Punjab University Journal of Mathematics
(ISSN 1016-2526)
Vol. 52(11)(2020) pp. 19-44

On Some Topological Polynomials of Dominating David Derived Graphs

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Received: 30 July, 2020 / Accepted: 18 September, 2020 / Published online: 12 November, 2020

Abstract.: In mathematical chemistry, molecular structure of any chemical substance can be expressed by a numeric number or polynomial or sequence of number which represent the whole graph is called topological index. An important branch of graph theory is the chemical graph theory. As a consequence of their worldwide uses, chemical networks have inspired researchers since their development. Determination of the expressions for topological indices of different derived graphs is a new and interesting problem in graph theory. In this article, some graphs which are derived from Honeycomb structure are studied, and found their exact results for Sum degree-based polynomials are obtained.

AMS (MOS) Subject Classification Codes: 05C09, 05C12, 05C90, 05C92
Key Words: Fifth M-Zagreb polynomials, Fifth generalized M-Zagreb polynomials and Fifth hyper M-Zagreb polynomials.

1. INTRODUCTION

The molecular structures are those in which atoms are connected by covalent bonds. In graph theory, atoms are considered as vertices and covalent bonds are as edges. Chem-informatics is a new area of research in which the subjects Chemistry, Mathematics, and Information science are combined. That is why it attain the highly attention of researchers around the world. In this paper, we are considering Dominating David Derived Networks which are derived from Honeycomb Structure. Honeycomb structures, inspired from bee honeycombs, had found widespread applications in various fields, including architecture, transportation, mechanical engineering, chemical engineering, nanofabrication, and recently biomedicine. A major challenge in this field is to understand the unique properties of honeycomb structures, which depended on their structures, scales, and the materials used [11].

A Topological index TI, sometimes also known as a graph-theoretic index, is a numerical invariant of a chemical graph [21]. There are many types of $TI's$ but most popular and authentic $TI's$ are Distance-based, Degree-based, Neighbourhood Degree-Based indices. These indices contains a lot of information within themselves.

The method of drawing Dominating David Derived graphs (dimension t) is as follows.
STEP 1:-Consider a Honeycomb graph $HC(t)$ dimension t.
STEP 2:-Split each edge into two by embedding another vertex.
STEP 3:-In each hexagon cell, connect the new vertices by an edge if they are at a distance of 4 units within the cell.
STEP 4:-Place vertices at new edge crossings.
STEP 5:-Remove initial vertices and edges of Honeycomb graph.
STEP 6:-Split each horizontal edge into two edges by inserting a new vertex. The resulting Graph is called Dominating David Derived system $DDD(t)$ of measurement t [3, 22].
The First type of Dominating David Derived graph $D_1(t)$ can be obtained by connecting vertices of degree two by an edge, which are not in the boundary.
The second type of Dominating David Derived graph $D_2(t)$ can be obtained by sub dividing once the new edge introduced in $D_1(t)$.
The Third type of Dominating David Derived graph $D_3(t)$ can be obtained from $D_1(t)$ by introducing parallel path of length 2 between the vertices of degree two which are not in the boundary. See the figure 5 for third type of dominating david derived graph of dimension 2, $D_3(2)$.

2. NOTATIONS AND PRELIMINARIES

In this article, Υ is considered a graph with a $V(\Upsilon)$ vertex set and an edge set of $E(\Upsilon)$, d_r is the degree of vertex $r \in V(\Upsilon)$. Let $S_\Upsilon(r)$ denote the sum of the degrees of all vertices adjacent to a vertex r. Graovac et al. defined fifth M-Zagreb indices as polynomials for a molecular graph [10] and these are characterized as

Let Υ be a graph. Then

$$M_1G_5(\Upsilon) = \sum_{rs \in E(\Upsilon)} (S_G(r) + S_G(s)), \tag{2. 1}$$

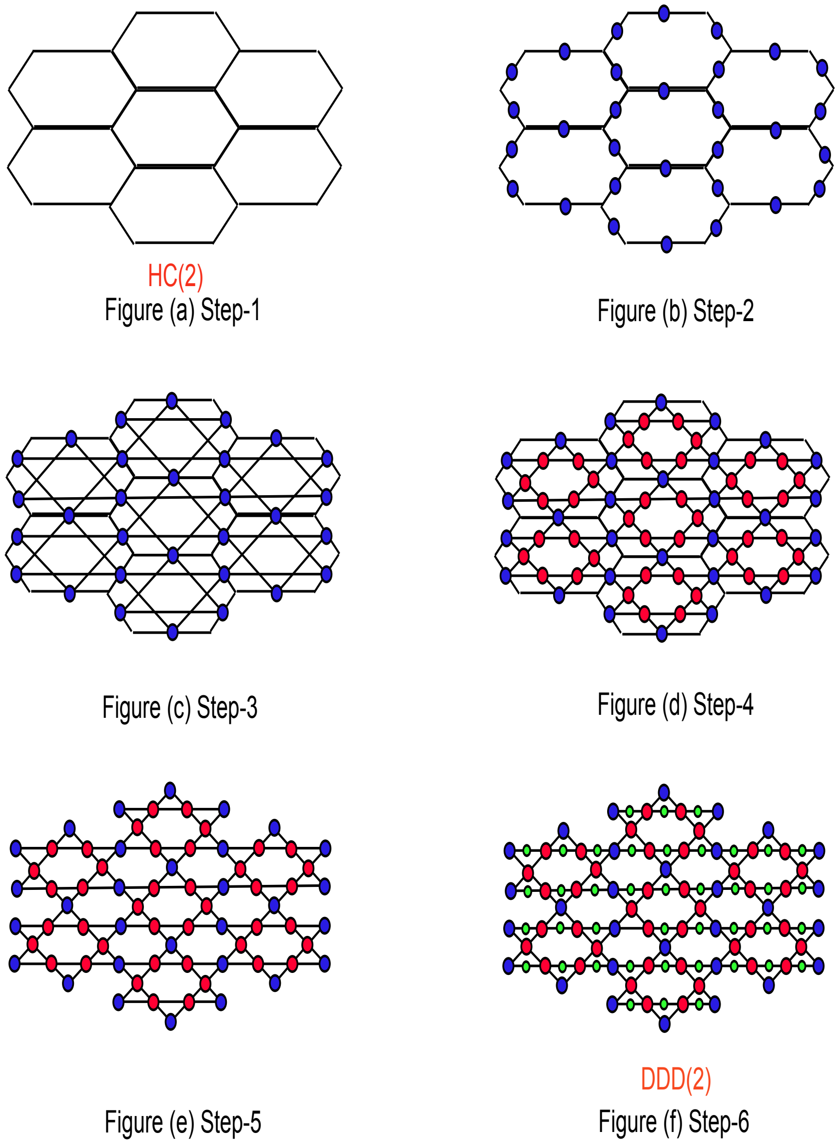

FIGURE 1. Construction Algorithm for Dominating David Derived graph $DDD(2)$.

$$M_2G_5(\Upsilon) = \sum_{rs \in E(\Upsilon)} (S_G(r) \times S_G(s)). \tag{2. 2}$$

V. R. Kulli [15] motivated by above indices and described some new topological polynomials and defined the general fifth M_1-Zagreb polynomial and the general fifth M_2-Zagreb

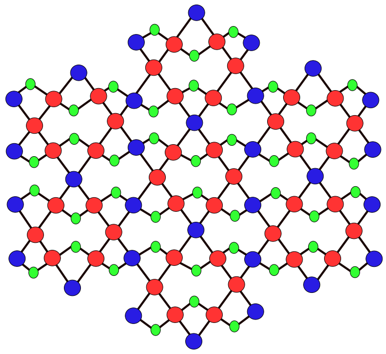

FIGURE 2. Isomorphic graph of DDD(2).

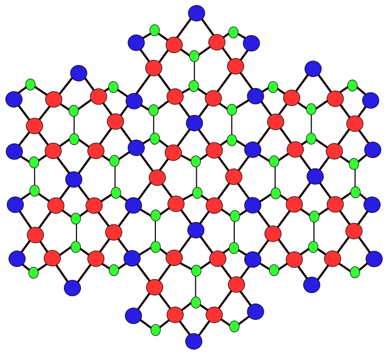

FIGURE 3. First type of Dominating David Derived graph $D_1(2)$.

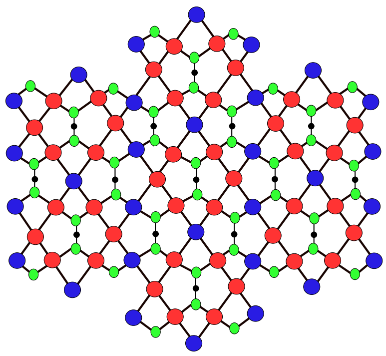

FIGURE 4. Second type of Dominating David Derived graph $D_2(2)$.

polynomial of a molecular graph Υ as:

$$M_1^a G_5(\Upsilon, x) = \sum_{rs \in E(\Upsilon)} x^{(S_G(r)+S_G(s))^a}, \tag{2. 3}$$

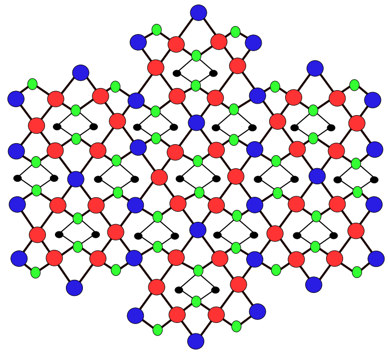

FIGURE 5. Third type of Dominating David Derived graph $D_3(2)$.

$$M_2^a G_5(\Upsilon, x) = \sum_{rs \in E(\Upsilon)} x^{(S_G(r) \times S_G(s))^a}. \quad (2.\ 4)$$

The fifth M_1 and M_2 Zagreb polynomials of a graph are defined as:

$$M_1 G_5(\Upsilon, x) = \sum_{rs \in E(\Upsilon)} x^{(S_G(r) + S_G(s))}, \quad (2.\ 5)$$

$$M_2 G_5(\Upsilon, x) = \sum_{rs \in E(\Upsilon)} x^{(S_G(r) \times S_G(s))}. \quad (2.\ 6)$$

The fifth HM_1 and HM_2 Zagreb polynomials of graph are defined as:

$$HM_1 G_5(\Upsilon, x) = \sum_{rs \in E(\Upsilon)} x^{(S_G(r) + S_G(s))^2}, \quad (2.\ 7)$$

$$HM_2 G_5(\Upsilon, x) = \sum_{rs \in E(\Upsilon)} x^{(S_G(r) \times S_G(s))^2}. \quad (2.\ 8)$$

3. RESULTS

We have study the new topological indices described by V. R. Kulli named as fifth M-Zagreb indices, fifth M-Zagreb polynomials and $M_3 - Zagreb$ index and give closed formulae of these indices for dominating david derived graphs. Haidar *et al.* studied degree based topological indices for various graphs [1]. For further study of topological indices of various graph families see, [1, 2, 3, 4, 5, 6, 8, 12, 13, 14, 16, 17, 18, 19, 23]. For the basic notations and definitions, see [7, 20, 24].

3.1. **Results for First Type of Dominating David Derived graphs.** In this section, we calculate degree-based topological indices of the dimension t for first type of Dominating David Derived graphs. In the coming theorems, we compute M-Zagreb indices and polynomials.

Corresponding the above indices, we are going to compute general fifth M-Zagreb polynomials for first type of Dominating David Derived graph $D_1(t)$.

Theorem 2.1.1. Let $\Upsilon_1 \cong D_1(t)$ be the first type of Dominating David Derived graph, then general fifth M-Zagreb polynomials of first and second type is equal to

$$\begin{aligned} M_1^a G_5(\Upsilon_1, x) &= 4tx^{12^a} + 4(t-1)x^{16^a} + 4tx^{17^a} + 2(t+1)x^{18^a} + 4(4t-3)x^{19^a} + \\ &\quad 4(t-1)x^{20^a} + 4(t-1)x^{21^a} + (9t^2-7t+3)x^{22^a} + 4tx^{23^a} + 4(t \\ &\quad -1)x^{24^a} + 4(t-1)(9t-8)x^{25^a} + 4(t-1)x^{26^a} + 8(t-1)x^{27^a} + \\ &\quad 4(t-1)x^{28^a} + 4(t-1)x^{29^a} + 4(t-1)(9t-10)x^{30^a}, \end{aligned}$$

$$\begin{aligned} M_2^a G_5(\Upsilon_1, x) &= 4tx^{36^a} + 4(t-1)x^{63^a} + 4tx^{66^a} + 4x^{72^a} + 2(t-1)x^{81^a} + 8(t-1) \\ &\quad x^{84^a} + 4(3t-2)x^{88^a} + 4(t-1)x^{104^a} + (9t^2-7t+3)x^{154^a} + 4(t \\ &\quad -1)x^{126^a} + 4x^{132^a} + 4(t-1)x^{143^a} + 4(t-1)(9t-8)x^{154^a} + 4(t \\ &\quad -1)x^{168^a} + 4(t-1)x^{176^a} + 4(t-1)x^{182^a} + 4(t-1)x^{196^a} + 4(t- \\ &\quad 1)(9t-10)x^{224^a}. \end{aligned}$$

Proof. We get the outcome with the edge partition in Table 1.
It follows from (1.3),

(S_r, S_s) where $rs \in E(\Upsilon_1)$	Number of edges	(S_r, S_s) where $rs \in E(\Upsilon_1)$	Number of edges
$(6,6)$	$4t$	$(11,11)$	$9t^2-7t+3$
$(6,11)$	$4t$	$(11,12)$	4
$(6,12)$	4	$(11,13)$	$4t-4$
$(6,14)$	$4t-4$	$(11,14)$	$36t^2-68t+32$
$(7,9)$	$4t-4$	$(11,16)$	$4t-4$
$(7,12)$	$4t-4$	$(12,14)$	$4t-4$
$(8,11)$	$12t-8$	$(13,14)$	$4t-4$
$(8,13)$	$4t-4$	$(13,16)$	$4t-4$
$(9,9)$	$2t-2$	$(14,14)$	$4t-4$
$(9,14)$	$4t-4$	$(14,16)$	$36t^2-76t+40$

TABLE 1. Edge partition of first type of Dominating David Derived graph ($D_1(t)$) based on sum of degrees of end vertices of each edge.

$$M_1^a G_5(\Upsilon_1, x) = \sum_{rs \in E(\Upsilon_1)} x^{(S_G(r)+S_G(s))^a}.$$

$$\begin{aligned} M_1^aG_5(\Upsilon_1,x) &= x^{(6+6)^a}|E_1(\Upsilon_1(t))| + x^{(6+11)^a}|E_2(\Upsilon_1(t))| + x^{(6+12)^a}|E_3(\Upsilon_1(t))| \\ &+x^{(6+14)^a}|E_4(\Upsilon_1(t))| + x^{(7+9)^a}|E_5(\Upsilon_1(t))| + x^{(7+12)^a}|E_6(\Upsilon_1(t)) \\ &| + x^{(8+11)^a}|E_7(\Upsilon_1(t))| + x^{(8+13)^a}|E_8(\Upsilon_1(t))| + x^{(9+9)^a}|E_9(\Upsilon_1(t \\ &))| + x^{(9+14)^a}|E_{10}(\Upsilon_1(t))| + x^{(11+11)^a}|E_{11}(\Upsilon_1(t))| + x^{(11+12)^a}| \\ &E_{12}(\Upsilon_1(t))| + x^{(11+13)^a}|E_{13}(\Upsilon_1(t))| + x^{(11+14)^a}|E_{14}(\Upsilon_1(t))| + \\ &x^{(11+16)^a}|E_{15}(\Upsilon_1(t))| + x^{(12+14)^a}|E_{16}(\Upsilon_1(t))| + x^{(13+14)^a}|E_{17}(\\ &\Upsilon_1(t))| + x^{(13+16)^a}|E_{18}(\Upsilon_1(t))| + x^{(14+14)^a}|E_{19}(\Upsilon_1(t))| + \\ &x^{(14+16)^a}|E_{20}(\Upsilon_1(t))|, \\ &= x^{(6+6)^a}(4t) + x^{(6+11)^a}(4t) + x^{(6+12)^a}(4) + x^{(6+14)^a}(4t-4) + \\ &x^{(7+9)^a}(4t-4) + x^{(7+12)^a}(4t-4) + x^{(8+11)^a}(12t-8) + x^{(8+13)^a} \\ &(4t-4) + x^{(9+9)^a}(2t-2) + x^{(9+14)^a}(4t-4) + x^{(11+11)^a}(9t^2-7 \\ &t+3) + x^{(11+12)^a}(4) + x^{(11+13)^a}(4t-4) + x^{(11+14)^a}(36t^2-68t \\ &+32) + x^{(11+16)^a}(4t-4) + x^{(12+14)^a}(4t-4) + x^{(13+14)^a}(4t-4) \\ &+x^{(13+16)^a}(4t-4) + x^{(14+14)^a}(4t-4) + x^{(14+16)^a}(36t^2-76t+ \\ &40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow M_1^aG_5(\Upsilon_1,x) &= 4tx^{12^a} + 4(t-1)x^{16^a} + 4tx^{17^a} + 2(t+1)x^{18^a} + 4(4t-3) \\ &x^{19^a} + 4(t-1)x^{20^a} + 4(t-1)x^{21^a} + (9t^2-7t+3)x^{22^a} + 4t \\ &x^{23^a} + 4(t-1)x^{24^a} + 4(t-1)(9t-8)x^{25^a} + 4(t-1)x^{26^a} \\ &+8(t-1)x^{27^a} + 4(t-1)x^{28^a} + 4(t-1)x^{29^a} + 4(t-1)(9t- \\ &10)x^{30^a}. \end{aligned}$$

Also from (1.4),

$$M_2^aG_5(\Upsilon_1,x) = \sum_{rs\in E(\Upsilon_1)} x^{(S_G(r)\times S_G(s))^a}.$$

$$\begin{aligned} M_2^aG_5(\Upsilon_1,x) &= x^{(6\times6)^a}|E_1(\Upsilon_1(t))| + x^{(6\times11)^a}|E_2(\Upsilon_1(t))| + x^{(6\times12)^a}|E_3(\Upsilon_1(t))| \\ &\quad +x^{(6\times14)^a}|E_4(\Upsilon_1(t))| + x^{(7\times9)^a}|E_5(\Upsilon_1(t))| + x^{(7\times12)^a}|E_6(\Upsilon_1(t)) \\ &\quad | + x^{(8\times11)^a}|E_7(\Upsilon_1(t))| + x^{(8\times13)^a}|E_8(\Upsilon_1(t))| + x^{(9\times9)^a}|E_9(\Upsilon_1(t \\ &\quad))| + x^{(9\times14)^a}|E_{10}(\Upsilon_1(t))| + x^{(11\times11)^a}|E_{11}(\Upsilon_1(t))| + x^{(11\times12)^a}|E \\ &\quad {}_{12}(\Upsilon_1(t))| + x^{(11\times13)^a}|E_{13}(\Upsilon_1(t))| + x^{(11\times14)^a}|E_{14}(\Upsilon_1(t))| + \\ &\quad x^{(11\times16)^a}|E_{15}(\Upsilon_1(t))| + x^{(12\times14)^a}|E_{16}(\Upsilon_1(t))| + x^{(13\times14)^a}|E_{17}(\Upsilon \\ &\quad {}_1(t))| + x^{(13\times16)^a}|E_{18}(\Upsilon_1(t))| + x^{(14\times14)^a}|E_{19}(\Upsilon_1(t))| + x^{(14\times16)^a} \\ &\quad |E_{20}(\Upsilon_1(t))|, \\ &= x^{(6\times6)^a}(4t) + x^{(6\times11)^a}(4t) + x^{(6\times12)^a}(4) + x^{(6\times14)^a}(4t-4) + \\ &\quad x^{(7\times9)^a}(4t-4) + x^{(7\times12)^a}(4t-4) + x^{(8\times11)^a}(12t-8) + x^{(8\times13)^a} \\ &\quad (4t-4) + x^{(9\times9)^a}(2t-2) + x^{(9\times14)^a}(4t-4) + x^{(11\times11)^a}(9t^2-7 \\ &\quad t+3) + x^{(11\times12)^a}(4) + x^{(11\times13)^a}(4t-4) + x^{(11\times14)^a}(36t^2-68t \\ &\quad +32) + x^{(11\times16)^a}(4t-4) + x^{(12\times14)^a}(4t-4) + x^{(13\times14)^a}(4t-4) \\ &\quad +x^{(13\times16)^a}(4t-4) + x^{(14\times14)^a}(4t-4) + x^{(14\times16)^a}(36t^2-76t+ \\ &\quad 40), \end{aligned}$$

By making some calculations, we get

$$\begin{aligned} \Longrightarrow M_2^aG_5(\Upsilon_1,x) &= 4tx^{36^a} + 4(t-1)x^{63^a} + 4tx^{66^a} + 4x^{72^a} + 2(t-1)x^{81^a} + 8(t \\ &\quad -1)x^{84^a} + 4(3t-2)x^{88^a} \\ &\quad +4(t-1)x^{104^a} + (9t^2-7t+3)x^{154^a} + 4(t-1)x^{126^a} + 4 \\ &\quad x^{132^a} + 4(t-1)x^{143^a} + 4(t-1)(9t-8)x^{154^a} + 4(t-1)x^{168^a} \\ &\quad +4(t-1)x^{176^a} + 4(t-1)x^{182^a} + 4(t-1)x^{196^a} + 4(t-1)(9t \\ &\quad -10)x^{224^a}. \end{aligned}$$

□

Corresponding the above indices, we are going to compute fifth M-Zagreb polynomials for first type of Dominating David Derived graph $D_1(t)$.

Theorem 2.1.2. Let $\Upsilon_1 \cong D_1(t)$ be the first type of Dominating David Derived graph, then fifth M-Zagreb polynomials of first and second type is equal to

$$\begin{aligned} M_1G_5(\Upsilon_1,x) &= 4tx^{12} + 4(t-1)x^{16} + 4tx^{17} + 2(t+1)x^{18} + 4(4t-3)x^{19} + 4(t \\ &\quad -1)x^{20} + 4(t-1)x^{21} + (9t^2-7t+3)x^{22} + 4tx^{23} + 4(t-1)x^{24} \\ &\quad +4(t-1)(9t-8)x^{25} + 4(t-1)x^{26} + 8(t-1)x^{27} + 4(t-1)x^{28} \\ &\quad +4(t-1)x^{29} + 4(t-1)(9t-10)x^{30}, \end{aligned}$$

$$\begin{aligned} M_2G_5(\Upsilon_1,x) &= 4tx^{36}+4(t-1)x^{63}+4tx^{66}+4x^{72}+2(t-1)x^{81}+8(t-1)x^{84}+\\ &\quad 4(3t-2)x^{88}+4(t-1)x^{104}+(9t^2-7t+3)x^{154}+4(t-1)x^{126}+\\ &\quad 4x^{132}+4(t-1)x^{143}+4(t-1)(9t-8)x^{154}+4(t-1)x^{168}+4(t\\ &\quad -1)x^{176}+4(t-1)x^{182}+4(t-1)x^{196}+4(t-1)(9t-10)x^{224}. \end{aligned}$$

Proof. We get the outcome with the edge partition in Table 1. It follows from (1.5),

$$M_1G_5(\Upsilon_1,x)=\sum_{rs\in E(\Upsilon_1)} x^{(S_G(r)+S_G(s))}.$$

$$\begin{aligned} M_1G_5(\Upsilon_1,x) &= x^{(6+6)}|E_1(\Upsilon_1(t))|+x^{(6+11)}|E_2(\Upsilon_1(t))|+x^{(6+12)}|E_3(\Upsilon_1(t))|+\\ &\quad x^{(6+14)}|E_4(\Upsilon_1(t))|+x^{(7+9)}|E_5(\Upsilon_1(t))|+x^{(7+12)}|E_6(\Upsilon_1(t))|+\\ &\quad x^{(8+11)}|E_7(\Upsilon_1(t))|+x^{(8+13)}|E_8(\Upsilon_1(t))|+x^{(9+9)}|E_9(\Upsilon_1(t))|+\\ &\quad x^{(9+14)}|E_{10}(\Upsilon_1(t))|+x^{(11+11)}|E_{11}(\Upsilon_1(t))|+x^{(11+12)}|E_{12}(\Upsilon_1(t)\\ &\quad)|+x^{(11+13)}|E_{13}(\Upsilon_1(t))|+x^{(11+14)}|E_{14}(\Upsilon_1(t))|+x^{(11+16)}|E_{15}(\\ &\quad \Upsilon_1(t))|+x^{(12+14)}|E_{16}(\Upsilon_1(t))|+x^{(13+14)}|E_{17}(\Upsilon_1(t))|+x^{(13+16)}\\ &\quad |E_{18}(\Upsilon_1(t))|+x^{(14+14)}|E_{19}(\Upsilon_1(t))|+x^{(14+16)}|E_{20}(\Upsilon_1(t))|,\\ &= x^{(6+6)}(4t)+x^{(6+11)}(4t)+x^{(6+12)}(4)+x^{(6+14)}(4t-4)+x^{(7+9)}(\\ &\quad 4t-4)+x^{(7+12)}(4t-4)+x^{(8+11)}(12t-8)+x^{(8+13)}(4t-4)+\\ &\quad x^{(9+9)}(2t-2)+x^{(9+14)}(4t-4)+x^{(11+11)}(9t^2-7t+3)+(4)\\ &\quad x^{(11+12)}+x^{(11+13)}(4t-4)+x^{(11+14)}(36t^2-68t+32)+x^{(11+16)}\\ &\quad (4t-4)+x^{(12+14)}(4t-4)+x^{(13+14)}(4t-4)+x^{(13+16)}(4t-4)+\\ &\quad x^{(14+14)}(4t-4)+x^{(14+16)}(36t^2-76t+40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow M_1G_5(\Upsilon_1,x) &= 4tx^{12}+4(t-1)x^{16}+4tx^{17}+2(t+1)x^{18}+4(4t-3)x^{19}+4\\ &\quad (t-1)x^{20}+4(t-1)x^{21}+(9t^2-7t+3)x^{22}+4tx^{23}+4(t-1\\ &\quad)x^{24}+4(t-1)(9t-8)x^{25}+4(t-1)x^{26}+8(t-1)x^{27}+4(t-\\ &\quad 1)x^{28}+4(t-1)x^{29}+4(t-1)(9t-10)x^{30}. \end{aligned}$$

Also from (1.6),

$$M_2G_5(\Upsilon_1,x)=\sum_{rs\in E(\Upsilon_1)} x^{(S_G(r)\times S_G(s))}.$$

$$\begin{aligned} M_2G_5(\Upsilon_1,x) &= x^{(6\times6)}|E_1(\Upsilon_1(t))| + x^{(6\times11)}|E_2(\Upsilon_1(t))| + x^{(6\times12)}|E_3(\Upsilon_1(t))| + \\ & x^{(6\times14)}|E_4(\Upsilon_1(t))| + x^{(7\times9)}|E_5(\Upsilon_1(t))| + x^{(7\times12)}|E_6(\Upsilon_1(t))| + \\ & x^{(8\times11)}|E_7(\Upsilon_1(t))| + x^{(8\times13)}|E_8(\Upsilon_1(t))| + x^{(9\times9)}|E_9(\Upsilon_1(t))| + \\ & x^{(9\times14)}|E_{10}(\Upsilon_1(t))| + x^{(11\times11)}|E_{11}(\Upsilon_1(t))| + x^{(11\times12)}|E_{12}(\Upsilon_1(t) \\ &)| + x^{(11\times13)}|E_{13}(\Upsilon_1(t))| + x^{(11\times14)}|E_{14}(\Upsilon_1(t))| + x^{(11\times16)}|E_{15}(\\ & \Upsilon_1(t))| + x^{(12\times14)}|E_{16}(\Upsilon_1(t))| + x^{(13\times14)}|E_{17}(\Upsilon_1(t))| + x^{(13\times16)} \\ & |E_{18}(\Upsilon_1(t))| + x^{(14\times14)}|E_{19}(\Upsilon_1(t))| + x^{(14\times16)}|E_{20}(\Upsilon_1(t))|, \\ &= x^{(6\times6)}(4t) + x^{(6\times11)}(4t) + x^{(6\times12)}(4) + x^{(6\times14)}(4t-4) + x^{(7\times9)} \\ & (4t-4) + x^{(7\times12)}(4t-4) + x^{(8\times11)}(12t-8) + x^{(8\times13)}(4t-4) + \\ & x^{(9\times9)}(2t-2) + x^{(9\times14)}(4t-4) + x^{(11\times11)}(9t^2-7t+3) + \\ & x^{(11\times12)}(4) + x^{(11\times13)}(4t-4) + x^{(11\times14)}(36t^2-68t+32) + \\ & x^{(11\times16)}(4t-4) + x^{(12\times14)}(4t-4) + x^{(13\times14)}(4t-4) + x^{(13\times16)} \\ & (4t-4) + x^{(14\times14)}(4t-4) + x^{(14\times16)}(36t^2-76t+40), \end{aligned}$$

By making some calculations, we get

$$\begin{aligned} \Longrightarrow M_2G_5(\Upsilon_1,x) &= 4tx^{36} + 4(t-1)x^{63} + 4tx^{66} + 4x^{72} + 2(t-1)x^{81} + 8(t-1) \\ & x^{84} + 4(3t-2)x^{88} + 4(t-1)x^{104} + (9t^2-7t+3)x^{154} + 4(t \\ & -1)x^{126} + 4x^{132} + 4(t-1)x^{143} + 4(t-1)(9t-8)x^{154} + 4(t \\ & -1)x^{168} + 4(t-1)x^{176} + 4(t-1)x^{182} + 4(t-1)x^{196} + 4(t \\ & -1)(9t-10)x^{224}. \end{aligned}$$

□ □

Theorem 2.1.3. Let $\Upsilon_1 \cong D_1(t)$ be the first type of Dominating David Derived graph, then hyper fifth M-Zagreb polynomials of first and second type is equal to

$$\begin{aligned} HM_1G_5(\Upsilon_1,x) &= 4t(x^{144} + x^{289}) + (4t-4)(x^{256} + x^{361} + x^{400} + x^{441} + x^{529} + \\ & x^{576} + x^{676} + 2x^{729} + x^{784} + x^{841}) + (2t+2)x^{324} + (12t-8) \\ & x^{361} + (9t^2-7t+3)x^{484} + 4x^{529} + (36t^2-68t+32)x^{625} + (\\ & 36t^2-76t+40)x^{900}, \end{aligned}$$

$$\begin{aligned} HM_2G_5(\Upsilon_1,x) &= 4t(x^{1296} + x^{4356}) + (4t-4)(x^{3969} + x^{7056} + x^{10816} + x^{15876} + \\ & x^{20449} + x^{28224} + x^{30976} + 2x^{33124} + x^{38416} + x^{43264}) + 4x^{5184} \\ & +(2t-2)x^{6561} + (12t-8)x^{7744} + (9t^2-7t+3) \times x^{14641} + 4 \\ & x^{17424} + (36t^2-68t+32)x^{23716} + (36t^2-76t+40)x^{50176}. \end{aligned}$$

Proof. We get the outcome with the edge partition in Table 1. It follows from (1.7),

$$HM_1G_5(\Upsilon_1,x) = \sum_{rs\in E(\Upsilon_1)} x^{(S_G(r)+S_G(s))^2}.$$

$$\begin{aligned}
HM_1G_5(\Upsilon_1,x) &= x^{(6+6)^2}|E_1(\Upsilon_1(t))| + x^{(6+11)^2}|E_2(\Upsilon_1(t))| + x^{(6+12)^2}|E_3(\Upsilon_1(t))| \\
&\quad + x^{(6+14)^2}|E_4(\Upsilon_1(t))| + x^{(7+9)^2}|E_5(\Upsilon_1(t))| + x^{(7+12)^2}|E_6(\Upsilon_1(t))| + x^{(8+11)^2}|E_7(\Upsilon_1(t))| + x^{(8+13)^2}|E_8(\Upsilon_1(t))| + x^{(9+9)^2}|E_9(\Upsilon_1(t))| + x^{(9+14)^2}|E_{10}(\Upsilon_1(t))| + x^{(11+11)^2}|E_{11}(\Upsilon_1(t))| + x^{(11+12)^2}|E_{12}(\Upsilon_1(t))| + x^{(11+13)^2}|E_{13}(\Upsilon_1(t))| + x^{(11+14)^2}|E_{14}(\Upsilon_1(t))| + x^{(11+16)^2}|E_{15}(\Upsilon_1(t))| + x^{(12+14)^2}|E_{16}(\Upsilon_1(t))| + x^{(13+14)^2}|E_{17}(\Upsilon_1(t))| + x^{(13+16)^2}|E_{18}(\Upsilon_1(t))| + x^{(14+14)^2}|E_{19}(\Upsilon_1(t))| + x^{(14+16)^2}|E_{20}(\Upsilon_1(t))|, \\
&= x^{(6+6)^2}(4t) + x^{(6+11)^2}(4t) + x^{(6+12)^2}(4) + x^{(6+14)^2}(4t-4) + x^{(7+9)^2}(4t-4) + x^{(7+12)^2}(4t-4) + x^{(8+11)^2}(12t-8) + x^{(8+13)^2}(4t-4) + x^{(9+9)^2}(2t-2) + x^{(9+14)^2}(4t-4) + x^{(11+11)^2}(9t^2-7t+3) + x^{(11+12)^2}(4) + x^{(11+13)^2}(4t-4) + x^{(11+14)^2}(36t^2-68t+32) + x^{(11+16)^2}(4t-4) + x^{(12+14)^2}(4t-4) + x^{(13+14)^2}(4t-4) + x^{(13+16)^2}(4t-4) + x^{(14+14)^2}(4t-4) + x^{(14+16)^2}(36t^2-76t+40),
\end{aligned}$$

By doing some calculations, we get

$$\begin{aligned}
\Longrightarrow HM_1G_5(\Upsilon_1,x) &= 4t(x^{144}+x^{289}) + (4t-4)(x^{256}+x^{361}+x^{400}+x^{441}+x^{529}+x^{576}+x^{676}+2x^{729}+x^{784}+x^{841}) + (2t+2)x^{324} \\
&\quad +(12t-8)x^{361} + (9t^2-7t+3)x^{484} + 4x^{529} + (36t^2-68t+32)x^{625} + (36t^2-76t+40)x^{900}.
\end{aligned}$$

Also from (1.8),

$$HM_2^aG_5(\Upsilon_1,x) = \sum_{rs\in E(\Upsilon_1)} x^{(S_G(r)\times S_G(s))^2}.$$

$$\begin{aligned} HM_2^aG_5(\Upsilon_1,x) &= x^{(6\times6)^2}|E_1(\Upsilon_1(t))|+x^{(6\times11)^2}|E_2(\Upsilon_1(t))|+x^{(6\times12)^2}|E_3(\Upsilon_1(t))|+x^{(6\times14)^2}|E_4(\Upsilon_1(t))|+x^{(7\times9)^2}|E_5(\Upsilon_1(t))|+x^{(7\times12)^2}|E_6(\Upsilon_1(t))|+x^{(8\times11)^2}|E_7(\Upsilon_1(t))|+x^{(8\times13)^2}|E_8(\Upsilon_1(t))|+x^{(9\times9)^2}|E_9(\Upsilon_1(t))|+x^{(9\times14)^2}|E_{10}(\Upsilon_1(t))|+x^{(11\times11)^2}|E_{11}(\Upsilon_1(t))|+x^{(11\times12)^2}|E_{12}(\Upsilon_1(t))|+x^{(11\times13)^2}|E_{13}(\Upsilon_1(t))|+x^{(11\times14)^2}|E_{14}(\Upsilon_1(t))|+x^{(11\times16)^2}|E_{15}(\Upsilon_1(t))|+x^{(12\times14)^2}|E_{16}(\Upsilon_1(t))|+x^{(13\times14)^2}|E_{17}(\Upsilon_1(t))|+x^{(13\times16)^2}|E_{18}(\Upsilon_1(t))|+x^{(14\times14)^2}|E_{19}(\Upsilon_1(t))|+x^{(14\times16)^2}|E_{20}(\Upsilon_1(t))|,\\ &= x^{(6\times6)^2}(4t)+x^{(6\times11)^a}(4t)+x^{(6\times12)^2}(4)+x^{(6\times14)^2}(4t-4)+x^{(7\times9)^2}(4t-4)+x^{(7\times12)^2}(4t-4)+x^{(8\times11)^2}(12t-8)+x^{(8\times13)^2}(4t-4)+x^{(9\times9)^2}(2t-2)+x^{(9\times14)^2}(4t-4)+x^{(11\times11)^2}(9t^2-7t+3)+x^{(11\times12)^2}(4)+x^{(11\times13)^2}(4t-4)+x^{(11\times14)^2}(36t^2-68t+32)+x^{(11\times16)^2}(4t-4)+x^{(12\times14)^2}(4t-4)+x^{(13\times14)^2}(4t-4)+x^{(13\times16)^2}(4t-4)+x^{(14\times14)^2}(4t-4)+x^{(14\times16)^2}(36t^2-76t+40), \end{aligned}$$

By making some calculations, we get

$$\begin{aligned} \Longrightarrow HM_2G_5(\Upsilon_1,x) &= 4t(x^{1296}+x^{4356})+(4t-4)(x^{3969}+x^{7056}+x^{10816}+x^{15876}+x^{20449}+x^{28224}+x^{30976}+2x^{33124}+x^{38416}+x^{43264})+4x^{5184}+(2t-2)x^{6561}+(12t-8)x^{7744}+(9t^2-7t+3)x^{14641}+4x^{17424}+(36t^2-68t+32)x^{23716}+(36t^2-76t+40)x^{50176}. \end{aligned}$$

□ □

3.2. **Results for Second Type of Dominating David Derived graph.** Now, we are calculating fifth M-Zagreb topological indices of the $\Upsilon_2 \cong D_2(t)$, where $t \in \mathbb{N}$ for second type of Dominating David Derived graph.

Theorem 2.2.1. Let $\Upsilon_2 \cong D_2(t)$ be the second type of DDD graph, then general fifth M-Zagreb polynomials of first and second type are equal to

$$\begin{aligned} M_1^aG_5(\Upsilon_2,x) &= 4tx^{12^a}+4(t-1)x^{14^a}+4(t-1)x^{15^a}+(18t^2-30t+14)x^{16^a}+4tx^{17^a}+4x^{18^a}+(4t-4)x^{19^a}+(12t-8)x^{19^a}+(4t-4)x^{20^a}+(4t-4)x^{21^a}+(8t-4)x^{21^a}+(4t-4)x^{22^a}+4x^{23^a}+(4t-4)x^{23^a}+(36t^2-72t+36)x^{24^a}+(4t-4)x^{25^a}+(4t-4)x^{26^a}+2(4t-4)x^{27^a}+(4t-4)x^{28^a}+(4t-4)x^{29^a}+(36t^2-76t+40)x^{30^a}, \end{aligned}$$

$$M_2^a G_5(\Upsilon_2, x) = 4tx^{36^a} + (4t-4)x^{48^a} + (4t-4)x^{56^a} + (18t^2 - 30t + 14)x^{60^a} + 4tx^{66^a} + 4x^{72^a} + 2(4t-4)x^{84^a} + (12t-8)x^{88^a} + (4t-4)x^{104^a} + (8t-4)x^{110^a} + (4t-4)x^{112^a} + (4t-4)x^{130^a} + 4x^{132^a} + (36t^2 - 72t + 36)x^{140^a} + (4t-4)x^{154^a} + (4t-4)x^{168^a} + (4t-4)x^{176^a} + (4t-4)x^{182^a} + (4t-4)x^{196^a} + (4t-4)x^{208^a} + (36t^2 - 76t + 40)x^{224^a}.$$

Proof. We get the outcome with the edge partition in Table 1.
It follows from (1.3),

(S_r, S_s) where $rs \in E(\Upsilon_2)$	Number of edges	(S_r, S_s) where $rs \in E(\Upsilon_2)$	Number of edges
$(6,6)$	$4t$	$(10,11)$	$8t-4$
$(6,8)$	$4t-4$	$(10,13)$	$4t-4$
$(6,10)$	$18t^2-30n+14$	$(10,14)$	$36t^2-72t+36$
$(6,11)$	$4t$	$(11,12)$	4
$(6,12)$	4	$(11,14)$	$4t-4$
$(6,14)$	$4t-4$	$(11,16)$	$4t-4$
$(7,8)$	$4t-4$	$(12,14)$	$4t-4$
$(7,12)$	$4t-4$	$(13,14)$	$4t-4$
$(8,11)$	$12t-8$	$(13,16)$	$4t-4$
$(8,13)$	$4t-4$	$(14,14)$	$4t-4$
$(8,14)$	$4t-4$	$(14,16)$	$36t^2-76t+40$

TABLE 2. Edge partition of second type of Dominating David Derived graph ($D_2(t)$) based on sum of degrees of end vertices of each edge.

$$M_1^a G_5(\Upsilon_2, x) = \sum_{rs \in E(\Upsilon_2)} x^{(S_G(r)+S_G(s))^a}.$$

$$\begin{aligned} M_1^aG_5(\Upsilon_2,x) &= x^{(6+6)^a}|E_1(\Upsilon_2(t))| + x^{(6+8)^a}|E_2(\Upsilon_2(t))| + x^{(6+10)^a}|E_3(\Upsilon_2(t))| + \\ & x^{(6+11)^a}|E_4(\Upsilon_2(t))| + x^{(6+12)^a}|E_5(\Upsilon_2(t))| + x^{(6+14)^a}|E_6(\Upsilon_2(t))| \\ & +x^{(7+8)^a}|E_7(\Upsilon_2(t))| + x^{(7+12)^a}|E_8(\Upsilon_2(t))| + x^{(8+11)^a}|E_9(\Upsilon_2(t)) \\ & | + x^{(8+13)^a}|E_{10}(\Upsilon_2(t))| + x^{(8+14)^a}|E_{11}(\Upsilon_2(t))| + x^{(10+11)^a}|E_{12}(\\ & \Upsilon_2(t))| + x^{(10+13)^a}|E_{13}(\Upsilon_2(t))| + x^{(10+14)^a}|E_{14}(\Upsilon_2(t))| + \\ & x^{(11+12)^a}|E_{15}(\Upsilon_2(t))| + x^{(11+14)^a}|E_{16}(\Upsilon_2(t))| + x^{(11+16)^a}|E_{17}(\Upsilon_2 \\ & (t))| + x^{(12+14)^a}|E_{18}(\Upsilon_2(t))| + x^{(13+14)^a}|E_{19}(\Upsilon_2(t))| + x^{(13+16)^a} \\ & |E_{20}(\Upsilon_2(t))| + x^{(14+14)^a}|E_{21}(\Upsilon_2(t))| + x^{(14+16)^a}|E_{22}(\Upsilon_2(t))|, \\ &= x^{(6+6)^a}(4t) + x^{(6+8)^a}(4t-4) + x^{(6+10)^a}(18t^2 - 30t + 14) + \\ & x^{(6+11)^a}(4t) + x^{(6+12)^a}(4) + x^{(6+14)^a}(4t-4) + x^{(7+8)^a}(4t-4) + \\ & x^{(7+12)^a}(4t-4) + x^{(8+11)^a}(12t-8) + x^{(8+13)^a}(4t-4) + x^{(8+14)^a} \\ & (4t-4) + x^{(10+11)^a}(8t-4) + x^{(10+13)^a}(4t-4) + x^{(10+14)^a}(36t^2 \\ & -72t + 36) + (4)x^{(11+12)^a} + x^{(11+14)^a}(4t-4) + x^{(11+16)^a}(4t-4) \\ & +x^{(12+14)^a}(4t-4) + x^{(13+14)^a}(4t-4) + x^{(13+16)^a}(4t-4) + \\ & x^{(14+14)^a}(4t-4) + x^{(14+16)^a}(36t^2 - 76t + 40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow M_1^aG_5(\Upsilon_2,x) &= 4tx^{12^a} + 4(t-1)x^{14^a} + 4(t-1)x^{15^a} + (18t^2 - 30t + 14)x^{16^a} \\ & +4tx^{17^a} + 4x^{18^a} + (4t-4)x^{19^a} + (12t-8)x^{19^a} + (4t-4) \\ & x^{20^a} + (4t-4)x^{21^a} + (8t-4)x^{21^a} + (4t-4)x^{22^a} + 4x^{23^a} + \\ & (4t-4)x^{23^a} + (36t^2 - 72t + 36)x^{24^a} + (4t-4)x^{25^a} + (4t- \\ & 4)x^{26^a} + 2(4t-4)x^{27^a} + (4t-4)x^{28^a} + (4t-4)x^{29^a} + (36t^2 \\ & -76t + 40)x^{30^a}. \end{aligned}$$

Also from (1.4),

$$M_2^aG_5(\Upsilon_1,x) = \sum_{rs \in E(\Upsilon_1)} x^{(S_G(r) \times S_G(s))^a}.$$

$$\begin{aligned}
M_2^aG_5(\Upsilon_1,x) &= x^{(6\times6)^a}|E_1(\Upsilon_2(t))| + x^{(6\times8)^a}|E_2(\Upsilon_2(t))| + x^{(6\times10)^a}|E_3(\Upsilon_2(t))| + \\
&\quad x^{(6\times11)^a}|E_4(\Upsilon_2(t))| + x^{(6\times12)^a}|E_5(\Upsilon_2(t))| + x^{(6\times14)^a}|E_6(\Upsilon_2(t))| \\
&\quad +x^{(7\times8)^a}|E_7(\Upsilon_2(t))| + x^{(7\times12)^a}|E_8(\Upsilon_2(t))| + x^{(8\times11)^a}|E_9(\Upsilon_2(t)) \\
&\quad | + x^{(8\times13)^a}|E_{10}(\Upsilon_2(t))| + x^{(8\times14)^a}|E_{11}(\Upsilon_2(t))| + x^{(10\times11)^a}|E_{12}(\\
&\quad \Upsilon_2(t))| + x^{(10\times13)^a}|E_{13}(\Upsilon_2(t))| + x^{(10\times14)^a}|E_{14}(\Upsilon_2(t))| + \\
&\quad x^{(11\times12)^a}|E_{15}(\Upsilon_2(t))| + x^{(11\times14)^a}|E_{16}(\Upsilon_2(t))| + x^{(11\times16)^a}|E_{17}(\Upsilon_2 \\
&\quad (t))| + x^{(12\times14)^a}|E_{18}(\Upsilon_2(t))| + x^{(13\times14)^a}|E_{19}(\Upsilon_2(t))| + x^{(13\times16)^a} \\
&\quad |E_{20}(\Upsilon_2(t))| + x^{(14\times14)^a}|E_{21}(\Upsilon_2(t))| + x^{(14\times16)^a}|E_{22}(\Upsilon_2(t))|, \\
&= x^{(6\times6)^a}(4t) + x^{(6\times8)^a}(4t-4) + x^{(6\times10)^a}(18t^2-30t+14) + \\
&\quad x^{(6\times11)^a}(4t) + x^{(6\times12)^a}(4) + x^{(6\times14)^a}(4t-4) + x^{(7\times8)^a}(4t-4) + \\
&\quad x^{(7\times12)^a}(4t-4) + x^{(8\times11)^a}(12t-8) + x^{(8\times13)^a}(4t-4) + x^{(8\times14)^a} \\
&\quad (4t-4) + x^{(10\times11)^a}(8t-4) + x^{(10\times13)^a}(4t-4) + x^{(10\times14)^a}(36t^2 \\
&\quad -72t+36) + (4)x^{(11\times12)^a} + x^{(11\times14)^a}(4t-4) + x^{(11\times16)^a}(4t-4) \\
&\quad +x^{(12\times14)^a}(4t-4) + x^{(13\times14)^a}(4t-4) + x^{(13\times16)^a}(4t-4) + \\
&\quad x^{(14\times14)^a}(4t-4) + x^{(14\times16)^a}(36t^2-76t+40),
\end{aligned}$$

By making some calculations, we get

$$\begin{aligned}
\Longrightarrow M_2^aG_5(\Upsilon_1,x) &= 4tx^{36^a} + (4t-4)x^{48^a} + (4t-4)x^{56^a} + (18t^2-30t+14)x^{60^a} \\
&\quad +4tx^{66^a} + 4x^{72^a} + 2(4t-4)x^{84^a} + (12t-8)x^{88^a} + (4t-4) \\
&\quad x^{104^a} + (8t-4)x^{110^a} + (4t-4)x^{112^a} + (4t-4)x^{130^a} + 4 \\
&\quad x^{132^a} + (36t^2-72t+36)x^{140^a} + (4t-4)x^{154^a} + (4t-4) \\
&\quad x^{168^a} + (4t-4)x^{176^a} + (4t-4)x^{182^a} + (4t-4)x^{196^a} + (4t \\
&\quad -4)x^{208^a} + (36t^2-76t+40)x^{224^a}.
\end{aligned}$$

□ □

Corresponding the above indices, we are going to compute fifth M-Zagreb polynomials for second type of Dominating David Derived graph $D_2(t)$.

Theorem 2.2.2. Let $\Upsilon_2 \cong D_2(t)$ be the second type of DDD graph, then fifth M-Zagreb polynomials of first and second type are equal to

$$\begin{aligned}
M_1^aG_5(\Upsilon_2,x) &= 4tx^{12} + 4(t-1)x^{14} + 4(t-1)x^{15} + (18t^2-30t+14)x^{16} + 4tx^{17} \\
&\quad +4x^{18} + (4t-4)x^{19} + (12t-8)x^{19} + (4t-4)x^{20} + (4t-4)x^{21} \\
&\quad +(8t-4)x^{21} + (4t-4)x^{22} + 4x^{23} + (4t-4)x^{23} + (36t^2-72t+ \\
&\quad 36)x^{24} + (4t-4)x^{25} + (4t-4)x^{26} + 2(4t-4)x^{27} + (4t-4)x^{28} \\
&\quad +(4t-4)x^{29} + (36t^2-76t+40)x^{30},
\end{aligned}$$

$$\begin{aligned} M_2G_5(\Upsilon_2,x) &= 4tx^{36}+(4t-4)x^{48}+(4t-4)x^{56}+(18t^2-30t+14)x^{60}+4tx^{66} \\ &\quad +4x^{72}+2(4t-4)x^{84}+(12t-8)x^{88}+(4t-4)x^{104}+(8t-4) \\ &\quad x^{110}+(4t-4)x^{112}+(4t-4)x^{130}+4x^{132}+(36t^2-72t+36) \\ &\quad x^{140}+(4t-4)x^{154}+(4t-4)x^{168}+(4t-4)x^{176}+(4t-4)x^{182} \\ &\quad +(4t-4)x^{196}+(4t-4)x^{208}+(36t^2-76t+40)x^{224}. \end{aligned}$$

Proof. We get the outcome with the edge partition in Table 2. It follows from (1.5),

$$M_1G_5(\Upsilon_1,x)=\sum_{rs\in E(\Upsilon_2)} x^{(S_G(r)+S_G(s))}.$$

$$\begin{aligned} M_1G_5(\Upsilon_2,x) &= x^{(6+6)}|E_1(\Upsilon_2(t))|+x^{(6+8)}|E_2(\Upsilon_2(t))|+x^{(6+10)}|E_3(\Upsilon_2(t))|+ \\ &\quad x^{(6+11)}|E_4(\Upsilon_2(t))|+x^{(6+12)}|E_5(\Upsilon_2(t))|+x^{(6+14)}|E_6(\Upsilon_2(t))|+ \\ &\quad x^{(7+8)}|E_7(\Upsilon_2(t))|+x^{(7+12)}|E_8(\Upsilon_2(t))|+x^{(8+11)}|E_9(\Upsilon_2(t))|+ \\ &\quad x^{(8+13)}|E_{10}(\Upsilon_2(t))|+x^{(8+14)}|E_{11}(\Upsilon_2(t))|+x^{(10+11)}|E_{12}(\Upsilon_2(t \\ &\quad))|+x^{(10+13)}|E_{13}(\Upsilon_2(t))|+x^{(10+14)}|E_{14}(\Upsilon_2(t))|+x^{(11+12)}|E_{15} \\ &\quad (\Upsilon_2(t))|+x^{(11+14)}|E_{16}(\Upsilon_2(t))|+x^{(11+16)}|E_{17}(\Upsilon_2(t))|+x^{(12+14)} \\ &\quad |E_{18}(\Upsilon_2(t))|+x^{(13+14)}|E_{19}(\Upsilon_2(t))|+x^{(13+16)}|E_{20}(\Upsilon_2(t))|+ \\ &\quad x^{(14+14)}|E_{21}(\Upsilon_2(t))|+x^{(14+16)}|E_{22}(\Upsilon_2(t))|, \\ &= x^{(6+6)}(4t)+x^{(6+8)}(4t-4)+x^{(6+10)}(18t^2-30t+14)+x^{(6+11)}(\\ &\quad 4t)+x^{(6+12)}(4)+x^{(6+14)}(4t-4)+x^{(7+8)}(4t-4)+x^{(7+12)}(4t- \\ &\quad 4)+x^{(8+11)}(12t-8)+x^{(8+13)}(4t-4)+x^{(8+14)}(4t-4)+x^{(10+11)} \\ &\quad (8t-4)+x^{(10+13)}(4t-4)+x^{(10+14)}(36t^2-72t+36)+(4)x^{(11+12)} \\ &\quad +x^{(11+14)}(4t-4)+x^{(11+16)}(4t-4)+x^{(12+14)}(4t-4)+x^{(13+14)} \\ &\quad (4t-4)+x^{(13+16)}(4t-4)+x^{(14+14)}(4t-4)+x^{(14+16)}(36t^2- \\ &\quad 76t+40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow M_1^aG_5(\Upsilon_2,x) &= 4tx^{12}+4(t-1)x^{14}+4(t-1)x^{15}+(18t^2-30t+14)x^{16}+4 \\ &\quad tx^{17}+4x^{18}+(4t-4)x^{19}+(12t-8)x^{19}+(4t-4)x^{20}+(4t \\ &\quad -4)x^{21}+(8t-4)x^{21}+(4t-4)x^{22}+4x^{23}+(4t-4)x^{23}+(\\ &\quad 36t^2-72t+36)x^{24}+(4t-4)x^{25}+(4t-4)x^{26}+2(4t-4) \\ &\quad x^{27}+(4t-4)x^{28}+(4t-4)x^{29}+(36t^2- \\ &\quad 76t+40)x^{30}. \end{aligned}$$

Also from (1.6),

$$M_2G_5(\Upsilon_2,x)=\sum_{rs\in E(\Upsilon_2)} x^{(S_G(r)\times S_G(s))}.$$

$$\begin{aligned}
M_2G_5(\Upsilon_2, x) &= x^{(6\times6)}|E_1(\Upsilon_2(t))| + x^{(6\times8)}|E_2(\Upsilon_2(t))| + x^{(6\times10)}|E_3(\Upsilon_2(t))| + \\
&\quad x^{(6\times11)}|E_4(\Upsilon_2(t))| + x^{(6\times12)}|E_5(\Upsilon_2(t))| + x^{(6\times14)}|E_6(\Upsilon_2(t))| + \\
&\quad x^{(7\times8)}|E_7(\Upsilon_2(t))| + x^{(7\times12)}|E_8(\Upsilon_2(t))| + x^{(8\times11)}|E_9(\Upsilon_2(t))| + \\
&\quad x^{(8\times13)}|E_{10}(\Upsilon_2(t))| + x^{(8\times14)}|E_{11}(\Upsilon_2(t))| + x^{(10\times11)}|E_{12}(\Upsilon_2(t))| \\
&\quad + x^{(10\times13)}|E_{13}(\Upsilon_2(t))| + x^{(10\times14)}|E_{14}(\Upsilon_2(t))| + x^{(11\times12)}|E_{15}(\\
&\quad \Upsilon_2(t))| + x^{(11\times14)}|E_{16}(\Upsilon_2(t))| + x^{(11\times16)}|E_{17}(\Upsilon_2(t))| + x^{(12\times14)} \\
&\quad |E_{18}(\Upsilon_2(t))| + x^{(13\times14)}|E_{19}(\Upsilon_2(t))| + x^{(13\times16)}|E_{20}(\Upsilon_2(t))| + \\
&\quad x^{(14\times14)}|E_{21}(\Upsilon_2(t))| + x^{(14\times16)}|E_{22}(\Upsilon_2(t))|, \\
&= x^{(6\times6)}(4t) + x^{(6\times8)}(4t-4) + x^{(6\times10)}(18t^2-30t+14) + x^{(6\times11)} \\
&\quad (4t) + x^{(6\times12)}(4) + x^{(6\times14)}(4t-4) + x^{(7\times8)}(4t-4) + x^{(7\times12)}(4t \\
&\quad -4) + x^{(8\times11)}(12t-8) + x^{(8\times13)}(4t-4) + x^{(8\times14)}(4t-4) + \\
&\quad x^{(10\times11)}(8t-4) + x^{(10\times13)}(4t-4) + x^{(10\times14)}(36t^2-72t+36) + \\
&\quad (4)x^{(11\times12)} + x^{(11\times14)}(4t-4) + x^{(11\times16)}(4t-4) + x^{(12\times14)}(4t- \\
&\quad 4) + x^{(13\times14)}(4t-4) + x^{(13\times16)}(4t-4) + x^{(14\times14)}(4t-4) + \\
&\quad x^{(14\times16)}(36t^2-76t+40),
\end{aligned}$$

By making some calculations, we get

$$\begin{aligned}
\Longrightarrow M_2G_5(\Upsilon_2, x) &= 4tx^{36} + (4t-4)x^{48} + (4t-4)x^{56} + (18t^2-30t+14)x^{60} + 4 \\
&\quad tx^{66} + 4x^{72} + 2(4t-4)x^{84} + (12t-8)x^{88} + (4t-4)x^{104} + (\\
&\quad 8t-4)x^{110} + (4t-4)x^{112} + (4t-4)x^{130} + 4x^{132} + (36t^2- \\
&\quad 72t+36)x^{140} + (4t-4)x^{154} + (4t-4)x^{168} + (4t-4)x^{176} \\
&\quad + (4t-4)x^{182} + (4t-4)x^{196} + (4t-4)x^{208} + (36t^2-76t+ \\
&\quad 40)x^{224}.
\end{aligned}$$

□ □

Theorem 2.2.3. Let $\Upsilon_2 \cong D_2(t)$ be the second type of DDD graph, then hyper fifth M-Zagreb polynomials of first and second type are equal to

$$\begin{aligned}
HM_1G_5(\Upsilon_2, x) &= 4tx^{144} + (4t-4)x^{196} + (4t-4)x^{225} + (18t^2-30t+14)x^{256} + \\
&\quad 4tx^{289} + 4x^{324} + (4t-4)x^{361} + (12t-8)x^{361} + (4t-4)x^{400} + \\
&\quad (4t-4)x^{441} + (8t-4)x^{441} + (4t-4)x^{484} + 4tx^{529} + (36t^2- \\
&\quad 72t+36)x^{576} + (4t-4)x^{625} + (4t-4)x^{676} + 2(4t-4)x^{729} + (\\
&\quad 4t-4)x^{784} + (4t-4)x^{841} + (36t^2-76t+40)x^{900},
\end{aligned}$$

$$HM_2G_5(\Upsilon_2,x) = 4tx^{1296}+(4t-4)x^{2304}+(4t-4)x^{3136}+(18t^2-30t+14) x^{3600}+4tx^{4356}+4x^{5184}+2(4t-4)x^{7056}+(12t-8)x^{7744}+(4t-4)x^{10816}+(8t-4)x^{12100}+(4t-4)x^{12544}+(4t-4)x^{16900} +4x^{17424}+(36t^2-72t+36)x^{19600}+(4t-4)x^{23716}+(4t-4) x^{28224}+(4t-4)x^{30976}+(4t-4)x^{33124}+(4t-4)x^{38416}+(4t -4)x^{43264}+(36t^2-76t+40)x^{50176}.$$

Proof. We get the outcome with the edge partition in Table 2. It follows from (1.7),

$$HM_1G_5(\Upsilon_2,x)=\sum_{rs\in E(\Upsilon_1)} x^{(S_G(r)+S_G(s))^2}.$$

$$\begin{aligned} HM_1G_5(\Upsilon_2,x) &= x^{(6+6)^2}|E_1(\Upsilon_2(t))|+x^{(6+8)^2}|E_2(\Upsilon_2(t))|+x^{(6+10)^2}|E_3(\Upsilon_2(t))| \\ &\quad +x^{(6+11)^2}|E_4(\Upsilon_2(t))|+x^{(6+12)^2}|E_5(\Upsilon_2(t))|+x^{(6+14)^2}|E_6(\Upsilon_2(\\ &\quad t))|+x^{(7+8)^2}|E_7(\Upsilon_2(t))|+x^{(7+12)^2}|E_8(\Upsilon_2(t))|+x^{(8+11)^2}|E_9(\\ &\quad \Upsilon_2(t))|+x^{(8+13)^2}|E_{10}(\Upsilon_2(t))|+x^{(8+14)^2}|E_{11}(\Upsilon_2(t))|+ \\ &\quad x^{(10+11)^2}|E_{12}(\Upsilon_2(t))|+x^{(10+13)^2}|E_{13}(\Upsilon_2(t))|+x^{(10+14)^2}|E_{14}(\\ &\quad \Upsilon_2(t))|+x^{(11+12)^2}|E_{15}(\Upsilon_2(t))|+x^{(11+14)^2}|E_{16}(\Upsilon_2(t))|+ \\ &\quad x^{(11+16)^2}|E_{17}(\Upsilon_2(t))|+x^{(12+14)^2}|E_{18}(\Upsilon_2(t))|+x^{(13+14)^2}|E_{19}(\\ &\quad \Upsilon_2(t))|+x^{(13+16)^2}|E_{20}(\Upsilon_2(t))|+x^{(14+14)^2}|E_{21}(\Upsilon_2(t))|+ \\ &\quad x^{(14+16)^a}|E_{22}(\Upsilon_2(t))|, \\ &= x^{(6+6)^2}(4t)+x^{(6+8)^2}(4t-4)+x^{(6+10)^2}(18t^2-30t+14)+ \\ &\quad x^{(6+11)^2}(4t)+x^{(6+12)^2}(4)+x^{(6+14)^2}(4t-4)+x^{(7+8)^2}(4t-4) \\ &\quad +x^{(7+12)^2}(4t-4)+x^{(8+11)^2}(12t-8)+x^{(8+13)^2}(4t-4)+ \\ &\quad x^{(8+14)^2}(4t-4)+x^{(10+11)^2}(8t-4)+x^{(10+13)^2}(4t-4)+ \\ &\quad x^{(10+14)^2}(36t^2-72t+36)+(4)x^{(11+12)^2}+x^{(11+14)^2}(4t-4) \\ &\quad +x^{(11+16)^2}(4t-4)+x^{(12+14)^2}(4t-4)+x^{(13+14)^2}(4t-4)+ \\ &\quad x^{(13+16)^2}(4t-4)+x^{(14+14)^2}(4t-4)+x^{(14+16)^2}(36t^2-76t+ \\ &\quad 40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow HM_1G_5(\Upsilon_2, x) &= 4tx^{144} + (4t-4)x^{196} + (4t-4)x^{225} + (18t^2 - 30t + 14) \\ & x^{256} + 4tx^{289} + 4x^{324} + (4t-4)x^{361} + (12t-8)x^{361} + (4t \\ & -4)x^{400} + (4t-4)x^{441} + (8t-4)x^{441} + (4t-4)x^{484} + 4t \\ & x^{529} + (36t^2 - 72t + 36)x^{576} + (4t-4)x^{625} + (4t-4)x^{676} \\ & +2(4t-4)x^{729} + (4t-4)x^{784} + (4t-4)x^{841} + (36t^2 - 76 \\ & t + 40)x^{900}. \end{aligned}$$

Also from (1.8),

$$HM_2^a G_5(\Upsilon_2, x) = \sum_{rs \in E(\Upsilon_2)} x^{(S_G(r) \times S_G(s))^2}.$$

$$\begin{aligned} HM_2G_5(\Upsilon_2, x) &= x^{(6\times6)^2}|E_1(\Upsilon_2(t))| + x^{(6\times8)^2}|E_2(\Upsilon_2(t))| + x^{(6\times10)^2}|E_3(\Upsilon_2(t))| \\ & +x^{(6\times11)^2}|E_4(\Upsilon_2(t))| + x^{(6\times12)^2}|E_5(\Upsilon_2(t))| + x^{(6\times14)^2}|E_6(\Upsilon_2(\\ & t))| + x^{(7\times8)^2}|E_7(\Upsilon_2(t))| + x^{(7\times12)^2}|E_8(\Upsilon_2(t))| + x^{(8\times11)^2}|E_9(\\ & \Upsilon_2(t))| + x^{(8\times13)^2}|E_{10}(\Upsilon_2(t))| + x^{(8\times14)^2}|E_{11}(\Upsilon_2(t))| + \\ & x^{(10\times11)^2}|E_{12}(\Upsilon_2(t))| + x^{(10\times13)^2}|E_{13}(\Upsilon_2(t))| + x^{(10\times14)^2}|E_{14}(\\ & \Upsilon_2(t))| + x^{(11\times12)^2}|E_{15}(\Upsilon_2(t))| + x^{(11\times14)^2}|E_{16}(\Upsilon_2(t))| + \\ & x^{(11\times16)^2}|E_{17}(\Upsilon_2(t))| + x^{(12\times14)^2}|E_{18}(\Upsilon_2(t))| + x^{(13\times14)^2}|E_{19}(\\ & \Upsilon_2(t))| + x^{(13\times16)^2}|E_{20}(\Upsilon_2(t))| + x^{(14\times14)^2}|E_{21}(\Upsilon_2(t))| + \\ & x^{(14\times16)^a}|E_{22}(\Upsilon_2(t))|, \\ &= x^{(6\times6)^2}(4t) + x^{(6\times8)^2}(4t-4) + x^{(6\times10)^2}(18t^2 - 30t + 14) + \\ & x^{(6\times11)^2}(4t) + x^{(6\times12)^2}(4) + x^{(6\times14)^2}(4t-4) + x^{(7\times8)^2}(4t-4) \\ & +x^{(7\times12)^2}(4t-4) + x^{(8\times11)^2}(12t-8) + x^{(8\times13)^2}(4t-4) + \\ & x^{(8\times14)^2}(4t-4) + x^{(10\times11)^2}(8t-4) + x^{(10\times13)^2}(4t-4) + \\ & x^{(10\times14)^2}(36t^2 - 72t + 36) + (4)x^{(11\times12)^2} + x^{(11\times14)^2}(4t-4) + \\ & x^{(11\times16)^2}(4t-4) + x^{(12\times14)^2}(4t-4) + x^{(13\times14)^2}(4t-4) + \\ & x^{(13\times16)^2}(4t-4) + x^{(14\times14)^2}(4t-4) + x^{(14\times16)^2}(36t^2 - 76t + \\ & 40), \end{aligned}$$

By making some calculations, we get

$$\implies HM_2G_5(\Upsilon_1,x) = 4tx^{1296}+(4t-4)x^{2304}+(4t-4)x^{3136}+(18t^2-30t+14)x^{3600}+4tx^{4356}+4x^{5184}+2(4t-4)x^{7056}+(12t-8)x^{7744}+(4t-4)x^{10816}+(8t-4)x^{12100}+(4t-4)x^{12544}+(4t-4)x^{16900}+4x^{17424}+(36t^2-72t+36)x^{19600}+(4t-4)x^{23716}+(4t-4)x^{28224}+(4t-4)x^{30976}+(4t-4)x^{33124}+(4t-4)x^{38416}+(4t-4)x^{43264}+(36t^2-76t+40)x^{50176}.$$

□ □

3.3. **Results for Third Type of Dominating David Derived Network.** In this section, we calculate degree-based topological indices of the dimension t for third type of Dominating David Derived graphs. In the coming theorems, we compute M-polynomials.

Theorem 2.3.1. Let $\Upsilon_3 \cong D_3(t)$ be the third type of DDD graph, then general fifth M-Zagreb polynomials of first and second type are equal to

$$M_1^aG_5(\Upsilon_3,x) = 4tx^{12a}+8(2t-1)x^{18a}+4(9t^2-10t+3)x^{20a}+(4t-4)x^{22a}+8tx^{24a}+12(t-1)x^{26a}+4(9t^2-14t+3)x^{28a}+(4t+4)x^{30a}+(4t-4)(9t-10)x^{32a},$$

$$M_2^aG_5(\Upsilon_3,x) = 4tx^{36a}+(4t+4)x^{72a}+(12t-12)x^{80a}+(4t-4)x^{84a}+(36t^2-44t+16)x^{96a}+(4t-4)x^{112a}+8tx^{144a}+(4t-4)x^{160a}+(8t-8)x^{168a}+(36t^2-60t+16)x^{192a}+(4t-4)x^{196a}+(36t^2-76t+40)x^{224a}.$$

Proof. We get the outcome with the edge partition in Table 3.
It follows from (1.3),

(S_r,S_s) where $rs \in E(\Upsilon_3)$	Number of edges	(S_r,S_s) where $rs \in E(\Upsilon_3)$	Number of edges
$(6,6)$	$4t$	$(12,12)$	$8t$
$(6,12)$	$4t+4$	$(12,14)$	$8t-8$
$(6,14)$	$4t-4$	$(12,16)$	$36t^2-60t+16$
$(8,10)$	$12t-12$	$(14,14)$	$4t-4$
$(8,12)$	$36t^2-44t+16$	$(14,16)$	$4t+4$
$(8,14)$	$4t-4$	$(16,16)$	$36t^2-76t+40$
$(10,16)$	$4t-4$		

TABLE 3. Edge partition of third type of Dominating David Derived graph ($D_3(t)$) based on sum of degrees of end vertices of each edge.

$$M_1^aG_5(\Upsilon_3,x) = \sum_{rs\in E(\Upsilon_3)} x^{(S_G(r)+S_G(s))^a}.$$

$$\begin{aligned} M_1^aG_5(\Upsilon_3,x) &= x^{(6+6)^a}|E_1(\Upsilon_3(t))| + x^{(6+12)^a}|E_2(\Upsilon_3(t))| + x^{(6+14)^a}|E_3(\Upsilon_3(t))| \\ &\quad +x^{(8+10)^a}|E_4(\Upsilon_3(t))| + x^{(8+12)^a}|E_5(\Upsilon_3(t))| + x^{(8+14)^a}|E_6(\Upsilon_3(t \\ &\quad))| + x^{(10+16)^a}|E_7(\Upsilon_3(t))| + x^{(12+12)^a}|E_8(\Upsilon_3(t))| + x^{(12+14)^a}|E_9 \\ &\quad (\Upsilon_3(t))| + x^{(12+16)^a}|E_{10}(\Upsilon_3(t))| + x^{(14+14)^a}|E_{11}(\Upsilon_3(t))| + \\ &\quad x^{(14+16)^a}|E_{12}(\Upsilon_3(t))| + x^{(16+16)^a}|E_{13}(\Upsilon_3(t))|, \\ &= x^{(6+6)^a}(4t) + x^{(6+12)^a}(4t+4) + x^{(6+14)^a}(4t-4) + x^{(8+10)^a}(12t \\ &\quad -12) + x^{(8+12)^a}(36t^2-44t+16) + x^{(8+14)^a}(4t-4) + x^{(10+16)^a} \\ &\quad (4t-4) + x^{(12+12)^a}(8t) + x^{(12+14)^a}(8t-8) + x^{(12+16)^a}(36t^2- \\ &\quad 60t+16) + x^{(14+14)^a}(4t-4) + x^{(14+16)^a}(4t+4) + x^{(16+16)^a}(36t^2 \\ &\quad -76t+40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow M_1^aG_5(\Upsilon_3,x) &= 4tx^{12a} + 8(2t-1)x^{18a} + 4(9t^2-10t+3)x^{20a} + (4t-4)x^{22a} \\ &\quad +8tx^{24a} + 12(t-1)x^{26a} + 4(9t^2-14t+3)x^{28a} + (4t+4) \\ &\quad x^{30a} + (4t-4)(9t-10)x^{32a}. \end{aligned}$$

Also from (1.4),

$$M_2^aG_5(\Upsilon_3,x) = \sum_{rs\in E(\Upsilon_3)} x^{(S_G(r)\times S_G(s))^a}.$$

$$\begin{aligned} M_2^aG_5(\Upsilon_3,x) &= x^{(6\times6)^a}|E_1(\Upsilon_3(t))| + x^{(6\times12)^a}|E_2(\Upsilon_3(t))| + x^{(6\times14)^a}|E_3(\Upsilon_3(t))| \\ &\quad +x^{(8\times10)^a}|E_4(\Upsilon_3(t))| + x^{(8\times12)^a}|E_5(\Upsilon_3(t))| + x^{(8\times14)^a}|E_6(\Upsilon_3(t \\ &\quad))| + x^{(10\times16)^a}|E_7(\Upsilon_3(t))| + x^{(12\times12)^a}|E_8(\Upsilon_3(t))| + x^{(12\times14)^a}|E_9 \\ &\quad (\Upsilon_3(t))| + x^{(12\times16)^a}|E_{10}(\Upsilon_3(t))| + x^{(14\times14)^a}|E_{11}(\Upsilon_3(t))| + \\ &\quad x^{(14\times16)^a}|E_{12}(\Upsilon_3(t))| + x^{(16\times16)^a}|E_{13}(\Upsilon_3(t))|, \\ &= x^{(6\times6)^a}(4t) + x^{(6\times12)^a}(4t+4) + x^{(6\times14)^a}(4t-4) + x^{(8\times10)^a}(12t \\ &\quad -12) + x^{(8\times12)^a}(36t^2-44t+16) + x^{(8\times14)^a}(4t-4) + x^{(10\times16)^a} \\ &\quad (4t-4) + x^{(12\times12)^a}(8t) + x^{(12\times14)^a}(8t-8) + x^{(12\times16)^a}(36t^2- \\ &\quad 60t+16) + x^{(14\times14)^a}(4t-4) + x^{(14\times16)^a}(4t+4) + x^{(16\times16)^a}(\\ &\quad 36t^2-76t+40), \end{aligned}$$

By making some calculations, we get

$$\begin{aligned} \Longrightarrow M_2^aG_5(\Upsilon_3,x) &= 4tx^{36a} + (4t+4)x^{72a} + (12t-12)x^{80a} + (4t-4)x^{84a} + (36 \\ &\quad t^2-44t+16)x^{96a} + (4t-4)x^{112a} + 8tx^{144a} + (4t-4)x^{160a} \\ &\quad +(8t-8)x^{168a} + (36t^2-60t+16)x^{192a} + (4t-4)x^{196a} + (\\ &\quad 36t^2-76t+40)x^{224a}. \end{aligned}$$

□ □

Theorem 2.3.2. Let $\Upsilon_3 \cong D_3(t)$ be the third type of DDD graph, then fifth M-Zagreb polynomials of first and second type are equal to

$$\begin{aligned} M_1G_5(\Upsilon_3, x) &= 4x^{12}\big[t + 2(2t-1)x^6 + (9t^2 - 10t + 3)x^8 + (t-1)x^{10} + 2t \\ & x^{12} + 3(t-1)x^{14} + (9t^2 - 14t + 3)x^{16} + (t+1)x^{18} + (t- \\ & 1)(9t-10)x^{20}\big], \end{aligned}$$

$$\begin{aligned} M_2G_5(\Upsilon_3, x) &= 4x^{36}\big[t + (t+1)x^{36} + 3(t-1)x^{44} + (t-1)x^{48} + (9t^2 - 11t + 4) \\ & x^{60} + (t-1)x^{76} + 2tx^{108} + (t-1)x^{124} + 2(t-1)x^{132} + (3t-4) \\ & (3t-1)x^{156} + (t-1)x^{160} + (t-1)(9t-10)x^{188}\big]. \end{aligned}$$

Proof. We get the outcome with the edge partition in Table 3. It follows from (1.5),

$$M_1G_5(\Upsilon_3, x) = \sum_{rs \in E(\Upsilon_3)} x^{(S_G(r)+S_G(s))}.$$

$$\begin{aligned} M_1G_5(\Upsilon_3, x) &= x^{(6+6)}|E_1(\Upsilon_3(t))| + x^{(6+12)}|E_2(\Upsilon_3(t))| + x^{(6+14)}|E_3(\Upsilon_3(t) \\ &)| + x^{(8+10)}|E_4(\Upsilon_3(t))| + x^{(8+12)}|E_5(\Upsilon_3(t))| + x^{(8+14)}|E_6(\\ & \Upsilon_3(t))| + x^{(10+16)}|E_7(\Upsilon_3(t))| + x^{(12+12)}|E_8(\Upsilon_3(t))| + \\ & x^{(12+14)}|E_9(\Upsilon_3(t))| + x^{(12+16)}|E_{10}(\Upsilon_3(t))| + x^{(14+14)}|E_{11}(\\ & \Upsilon_3(t))| + x^{(14+16)}|E_{12}(\Upsilon_3(t))| + x^{(16+16)}|E_{13}(\Upsilon_3(t))|, \\ &= x^{(6+6)}(4t) + x^{(6+12)}(4t+4) + x^{(6+14)}(4t-4) + x^{(8+10)}(12t \\ & -12) + x^{(8+12)}(36t^2 - 44t + 16) + x^{(8+14)}(4t-4) + x^{(10+16)} \\ & (4t-4) + x^{(12+12)}(8t) + x^{(12+14)}(8t-8) + x^{(12+16)}(36t^2 - \\ & 60t + 16) + x^{(14+14)}(4t-4) + x^{(14+16)}(4t+4) + x^{(16+16)}(36 \\ & t^2 - 76t + 40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow M_1G_5(\Upsilon_3, x) &= 4x^{12}\big[t + 2(2t-1)x^6 + (9t^2 - 10t + 3)x^8 + (t-1)x^{10} + 2t \\ & x^{12} + 3(t-1)x^{14} + (9t^2 - 14t + 3)x^{16} + (t+1)x^{18} + (t- \\ & 1)(9t-10)x^{20}\big]. \end{aligned}$$

Also from (1.6),

$$M_2G_5(\Upsilon_3, x) = \sum_{rs \in E(\Upsilon_3)} x^{(S_G(r) \times S_G(s))}.$$

$$\begin{aligned}
M_2G_5(\Upsilon_3,x) &= x^{(6\times6)}|E_1(\Upsilon_3(t))| + x^{(6\times12)}|E_2(\Upsilon_3(t))| + x^{(6\times14)}|E_3(\Upsilon_3(t))| + \\
&\quad x^{(8\times10)}|E_4(\Upsilon_3(t))| + x^{(8\times12)}|E_5(\Upsilon_3(t))| + x^{(8\times14)}|E_6(\Upsilon_3(t))| + \\
&\quad x^{(10\times16)}|E_7(\Upsilon_3(t))| + x^{(12\times12)}|E_8(\Upsilon_3(t))| + x^{(12\times14)}|E_9(\Upsilon_3(t))| \\
&\quad + x^{(12\times16)}|E_{10}(\Upsilon_3(t))| + x^{(14\times14)}|E_{11}(\Upsilon_3(t))| + x^{(14\times16)}|E_{12}(\Upsilon_3 \\
&\quad (t))| + x^{(16\times16)}|E_{13}(\Upsilon_3(t))|, \\
&= x^{(6\times6)}(4t) + x^{(6\times12)}(4t+4) + x^{(6\times14)}(4t-4) + x^{(8\times10)}(12t-12 \\
&\quad) + x^{(8\times12)}(36t^2-44t+16) + x^{(8\times14)}(4t-4) + x^{(10\times16)}(4t-4) \\
&\quad + x^{(12\times12)}(8t) + x^{(12\times14)}(8t-8) + x^{(12\times16)}(36t^2-60t+16) + \\
&\quad x^{(14\times14)}(4t-4) + x^{(14\times16)}(4t+4) + x^{(16\times16)}(36t^2-76t+40),
\end{aligned}$$

By making some calculations, we get

$$\begin{aligned}
\Longrightarrow M_2G_5(\Upsilon_3,x) &= 4x^{36}\big[t + (t+1)x^{36} + 3(t-1)x^{44} + (t-1)x^{48} + (9t^2-11t+ \\
&\quad 4)x^{60} + (t-1)x^{76} + 2tx^{108} + (t-1)x^{124} + 2(t-1)x^{132} + (\\
&\quad 3t-4)(3t-1)x^{156} + (t-1)x^{160} + (t-1)(9t-10)x^{188}\big].
\end{aligned}$$

□ □

Theorem 2.3.3. Let $\Upsilon_3 \cong D_3(t)$ be the third type of Dominating David Derived graph, then hyper fifth M-Zagreb polynomials of first and second type are equal to

$$\begin{aligned}
HM_1G_5(\Upsilon_3,x) &= 4x^{144}\big[t + (4t-2)x^{180} + (9t^2-10t+3)x^{256} + (t-1)x^{340} + 2t \\
&\quad x^{432} + 3(t-1)x^{532} + (9t^2-14t+3)x^{640} + (t+1)x^{756} + (t- \\
&\quad 1)(9t-10)x^{880}\big],
\end{aligned}$$

$$\begin{aligned}
HM_2G_5(\Upsilon_3,x) &= 4tx^{1296} + (4t+4)x^{5184} + (12t-12)x^{6400} + (4t-4)x^{7056} + (36 \\
&\quad t^2-44t+16)x^{9216} + (4t-4)x^{12544} + 8tx^{20736} + (4t-4)x^{25600} \\
&\quad + (8t-8)x^{28224} + (36t^2-60t+16)x^{36864} + (4t-4)x^{38416} + (4t \\
&\quad +4)x^{50176} + (36t^2-76t+40)x^{65536}.
\end{aligned}$$

Proof. We get the outcome with the edge partition in Table 3. It follows from (1.7),

$$HM_1G_5(\Upsilon_3,x) = \sum_{rs\in E(\Upsilon_3)} x^{(S_G(r)+S_G(s))^2}.$$

$$\begin{aligned} HM_1G_5(\Upsilon_3,x) &= x^{(6+6)^2}|E_1(\Upsilon_3(t))| + x^{(6+12)^2}|E_2(\Upsilon_3(t))| + x^{(6+14)^2}|E_3(\Upsilon_3(t))| \\ &+x^{(8+10)^2}|E_4(\Upsilon_3(t))| + x^{(8+12)^2}|E_5(\Upsilon_3(t))| + x^{(8+14)^2}|E_6(\Upsilon_3(t))| + x^{(10+16)^2}|E_7(\Upsilon_3(t))| + x^{(12+12)^2}|E_8(\Upsilon_3(t))| + x^{(12+14)^2}| \\ &E_9(\Upsilon_3(t))| + x^{(12+16)^2}|E_{10}(\Upsilon_3(t))| + x^{(14+14)^2}|E_{11}(\Upsilon_3(t))| + \\ &x^{(14+16)^2}|E_{12}(\Upsilon_3(t))| + x^{(16+16)^2}|E_{13}(\Upsilon_3(t))|, \\ &= x^{(6+6)^2}(4t) + x^{(6+12)^2}(4t+4) + x^{(6+14)^2}(4t-4) + x^{(8+10)^2}(12t \\ &-12) + x^{(8+12)^2}(36t^2-44t+16) + x^{(8+14)^2}(4t-4) + x^{(10+16)^2} \\ &(4t-4) + x^{(12+12)^2}(8t) + x^{(12+14)^2}(8t-8) + x^{(12+16)^2}(36t^2- \\ &60t+16) + x^{(14+14)^2}(4t-4) + x^{(14+16)^2}(4t+4) + x^{(16+16)^2}(36 \\ &t^2-76t+40), \end{aligned}$$

By doing some calculations, we get

$$\begin{aligned} \Longrightarrow HM_1G_5(\Upsilon_3,x) &= 4x^{144}\big[t + (4t-2)x^{180} + (9t^2-10t+3)x^{256} + (t-1)x^{340} \\ &+2tx^{432} + 3(t-1)x^{532} + (9t^2-14t+3)x^{640} + (t+1)x^{756} \\ &+(t-1)(9t-10)x^{880}\big]. \end{aligned}$$

Also from (1.8),

$$HM_2^aG_5(\Upsilon_3,x) = \sum_{rs\in E(\Upsilon_3)} x^{(S_G(r)\times S_G(s))^2}.$$

$$\begin{aligned} HM_2^aG_5(\Upsilon_3,x) &= x^{(6\times6)^2}|E_1(\Upsilon_3(t))| + x^{(6\times12)^2}|E_2(\Upsilon_3(t))| + x^{(6\times14)^2}|E_3(\Upsilon_3(t) \\ &)| + x^{(8\times10)^2}|E_4(\Upsilon_3(t))| + x^{(8\times12)^2}|E_5(\Upsilon_3(t))| + x^{(8\times14)^2}|E_6(\\ &\Upsilon_3(t))| + x^{(10\times16)^2}|E_7(\Upsilon_3(t))| + x^{(12\times12)^2}|E_8(\Upsilon_3(t))| + \\ &x^{(12\times14)^2}|E_9(\Upsilon_3(t))| + x^{(12\times16)^2}|E_{10}(\Upsilon_3(t))| + x^{(14\times14)^2}|E_{11}(\\ &\Upsilon_3(t))| + x^{(14\times16)^2}|E_{12}(\Upsilon_3(t))| + x^{(16\times16)^2}|E_{13}(\Upsilon_3(t))|, \\ &= x^{(6\times6)^2}(4t) + x^{(6\times12)^2}(4t+4) + x^{(6\times14)^2}(4t-4) + x^{(8\times10)^2}(12 \\ &t-12) + x^{(8\times12)^2}(36t^2-44t+16) + x^{(8\times14)^2}(4t-4) + \\ &x^{(10\times16)^2}(4t-4) + x^{(12\times12)^2}(8t) + x^{(12\times14)^2}(8t-8) + x^{(12\times16)^2} \\ &(36t^2-60t+16) + x^{(14\times14)^2}(4t-4) + x^{(14\times16)^2}(4t+4) + \\ &x^{(16\times16)^2}(36t^2-76t+40), \end{aligned}$$

By making some calculations, we get

$$\begin{aligned} \Longrightarrow HM_2G_5(\Upsilon_3,x) &= 4tx^{1296} + (4t+4)x^{5184} + (12t-12)x^{6400} + (4t-4)x^{7056} \\ &+(36t^2-44t+16)x^{9216} + (4t-4)x^{12544} + 8tx^{20736} + (4t \\ &-4)x^{25600} + (8t-8)x^{28224} + (36t^2-60t+16)x^{36864} + (4t \\ &-4)x^{38416} + (4t+4)x^{50176} + (36t^2-76t+40)x^{65536}. \end{aligned}$$

□ □

4. Conclusion

Topological index is a numeric quantity which represents the structure of the graph. Topological indices underlying specific physical and chemical properties of chemical compounds are those which connect.In this article, we computed sum of degree-based indices for some derived graphs of honeycomb structure. We also computed certain sum of degree-based polynomials such as fifth M-Zagreb, fifth hyper M-Zagreb, Generalized fifth M-Zagreb indices for all types of Dominating David Derived graphs. These results are very useful for researchers working in computer science and chemistry, who encounter honeycomb graphs. These results can also play a vital part in the determination of the significance of honeycomb derived graphs. Like certain other topological indices, determining the representations of derived graphs like these is an open problem.

5. Acknowledgments

The authors are very grateful to the referees for their careful reading with corrections and useful comments, which improved this work very much.

References

[1] H. Ali, M. A. Binyamin, M. K. Shafiq, W. Gao, *On the Degree-Based Topological Indices of Some Derived Networks*, Mathematics, **7**, (2019) 612.

[2] H. Ali, A. Sajjad, *On further results of hex derived networks*, Open J. Discret. Appl. Math. **2** No. 2 (2019), 32-40.

[3] U. Baber, H. Ali, S. H. Arshad, U. Sheikh, *Multiplicative Topological Properties of Graphs Derived from Honeycomb Structure*, Aims Mathematics, **5**, No. 2 (2020) 1562-1587.

[4] A. Q. Baig, M. Imran, H. Ali, *Computing Omega, Sadhana and PI polynomials of benzoid carbon nanotubes*, Optoelectronics and advanced materials-Rapid Communications, **9**, No. 1-2 (2015), 248-255.

[5] A. Q. Baig, M. Imran, H. Ali, *On Topological Indices of Poly Oxide, Poly Silicate, DOX and DSL Networks*, Can. J. Chem. **93**, (2015) 730-739.

[6] A. Q. Baig, M. Imran, H. Ali, S. U. Rehman, *Computing topological polynomials of certain nanostructures*, J. Optoelectron. Adv. M., **17**, No. 5-6 (2015), 877-883.

[7] M. V. Diudea, I. Gutman, J. Lorentz, *Molecular Topology*, Nova Science Publishers: Huntington, NY, USA, 2001.

[8] M. Eliasi, A. Iranmanesh, I. Gutman, *Multiplicative versions of first Zagreb index*, Match-Communications in Mathematical and Computer Chemistry, **68**, No. 1 (2012) 217.

[9] W. Gao, M. K. Siddiqui, M. Naeem, M. Imran, *Computing multiple ABC index and multiple GA index of some grid graphs*, Open Physics, **16**, No. 1 (2018) 588-98.

[10] A. Graovac, M. Ghorbani, M. A. Hosseinzadeh, *Computing Degree Based Topological Properties of Third Type of Hex-Derived Networks*, J. of Math. Nanosci. **1**, (2011) 33-42.

[11] HoneyComb Structure [online]. Available from https://www.sciencedirect.com/topics/materials-science/honeycomb-structure

[12] M. Imran, A. Q. Baig, H. Ali, *On topological properties of dominating David derived networks*, Can. J. Chem. **94**, (2015), 137-148.

[13] M. Imran, A. Q. Baig, S. U. Rehman, H. Ali, R. Hasni, *Computing topological polynomials of mesh-derived networks*, Discret. Math. Algorithms Appl. **10**, (2018) 1850077.

[14] M. Imran, A. Q. Baig, H. M. A. Siddiqui, R. Sarwar, *On molecular topological properties of diamond like networks*, Can. J. Chem. **95**, (2017) 758-770.

[15] V. R. Kulli *General Fifth M-Zagreb Indices and Fifth M-Zagreb Polynomials of PAMAM Dendrimers*, Intern. J. Fuzzy Mathematical Archive, **22**, (2017), 99-103.

[16] V. R. Kulli, *Multiplicative hyper-Zagreb indices and coindices of graphs: Computing these indices of some nanostructures*, *International Research Journal of Pure Algebra.* **6**, No. 7 (2016) 342-347.
[17] J. B. Liu, H. Ali, M. K. Shafiq, U. Munir, On Degree-Based Topological Indices of Symmetric Chemical Structures. Symmetry, **10**, No. 11 (2018) 619.
[18] J. B. Liu, A. Q. Baig, W. Khalid, M. R. Farahni *Multiplicative indices of carbon graphite t-levels*, Comptes rendus de lAcadmie bulgare des Sciences, **17**, No. 1 (2018) 10-21.
[19] J. B. Liu, M. K. Shafiq, H. Ali, A. Naseem, N. Maryam, S. S. Asghar, *Topological Indices of mth Chain Silicate Graphs*, Mathematics, **7**, No. 1 (2019) 42.
[20] A. Nayak, I. Stojmenovic, *Hand Book of Applied Algorithms: Solving Scientific, Engineering, and Practical Problems*, John Wiley and Sons: Hoboken, NJ, USA, 2007.
[21] D. Plavšić, S. Nokolić, N. Trinajstić, Z. Mihalić, *On the Haray index for the Characterization of Chemical Graphs*, J. Math. Chem. **12**, (1993) 235-250.
[22] F. Simonraj, A. George, *Embedding of poly honeycomb networks and the metric dimension of star of david network*, GRAPH-HOC. **4**, (2012) 11-28.
[23] C. C. Wei, H. Ali, M. A. Binyamin, M. N. Naeem, J. B. Liu, *Computing Degree Based Topological Properties of Third Type of Hex-Derived Networks*, Mathematics, **7**, (2019), 368.
[24] H. Wiener, *Structural determination of paraffin boiling points*, J. Am. Chem. Soc. **69**, (1947) 17-20.