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A Study of Third-order KdV and mKdV Equations by Laplace Decomposition Method

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Abstract.: In this article, the Laplace decomposition method is implemented to solve nonlinear partial differential equations. Third-order KdV and mKdV equations with initial conditions have been considered to check the validity of the proposed method. Results obtained by this method are compared with the exact solutions in literature numerically as well as graphically and are found to be in good agreement with each other. The proposed method finds the solutions without any discretization, perturbation, linearization, or restrictive assumptions. Obtained results show that the LDM is highly accurate and easy to apply for NLPDEs in various fields.

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Key Words: Laplace Decomposition Method, KdV and mKdV equations, Adomian polynomial, nonlinear partial differential equation.

1. Introduction

Due to the wide applicability of NLPDEs in various fields of Mathematics, Engineering, and Physics, NLPDEs have achieved more surveillance by many researchers. NLPDEs play a vital role in describing many important physical phenomena. These phenomena include shallow-water waves, finance, biology, chemistry, plasma physics, elastic rods, anharmonic lattices, DNA excitation, matter waves in Bose-Einstein condensation, ultrapulses in nonlinear optics, etc. Therefore, the solution of NLPDEs plays a significant role in understanding different physical phenomena. To find the solution to NLPDEs, many effective methods have been used such as the Tanh method [15], Lie symmetry analysis [16], Inverse scattering method [27], Hirota bilinear method [7], Bcklund transformation [23], Darboux transformation method [5], Differential transformation method [2], ADM

[22], LDM [10,6] and so on.

In the present paper, LDM is used to find the solutions of KdV and mKdV equations with different initial conditions. LDM was introduced by Khuri to solve nonlinear differential equations [13]. Further, this method was modified by Khan [12]. Further, the LDM was combined with VIM, HPM, and ADM to build a powerful technique for handling many nonlinear problems. H. Jafari found the solution of linear and nonlinear fractional diffusion-wave equations by using LDM [10]. E. Yusufolu used LDM to solve Duffing equation [26]. Hosseinzadeh and Jafari applied LDM for the solution of the Klein-Gordon equation [8]. E. Akgl and Co-authors used MLDM and Sumudu transform to investigate the financial models based on market equilibrium and option pricing [1]. In [24] M. Yavuz and co-authors obtained a numerical solution of fractional order Schrdinger-KdV equation with Mittag-Leffler nonsingular kernel by using MLDM. M. Yavuz and N. Sene combine the ρ-Laplace homotopy perturbation transform method with HBIM to obtain the solution of the incompressible second-grade fluid model [25].

The KdV equation is a nonlinear partial differential equation that was derived by Diederik Korteweg and G. de Vries in 1895, which depicts waves on shallow water surfaces [14]. In various branches of Mathematics, Engineering, and Physics, the KdV equation plays an elementary role. KdV equation has been used to describe various physical phenomena like a magnetohydrodynamic wave in a warm plasma, fluid dynamics, aerodynamics, continuum mechanics as a model for shock wave formation, turbulence, mass transport, acoustic waves in an anharmonic crystal, surface gravity waves, nonlinear acoustic of bubbly liquids, magma flow and conduit waves, and cosmology, etc. KdV equation possesses many important properties such as infinitely many conservative laws, bi-Hamiltonian structures, symmetries, equations, and Lax pair. Many researchers investigate different aspects of the KdV equation. G. Wang and Xu. T. studied symmetric properties of the time-fractional KdV equation by using the Lie group analysis method [17]. Recently G. Wang and A. Kara introduced (2+1) dimensional KdV and mKdV equation and also studied its symmetries, group invariant solutions, and conservative laws [18]. Along with some traveling wave solutions, bifurcation and phase portrait of (2+1)-dimensional KdV equation are investigated by A. Elmandouha and A. Ibrahim [4]. In [19] authors studied the generalized double dispersion Boussinesq equation by reducing it to the KdV equation. Lie point symmetries and conservation laws of the time-fractional nonlinear dispersive equation are constructed by G. Wang and Co-authors [20]. In [21] the generalized fifth-order KdV equation is investigated by using group methods also conservative laws and some soliton solutions are obtained by G. Wang and co-authors.

This paper is organized into five sections. In section 2, LDM is explained. Section 3 contain applications, graphical presentation and numerical illustration are given in section 4. Section 5 is about the conclusion.

2. LAPLACE DECOMPOSITION METHOD

Consider the homogeneous NLPDE in general form

$$Lu(x,t) + Ru(x,t) + Nu(x,t) = 0\ , \tag{2. 1}$$

with initial condition

$$u\left(x,0\right) = f\left(x\right),$$

where $L = \frac{\partial}{\partial t}$, R is general linear operator Nu is nonlinear term.
Taking Laplace transform of (1) w.r.t.t

$$\begin{aligned} su\left(x,s\right) - u\left(x,0\right) &= -\mathcal{L}_t\left\{Ru\left(x,t\right) + Nu\left(x,t\right)\right\}, \\ u\left(x,s\right) &= \frac{f\left(x\right)}{s} - \frac{1}{s}\mathcal{L}_t\left\{Ru\left(x,t\right) + Nu\left(x,t\right)\right\}. \end{aligned}$$

Taking inverse Laplace transform,

$$u\left(x,t\right) = f\left(x\right) - \mathcal{L}_t^{-1}\left\{\frac{1}{s}\mathcal{L}_t Ru\left(x,t\right) + Nu\left(x,t\right)\right\}. \tag{2. 2}$$

Represent solution of the equation (2.1) in an infinite series form,

$$u(x,t) = \sum_{n=0}^{\infty} u_n\left(x,t\right). \tag{2. 3}$$

A nonlinear term is represented by an infinite series of the so-called Adomian polynomials

$$Nu = \sum_{n=0}^{\infty} \mathcal{A}_n\ . \tag{2. 4}$$

The Adomian polynomials, $\mathcal{A}_n$'s are generated with the help of following relation

$$\mathcal{A}_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty}\lambda^k u_k\right)\right]_{\lambda=0}\ ,\quad n \geq 0. \tag{2. 5}$$

From equation (2.2) to (2.4),

$$\sum_{n=0}^{\infty} u_n\left(x,t\right) = f\left(x\right) - \mathcal{L}_t^{-1}\left\{\frac{1}{s}\mathcal{L}_t\left\{R\sum_{n=0}^{\infty} u_n\left(x,t\right) + \sum_{n=0}^{\infty}\mathcal{A}_n\right\}\right\}, \tag{2. 6}$$

where components of the series are usually determined recursively by

$$\begin{aligned} u_0\left(x,t\right) &= f\left(x\right), \\ u_{n+1}\left(x,t\right) &= \mathcal{L}_t^{-1}\left\{\frac{1}{s}\mathcal{L}_t\left\{R\left(u_n\right) + \mathcal{A}_n\right\}\right\},\quad n = 0,1,2,3,\ldots\ . \end{aligned}$$

N term approximate solution is given by

$$\phi_N(x,t) = \sum_{n=0}^{N-1} u_n\left(x,t\right),\qquad where\quad N \geq 1.$$

And exact solution is

$$u\left(x,t\right) = \lim_{N\to\infty}\phi_N\left(x,t\right).$$

3. Application of LDM

Example 1: Consider third order KdV equation, [11]

$$u_t - 6uu_x + u_{xxx} = 0\ , \tag{3.1}$$

with initial condition

$$u\left(x,0\right) = -2\frac{k^2 e^{kx}}{\left(1+e^{kx}\right)^2},$$

where $u(x,t)$ is displacement,uu_x and u_{xxx} represent distortion and dispersion of the wave respectively.
Taking Laplace transform of equation (3.7),

$$\begin{aligned} su\left(x,s\right) - u\left(x,0\right) &= L_t\left\{6uu_x - u_{xxx}\right\}, \\ u\left(x,s\right) &= \frac{1}{s}u\left(x,0\right) + \frac{1}{s}L_t\left\{6uu_x - u_{xxx}\right\}, \\ u\left(x,s\right) &= -\frac{2}{s}\frac{k^2 e^{kx}}{\left(1+e^{kx}\right)^2} + \frac{1}{s}L_t\left\{6uu_x - u_{xxx}\right\}. \end{aligned}$$

Taking inverse Laplace transform,

$$u\left(x,t\right) = -2\frac{k^2 e^{kx}}{\left(1+e^{kx}\right)^2} + L_t^{-1}\left\{\frac{1}{s}L_t\left\{6uu_x - u_{xxx}\right\}\right\}.$$

Based on the LDM algorithm, we get

$$\sum_{n=0}^{\infty} u_n\left(x,t\right) = -2\frac{k^2 e^{kx}}{\left(1+e^{kx}\right)^2} + L_t^{-1}\left\{\frac{1}{s}L_t\left\{6\sum_{n=0}^{\infty}\mathcal{A}_n - \sum_{n=0}^{\infty}u_{nxxx}\right\}\right\}, n \geq 0. \tag{3.2}$$

Which yields the following recurrence relation

$$\begin{aligned} u_0\left(x,t\right) &= -2\frac{k^2 e^{kx}}{\left(1+e^{kx}\right)^2}\ , \\ u_{n+1}\left(x,t\right) &= L_t^{-1}\left\{\frac{1}{s}L_t\left\{6\mathcal{A}_n - u_{nxxx}\right\}\right\}, \qquad n = 0,1,2,3,... \end{aligned} \tag{3.3}$$

Where $\mathcal{A}'_n s$ are Adomian polynomials, few Adomian polynomials are:

$$\begin{aligned} \mathcal{A}_0 &= u_0 u_{0x}, \\ \mathcal{A}_1 &= u_0 u_{1x} + u_1 u_{0x}, \\ \mathcal{A}_2 &= u_0 u_{2x} + u_1 u_{1x} + u_2 u_{0x}, \\ \mathcal{A}_3 &= u_0 u_{3x} + u_1 u_{2x} + u_2 u_{1x} + u_3 u_{0x} \quad ......\ so\ on. \end{aligned}$$

By using the recurrence relation (3.3), the first few components of the series solution are

$$\begin{aligned}
u_1(x,t) &= L_t^{-1}\left\{\frac{1}{s}L_t\left\{6\mathcal{A}_0 - u_{0xxx}\right\}\right\},\\
u_1(x,t) &= \frac{-2k^5e^kx(e^kx-1)}{(1+e^kx)^3}t,\\
u_2(x,t) &= L_t^{-1}\left\{\frac{1}{s}L_t\left\{6\mathcal{A}_1 - u_{1xxx}\right\}\right\},\\
u_2(x,t) &= \frac{-2k^8e^kx(e^2kx-4e^kx+1)}{(1+e^kx)^4}\frac{t^2}{2!},\\
u_3(x,t) &= L_t^{-1}\left\{\frac{1}{s}L_t\left\{6\mathcal{A}_2 - u_{2xxx}\right\}\right\},\\
u_3(x,t) &= \frac{-2k^11e^kx(e^3kx-11e^2kx+11e^kx-1)}{(1+e^kx)^5}\frac{t^3}{3!},
\end{aligned}$$

In the same manner, the rest components of the series of solution can be easily obtained. Then the solution in series form is

$$\begin{aligned}
u(x,t) = -2\frac{k^2e^{kx}}{(1+e^{kx})^2} - \frac{2k^5e^kx(e^kx-1)}{(1+e^kx)^3}t - \frac{2k^8e^kx(e^2kx-4e^kx+1)}{(1+e^kx)^4}\frac{t^2}{2!}\\
-\frac{2k^11e^kx(e^3kx-11e^2kx+11e^kx-1)}{(1+e^kx)^5}\frac{t^3}{3!} + \ldots
\end{aligned} \tag{3.4}$$

Closed form of the series is,

$$u(x,t) = -2\frac{k^2e^k(x-k^2t)}{(1+e^k(x-k^2t))^2}. \tag{3.5}$$

Which is the exact solution of equation (3.1). The result is verified through substitution.

Example 2: Consider the following KdV equation [3]

$$u_t + 6uu_x + u_xxx = 0, \tag{3.6}$$

with initial condition

$$u(x,0) = x.$$

The LDM leads to the following scheme:

$$\begin{aligned}
u_0(x,t) &= x,\\
u_n(x,t) &= -L_t^{-1}\left\{\frac{1}{s}L_t\left\{6\mathcal{A}_n + u_{nxxx}\right\}\right\}, \qquad n = 0,1,2,3,.
\end{aligned} \tag{3. 7}$$

By using above relation, we get the following first few components of the series of solution,

$$\begin{aligned}
u_1(x,t) &= -6xt,\\
u_2(x,t) &= 36xt^2,\\
u_3(x,t) &= -216xt^3.
\end{aligned}$$

In the same manner, rest components of the series of solution are easily obtained. Then the solution in series form is

$$\begin{aligned} u(x,t) &= x - 6xt + 36xt^2 - 216xt^3 + ... \\ u(x,t) &= x\sum_{n=0}^{\infty}(-1)^n(6t)^n. \end{aligned} \quad (3.\ 8)$$

Solution in closed form is,

$$u(x,t) = \frac{x}{1+6t}, \qquad provided \quad t \neq -\frac{1}{6}.$$

Which is the exact solution of equation (3.6) which can be verified through substitution.

Example 3: Consider modified KdV equation

$$u_t - \frac{1}{2}u^2u_x + u_{xxx} = 0, \quad (3.\ 9)$$

with initial condition

$$u(x,0) = 3tanh\left(\frac{\sqrt{3}}{2}x\right).$$

The LDM leads to the following recurrence relation:

$$\begin{aligned} u_0(x,t) &= 3tanh\left(\frac{\sqrt{3}}{2}x\right), \\ u_{n+1}(x,t) &= L_t^{-1}\left\{\frac{1}{s}L_t\left\{\frac{1}{2}\mathcal{B}_n + u_{nxxx}\right\}\right\}, \quad n = 0,1,2, \end{aligned} \quad (3.\ 10)$$

Where $\mathcal{B}_n's$ are Adomian polynomials, few Adomian polynomials are:

$$\begin{aligned} \mathcal{B}_0 &= u_0^2u_{0x}, \\ \mathcal{B}_1 &= 2u_0u_1u_0x + u_0^2u_1x, \\ \mathcal{B}_2 &= u_1^2u_0x + 2u_0u_2u_0x + 2u_0u_1u_1x + u_0^2u_2x, \\ \mathcal{B}_3 &= u_0^2u_3x + 2u_0u_1u_2x + 2u_0u_2u_1x + 2u_0u_3u_0x + 2u_1u_2u_0x + u_1^2u_1x, ... \quad so \quad on. \end{aligned}$$

Proceeding as before, the first some components of the series are

$$\begin{aligned} u_1(x,t) &= \frac{9\sqrt{3}}{2}\left(1 - tanh^2\left(\frac{\sqrt{3}}{2}x\right)\right)t, \\ u_2(x,t) &= -\frac{81}{16}\left(tanh\left(\frac{\sqrt{3}}{2}x\right) - tanh^3\left(\frac{\sqrt{3}}{2}x\right)\right)t^2, \\ u_3(x,t) &= -\frac{81\sqrt{3}}{64}\left(3tanh^4\left(\frac{\sqrt{3}}{2}x\right) - 4tanh^2\left(\frac{\sqrt{3}}{2}x\right) + 1\right)t^3. \end{aligned}$$

We can easily find the other components of the series and then solution in series form of equation (3.9) is

$$u(x,t) = 3tanh\left(\frac{\sqrt{3}}{2}x\right) + \frac{9\sqrt{3}}{2}\left(1 - tanh^2\left(\frac{\sqrt{3}}{2}x\right)\right)t$$
$$-\frac{81}{16}\left(tanh\left(\frac{\sqrt{3}}{2}x\right) - tanh^3\left(\frac{\sqrt{3}}{2}x\right)\right)t^2$$
$$-\frac{81\sqrt{3}}{64}\left(3tanh^4\left(\frac{\sqrt{3}}{2}x\right) - 4tanh^2\left(\frac{\sqrt{3}}{2}x\right) + 1\right)t^3 + ... \quad .$$

The closed form solution of equation (3.9) can be obtained as

$$u(x,t) = 3tanh\left(\frac{\sqrt{3}}{2}\left(x + \frac{3}{2}t\right)\right). \tag{3. 11}$$

This is the kink soliton solution of equation (3.9) is in full agreement with the result obtained by Tanh-Coth method [9].

4. Error estimation and Graphical Presentation

TABLE 1. Absolute error for example 1, at k = 2, N=4

x \ t	0.01	0.02	0.03	0.04
-2	1.2256E-7	1.9499E-6	9.8418E-6	3.0998E-5
-1	1.1620E-6	1.8216E-5	9.0308E-5	2.7936E-4
0	3.4102E-6	5.4415E-5	2.7424E-4	8.6130E-4
1	1.2085E-6	1.9700E-5	1.0153E-4	3.2640E-4
2	1.2255E-7	1.9620E-6	9.9319E-6	3.1361E-5

TABLE 2. Absolute error for example 2, N=21

x \ t	0.025	0.05	0.075	0.1
1	1.1102E-16	8.0464E-12	3.5983E-8	1.3710E-5
2	2.2204E-16	1.6093E-11	7.1966E-8	2.7421E-5
3	4.4409E-16	2.4140E-11	1.0795E-7	4.1132E-5
4	4.4408E-16	3.2186E-11	1.4393E-7	5.4842E-5

TABLE 3. Absolute error for example 3, N=7

x \ t	0.05	0.1	0.2	0.3
1	1.6240E-8	1.9592E-8	2.2178E-6	3.3340E-5
2	2.9372E-11	3.6548E-9	4.4221E-7	7.1464E-6
3	1.4769E-12	1.9057E-10	2.4681E-8	4.2471E-7
4	6.2972E-13	7.9462E-11	9.8957E-8	1.6456E-8

FIGURE 1. 2D graph of approximate solution for N=4 and exact solution of example 1 for $k = 2, -4 \leq x \leq 4$ at $t = -0.05, -0.1, 0, 0.05, 0.1$

FIGURE 2. Graph of approximate solution for N=4, (b) exact solution of example 1 for $k = 2, -4 \leq x \leq 4$ and$-0.1 \leq t \leq 0.1$

FIGURE 3. Graph of approximate solution of example 1 for $k = 1, 2, 3, 4att = 0$ and$-4 \leq x \leq 4$

FIGURE 4. 2D graph of LDM solution for N=21 and exact solution of example 2 at $t = -0.1, -0.05, 0, 0.05, 0.1$ for $-4 \leq x \leq 4$

FIGURE 5. Graph of LDM solution for N=21 and exact solution of example 2 at $t = -0.1, -0.05, 0, 0.05, 0.1$ for $-4 \leq x \leq 4$

The two-dimensional visualization of the Laplace decomposition method solutions and exact solutions are given in figures 1,4, and 6. Tables 1-3 represent absolute errors between approximate solutions and exact solutions. Solitary wave solution is obtained for examples

FIGURE 6. 2D graph of approximate solution for N=7 and exact solution of example 3 at t=-0.5,-0.3,-0.1,0.1,0.3,0.5 for $-4 \leq x \leq 4$

FIGURE 7. Graph of approximate solution for N=7 and exact solution of example 3 at t=-0.5,-0.3,-0.1,0.1,0.3,0.5 for $-4 \leq x \leq 4$

1 and 3, and rational solution for example 2. Figure 1 represents the negative pulse soliton solution of example 1 propagating to the right. In figure 2, graph of approximate solution for k=1,2,3 and 4 at t=0 is plotted, which shows that the depth of the soliton is $\frac{k^2}{2}$. The kink-soliton solution of example 3 which propagates towards the left is shown in figure 6.

5. CONCLUSION

LDM has been effectively implemented to obtain the solution of third-order KdV and mKdV equations. Two traveling wave solutions: negative pulse soliton and kink-soliton solution and one rational solution are obtained for test problems. The solutions obtained are in series form and the closed-form of the series gives the exact solution. All the obtained solutions are fully agreed with the solutions obtained by other methods in the literature. Two and three-dimensional graphs of approximate solutions and exact solutions are plotted which shows the accuracy of the proposed method and also shows the behaviour of the solutions. To verify the accuracy of the proposed method numerically, absolute error tables are prepared. We achieved a very good approximation with exact solutions by using few terms of the series. Errors can be made smaller by adding more terms of series. Graphical and numerical results justify the advantages of LDM. We conclude that this method is accurate, efficient, and avoids massive computation works. Thus, LDM is a very powerful and efficient technique to find the solution of a wide class of linear and NLPDEs in various fields.

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