varsigma, \sigma \in S$, the mapping $\vartheta : X \times X \to \mathbb{C}$ satisfies:*
(i) $0 \precsim \vartheta(\varrho, \varsigma)$ *and* $\vartheta(\varrho, \varsigma) = 0 \Rightarrow \varrho = \varsigma$.
(ii) $\vartheta(\varrho, \varsigma) = \vartheta(\varsigma, \varrho)$
(iii) $\vartheta(\varrho, \varsigma) \precsim \vartheta(\varrho, \sigma) + \vartheta(\sigma, \varsigma)$

Then ϑ is called a complex valued d-metric on S, and (S, ϑ) is called a complex valued metric space.

Example 1.4. [18] *Let $\vartheta : S \times S \to \mathbb{C}$ be defined by*

$$\vartheta(\varrho, \varsigma) = \max\{\varrho, \varsigma\},$$

where $S = \mathbb{C}$. It is clear that ϑ is a complex valued dislocated metric.

Example 1.5. *Let $\vartheta : S \times S \to \mathbb{C}$ be defined by*

$$\vartheta(\varrho, \varsigma) = \begin{cases} 1, & \varrho = \varsigma \\ \max\{\varrho, \varsigma\}, & \varrho \neq \varsigma \end{cases}$$

where $S = \mathbb{C}$.

2. Main results

In this section, we give the completion theorem for existence and uniqueness of complex valued dislocated type metric spaces.

Theorem 2.1. ***(Completion.)*** *Let (S, ϑ) be a complex valued dislocated metric space. Then there exists a complete complex valued dislocated metric space (S^*, ϑ^*) and an isodistance $f : S \to S^*$ such that $f(S)$ is dense in S^*.*

Proof. Let A be the collection of points of S whose self distance is non zero and let $B = S - A$. Let $\bar{A}$ be the collection of sequences in S which are ultimately a constant complex element lying in $\bar{A}$ and $\bar{B}$ denote the class of Cauchy sequences in B. We define relations $\sim_{\bar{A}}$ and $\sim_{\bar{B}}$, respectively, on $\bar{A}$ and $\bar{B}$ as follows.

If $(\varrho_n)(\varsigma_n)$ are sequences in $\bar{A}$ then $(\varrho_n) \sim_{\bar{A}} (\varsigma_n)$ iff the ultimately constant value of (ϱ_n) coincides with that of (ς_n). If (ϱ_n) (ς_n) are sequences in $\bar{B}$ then $(\varrho_n) \sim_{\bar{B}} (\varsigma_n)$ iff $\lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_n)| = 0$. Clearly $\sim_{\bar{A}}$ is an equivalence relation. We verify that $\sim_{\bar{B}}$ is an equivalence relation. Suppose $(\varrho_n) \in B$. Since (ϱ_n) is a Cauchy sequence in $\bar{B}$, $\lim_{n\to\infty} |\vartheta(\varrho_n, \varrho_n)| = 0$ and hence $\sim_{\bar{B}}$ is reflexive. Suppose $(\varrho_n) \sim_{\bar{B}} (\varsigma_n)$ for (ϱ_n), $(\varsigma_n) \in \bar{B}$. Then $\lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_n)| = \lim_{n\to\infty} |\vartheta(\varsigma_n, \varrho_n)| = 0$. Hence $\sim_{\bar{B}}$ is symmetric.

If (ϱ_n),(ς_n),$(\sigma_n) \in \bar{B}$,$(\varrho_n) \sim_{\bar{B}} (\varsigma_n)$ and $(\varsigma_n) \sim_{\bar{B}} (\sigma_n)$.

$$\vartheta(\varrho_n, \sigma_n) \preceq \vartheta(\varrho_n, \varsigma_n) + \vartheta(\varsigma_n, \sigma_n). \tag{2. 1}$$

Taking limit on both sides

$$\lim |\vartheta(\varrho_n, \sigma_n)| \preceq \lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_n)| + \lim |\vartheta(\varsigma_n, \sigma_n)|$$

$$\Rightarrow \lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_n)| = 0. \tag{2. 2}$$

This proves that $\sim_{\bar{B}}$ is transitive and hence an equivalence relation.

Let $\bar{S} = \bar{A} \cup \bar{B}$.Then $\backsim = \sim_{\bar{A}} \cup \sim_{\bar{B}}$ is an equivalence relation on $\bar{S}$. Let S^* denote the $\bar{S}/\sim$. If $(\varrho_n) \in \bar{B}$,$[(\varrho_n)]$ denotes the equivalence class in S^* containing the sequence (ϱ_n). If $\varrho \in S$ let (ϱ) be the constant sequence (ϱ_n) where $\varrho_n = \varrho$, $\forall n$ and $\hat{\varrho} = [(\varrho)]$ the equivalence class containing (ϱ).

For cauchy sequence (ϱ_n) and (ς_n) in $\bar{B}$,

$$\lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_{n+m})| = 0 \quad \text{and} \quad \lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_{n+m})| = 0.$$

Consider

$$\vartheta(\varrho_n, \varsigma_n) \leq \vartheta(\varrho_n, \varrho_{n+m}) + \vartheta(\varrho_{n+m}, \varsigma_{n+m}) + \vartheta(\varsigma_{n+m}, \varsigma_n)$$

$$|\vartheta(\varrho_n, \varsigma_n) - d(\varrho_{n+m}, \varsigma_{n+m})| \leq |\vartheta(\varrho_n, \varsigma_m)| + |\vartheta(\varsigma_n, \varsigma_m)|.$$

Taking limit implies that

$$\lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_n) - \vartheta(\varrho_{n+m}, \varsigma_{n+m})| = 0 \tag{2. 3}$$

proving that $\vartheta(\varrho_n, \varsigma_n)$ is a Cauchy sequence of complex numbers. By the completeness of $\mathbb{C}$ this sequence converges.

The definition of $\sim_{\bar{B}}$ makes it obvious that $\lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_n)|$ is independent of the choice of the representative sequences(ϱ_n), (ς_n) respectively, from the classes $[(\varrho_n)]$,$[(\varsigma_n)]$.

We can prove similarly if $\varrho \in S$ and $(\varsigma_n) \in \bar{B}$, $(\sigma_n) \in \bar{B}$,$\lim \vartheta(\varrho, \varsigma_n)$ or $\lim \vartheta(\varrho, \sigma_n)$ exist or equal. Provided (ς_n) and (σ_n) belong to the same equivalence class.

We define $\vartheta^* : S \times S \to \mathbb{C}$ as follows.

$\vartheta^*(\varrho^*, \varsigma^*) = \vartheta^*([(\varrho_n)], [(\varsigma_n)]) = \vartheta(\varrho, \varsigma)$ if $(\varrho_n), (\varsigma_n) \in \bar{A}$ and ϱ and ς respectively the ultimate constants term of (ϱ_n) (ς_n)

$\vartheta^*(\varrho^*, \varsigma^*) = \vartheta^*([(\varrho_n)], [(\varsigma_n)]) = \lim_{n\to\infty} \vartheta(\varrho, \varsigma_n)$ if $(\varrho_n) \in \bar{A}$, $(\varsigma_n) \in \bar{B}$ and $\varrho_n = \varrho$ eventually.

if $(\varrho_n) \in \bar{B}$, $(\varsigma_n) \in \bar{A}$, then define $\vartheta^*(\varrho^*, \varsigma^*) = \vartheta^*([(\varrho_n)], [(\varsigma_n)])$=$\vartheta^*([(T_n)], [(S_n)])$.

If $(\varrho_n) \in \bar{B}$, $(\varsigma_n) \in \bar{B}$ then define $\vartheta^*(\varrho^*, \varsigma^*) = \vartheta^*([(\varrho_n)], [(\varsigma_n)])$=$\lim_{n\to\infty} \vartheta(\varrho_n, \varsigma_n)$

Verification that ϑ^* Is a d-Metric on S^*. Clearly $\vartheta^*(\varrho^*, \varsigma^*) \geq 0$ and $\vartheta^*(\varrho^*, \varsigma^*) = \vartheta^*(\varsigma^*, \varrho^*)$ for $\varrho^*, \varsigma^* \in S^*$. Suppose $\vartheta^*(\varrho^*, \varsigma^*) = 0$. Let $(\varrho_n) \in \varrho^*$ and $(\varsigma_n) \in \varsigma^*$. We first see that (ϱ_n), (ς_n) either are both in $\bar{A}$ or are both in $\bar{B}$. Suppose, on the contrary, $(\varrho_n) \in \bar{A}$ and $(\varsigma_n) \in \bar{B}$. Let ϱ be the ultimately constant value of (ϱ_n). Consider

$$0 \precsim \vartheta(\varrho, \varrho) \preceq \vartheta(\varrho, \varsigma) + \vartheta(\varsigma, \varrho) = 2\vartheta(\varrho, \varsigma) \quad \forall n$$

$$\Rightarrow 0 = \vartheta^*(\varrho^*, \varsigma^*) = lim_{n\to\infty} |\vartheta(\varrho, \varsigma_n)|. \tag{2. 4}$$

Hence $0 \precsim \vartheta(\varrho, \varsigma) \precsim lim_{n\to\infty} |\vartheta(\varrho, \varsigma_n)| = 0$, contrary to the fact that $\varrho \in A$. Suppose ϱ^*,$\varsigma^* \in A$, $(\varrho_n) \in \varrho^*$, and $(\varsigma_n) \in \varsigma^*$ with ϱ, ς the ultimately constant values of (ϱ_n) and

(ς_n), respectively. Then $\vartheta^*(\varrho^*, \varsigma^*) = 0 \Rightarrow \vartheta(\varrho, \varsigma) = 0 \Rightarrow \varrho = \varsigma \Rightarrow (\varrho_n) \sim (\varsigma_n) \Rightarrow \varrho^* = \varsigma^*$.

Suppose $\varrho^*, \varsigma^* \in \bar{B}$, $(\varrho_n) \in \varrho^*$ and $(\varsigma_n) \in \varsigma^*$. Consider
$\vartheta^*(\varrho^*, \varsigma^*) = 0 \Rightarrow lim_{n\to\infty}\vartheta(\varrho_n, \varsigma_n) = 0$
$\Rightarrow (\varrho_n) \sim (\varsigma_n)$
$\Rightarrow \varrho^* = \varsigma^*$.
Since

$$\vartheta^*(\varrho^*, \varsigma^*) = \lim |\vartheta(\varrho_n, \varsigma_n)|$$

Consider

$$\vartheta(_n, \varsigma_n) \preceq \vartheta(\varrho_n, \sigma_n) + \vartheta(\sigma_n, \varsigma_n)$$

Taking limit implies that

$$\lim_{n\to\infty} |\vartheta(\varrho_n, \varsigma_n)| \preceq \lim_{n\to\infty} |\vartheta(\varrho_n, \sigma_n)| + \lim_{n\to\infty} |\vartheta(\sigma_n, \varsigma_n)|$$

$$\Rightarrow \vartheta^*(\varrho^*, \varsigma^*) \preceq \vartheta^*(\varrho^*, \sigma^*) + \vartheta^*(\sigma^*, \varsigma^*). \tag{2. 5}$$

So (S^*, ϑ^*) is a complex valued dislocated metric. Embedding of S in S^*. Define $f : S \to S*$ by $f(\varrho) = \hat{\varrho}$. It is clear that f is an isodistance.We now verify that $f(\varrho)$ is dense in S^*. Let $[(\varrho_n)] \in S^*$

Case (i) $(\varrho_n) \in \bar{A}$. In this case let ϱ be the ultimately constant value of (ϱ_n). Then by the definition of f, $\hat{\varrho} = [(\varrho_n)] \in f(x)$. Then $\hat{\varrho} = [(\varrho_n)]$. Thus $[(\varrho_n] \in f(\varrho)$ in this case.

Case (ii) $((\varrho_n) \in \bar{B}$ such that $\lim_{n\to\infty} |\vartheta(\varrho_n, \varrho_{n+m})| = 0$. Then since $\varrho \in B$,$\vartheta(\varrho, \varrho) = 0$

$$\vartheta^*([(\varrho)], \hat{\varrho}) = \lim_{n\to\infty} |\vartheta(\varrho_n, \varrho)| = 0. \tag{2. 6}$$

Hence $f(S)$ is dense in S^*.(S^*, ϑ^*) is Complete. let $\varrho_n \in x^*$ such that
$\vartheta^*(\varrho_{n+m}, \varrho^*) = \vartheta(\varrho_{n+m}, \varrho_n) \Rightarrow \lim_{n\to\infty} \vartheta^*(\varrho_{n+m}, \varrho_n) = 0$
Let ϱ_n^* be cauchy sequence.i.e

$$\lim_{n\to\infty} |\vartheta^*(\varrho_n^*, \varrho_{n+m}^*)| = 0$$

$$|\vartheta^*(\varrho_n^*, \varrho^*)| \preceq |\vartheta^*(\varrho_n^*, \varrho_{n+m})| + |\vartheta^*(\varrho_{n+m}, \varrho^*)|$$

Taking limit implies that

$$\Rightarrow \lim_{n\to\infty} |\vartheta^*(\varrho_n^*, \varrho^*)| = 0 \tag{2. 7}$$

This implies that (ϱ_n^*) converges to ϱ^* proving that (S^*, ϑ^*) is complete. □

Definition 2.2. *Let* (S, ϑ) *and* (S_1, ϑ_1) *be complex valued metric spaces.* (S_1, ϑ_1) *is said to be a completion of* (S_1, ϑ_1) *if (i)* (S_1, ϑ_1) *is complete; (ii) there is an isodistance* f*:*$(S, \vartheta) \to (S_1, \vartheta_1)$ *such that* $f(\varrho)$ *is dense in* S_1 *.*

Theorem 2.3. *The completion* (S_1, ϑ_1) *of a complex valued* d*-metric space* (S, ϑ) *is unique with respect to isometry under denseness.*

Proof. Consider $f_1 : (S, \vartheta) \to (S_1, \vartheta_1)$, $f_2 : (S, \vartheta) \to (S_2, \vartheta_2)$, and $f : (S_1, \vartheta_1) \to (S_2, \vartheta_2)$
Definition of f. If $\varrho \in S_1$ and ϱ is a point of S_1 such that $\vartheta(\varrho, \varrho) \neq 0$, then $f_1^{-1}(\varrho)$ is a point of S whose self distance is non-zero; hence $f_2(f_1^{-1}(\varrho)$ is a point of S_2 whose self distance is also non-zero.
Define $f(\varrho) = f_2(f_1^{1}(\varrho))$. If $\varrho \in S_1$ is a point whose self distance is zero then, there exists a sequence (z_n) in S such that $\{f_1 z_n\}$ converges to ϱ in (S_1, ϑ_1).
Since f_1 is an isodistance and $\{f_1 z_n\}$ is convergent and hence a Cauchy sequence, it follows that $\{z_n\}$ is a Cauchy sequence in S. Since f_2 is an isodistance and $\{z_n\}$ is a Cauchy sequence, it follows that $\{f_2 z_n\}$ is a Cauchy sequence in (S_2, ϑ_2). Since (S_2, ϑ_2) is complete, there exists $z \in S_2$ such that $\lim |\vartheta_2(f_2 z_n, z)| = 0$. Clearly this z is independent of the choice of the sequence $\{z_n\}$ in S.
Define $f(\varrho) = z$. Clearly $ff_1 = f_2$ and bijection.
f is an Isodistance. If $\varrho, \varsigma \in S$, $f(f_1(\varrho)) = f_2(\varrho)$ and $f(f_1(\varsigma)) = f_2(\varsigma)$.
So $\vartheta_2(f(f_1(\varrho)), f(f_1(\varsigma))) = \vartheta_2((f_2(\varrho)), f_2(\varsigma)) = \vartheta_2(\varrho, \varsigma) = \vartheta_1((f_1(\varrho)), f_1(\varsigma))$.

If $\varrho, \varsigma \in S_1 - S$ and $\varrho = \lim f_1 \varrho_n$, $\varsigma = \lim f_1 \varsigma_n$ where $\varrho_n, \varsigma_m \in S$, then

$$\begin{aligned}
\vartheta_2(f\varrho, f\varsigma) &= \vartheta_2(\lim f_2\varrho_n, \lim f_2\varsigma_n) && (2.\ 8)\\
&= \lim \vartheta_2(f_2\varrho_n, f_2\varsigma_n) && (2.\ 9)\\
&= \lim \vartheta(\varrho_n, \varsigma_n) && (2.\ 10)\\
&= \vartheta_1(\lim f_1\varrho_n, \lim f_1\varsigma_n) && (2.\ 11)\\
&= \vartheta_1(\varrho, \varsigma) && (2.\ 12)
\end{aligned}$$

The arguments for the cases when $\varrho \in S_1 - S$ and $\varsigma \in S$ or $\varrho \in S$ and $\varsigma \in S_1 - S$ are similar. Hence f is an isodistance. Interchanging the places of S_1 and S_2 , we get in a similar way an isodistance $g : S_2 \to S_1$ such that $gf_2 = f_1$. Since $gf_2 = f_1$ and $ff_1 = f_2$, we have $fgf_2 = ff_1$ and $gff_1 = gf_2 = f_1$

Since $f(\varrho)$ is dense in S_1 and $f_2(\varrho) \in S_2$, we get $fg =$ identity on S_1 and gf is identity on S_2 . Hence g and f are bijections. □

3. Discussion

We used the classical technique of equivalence classes of Cauchy sequences to prove the completion of complex-valued dislocated metric spaces in this paper. We provide the uniqueness of completion of dislocated type metric space. It is still a question that a dislocated b-metric space has a completion?

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REFERENCES

[1] T. Abdeljawad, *Completion of cone metric spaces*, Hacettepe Journal of Mathematics and Statistics, **39**(2010) 67-74.

[2] T.V. An and N.V. Dung, *Answers to Kirk Shahzads questions on strong b-metric spaces*, Taiwanese J. Math. 20 (2016) 11751184. doi:10.11650/tjm.20.201

[3] A. Andrikopoulos, *A solution to the completion problem for quasi-pseudometric spaces*, International Journal of Mathematics and Mathematical Sciences, 2013.

[4] W.M. Alfaqih, M. Imdad and F. Rouzkard, *Unified common fixed point theorems in complex valued metric spaces via an implicit relation with applications*, Boletim da Sociedade Paranaense de Matemtica, **38,** No.4,(2020) 9-29.

[5] M. A. Alghamdi, Hussain, N., and Salimi, P. *Fixed point and coupled fixed point theorems on b-metric-like spaces*, Journal of Inequalities and Applications, **402**(2013).

[6] A.H. Ansari, O. Ege and S. Radenovic, *Some fixed point results on complex valued G_b-metric spaces*, Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales, Seria A. Matematicas, **112,** No.2 (2018) 463-472.

[7] A. Azam, B. Fisher and M. Khan, *Common fixed point theorems in complex valued metric spaces*, Numerical Functional Analysis and Optimization, **32**(2011) 243-253.

[8] I. Beg, M. Tahir, and F. Rashied, *Completion of complex valued strong b-metric spaces*, Discussiones Mathematicae: Differential Inclusions, Control & Optimization, **40**(2020).

[9] S. Chandok and D. Kumar, *Some common fixed point results for rational type contraction mappings in complex valued metric spaces*. Journal of Operators, **2013** (2013).

[10] N. V. Dung, *On the completion of partial metric spaces*, Quaestiones Mathematicae, **40**(2017) 589-597.

[11] V. Dung, and v. L. Hang, *On the completion of b-metric spaces*,Bulletin of the Australian Mathematical Society,98, No. 2 (2018) 298-304.

[12] A. Eskandar, H. Aydi, M. Arshad, and M. D. l. Sen. *Hybrid iri type graphic Y, Λ-contraction mappings with applications to electric circuit and fractional differential equations*, Symmetry, **12,** No. 3 (2020) 467.

[13] O. Ege, *Complex valued rectangular b-metric spaces and an application to linear equations*, Journal of Nonlinear Science and Applications, **8,** No. 6 (2015) 1014-1021.

[14] O. Ege, *Complex valued G_b-metric spaces*, Journal of Computational Analysis and Applica- tions, **21,** No.2 (2016) 363-368.

[15] O. Ege, *Some fixed point theorems in complex valued G_b-metric spaces*, Journal of Nonlinear and Convex Analysis, **18,** No. 11 (2017) 1997-2005.

[16] O. Ege and I. Karaca, *Common fixed point results on complex valued G_b-metric spaces*, Thai Journal of Mathematics, **16,** No. 3 (2018) 775-787.

[17] O. Ege, C. Park, A.H. Ansari, *A diferent approach to complex valued G_b-metric spaces*, Ad- vances in Difference Equations, 2020:**152** (2020) 1-13 https://doi.org/10.1186/s13662-020-02605-0 (2020).

[18] O. Ege, and I. Karaca, *Complex valued dislocated metric spaces*,The Korean Journal of Mathematics,**26**(2018) 809-822.

[19] X. Ge and S. Lin, *Completions of partial metric spaces*, Topology and its Applications, **182**(2015), 16-23.

[20] D. Hasanah, and I. Supeno, *The existence and uniqueness of completion of complex valued metric spaces*, E-Jurnal Matematika,**7,** No. 2 (2018) 187-193.

[21] Y. J. Jeon, and I. K. Chang.*Lipschitz mapping in metric like space*, Journal of the Chungcheong Mathematical Society, *32*, No. 4 (2019) 393-400.

[22] P. S. Kumari, S. Ramabhadra, I., J. M. Rao, and D. Panthi, *Completion of a dislocated metric space*. In Abstract and Applied Analysis (2015).

[23] P. S. Kumari, C. V. Ramana, K. Zoto, and D. Panthi, *Fixed point theorems and generalizations of dislocated metric spaces*, Indian Journal of Science and Technology,**8,** No. 3 (2015) 154-158.

[24] S. G. Matthews, *Partial metric topology*, Annals of the New York Academy of Sciences-Paper Edition, **728** 183-197(1994).

[25] M, Nabil. *Double controlled metric-like spaces*, Journal of Inequalities and Applications, **2020,** 189 (2020): 1-12.

[26] K. P. R. Rao, P. R. Swamy, and J. R. Prasad, *A common fixed point theorem in complex valued b-metric spaces*,Bulletin of Mathematics and Statistics Research,**1**, No. 1 (2013) 1-8.

[27] M. U. Rahman, and M. Sarwar, *Fixed point theorems in generalized types of b-dislocated metric spaces*, Elec. J. Math. Anal. Appl,**5**, No.2 (2017) 10-17

[28] W. Sintunavarat, Y.J. Cho and P. Kumam, *Urysohn integral equations approach by common fixed points in complex-valued metric spaces*, Advances in Difference Equations, **2013,** No. 1 (2013):49.

[29] W. Sintunavarat, M.B. Zada and M. Sarwar. *Common solution of urysohn integral equations with the help of common fixed point results in complex valued metric spaces*, Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas, **111**, No. 2 (2017): 531545.