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An Efficient Method for Solving Intuitionistic Fuzzy Multi-objective Optimization Problems

Habtamu Tsegaye
Department of Mathematics,
Arba Minch University, Ethiopia,
Email: hbtmtsgy@gmail.com

Natesan Thillaigovindan
Department of Mathematics,
Arba Minch University, Ethiopia,
Email: thillaigovindan.natesan@gmail.com

Getinet Alemayehu
Department of Mathematics,
Haramaya University, Ethiopia,
Email: getalem2014@gmail.com

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Abstract.: Decision-making is one of the contemporary issues in this modern era due to the interaction of risk and uncertainties in every aspect of the daily lives of human beings. Accordingly, solving practical optimization problems tends to be more challenging. The fundamental reason for this investigation is to explore an effective solution technique for multi-objective optimization problems (MOOPs) in an intuitionistic fuzzy environment (IFE) addressing the issue of determining proper violation parameters and tolerances to the objectives and constraints. The other significant characteristic of this study is the consideration of the decision-maker's perspective, namely, optimistic, pessimistic and mixed views in the solution procedure. In the proposed method, compared to the existing study, the required number of iterations and stages are considerably reduced in solving intuitionistic fuzzy multi-objective optimization problems (IFMOOPs). Hence it has imperative advantages in solving complex real-world problems without much difficulty. One problem is solved to demonstrate the competency of the planned approach. A comparative analysis is also undertaken to ascertain the efficiency of the technique.

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1. INTRODUCTION

Most of the real-world issues are multi-objective in nature [19]. As a result, several classical optimization tools are proposed in operations research to manage MOOPs. Multi-objective optimization (MOO) techniques are successfully utilized in different areas to solve practical problems with a well-defined structure. But most real-world problems are often ill-defined, imprecise and uncertain due to inconsistent natural systems, errors of measurement, deficiency in statistical data, incomplete knowledge, lack of adequate information about the problem, subjectivity and preference of human judgment [5]. Hence, to cope with such situations in MOOPs, fuzzy set theory (FST) was applied.

FST initially proposed by Zadeh [22] in 1965. Later, Bellman and Zadeh [5] employed FST for issues of decision-making. Subsequently, Zimmermann [24] propounded the model of fuzzy linear programming problem. After practical investigation, Zimmermann and Zysno [23] explained that the geometric mean is an efficient tool for aggregating MOOPs in fuzzy environment compared to max-min and algebraic sum operators for its compensatory nature and in providing a better solution to the problems.

In fuzzy optimization, the best solution is identified utilizing solely membership degree of a variable to a set, considering non-membership degree as the complement of degree of membership. Although decision-making issues in a fuzzy environment were examined by many investigators, their studies were limited in scope and did not represent the real problems in their true nature. This is due to insufficient information, uncertainties and vagueness involved in every aspect of multi-objective problems [1]. Accordingly, it has been identified that there is an undetermined aspect of belongings of an element to a set which lead to several higher-order extensions. Among them, the most influential extension and development of fuzzy set is intuitionistic fuzzy set (IFS) presented by Atanassov [3] in 1983 and utilized in broad practical problems due to its capability to address uncertainties and vagueness of real-life situations as it comprises a degree of hesitancy in addition to degrees of acceptance and rejection.

The usage of an IFS to MOOPs was initially proposed by Angelov [2] in 1995, who later gave a detailed definition of the optimization procedure in an IFE. This technique depends on the maximization of the least membership degree and minimization of the highest non-membership degree, using the classical optimization model, as an intuitionistic fuzzy aggregation operator. Angelov's model [2] is an extension of Bellman and Zadeh's approach [5]. Later, Yager [21] pointed out the drawback of Angelov's intuitionistic fuzzy optimization (IFO) method [2] and developed a better model that conquers this issue. Soon after, Dubey et al. [7] formulated a model employing Yager's approach [21] that overcomes the apparent disadvantages in the existing methods.

Singh and Yadav [18] presented a framework to handle IFMOOPs by transforming the problems into their analogous crisp MOOPs utilizing accuracy function and then the problems changed into the fuzzy goal programming problems to find the solution. The result of their study showed that employing hyperbolic membership function provides a better solution compared to parabolic and linear membership functions.

Rani et al. [16] devised an IFO model incorporating Agelov's optimization method [2] and Yager's aggregation operator [21] to solve MOOP in an IFE under optimistic and pessimistic viewpoints. The centroid method is applied to defuzzify parabolic fuzzy numbers

which are used as coefficients of the functions in the problem.
A promising IFO model was formulated by Razmi et al. [17] comprising IFS, goal programming and interactive procedure to handle the difficulties of the stated strategies. Their aggregation operator is devised integrating Yager's operator [21], Chang's membership function approximation approach [6] and goal programming. The compromise solutions obtained by this technique fulfill the Pareto-optimality conditions.
Singh and Yadav [20] developed a novel model for solving IFMOOPs under optimistic, pessimistic and mixed decision-maker's viewpoints. In their study, coefficients of objectives and constraints of the problem are assumed to be left and right-type intuitionistic fuzzy numbers (IFN) and reference functions are used to describe the levels of satisfaction and dissatisfaction.
Recently Jafarian et al. [9] proposed an effective solution algorithm for multi-objective nonlinear programming problems in an IFE resolving the limitations of the existing methods. Their model integrated IFS, interactive optimization and geometric programming to find a compromise solution that satisfies the conditions of intuitionistic fuzzy efficiency and Pareto-optimality.

Several shortcomings of the methods in the literature are well addressed and resolved by Jafarian et al. [9]. But it has an apparent drawback in the determination of the values of violation parameters and tolerances for constraints and objectives in the solution procedure of IFMOOPs. Moreover, in most of the existing studies, the main standpoints of the decision-makers are not considered towards the optimization method of real-life problems. Motivated by these limitations, this study proposes to develop a method to solve MOOPs in an IFE in which the decision-makers can effectively fix the optional violation parameters and tolerance values that give an optimal solution of the problem as per his interest. Furthermore, efforts have been made to modify the existing approach by incorporating an IFO, interactive method and weighted geometric aggregation operator in the solution procedure from both methodological and computational points of view.

The remainder of the paper is structured as follows. In Section 2, fundamental concepts of IFMOOP are discussed briefly. In Section 3, the existing models with their respective drawbacks are described. Section 4 presents the proposed approach and states its advantages compared to the existing methods. To demonstrate the competency of the proposed approach, an illustrative example is presented in Section 5. In Section 6, a comparative study is given. Section 7 draws conclusions.

2. Preliminaries

MOO entails the concurrent optimization of more than one objective which are incommensurate and usually conflicting with each other under a set of constraints. In classical optimization, a MOOP represented as vector optimization can be modeled as [16]:

$$\begin{aligned}
&\max \{f_1(X), f_2(X), ..., f_{k_1}(X)\},\\
&\min \{f_{k_1+1}(X), f_{k_1+2}(X), ..., f_k(X)\}\\
&\qquad\qquad \text{subject to}\\
&g_i(X) \le c_i,\ i = 1, 2, \dots, m_1,\\
&g_i(X) \ge c_i,\ i = m_1 + 1, m_1 + 2, \dots, m_2,\\
&g_i(X) = c_i,\ i = m_2 + 1, m_2 + 2, ..., m,\\
&X \ge 0,
\end{aligned} \tag{2. 1}$$

where $f_j(X),\ j = 1, 2, ..., k$ and $g_i(X),\ i = 1, 2, ..., m$ are real-valued linear or nonlinear functions and they represent objectives and constraints respectively, X is n-dimensional decision vector $X = (x_1, x_2, ..., x_n)$, $\forall j = 1, 2, ..., k$ and $i = 1, 2, ..., m$.

Definition 2.1. [3] *Let X be a crisp collection of objects called the universe and x be an element of X. An IFS $\tilde{A}$ in X is given by*

$$\tilde{A} = \{< x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) > : x \in X\}$$

where $\mu_{\tilde{A}}(x), v_{\tilde{A}}(x), \pi_{\tilde{A}}(x)\colon X \to [0, 1]$ such that $0 \le \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \le 1$, $\pi_{\tilde{A}}(x) = 1-(\mu_{\tilde{A}}(x)+v_{\tilde{A}}(x))$ for every $x \in X$. The value $\mu_{\tilde{A}}(x)$ indicates the degree of membership of x in $\tilde{A}$, $v_{\tilde{A}}(x)$ indicates the degree of non-membership of x in $\tilde{A}$ and $\pi_{\tilde{A}}(x)$ indicates the degree of indeterminacy of x being in $\tilde{A}$.

Definition 2.2. [14] *An IFS $\tilde{A}$ is said to be an intuitionistic fuzzy number (IFN) if*

(i) *$\tilde{A}$ is an intuitionistic fuzzy subset of $\mathbb{R}$,*
(ii) *$\exists\, x_1, x_2 \in \mathbb{R}$: $\mu_{\tilde{A}}(x_1) = 1$ and $v_{\tilde{A}}(x_2) = 1$,*
(iii) *$\tilde{A}$ is an intuitionistic fuzzy convex; i.e., $\forall x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1]$*

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \ge \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \quad \textit{and}$$

$$v_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \le \max(v_{\tilde{A}}(x_1), v_{\tilde{A}}(x_2)),$$

(iv) *$\mu_{\tilde{A}}(x), v_{\tilde{A}}(x) : \mathbb{R} \to [0, 1]$ are upper and lower semi-continuous functions, respectively.*

Definition 2.3. [19] *A triangular IFN $\tilde{A}$ is an IFS and denoted by $\tilde{A} =< a_1, a_2, a_3; a'_1, a_2, a'_3 >$, where $a'_1 \le a_1 \le a_2 \le a_3 \le a'_3$, with the following membership and non-membership functions:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \textit{if } a_1 < x \le a_2\\ \frac{a_3-x}{a_3-a_2}, & \textit{if } a_2 \le x < a_3\\ 0, & \textit{otherwise}, \end{cases}$$

and

$$v_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1}, & \textit{if } a'_1 < x \le a_2\\ \frac{x-a_2}{a'_3-a_2}, & \textit{if } a_2 \le x < a'_3\\ 1, & \textit{otherwise}. \end{cases}$$

Definition 2.4. [15] *Let $\tilde{A}$ be a triangular IFN, then the score function value for membership function $\mu_{\tilde{A}}$ is denoted by $\mathfrak{s}(\mu_{\tilde{A}})$ and defined as $\mathfrak{s}(\mu_{\tilde{A}}) = \frac{a_1+2a_2+a_3}{4}$ and also the score function value for non-membership function $v_{\tilde{A}}$ is denoted by $\mathfrak{s}(v_{\tilde{A}})$ and defined as $\mathfrak{s}(v_{\tilde{A}}) = \frac{a'_1+2a_2+a'_3}{4}$.*
The accuracy function of $\tilde{A}$ is designated by $\Gamma(\tilde{A})$ and described as $\Gamma(\tilde{A}) = \frac{\mathfrak{s}(\mu_{\tilde{A}})+\mathfrak{s}(v_{\tilde{A}})}{2} = \frac{(a_1+2a_2+a_3)+(a'_1+2a_2+a'_3)}{8}$. The accuracy value is used to defuzzify the given triangular IFN.

The intuitionistic fuzzy version of the above classical MOOP in (2.1) is called intuitionistic fuzzy multi-objective optimization problem (IFMOOP) and represented as [16]:

$$\begin{aligned} &\max\{\widetilde{f}_1(X), \widetilde{f}_2(X), ..., \widetilde{f}_{k_1}(X)\}, \\ &\min\{\widetilde{f}_{k_1+1}(X), \widetilde{f}_{k_1+2}(X), ..., \widetilde{f}_k(X)\} \\ &\qquad\qquad \text{subject to} \\ &\widetilde{g}_i(X) \precsim \widetilde{c}_i,\ i = 1, 2, \ldots, m_1, \\ &\widetilde{g}_i(X) \succsim \widetilde{c}_i,\ i = m_1+1, m_1+2, \ldots, m_2, \\ &\widetilde{g}_i(X) \approx \widetilde{c}_i,\ i = m_2+1, m_2+2, \ldots, m, \\ &X \geq 0, \end{aligned} \tag{2. 2}$$

where the functions $\widetilde{f}_j(X)$ and $\widetilde{g}_i(X)$ are objectives and constraints respectively for all vector of decision variables $X \subseteq \mathbb{R}^n$, $\forall j = 1, 2, \ldots, n$ and $\forall i = 1, 2, \ldots, m$ in an IFE and $\precsim$, $\succsim$ and $\approx$ respectively denote the intuitionistic fuzzy version of $\leq$, $\geq$ and $=$ in the classical optimization.

For the sake of a simple presentation of the subsequent definitions (Definitions 2.5 - 2.7), we rewrite problem (2.2) as a maximization problem constrained by less than or equal to intuitionistic fuzzy inequalities to get the IFMOOP as

$$\begin{aligned} &\max\{\widetilde{f}_1(X), \widetilde{f}_2(X), ..., \widetilde{f}_{k_1}(X)\} \\ &\qquad\qquad \text{subject to} \\ &\widetilde{g}_i(X) \precsim \widetilde{c}_i,\ i = 1, 2, \ldots, m_1, \\ &X \geq 0. \end{aligned} \tag{2. 3}$$

Let $\mathbb{S}$ be the set of all feasible solutions of problem (2.3), then the solutions of this problem can be categorized depending on the conditions they fulfill. These are presented in Definitions 2.5 to 2.7.

Definition 2.5. [9] *A solution $X^* \in \mathbb{S}$ is said to be a complete optimal solution of problem (2.3) provided that there is no another $X \in \mathbb{S}$ such that*
$\widetilde{f}_j(X^*) \precsim \widetilde{f}_j(X) \wedge (\mu_i(g_i(X)) \geq \mu_i(g_i(X^*)) \wedge v_i(g_j(X)) \leq v_i(g_i(X^*)), \forall j = 1, 2, \ldots, k_1,$ $\forall i = 1, 2, \ldots, m_1$*, where the functions $\mu_i(g_i(X^*)$ and $v_i(g_i(X^*)$ describe the membership and non-membership of the i^{th} constraint at X^*, respectively.*

But since there are two or more contradictory objectives in MOOP, it is not possible to get such a complete optimal solution that concurrently optimizes all objectives.

Definition 2.6. [9] *A solution $X^* \in \mathbb{S}$ is said to be a Pareto-optimal solution (POS) of problem (2.3) if there is no another $X \in \mathbb{S}$ such that*
$\widetilde{f}_j(X^*) \precsim \widetilde{f}_j(X) \wedge (\mu_i(g_i(X)) \geq \mu_i(g_i(X^*)) \wedge v_i(g_i(X)) \leq v_i(g_i(X^*))), \forall j = 1, 2, \ldots, k_1$, $\forall i = 1, 2, \ldots, m_1$ *and* $\widetilde{f}_j(X^*) \prec \widetilde{f}_j(X) \vee (\mu_i(g_i(X)) > \mu_i(g_i(X^*)) \vee v_i(g_i(X)) < v_i(g_i(X^*)))$, $\exists j \in \{1, 2, \ldots, k_1\}, \exists i \in \{1, 2, \ldots, m_1\}$.

That means, a solution is said to be POS, if the functional value(s) at POS cannot be improved without worsening the value(s) of other function(s) at that point.

Definition 2.7. [17] *A solution $X^* \in \mathbb{S}$ is called an intuitionistic fuzzy efficient solution (IFES) of problem (2.3) if there is no another $X \in \mathbb{S}$ such that*
$(\mu_j(f_j(X^*)) \leq \mu_j(f_j(X)) \wedge v_j(f_j(X^*)) \geq v_j(f_j(X))) \wedge (\mu_i(g_i(X)) \geq \mu_i(g_i(X^*)) \wedge v_i(g_j(X)) \leq v_i(g_i(X^*)))$, $\forall j = 1, 2, \ldots, k_1, \forall i = 1, 2, \ldots, m_1$ *and*
$(\mu_j(f_j(X^*)) < \mu_j(f_j(X)) \vee v_j(f_j(X^*)) > v_j(f_j(X))) \vee (\mu_i(g_i(X)) > \mu_i(g_i(X^*)) \vee v_i(g_j(X)) < v_i(g_i(X^*)))$, $\exists j \in \{1, 2, \ldots, k_1\}, \exists i \in \{1, 2, \ldots, m_1\}$,
where the functions $\mu_j(f_j(X^*)$ and $v_j(f_j(X^*)$ describe the membership and non-membership of the j^{th} objective at X^*, respectively.

An IFES describes the point at which the degrees of acceptances and rejections of the problem are optimized, hence several alternative points can be obtained which satisfy these conditions when one or more degree(s) is(are) fully achieved.
Every POS is an IFES. However, for the case when the value(s) of membership degree is (are) one, the converse need not necessarily be true [10].

3. Existing Models for Solving IFMOOP

As mentioned, most of the existing methods designed to solve an IFMOOP are devoted to search an IFES neglecting the aspect of Pareto-optimality. Thereby the existing models presented below are characterized by this major defect until Razmi et al. [17] pointed out lately. Like fuzzy optimization, the process of solving IFO problem comprises two stages, namely, aggregation of objectives and constraints and then defuzzification to form a crisp optimization problem. Most techniques in the literature aim to maximize the minimum membership degree and minimize the maximum non-membership degree of the intuitionistic fuzzy objectives and constraints simultaneously. This concept was initially presented by Angelov [2] using crisp optimization model as presented in (3.4).
The objective of problem (3.4) is maximizing the minimum satisfaction and minimizing the maximum dissatisfaction of the objectives and constraints simultaneously. Since it aims to optimize the worst circumstances, it doesn't guarantee compromise and POS.

$$\begin{aligned}
&\max\ (\alpha-\beta)\\
&\quad\text{subject to}\\
&\mu_j(f_j(X))\geq\alpha,\\
&v_j(f_j(X))\leq\beta,\\
&\mu_i(g_i(X))\geq\alpha,\\
&v_i(g_i(X))\leq\beta,\\
&\alpha+\beta\leq 1,\\
&\alpha\geq\beta,\\
&\beta\geq 0,\\
&X\geq 0,
\end{aligned}\tag{3. 4}$$

where $\alpha=\min\{\mu_j(f_j(X)),\mu_i(g_i(X))\},\ \beta=\max\{v_j(f_j(X)),v_i(g_i(X))\},\ j=1,2,...,k$ and $i=1,2,...,m$.

Yager [21] suggested an alternative approach that overcomes drawbacks of Angelov's IFO method [2] and proposed a value function that takes indeterminacy into account and defined as:

$$\psi_{\tilde{D}}(X)=\mu_{\tilde{D}}(X)+\Lambda\pi_{\tilde{D}}(X) \tag{3. 5}$$

where $\Lambda\in[0,1]$.
Though this model resolved the main demerits of the approach in (3.4), it is not a compensatory value function.

Employing the aggregation operators in (3.4) and (3.5), Rani et al. [16] developed an IFO model that overcomes the difficulties of these approaches. Their model, for an optimistic viewpoint, is as follows:

$$\max\ \phi$$
subject to
$$
\begin{aligned}
&(1-\lambda)(\frac{(f_j(X))^t - L_j^t}{U_j^t - L_j^t}) - \lambda(\frac{U_j^t-(f_j(X))^t}{U_j^t-(L_j-\lambda_j)^t}) + \lambda \geq \phi,\ j = 1,2,...,k_1,\\
&\lambda - \lambda(\frac{U_j^t-(f_j(X))^t}{U_j^t-(L_j-\alpha_j)^t}) \geq \phi,\ j=1,2,...,k_1,\\
&(1-\lambda)(\frac{U_j^t-(f_j(X))^t}{U_j^t - L_j^t}) - \lambda(\frac{(f_j(X))^t - L_j^t}{(U_j+\lambda_j)^t - L_j^t}) + \lambda \geq \phi,\\
&\qquad\qquad\qquad\qquad j = k_1+1, k_1+2, ..., k,\\
&\lambda - \lambda(\frac{(f_j(X))^t - L_j^t}{(U_j+\lambda_j)^t - L_j^t}) \geq \phi,\ j = k_1+1,k_1+2,...,k,\\
&g_i(X) \leq c_i,\ i = 1,2,\cdots,m_1,\\
&g_i(X) \geq c_i,\ i = m_1+1, m_1+2,\cdots,m_2,\\
&g_i(X) = c_i,\ i = m_2+1, m_2+2,\cdots,m,\\
&X \geq 0,
\end{aligned}
\tag{3. 6}
$$

where $\phi = \min\{\psi_{L_j}(f_j(X)), \psi_{U_j}(f_j(X))\}, \lambda \in (0,1], j = 1,2,...,k$.

Singh and Yadev [20] explained the shortcoming of functional value in the model (3.5) and proposed a new aggregation operator by extending Atanassov's point operator [4] to conquer the negative aspects of the above methods. They gave the following model for solving IFMOOP:

$$\max\ \eta$$
subject to
$$
\begin{aligned}
&\alpha + \frac{\lambda(1-\alpha-\beta)}{2-\alpha-\beta} \geq \eta,\\
&\mu_{U_j}(f_j(X)) \geq \alpha,\ j=1,2,...,k_1,\\
&v_{U_j}(f_j(X)) \leq \beta,\ j=1,2,...,k_1,\\
&\mu_{L_j}(f_j(X)) \geq \alpha,\ j=k_1+1,k_1+2,...,k_1,\\
&v_{L_j}(f_j(X)) \leq \beta,\ j=k_1+1,k_1+2,...,k_1,\\
&g_i(X) \leq c_i,\ i = 1,2,\cdots,m_1,\\
&g_i(X) \geq c_i,\ i = m_1+1, m_1+2,\cdots,m_2,\\
&g_i(X) = c_i,\ i = m_2+1, m_2+2,\cdots,m,\\
&X \geq 0,
\end{aligned}
\tag{3. 7}
$$

where $\alpha = \min(\mu_j(f_j(X))$ and $\beta = \max(v_j(f_j(X)), \lambda \in (0,1],\ j=1,2,...,k$.
The main disadvantages of the approaches in (3.6) and (3.7) are the use of non-compensatory operators and the degrees of satisfaction and dissatisfaction of the constraints are not considered in the objective functions.

Although the above mentioned methods have their own important advantages, their main pitfall is that they consider the worst situation among the levels of acceptances and rejections in the IFMOOP as objectives and based on this the optimal solution is found. Thus the obtained solution couldn't be a compromise solution.

Jafarian et al. [9] formulated an effectual IFO model and designed a two-stage solution procedure to find a compromised solution that satisfies both the intuitionistic fuzzy efficiency and Pareto-optimality conditions. Their model is presented as follows:

$$\begin{aligned}
&\min \left(\prod_{j=1}^{k}\rho_j^{w_j}\prod_{i=1}^{m}\rho_i^{w_i}\right)^{\Lambda}\left(\prod_{j=1}^{k}\varepsilon_j^{w_j}\prod_{i=1}^{m}\varepsilon_i^{w_i}\right)^{1-\Lambda}\\
&\quad\text{subject to}\\
&\mu_j(f_j(X)) \geq \rho_j^{-1},\\
&v_j(f_j(X)) + \varepsilon_j^{-1} \leq 1,\\
&\mu_i(g_i(X)) \geq \rho_i^{-1},\\
&v_i(g_i(X)) + \varepsilon_i^{-1} \leq 1,\\
&\rho_j^{-1} \geq 1,\\
&\varepsilon_j^{-1} \geq 1,\\
&\rho_i^{-1} \geq 1,\\
&\varepsilon_i^{-1} \geq 1,\\
&X > 0,
\end{aligned} \tag{3. 8}$$

where $\rho_j = \frac{1}{\alpha_j},\ \varepsilon_j = \frac{1}{1-\beta_j},\ \rho_i = \frac{1}{\alpha_i},\ \varepsilon_i = \frac{1}{1-\beta_i},\ \rho_j \geq 1,\ \varepsilon_j \geq 1,\ \rho_i \geq 1,\ \varepsilon_i \geq 1,$ w_i and w_j are weights assigned to objectives and constraints respectively, $j = 1, 2, ..., k$ and $i = 1, 2, ..., m$ and $\Lambda \in [0, 1]$ is the fuzzification parameter.

Model (3.8) has several important advantages and overcame many of the shortcomings mentioned in the existing methods in the literature. But it involves computational burden and critical analysis of the problem to determine suitable violation parameters and tolerances for the objectives and constraints in order to find a Pareto-optimal solution to the problem.

4. THE PROPOSED SOLUTION APPROACH

To address the detailed realistic features of physical world problems, we are considering decision-making problems under three points of view depending on the attitude of the decision-maker, namely, optimistic, pessimistic and mixed in which the decision maker realizes the problem and accordingly the levels of acceptance, rejection and hesitation may vary to some extent in the solution process. Thus, based on views of a decision-maker, membership and non-membership functions corresponding to each constraint and objective functions are described by introducing new parameters of violations and tolerance values as described below.

Inclusion of the levels of acceptance and rejection of each constraint besides the levels of acceptance and rejection of each objective into the aggregation operator of IFO helps to pursue better utilization of each constrained resource. However, the majority of the existing approaches didn't consider the issue of reaching a better utilization of limited resources. Thus, in this study, we discuss this aspect as well in detail.

In the mathematical expression of IFMOOP, all or some of the coefficients of objectives and constraints are IFN and also intuitionistic fuzzy inequalities and equalities are used in the description of constraints to make flexible feasible regions. As a result of these, firstly each IFN, triangular IFN in our case, has to be defuzzified using accuracy function (Definition 2.4) and each flexible inequities and equalities replaced by analogous rigid inequities and equalities to reformulate the IFMOOP into its equivalent crisp MOOP (2.1). Then the degrees of acceptance and rejection of each objective and constraint are described to form a single objective classical optimization problem. The solution of the IFMOOP can be found by solving the analogous classical optimization problem.

Since the solution of IFMOOP is greatly affected by the set of constraints, characterization of constraints through membership and non-membership functions are vital in the solution process to employ it into the solution mix beyond identifying the optimal level of utilized resources. In order to describe efficiently the membership and non-membership functions for constraints of IFMOOP, an effective method is proposed to generate different alternative violation parameters and tolerance values to each constraint. The procedure is presented below.

1. Determine an appropriate non-negative violation parameter ℓ_i for the i^{th} constraint $g_i(X)$.
2. Generate different alternative tolerance values ξ_i and κ_i for each constraint, where $\xi_i = \lambda(\ell_i)$, $\kappa_i = \lambda(\ell_i + \xi_i)$, $\lambda \in (0, 1)$ and c_i is the aspersion level of the i^{th} constraint.
3. Construct the membership and non-membership functions for each constraint based on the viewpoint of decision-maker.

The description of membership and non-membership functions for constraints of IFMOOP with regard to decision-maker's viewpoints are presented in the subsequent subsections.

The general procedure for construction of membership and non-membership functions for the objective functions are listed below:

1. Solve the MOOP (2.1) using an appropriate classical optimization method considering only one objective function, ignoring all other objective functions, at a time subject to the given constraints. Repeat the procedure for each objective. Let the optimal solutions obtained for each objective function be $X_1, X_2, ..., X_k$ respectively, so that $\mathbb{X} = \{X_1, X_2, ..., X_k\}$.
2. Employing the resulting solution vectors in Step 1, find the values of objective functions $f_j(X), j = 1, 2, ..., k$, at each point $X_1, X_2, ..., X_k$ respectively as illustrated in Table 1. In case that no optimal solution can be found for at least one of the objectives for the reason that it is not bounded or doesn't have a feasible solution, the decision-maker needs to remodel the problem.
3. Find the least and highest values of each objective function in the tabulated values of objective functions in Step 2. For maximization type MOOP, the diagonal values of the payoff table contain the maximum achievable values of objective functions, whereas the minimum value of each objective function is determined by choosing

the minimum value in each column of the table of results. Let L_j be the least and U_j be the highest values of $f_j(X)$, i.e., $L_j = \min\{f_j(X) : X \in \mathbb{X}\}$ and $U_j = \max\{f_j(X) : X \in \mathbb{X}\}$.

4. Construct the membership and non-membership functions for each objective with the help of newly introduced tolerance variables δ_j and ζ_j, where $\delta_j = \lambda(U_j - L_j)$, $\zeta_j = \lambda(U_j - (L_j - \delta_j))$ and $\lambda \in (0,1)$ based on the view of a decision-maker.

TABLE 1. Payoff table.

Solution	Objectives				expended resources			
X	$f_1(X)$	$f_2(X)$	$\dots$	$f_k(X)$	c_1	c_2	...	c_m
X_1	$f_1(X_1)$	$f_2(X_1)$	$\dots$	$f_k(X_1)$	$g_1(X_1)$	$g_2(X_1)$	...	$g_m(X_1)$
X_2	$f_1(X_2)$	$f_2(X_2)$	$\dots$	$f_k(X_2)$	$g_1(X_2)$	$g_2(X_2)$	...	$g_m(X_2)$
.	.	.	$\dots$	.	.	.	...	.
.	.	.	$\dots$	.	.	.	...	.
.	.	.	$\dots$	.	.	.	...	.
X_k	$f_1(X_k)$	$f_2(X_k)$	$\dots$	$f_k(X_k)$	$g_1(X_2)$	$g_2(X_2)$	...	$g_m(X_k)$

Since $f_j(X) : j = 1, 2, ..., k_1$ represent maximization of objectives, the level of acceptance increases as each functional value approaches its respective highest value U_j and hence decision-maker is fully satisfied if all objective functions attain their respective highest values. But mostly that attainment of exact value of these highest values is doubtful. As a result of this depending on the decision-maker's verdict the attainability degree ($\mu_{U_j}(f_j(X))$) and non-attainability degree ($v_{U_j}(f_j(X))$) of the utmost value can be elucidated using optimistic, pessimistic and mixed point of views. On the other hand, since $f_j(X) : j = k_1 + 1, k_1 + 2, ..., k$ represent minimization of objectives, the level of satisfaction increases as each functional value approaches its respective least value L_j and if all the objectives attain their respective least values then the decision-maker is fully satisfied. However, attaining these least values is doubtful. Because of this the degrees of attainability ($\mu_{L_j}(f_j(X))$) and non-attainability ($v_{L_j}(f_j(X))$) of the lower bound L_j for minimization problem also interpreted based on the attitude of decision-makers like maximization problem.

The expression of the membership function is the same for the three viewpoints, whereas the non-membership function expression is dissimilar under each approach as discussed in the succeeding subsections.

In MOOP degrees of satisfaction and dissatisfaction between the values of lower and upper bounds are characterized by nonlinear functions since the judgment and interest of decision maker instantly vary at a specific point of these values. Accordingly, the shape of membership and non-membership functions of actual problems are most likely nonlinear. Thus, in the present study nonlinear membership and non-membership functions are employed to describe the satisfaction and dissatisfaction levels of decision-maker in the process of achieving the aspired values and effective utilization of limited resources.

4.1. **The optimistic approach.** In this approach, a decision-maker takes a liberal view about rejection. It implies that even if the degree of acceptance of X is zero, the decision-maker is not reject it fully. Therefore, for certain tolerances, membership function ($\mu_{c_i}(g_i(X))$)

and non-membership function ($v_{c_i}(g_i(X))$) of the i^{th} constraint $g_i(X)$ in optimistic approach for less than or equal to $(\leq)$ type constraints are respectively defined as follows:

$$\mu_{c_i}(g_i(X)) = \begin{cases} 1, & \text{if } g_i(X) \leq c_i \\ \frac{(c_i+\ell_i)^t - (g_i(X))^t}{(c_i+\ell_i)^t - c_i^t}, & \text{if } c_i < g_i(X) < c_i + \ell_i \\ 0, & \text{if } g_i(X) \geq c_i + \ell_i, \end{cases} \quad (4.\ 9)$$

and

$$v_{c_i}(g_i(X)) = \begin{cases} 0, & \text{if } g_i(X) \leq c_i \\ \frac{(g_i(X))^t - c_i^t}{(c_i+\ell_i+\xi_i)^t - c_i^t}, & \text{if } c_i < g_i(X) < c_i + \ell_i + \xi_i \\ 1, & \text{if } g_i(X) \geq c_i + \ell_i + \xi_i, \end{cases} \quad (4.\ 10)$$

where c_i is the aspiration level for the i^{th} constraint, ℓ_i is subjectively chosen non-negative constant violation of the i^{th} constraint, t is positive real number assigned by decision-maker and used to describe the non-linearity of membership and non-membership functions, ξ_i is tolerance value of the i^{th} constraint and defined as $\xi_i = \lambda(\ell_i)$, $\lambda \in (0,1)$ and $i = 1,2,...,m_1$.

Likewise, membership function ($\mu_{c_i}(g_i(X))$) and non-membership function $(v_{c_i}(g_i(X)))$ of the i^{th} constraint function $g_i(X)$ for greater than or equal to $(\geq)$ type constraints are respectively defined as follows:

$$\mu_{c_i}(g_i(X)) = \begin{cases} 0, & \text{if } g_i(X) \leq c_i - \ell_i \\ \frac{(g_i(X))^t - (c_i-\ell_i)^t}{c_i^t - (c_i-\ell_i)^t}, & \text{if } c_i - \ell_i < g_i(X) < c_i \\ 1, & \text{if } g_i(X) \geq c_i, \end{cases} \quad (4.\ 11)$$

and

$$v_{c_i}(g_i(X)) = \begin{cases} 1, & \text{if } g_i(X) \leq c_i - \ell_i - \xi_i \\ \frac{c_i^t - (g_i(X))^t}{c_i^t - (c_i-\ell_i-\xi_i)^t}, & \text{if } c_i - \ell_i - \xi_i < g_i(X) < c_i \\ 0, & \text{if } g_i(X) \geq c_i, \end{cases} \quad (4.\ 12)$$

where c_i is the aspiration level for the i^{th} constraint, ℓ_i is subjectively chosen non-negative constant violation of the i^{th} constraint, ξ_i is tolerance value of the i^{th} constraint and defined as $\xi_i = \lambda(\ell_i)$, $\lambda \in (0,1)$ and $i = m_1 + 1, m_1 + 2, ..., m_2$.

The membership function ($\mu_{U_j}(f_j(X))$) and non-membership function ($v_{U_j}(f_j(X))$) of the j^{th} objective function $f_j(X)$ in optimistic approach to maximization problem are respectively defined as follows:

$$\mu_{U_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq L_j \\ \frac{(f_j(X))^t - L_j^t}{U_j^t - L_j^t}, & \text{if } L_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \geq U_j, \end{cases} \quad (4.\ 13)$$

and

$$v_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j - \delta_j \\ \frac{U_j^t - (f_j(X))^t}{U_j^t - (L_j-\delta_j)^t}, & \text{if } L_j - \delta_j < f_j(X) < U_j \\ 0, & \text{if } f_j(X) \geq U_j, \end{cases} \quad (4.\ 14)$$

where δ_j is a tolerance value of the j^{th} objective and defined as $\delta_j = \lambda(U_j - L_j)$, $\lambda \in (0,1)$ and $j = 1, 2, 3, ..., k_1$. Here it is important to notice that if $U_j = L_j$, then we define $\mu_{U_j}(f_j(X)) = 1$ for any j.

Their possible general shape is shown in Fig. 1(a).

Similarly, the membership function $(\mu_{L_j}(f_j(X)))$ and non-membership function $(v_{L_j}(f_j(X)))$ of the j^{th} objective function $f_j(X)$ for minimization problem are respectively defined as follows:

$$\mu_{L_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j \\ \frac{U_j^t - (f_j(X))^t}{U_j^t - L_j^t}, & \text{if } L_j < f_j(X) < U_j \\ 0, & \text{if } f_j(X) \geq U_j, \end{cases} \tag{4. 15}$$

and

$$v_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq L_j \\ \frac{(f_j(X))^t - L_j^t}{(U_j + \delta_j)^t - L_j^t}, & \text{if } L_j < f_j(X) < U_j + \delta_j \\ 1, & \text{if } f_j(X) \geq U_j + \delta_j, \end{cases} \tag{4. 16}$$

where δ_j is a tolerance value of the j^{th} objective and defined as $\delta_j = \lambda(U_j - L_j)$, $\lambda \in (0,1)$ and $j = k_1 + 1, k_1 + 2, k_1 + 3, ..., k$. Their possible general shape is shown in Fig. 1(b).

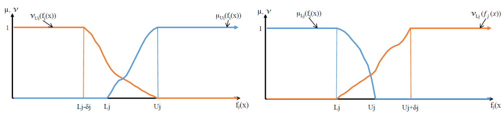


Fig. 1. Membership and non-membership functions for maximization (a) and minimization (b) objectives.

4.2. **The pessimistic approach.** In a pessimistic approach, a decision-maker is presumably extra cautious for acceptance. That is, even if the degree of rejection of X is zero, the decision-maker is not ready to accept it fully. In this approach, the membership function $(\mu_{c_i}(g_i(X)))$ and non-membership function $v_{c_i}(g_i(X)))$ of the i^{th} constraint function $g_i(X)$ for less than or equal to $(\leq)$ type constraints are respectively defined as follows:

$$\mu_{c_i}(g_i(X)) = \begin{cases} 1, & \text{if } g_i(X) \leq c_i \\ \frac{(c_i + \ell_i)^t - (g_i(X))^t}{(c_i + \ell_i)^t - c_i^t}, & \text{if } c_i < g_i(X) < c_i + \ell_i \\ 0, & \text{if } g_i(X) \geq c_i + \ell_i, \end{cases} \tag{4. 17}$$

and

$$v_{c_i}(g_i(X)) = \begin{cases} 0, & \text{if } g_i(X) \leq c_i + \ell_i - \xi_i \\ \frac{(g_i(X))^t - (c_i + \ell_i - \xi_i)^t}{(c_i + \ell_i)^t - (c_i + \ell_i - \xi_i)^t}, & \text{if } c_i + \ell_i - \xi_i < g_i(X) < c_i + \ell_i \\ 1, & \text{if } g_i(X) \geq c_i + \ell_i, \end{cases} \tag{4. 18}$$

where c_i is the aspiration level for the i^{th} constraint, ℓ_i is subjectively chosen non-negative constant violation of the i^{th} constraint, ξ_i is tolerance value of the i^{th} constraint and defined as $\xi_i = \lambda(\ell_i)$, $\lambda \in (0,1)$ and $i = 1, 2, ..., m_1$.

In the same way, membership function $(\mu_{c_i}(g_i(X)))$ and non-membership function

$(v_{c_i}(g_i(X)))$ of the i^{th} constraint function $g_i(X)$ for greater than or equal to $(\geq)$ type constraints are respectively defined as follows:

$$\mu_{c_i}(g_i(X)) = \begin{cases} 0, & \text{if } g_i(X) \leq c_i - \ell_i \\ \frac{(g_i(X))^t - (c_i-\ell_i)^t}{c_i^t - (c_i-\ell_i)^t}, & \text{if } c_i - \ell_i < g_i(X) < c_i \\ 1, & \text{if } g_i(X) \geq c_i, \end{cases} \tag{4. 19}$$

and

$$v_{c_i}(g_i(X)) = \begin{cases} 1, & \text{if } g_i(X) \leq c_i - \ell_i \\ \frac{(c_i-\ell_i+\xi_i)^t - (g_i(X))^t}{(c_i-\ell_i+\xi_i)^t - (c_i-\ell_i)^t}, & \text{if } c_i - \ell_i < g_i(X) < c_i - \ell_i + \xi_i \\ 0, & \text{if } g_i(X) \geq c_i - \ell_i + \xi_i, \end{cases} \tag{4. 20}$$

where c_i is the aspiration level for the i^{th} constraint, ℓ_i is subjectively chosen non-negative violation parameter of the i^{th} constraint, ξ_i is the tolerance value of the i^{th} constraint and defined as $\xi_i = \lambda(\ell_i)$, $\lambda \in (0,1)$ and $i = m_1+1, m_1+2, ..., m_2$.

The membership function ($\mu_{U_j}(f_j(X))$) and non-membership function $(v_{U_j}(f_j(X)))$ of the j^{th} objective function $f_j(X)$ in pessimistic approach to maximization problem are respectively defined as follows:

$$\mu_{U_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq L_j \\ \frac{(f_j(X))^t - L_j^t}{U_j^t - L_j^t}, & \text{if } L_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \geq U_j, \end{cases} \tag{4. 21}$$

and

$$v_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j \\ \frac{(L_j+\delta_j)^t - (f_j(X))^t}{(L_j+\delta_j)^t - L_j^t}, & \text{if } L_j < f_j(X) < L_j + \delta_j \\ 0, & \text{if } f_j(X) \geq L_j + \delta_j, \end{cases} \tag{4. 22}$$

where δ_j is a tolerance value of the j^{th} objective and defined as $\delta_j = \lambda(U_j - L_j)$, $\lambda \in (0,1)$ and $j = 1, 2, 3, ..., k_1$. Their possible general shape is shown in Fig. 2(a).
On the other hand, the membership function ($\mu_{L_j}(f_j(X))$) and non-membership function $(v_{L_j}(f_j(X)))$ of the j^{th} objective function $f_j(X)$ for minimization problem are respectively defined as follows:

$$\mu_{L_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j \\ \frac{U_j^t - (f_j(X))^t}{U_j^t - L_j^t}, & \text{if } L_j < f_j(X) < U_j \\ 0, & \text{if } f_j(X) \geq U_j, \end{cases} \tag{4. 23}$$

and

$$v_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq U_j - \delta_j \\ \frac{(f_j(X))^t - (U_j-\delta_j)^t}{U_j^t - (U_j-\delta_j)^t}, & \text{if } U_j - \delta_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \geq U_j, \end{cases} \tag{4. 24}$$

where δ_j is a tolerance value of the j^{th} objective and defined as $\delta_j = \lambda(U_j - L_j)$, $\lambda \in (0,1)$ and $j = k_1+1, k_1+2, k_1+3, ..., k$. Their possible general shape is shown in Fig. 2(b).

4.3. **The mixed approach.** In a mixed approach, a decision-maker is not flexible to reject and also not capable for extra acceptance. The membership function $(\mu_{c_i}(g_i(X)))$ and non-membership function $(v_{c_i}(g_i(X)))$ of the i^{th} constraint function $g_i(X)$ in this approach for less than or equal to $(\leq)$ type constraints are respectively defined as follows:

$$\mu_{c_i}(g_i(X)) = \begin{cases} 1, & \text{if } g_i(X) \leq c_i \\ \frac{(c_i+\ell_i)^t-(g_i(X))^t}{(c_i+\ell_i)^t-c_i^t}, & \text{if } c_i < g_i(X) < c_i+\ell_i \\ 0, & \text{if } g_i(X) \geq c_i+\ell_i, \end{cases} \tag{4. 25}$$

and

$$v_{c_i}(g_i(X)) = \begin{cases} 0, & \text{if } g_i(X) \leq (c_i+\ell_i)-\kappa_i \\ \frac{(g_i(X))^t-((c_i+\ell_i)-\kappa_i)^t}{(c_i+\ell_i+\xi_i)^t-(c_i+\ell_i-\kappa_i)^t}, & \text{if } (c_i+\ell_i)-\kappa_i < g_i(X) < c_i+\ell_i+\xi_i \\ 1, & \text{if } g_i(X) \geq c_i+\ell_i+\xi_i, \end{cases} \tag{4. 26}$$

where c_i is the aspiration level of the i^{th} constraint, ℓ_i is subjectively chosen non-negative violation parameter of the i^{th} constraint, ξ_i and κ_i are tolerance values of the i^{th} constraint and defined as $\xi_i = \lambda(\ell_i)$ and $\kappa_i = \lambda((c_i+\ell_i+\xi_i)-c_i) = \lambda(\ell_i+\xi_i)$, $\lambda \in (0,1)$ and $i = 1, 2, ..., m_1$.

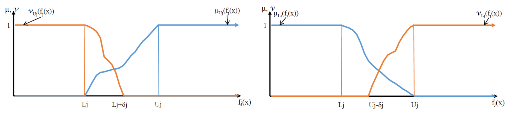
Similarly, membership function $(\mu_{c_i}(g_i(X)))$ and non-membership function $(v_{c_i}(g_i(X)))$ of the i^{th} constraint $g_i(X)$ for greater than or equal to $(\geq)$ type constraints are respectively defined as follows:

$$\mu_{c_i}(g_i(X)) = \begin{cases} 0, & \text{if } g_i(X) \leq c_i-\ell_i \\ \frac{(g_i(X))^t-(c_i-\ell_i)^t}{c_i^t-(c_i-\ell_i)^t}, & \text{if } c_i-\ell_i < g_i(X) < c_i \\ 1, & \text{if } g_i(X) \geq c_i, \end{cases} \tag{4. 27}$$

and

$$v_{c_i}(g_i(X)) = \begin{cases} 1, & \text{if } g_i(X) \leq c_i-\ell_i-\xi_i \\ \frac{(c_i-\ell_i+\kappa_i)^t-(g_i(X))^t}{(c_i-\ell_i+\kappa_i)^t-(c_i-\ell_i-\xi_i)^t}, & \text{if } c_i-\ell_i-\xi_i < g_i(X) < c_i-\ell_i+\kappa_i \\ 0, & \text{if } g_i(X) \geq c_i-\ell_i+\kappa_i, \end{cases} \tag{4. 28}$$

where c_i is the aspiration level of the i^{th} constraint, ℓ_i is subjectively chosen non-negative violation parameter of the i^{th} constraint, ξ_i and κ_i are tolerance values of the i^{th} constraint function and defined as $\xi_i = \lambda(\ell_i)$ and $\kappa_i = \lambda(c_i-(c_i-\ell_i-\xi_i)) = \lambda(\ell_i+\xi_i)$, $\lambda \in (0,1)$, $i = m_1+1, m_1+2, ..., m_2$.



(a) (b)

Fig. 2. Membership and non-membership functions for maximization (a) and minimization (b) objectives.

The membership function ($\mu_{U_j}(f_j(X))$) and non-membership function ($v_{U_j}(f_j(X))$) of the j^{th} objective function $f_j(X)$ in mixed approach to maximization problem are respectively defined as follows:

$$\mu_{U_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq L_j \\ \frac{(f_j(X))^t - L_j^t}{U_j^t - L_j^t}, & \text{if } L_j < f_j(X) < U_j \\ 1, & \text{if } f_j(X) \geq U_j, \end{cases} \tag{4. 29}$$

and

$$v_{U_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j - \delta_j \\ \frac{(L_j+\zeta_j)^t - (f_j(X))^t}{(L_j+\zeta_j)^t - (L_j-\delta_j)^t}, & \text{if } L_j - \delta_j < f_j(X) < L_j + \zeta_j \\ 0, & \text{if } f_j(X) \geq L_j + \zeta_j, \end{cases} \tag{4. 30}$$

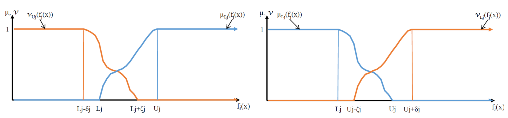
where δ_j and ζ_j are tolerance values of the j^{th} objective and defined as $\delta_j = \lambda(U_j - L_j)$, $\zeta_j = \lambda(U_j - (L_j - \delta_j))$, $\lambda \in (0,1)$, $j = 1,2,3,...,k_1$. Their possible general shape is shown in Fig. 3(a).
Similarly, the membership function $(\mu_{L_j}(f_j(X)))$ and non-membership function $(v_{L_j}(f_j(X)))$ of the j^{th} objective function $f_j(X)$ for minimization problem are respectively defined as follows:

$$\mu_{L_j}(f_j(X)) = \begin{cases} 1, & \text{if } f_j(X) \leq L_j \\ \frac{U_j^t - (f_j(X))^t}{U_j^t - L_j^t}, & \text{if } L_j < f_j(X) < U_j \\ 0, & \text{if } f_j(X) \geq U_j, \end{cases} \tag{4. 31}$$

and

$$v_{L_j}(f_j(X)) = \begin{cases} 0, & \text{if } f_j(X) \leq U_j - \zeta_j \\ \frac{(f_j(X))^t - (U_j-\zeta_j)^t}{(U_j+\delta_j)^t - (U_j-\zeta_j)^t}, & \text{if } U_j - \zeta_j < f_j(X) < U_j + \delta_j \\ 1, & \text{if } f_j(X) \geq U_j + \delta_j, \end{cases} \tag{4. 32}$$

where δ_j and ζ_j are tolerance values of the j^{th} objective and defined as $\delta_j = \lambda(U_j - L_j)$, $\zeta_j = \lambda((U_j + \delta_j) - L_j)$, $\lambda \in (0,1)$, $j = k_1+1, k_1+2, k_1+3, ..., k$.
Their possible general shape is shown in Fig. 3(b).



(a) (b)

Fig. 3. Membership and non-membership functions for maximization (a) and minimization (b) objectives.

Following the approaches of Luhandjula [13] and Jafarian et al. [9], employing the weighted geometric mean as aggregation operator, IFMOOP (2.2) can be solved using an equivalent crisp model involving each membership and non-membership function of the respective constraints and objectives in the problem as follows:

$$\max \left(\prod_{j=1}^{k} \alpha_j^{w_j} \prod_{i=1}^{m_2} \alpha_i^{w_i}\right)^{\Lambda} \left(\prod_{j=1}^{k}(1-\beta_j)^{w_j} \prod_{i=1}^{m_2}(1-\beta_i)^{w_i}\right)^{1-\Lambda}$$

subject to

$$\begin{aligned}
&\mu_{U_j}(f_j(X)) \geq \alpha_j,\ j=1,2,...,k_1,\\
&v_{U_j}(f_j(X)) \leq \beta_j,\ j=1,2,...,k_1,\\
&\mu_{L_j}(f_j(X)) \geq \alpha_j,\ j=k_1+1,k_1+2,...,k,\\
&v_{L_j}(f_j(X)) \leq \beta_j,\ j=k_1+1,k_1+2,...,k,\\
&\mu_{c_i}(g_i(X)) \geq \alpha_i,\ i=1,2,\ldots,m_1,\\
&v_{c_i}(g_i(X)) \leq \beta_i,\ i=1,2,\ldots,m_1,\\
&\mu_{c_i}(g_i(X)) \geq \alpha_i,\ i=m_1+1,m_1+2,\ldots,m_2,\\
&v_{c_i}(g_i(X)) \leq \beta_i,\ i=m_1+1,m_1+2,\ldots,m_2,\\
&0 \leq \alpha_j+\beta_j \leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&0 \leq \alpha_i+\beta_i \leq 1,\ i=1,2,...,m_1,m_1+1,m_1+2,\ldots,m_2,\\
&\alpha_j \geq 0,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\beta_j \geq 0,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\alpha_i \geq 0,\ i=1,2,\ldots,m_1,m_1+1,m_1+2,\ldots,m_2,\\
&\beta_i \geq 0,\ i=1,2,\ldots,m_1,m_1+1,m_1+2,\ldots,m_2,\\
&X \geq 0,
\end{aligned} \tag{4. 33}$$

where $\alpha_j = \min(\mu_j(f_j(X)))$, $\beta_j = \max(v_j(f_j(X))), \alpha_i = \min(\mu_i(g_i(X)))$, $\beta_i = \max(v_i(g_i(X))), w_j > 0$ and $w_i > 0$, such that $\sum_{j=1}^{k} w_j + \sum_{i=1}^{m_2} w_i = 1$, are weights assigned to the j^{th} objective function and the i^{th} constraint function, respectively and $\Lambda \in (0,1]$ is fuzzification parameter.

The value of a fuzzification parameter indicates the level of emphasis given to the membership and non-membership functions of objectives and constraints [9]. Since the model (4.33) is in the form of a geometric programming problem, variables in the objective and constraints are expressed using positive variables for better outcomes. Hence, using the Jafarian et al. [9] transformation strategy α and β can be expressed as $\rho_j = \frac{1}{\alpha_j}$, $\rho_i = \frac{1}{\alpha_i}$, $\varepsilon_j = \frac{1}{1-\beta_j}$ and $\varepsilon_i = \frac{1}{1-\beta_i}$, where $\rho_j \geq 1$, $\rho_i \geq 1$, $\varepsilon_j \geq 1$ and $\varepsilon_i \geq 1$, $\forall j = 1,2,...,k$ and $\forall i = 1,2,...,m$.

Taking the perspectives of decision-maker into consideration as described above, with the transformed representation of α and β, there are three different scenarios in which the crisp model (4.33) can be expressed as follows.

For the optimistic decision-maker, crisp programming problem (4.33) takes the following form:

$$\begin{aligned}
&\min \ \left(\prod_{j=1}^{k}\rho_j^{w_j}\prod_{i=1}^{m_2}\rho_i^{w_i}\right)^{\Lambda}\left(\prod_{j=1}^{k}\varepsilon_j^{w_j}\prod_{i=1}^{m_2}\varepsilon_i^{w_i}\right)^{1-\Lambda}\\
&\qquad\qquad \text{subject to}\\
&\frac{(f_j(X))^t+\rho_j^{-1}(L_j^t-U_j^t)}{L_j^t}\geq 1,\ j=1,2,...,k_1,\\
&\frac{(f_j(X))^t+\varepsilon_j^{-1}((L_j-\delta_j)^t-U_j^t)}{(L_j-\delta_j)^t}\geq 1,\ j=1,2,...,k_1,\\
&\frac{(f_j(X))^t+\rho_j^{-1}(U_j^t-L_j^t)}{U_j^t}\leq 1,\ j=k_1+1,k_1+2,...,k,\\
&\frac{(f_j(X))^t+\varepsilon_j^{-1}((U_j+\delta_j)^t-L_j^t)}{(U_j+\delta_j)^t}\leq 1,\ j=k_1+1,k_1+2,...,k,\\
&\frac{(g_i(X))^t+\rho_i^{-1}((c_i+\ell_i)^t-c_i^t)}{(c_i+\ell_i)^t}\leq 1,\ i=1,2,...,m_1,\\
&\frac{(g_i(X))^t+\varepsilon_i^{-1}((c_i+\ell_i+\xi_i)^t-c_i^t)}{(c_i+\ell_i+\xi_i)^t)}\leq 1,\ i=1,2,...,m_1,\\
&\frac{(g_i(X))^t+\rho_i^{-1}((c_i-\ell_i)^t-c_i^t)}{(c_i-\ell_i)^t}\geq 1,\ i=m_1+1,m_1+2,...,m_2,\\
&\frac{(g_i(X))^t+\varepsilon_i^{-1}((c_i-\ell_i-\xi_i)^t-c_i^t)}{(c_i-\ell_i-\xi_i)^t}\geq 1,\ i=m_1+1,m_1+2,...,m_2,\\
&0\leq 1+\rho_j^{-1}-\varepsilon_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&0\leq 1+\rho_i^{-1}-\varepsilon_i^{-1}\leq 1,\ i=1,2,...,m_1,m_1+1,m_1+2,...,m_2,\\
&\rho_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\varepsilon_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\rho_i^{-1}\leq 1,\ i=1,2,\cdots,m_1,m_1+1,m_1+2,...,m_2,\\
&\varepsilon_i^{-1}\leq 1,\ i=1,2,\cdots,m_1,m_1+1,m_1+2,...,m_2,\\
&X\geq 0.
\end{aligned} \tag{4. 34}$$

Using the pessimistic approach, crisp programming problem (4.33) becomes:

$$\min \left(\prod_{j=1}^{k}\rho_j^{w_j}\prod_{i=1}^{m_2}\rho_i^{w_i}\right)^{\Lambda}\left(\prod_{j=1}^{k}\varepsilon_j^{w_j}\prod_{i=1}^{m_2}\varepsilon_i^{w_i}\right)^{1-\Lambda}$$

subject to

$$\begin{aligned}
&\frac{(f_j(X))^t+\rho_j^{-1}(L_j^t-U_j^t)}{L_j^t}\geq 1,\ j=1,2,...,k_1,\\
&\frac{(f_j(X))^t+\varepsilon_i^{-1}(L_j^t-(L_j+\delta_j)^t)}{L_j^t}\geq 1,\ j=1,2,...,k_1,\\
&\frac{(f_j(X))^t+\rho_j^{-1}(U_j^t-L_j^t)}{U_j^t}\leq 1,\ j=k_1+1,k_1+2,...,k,\\
&\frac{(f_j(X))^t+\varepsilon_i^{-1}(U_j^t-(U_j-\delta_j)^t)}{U_j^t}\leq 1,\ j=k_1+1,k_1+2,...,k,\\
&\frac{(g_i(X))^t+\rho_j^{-1}((c_i+\ell_i)^t-c_i^t)}{(c_i+\ell_i)^t}\leq 1,\ i=1,2,...,m_1,\\
&\frac{(g_i(X))^t+\varepsilon_i^{-1}((c_i+\ell_i)^t-(c_i+\ell_i-\xi_i)^t)}{(c_i+\ell_i)^t}\leq 1,\ i=1,2,...,m_1,\\
&\frac{(g_i(X))^t+\rho_j^{-1}((c_i-\ell_i)^t-c_i^t)}{(c_i-\ell_i)^t}\geq 1,\ i=m_1+1,m_1+2,....,m_2,\\
&\frac{(g_i(X))^t+\varepsilon_i^{-1}((c_i-\ell_i)^t-(c_i-\ell_i+\xi_i)^t)}{(c_i-\ell_i)^t}\geq 1,\ i=m_1+1,m_1+2,...,m_2,\\
&0\leq 1+\rho_j^{-1}-\varepsilon_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&0\leq 1+\rho_i^{-1}-\varepsilon_i^{-1}\leq 1,\ i=1,2,...,m_1,m_1+1,m_1+2,...,m_2,\\
&\rho_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\varepsilon_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\rho_i^{-1}\leq 1,\ i=1,2,\cdots,m_1,m_1+1,m_1+2,...,m_2,\\
&\varepsilon_i^{-1}\leq 1,\ i=1,2,\cdots,m_1,m_1+1,m_1+2,...,m_2,\\
&X\geq 0.
\end{aligned}$$

(4. 35)

For mixed decision maker's point of view, the crisp programming problem (4.33) can be expressed as:

$$
\begin{aligned}
&\min \left(\prod_{j=1}^{k}\rho_j^{w_j}\prod_{i=1}^{m_2}\rho_i^{w_i}\right)^{\Lambda}\left(\prod_{j=1}^{k}\varepsilon_j^{w_j}\prod_{i=1}^{m_2}\varepsilon_i^{w_i}\right)^{1-\Lambda}\\
&\qquad\qquad \text{subject to}\\
&\frac{(f_j(X))^t+\rho_j^{-1}(L_j^t-U_j^t)}{L_j^t}\geq 1,\ j=1,2,...,k_1,\\
&\frac{(f_j(X))^t+\varepsilon_j^{-1}((L_j-\delta_j)^t-(L_j+\zeta_j)^t)}{(L_j-\delta_j)^t}\geq 1,\ j=1,2,...,k_1,\\
&\frac{(f_j(X))^t+\rho_j^{-1}(U_j^t-L_j^t)}{U_j^t}\leq 1,\ j=k_1+1,k_1+2,...,k,\\
&\frac{(f_j(X))^t+\varepsilon_j^{-1}((U_j+\delta_j)^t-(U_j-\zeta_j)^t)}{(U_j+\delta_j)^t}\leq 1,\ j=k_1+1,k_1+2,...,k,\\
&\frac{(g_i(X))^t+\rho_i^{-1}((c_i+\ell_i)^t-c_i^t)}{(c_i+\ell_i)^t}\leq 1,\ i=1,2,...,m_1,\\
&\frac{(g_i(X))^t+\varepsilon_i^{-1}((c_i+\ell_i+\xi_i)^t-(c_i+\ell_i-\kappa_i)^t)}{(c_i+\ell_i+\xi_i)^t}\leq 1,\ i=1,2,...,m_1,\\
&\frac{(g_i(X))^t+\rho_i^{-1}((c_i-\ell_i)^t-c_i^t)}{(c_i-\ell_i)^t}\geq 1,\ i=m_1+1,m_1+2,...,m_2,\\
&\frac{(g_i(X))^t+\varepsilon_i^{-1}((c_i-\ell_i-\xi_i)^t-(c_i-\ell_i+\kappa_i)^t)}{(c_i-\ell_i-\xi_i)^t}\geq 1,\ i=m_1+1,m_1+2,...,m_2,\\
&0\leq 1+\rho_j^{-1}-\varepsilon_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&0\leq 1+\rho_i^{-1}-\varepsilon_i^{-1}\leq 1,\ i=1,2,...,m_1,m_1+1,m_1+2,...,m_2,\\
&\rho_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\varepsilon_j^{-1}\leq 1,\ j=1,2,...,k_1,k_1+1,k_1+2,...,k,\\
&\rho_i^{-1}\leq 1,\ i=1,2,\cdots,m_1,m_1+1,m_1+2,...,m_2,\\
&\varepsilon_i^{-1}\leq 1,\ i=1,2,\cdots,m_1,m_1+1,m_1+2,...,m_2,\\
&X\geq 0.
\end{aligned}
\tag{4. 36}
$$

As indicated above, in the proposed approach, a decision-maker considers one of the three types of degrees of acceptances and rejections depending on his opinion. The model in each approach incorporates IFO, weighted geometric programming and interactive methods of optimization. The solution obtained by this method is compensatory and efficient as each membership and non-membership functions of objectives and constraints are involved in the solution procedure. The decision-maker has also ample opportunity to get several optional solutions depending on the value of $\lambda \in (0,1)$ for violations and tolerances. After

post optimality analysis, if the decision-maker is not satisfied with the solution, then the problem can be solved considering other preferences or the problem may be remodeled until a POS is obtained.
Unlike Jafarian et al. [9] solution method, in the proposed method the decision-maker is asked to give only one violation parameter for each constraint. Based on the given one, the rest of the optional values of violations and tolerances can be generated as described above. In this way, the problem is treated under different values of violations parameters until the decision-maker is satisfied with the solution obtained.

Most of the existing studies have used non-compensatory aggregation operators and hence their method does not guarantee POS. Since an IFES may not be a non-dominated solution, the two-phase method is developed by a few prominent researchers to solve fuzzy and IFO problems effectively.
Lee and Li [11] and then Guu and Wu [8] proposed the max-min operator in the first phase of solving a fuzzy optimization problem, then averaging operator in combination with the max-min operator is used in the second phase to find a non-dominated and compensatory solution. Their method intends to simultaneously maximize the minimum degree of satisfaction and total satisfaction for achieving the desired goals.
Jimenez and Bilbao [10] demonstrated that a fuzzy efficient solution may not be a POS whenever at least one of the fuzzy goals is completely achieved. In their solution procedure, a conventional goal programming problem is proposed to find a POS by maximizing the sum of underachievement of the objective functions that attain the maximum grade of satisfaction.
Razmi et al. [17] developed a goal programming model to obtain POS in the second phase when the IFES fails to satisfy Pareto-optimality conditions. Lately, Jafarian et al. [9] put forward the use of goal programming approach in the form of geometric programming problems to find a POS for intuitionistic fuzzy multi-objective non-linear programming problems.
In this study, we have improved the model in the second phase of the Jafarian et al. approach [9] given in (4.37) to find the POS for IFMOOP, in the case at least one IFES fails to satisfy the Pareto-optimality conditions, as follows:

$$\begin{aligned} &\max \sum_{s\in S} w_s \frac{d_s}{|f_s(X^*)|} \\ &\qquad\quad \text{subject to} \\ &f_s(X)+d_s \le f_s(X^*),\ s\in S \\ &\mu(f_q(X)) \ge \mu(f_q(X^*)),\ q\in Q \\ &\mu(g_i(X)) \ge \mu(g_i(X^*)),\ i=1,2,3,...,m \\ &d_s \ge 0,\ s\in S \\ &X\ge 0, \end{aligned} \tag{4. 37}$$

where $w_s > 0$ is the weight assigned to the s^{th} objective function, X^* is an IFES which is obtained using the model (4.34) or (4.35) or (4.36) (in the phase *I*), depending on the views of decision-maker, s and q are subscripts which refer to objective functions such that $\mu(f_s(X)) = 1$ and $\mu(f_q(X)) < 1$, respectively, and d_s is the deviation of the s^{th} objective

function from $f_s(X^*)$, which has the same importance as the negative deviation in classical goal programming.
The above model (4.37) can be reformulated in the form of the geometric programming problem as follows:

$$\begin{aligned}
&\max \sum_{s\in S} w_s r_s^{-1}\\
&\qquad\text{subject to}\\
&\frac{(r_s^{-1})^t (f_s(X))^t}{(f_s(X^*))^t} \le 1,\ s\in S\\
&\frac{(f_q(X))^t}{U_j^t - \mu(f_q(X^*))(U_j^t - L_j^t)} \le 1,\ q\in Q\\
&\frac{(g_i(X))^t}{(c_i+\ell_i)^t - \mu(g_i(X^*))((c_i+\ell_i)^t - c_i^t)} \le 1,\ i=1,2,...,m_1\\
&r_s \le 1,\ s\in S\\
&X \ge 0,
\end{aligned} \tag{4. 38}$$

where $r_s = 1 - \frac{d_s}{f_s(X^*)}$ which is derived from the first constraint of model (4.37) and t is a positive real number prescribed by the decision-maker. Here, it is important to note that the model (4.38) is employed to solve the second phase of minimization type IFMOOP with less than or equal to type constraints. In the same manner, one can reformulate for the counterpart problem as well.

As discussed above, the overall solution procedure of the proposed method to solve an IFMOOP can be recapitulated as follows:

Step 1. Formulate the problem as an IFMOOP.

Step 2. Represent the IFMOOP by its equivalent crisp MOOP using the accuracy function.

Step 3. Describe the membership and non-membership functions of each objective and constraint of the problem regarding the viewpoint of decision-maker.

Step 4. Employ model (4.34) or (4.35) or (4.36) to find an IFES depending on the case.

Step 5. If the obtained IFES satisfies the Pareto-optimality conditions, then the solution is POS otherwise proceed to Step 6.

Step 6. Use the model in the second phase (for eg., model (4.38) for optimistic decision-maker) to improve the obtained solution in Step 4.

Step 7. If the decision-maker is satisfied with the obtained solution in Step 4 or Step 6, then the required solution is obtained and terminate the solving process. Otherwise, repeat the process by reformulating the IFMOOP or changing the violation parameters, tolerance values, fuzzification parameters and weights assigned to the objectives and constraints.

5. NUMERICAL EXAMPLE

Example 5.1. *Solve*

$$\begin{aligned}
&\max \widetilde{f}_1(x) = \widetilde{5.5}x_1 \oplus \widetilde{3.3}x_2^{0.5} \oplus \widetilde{5}x_3,\\
&\max \widetilde{f}_2(x) = \widetilde{3}x_1^{2.5} \oplus \widetilde{2.3}x_2 \oplus \widetilde{1}x_3{}^2\\
&\qquad\qquad \textit{subject to}\\
&\widetilde{2.7}x_1 \oplus \widetilde{1}x_2 \oplus \widetilde{3.3}x_3 \precsim \widetilde{15},\\
&\widetilde{1.7}x_1 \ominus \widetilde{2}x_2 \oplus \widetilde{3.5}x_3 \succsim \widetilde{5.5},\\
&X = (x_1, x_2, x_3) \geq 0,
\end{aligned} \tag{5. 39}$$

where $\widetilde{15} =< 14.5, 15, 15.5; 14, 15, 16 >$, $\widetilde{5.5} =< 4, 5, 6; 3.5, 5, 6.5 >$,
$\widetilde{5} =< 3.5, 5, 5.7; 3, 5, 6.8 >$, $\widetilde{3.5} =< 3, 3.2, 4; 2, 3.2, 4.2 >$,
$\widetilde{3.3} =< 2, 3, 4; 1.5, 3, 5.5 >$, $\widetilde{3} =< 2.5, 3, 3.5; 2, 3, 4 >$,
$\widetilde{2.7} =< 2, 2.5, 3; 1.5, 2.5, 3.5 >$, $\widetilde{2.3} =< 1.5, 2, 3; 1, 2, 3.5 >$,
$\widetilde{2} =< 1.5, 2, 2.5; 1, 2, 3 >$, $\widetilde{1.7} =< 1, 1.5, 2; 0.5, 1.5, 2.5 >$, $\widetilde{1} =< 0.3, 0.5, 0.7; 0.2, 0.5, 0.8 >$.

Using accuracy function (Definition 2.4), problem (5.39) is transformed into the following equivalent crisp multi-objective non-linear programming problem.

$$\begin{aligned}
&\max f_1(x) = 5x_1 + 3.125x_2^{0.5} + 4.875x_3,\\
&\max f_2(x) = 3x_1^{2.5} + 2.125x_2 + 0.5x_3^2\\
&\qquad\qquad \text{subject to}\\
&2.5x_1 + 0.5x_2 + 3.125x_3 \leq 15,\\
&1.5x_1 - 2x_2 + 3.25x_3 \geq 5,\\
&x_1, x_2, x_3 \geq 0.
\end{aligned} \tag{5. 40}$$

The extreme solutions of the this problem are given in Table 2.

TABLE 2. Extreme solutions.

$X = (x_1, x_2, x_3)$	$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$
$X_1 = (5.6522, 1.7391, 0.0000)$	32.3820	231.5514	15.0000	5.0000
$X_2 = (6.0000, 0.0000, 0.0000)$	30.0000	264.5449	15.0000	9.0000

Assuming $\ell_1 = 0.5$, $\ell_2 = 0.5$, $\Lambda = 0.5$ for both membership and non-membership functions and taking equal weights for objectives and constraints, the problem is solved considering the three points of view employing models (4.34), (4.35) and (4.36), respectively. Using different values of $\lambda \in (0, 1)$, several possible solutions can be obtained but for the sake simple presentation we utilize $\lambda = 0.25$, $\lambda = 0.5$ and $\lambda = 0.75$ for solving this problem.

(a) The optimistic view

Case I: when $\lambda = 0.25, t = 1$.

$$
\begin{aligned}
&\min\ \rho_1^{0.125}\rho_2^{0.125}\rho_3^{0.125}\rho_4^{0.125}\varepsilon_1^{0.125}\varepsilon_2^{0.125}\varepsilon_3^{0.125}\varepsilon_4^{0.125}\\
&\qquad\qquad \text{subject to}\\
&\frac{5x_1 + 3.125x_2^{0.5} + 4.875x_3 + (30 - 32.3820)\rho_1^{-1}}{30} \geq 1,\\
&\frac{3x_1^{2.5} + 2.125x_2 + 0.5x_3^2 + (231.5514 - 264.5449)\rho_2^{-1}}{231.5514} \geq 1,\\
&\frac{2.5x_1 + 0.5x_2 + 3.125x_3 + (15.5 - 15)\rho_3^{-1}}{15.5} \leq 1,\\
&\frac{1.5x_1 - 2x_2 + 3.25x_3 + (4.5 - 5)\rho_4^{-1}}{4.5} \geq 1,\\
&\frac{5x_1 + 3.125x_2^{0.5} + 4.875x_3 + (29.4045 - 32.3820)\varepsilon_1^{-1}}{29.4045} \geq 1,\\
&\frac{3x_1^{2.5} + 2.125x_2 + 0.5x_3^2 + (223.3030 - 264.5449)\varepsilon_2^{-1}}{223.3030} \geq 1,\\
&\frac{2.5x_1 + 0.5x_2 + 3.125x_3 + (15.625 - 15)\varepsilon_3^{-1}}{15.625} \leq 1,\\
&\frac{1.5x_1 - 2x_2 + 3.25x_3 + (4.375 - 5)\varepsilon_4^{-1}}{4.375} \geq 1,\\
&1 + \rho_1^{-1} - \varepsilon_1^{-1} \leq 1,\\
&1 + \rho_2^{-1} - \varepsilon_2^{-1} \leq 1,\\
&1 + \rho_3^{-1} - \varepsilon_3^{-1} \leq 1,\\
&1 + \rho_4^{-1} - \varepsilon_4^{-1} \leq 1,\\
&\rho_j^{-1} \leq 1,\ j = 1,2,3,4\\
&\varepsilon_j^{-1} \leq 1,\ j = 1,2,3,4\\
&x_j \geq 0,\ j = 1,2,3.
\end{aligned}
\tag{5. 41}
$$

In a similar way, the problem can be solved using the following cases: when $\lambda = 0.5, t = 1$ as case II, when $\lambda = 0.75, t = 1$ as case III, then these three cases are also solved for $t = 2$. The solutions based on each case for $t = 1$ and $t = 2$ are presented in the upper and lower parts of Table 3 respectively.

TABLE 3. Solutions for optimistic view.

λ	$X=(x_1,x_2,x_3)$		$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$
0.25	(5.9533, 0.4614, 0)		31.8894	260.4095	15.1140	8.0070
		μ	0.7932	0.8746	0.7719	1.0000
		v	0.1654	0.1002	0.1824	0.0000
0.50	(5.9470, 0.4767, 0)		31.8930	259.7642	15.1060	7.9672
		μ	0.7947	0.7367	0.7878	1.0000
		v	0.1368	0.2632	0.1414	0.0000
0.75	(5.9543, 0.4596, 0)		31.8902	260.5169	15.1156	8.0123
		μ	0.7935	0.8779	0.7687	1.0000
		v	0.1179	0.0697	0.1321	0.0000
0.25	(5.9626, 0.4530, 0)		31.9164	261.4060	15.1330	8.0378
		μ	0.7985	0.8991	0.7370	1.0000
		v	0.1627	0.0820	0.2094	0.0000
0.5	(5.9585, 0.4654, 0)		31.9248	260.9893	15.1291	8.0068
		μ	0.8021	0.7689	0.7448	1.0000
		v	0.1344	0.2310	0.1687	0.0000
0.75	(5.9659, 0.4491, 0)		31.9241	261.7623	15.1394	8.0505
		μ	0.8018	0.9105	0.7243	1.0000
		v	0.1165	0.0538	0.1555	0.0000

(b) The pessimistic view
Case I: when $\lambda = 0.25, t = 1$

$$
\begin{aligned}
&\min\ \rho_1^{0.125}\rho_2^{0.125}\rho_3^{0.125}\rho_4^{0.125}\varepsilon_1^{0.125}\varepsilon_2^{0.125}\varepsilon_3^{0.125}\varepsilon_4^{0.125}\\
&\qquad\qquad \text{subject to}\\
&\frac{5x_1+3.125x_2^{0.5}+4.875x_3+(30-32.3820)\rho_1^{-1}}{30}\geq 1,\\
&\frac{3x_1^{2.5}+2.125x_2+0.5x_3^2+(231.5514-264.5449)\rho_2^{-1}}{231.5514}\geq 1,\\
&\frac{2.5x_1+0.5x_2+3.125x_3+(15.5-15)\rho_3^{-1}}{15.5}\leq 1,\\
&\frac{1.5x_1-2x_2+3.25x_3+(4.5-5)\rho_4^{-1}}{4.5}\geq 1,\\
&\frac{5x_1+3.125x_2^{0.5}+4.875x_3+(30-30.5955)\varepsilon_1^{-1}}{30}\geq 1,\\
&\frac{3x_1^{2.5}+2.125x_2+0.5x_3^2+(231.5514-239.7998)\varepsilon_2^{-1}}{231.5514}\geq 1,\\
&\frac{2.5x_1+0.5x_2+3.125x_3+(15.5-15.375)\varepsilon_3^{-1}}{15.5}\leq 1,\\
&\frac{1.5x_1-2x_2+3.25x_3+(4.5-4.625)\varepsilon_4^{-1}}{4.5}\geq 1,\\
&1+\rho_1^{-1}-\varepsilon_1^{-1}\leq 1,\\
&1+\rho_2^{-1}-\varepsilon_2^{-1}\leq 1,\\
&1+\rho_3^{-1}-\varepsilon_3^{-1}\leq 1,\\
&1+\rho_4^{-1}-\varepsilon_4^{-1}\leq 1,\\
&\rho_j^{-1}\leq 1,\ j=1,2,3,4\\
&\varepsilon_j^{-1}\leq 1,\ j=1,2,3,4\\
&x_j\geq 0,\ j=1,2,3.
\end{aligned}
\tag{5. 42}
$$

TABLE 4. Solutions for pessimistic view.

λ	$X=(x_1,x_2,x_3)$		$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$
0.25	(5.9518, 0.4642, 0)		31.8887	260.2596	15.1118	7.9992
		μ	0.7929	0.8701	0.7763	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.50	(5.9518, 0.4642, 0)		31.8887	260.2596	15.1118	7.9992
		μ	0.7929	0.8701	0.7763	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.75	(5.9518, 0.4642, 0)		31.8887	260.2596	15.1118	7.9992
		μ	0.7929	0.8701	0.7763	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.25	(5.9593, 0.4572, 0)		31.9096	261.0544	15.1268	8.0245
		μ	0.7956	0.8879	0.7493	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.50	(5.9593, 0.4572, 0)		31.9096	261.0544	15.1268	8.0245
		μ	0.7956	0.8879	0.7493	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.75	(5.9586, 0.4570, 0)		32.3332	264.5449	15.2641	7.8136
		μ	0.7938	0.8854	0.7531	1.0000
		v	0.0000	0.0000	0.0000	0.0000

Likewise, the problem can be solved for the following cases: when $\lambda = 0.5, t = 1$, when $\lambda = 0.75, t = 1$ then these three cases are again solved for $t = 2$. The solutions for these three cases for $t = 1$ and $t = 2$ are presented in the upper and lower parts of Table 4 respectively.

(c) The mixed view

Case I: when $\lambda = 0.25, t = 1$

$$\begin{aligned}
&\min\ \rho_1^{0.125}\rho_2^{0.125}\rho_3^{0.125}\rho_4^{0.125}\varepsilon_1^{0.125}\varepsilon_2^{0.125}\varepsilon_3^{0.125}\varepsilon_4^{0.125}\\
&\qquad\qquad \text{subject to}\\
&\frac{5x_1 + 3.125x_2^{0.5} + 4.875x_3 + (30 - 32.3820)\rho_1^{-1}}{30} \geq 1,\\
&\frac{3x_1^{2.5} + 2.125x_2 + 0.5x_3^2 + (231.5514 - 264.5449)\rho_2^{-1}}{231.5514} \geq 1,\\
&\frac{2.5x_1 + 0.5x_2 + 3.125x_3 + (15.5 - 15)\rho_3^{-1}}{15.5} \leq 1,\\
&\frac{1.5x_1 - 2x_2 + 3.25x_3 + (4.5 - 5)\rho_4^{-1}}{4.5} \geq 1,\\
&\frac{5x_1 + 3.125x_2^{0.5} + 4.875x_3 + (29.4045 - 30.7444)\varepsilon_1^{-1}}{29.4045} \geq 1,\\
&\frac{3x_1^{2.5} + 2.125x_2 + 0.5x_3^2 + (223.303 - 241.8618)\varepsilon_2^{-1}}{223.303} \geq 1,\\
&\frac{2.5x_1 + 0.5x_2 + 3.125x_3 + (15.625 - 15.3437)\varepsilon_3^{-1}}{15.625} \leq 1,\\
&\frac{1.5x_1 - 2x_2 + 3.25x_3 + (4.375 - 4.6562)\varepsilon_4^{-1}}{4.375} \geq 1,\\
&1 + \rho_1^{-1} - \varepsilon_1^{-1} \leq 1,\\
&1 + \rho_2^{-1} - \varepsilon_2^{-1} \leq 1,\\
&1 + \rho_3^{-1} - \varepsilon_3^{-1} \leq 1,\\
&1 + \rho_4^{-1} - \varepsilon_4^{-1} \leq 1,\\
&\rho_j^{-1} \leq 1,\ j = 1, 2, 3, 4\\
&\varepsilon_j^{-1} \leq 1,\ j = 1, 2, 3, 4\\
&x_j \geq 0,\ j = 1, 2, 3.
\end{aligned} \tag{5. 43}$$

Similarly, the problem is solved for $\lambda = 0.25$ and $t = 1$, $\lambda = 0.5$ and $t = 1$ and $\lambda = 0.75$ and $t = 1$ then these three cases are also solved when t=2. The solutions to all the above cases are presented in the upper and lower parts of Table 5 for $t = 1$ and $t = 2$ respectively. As shown above, the solution to this problem has no significant differences under optimistic, pessimistic and mixed views for the corresponding values of λ and t. The solution obtained satisfies the Pareto-optimality conditions under each case and the objectives of the problem are achieved.

TABLE 5. Solutions for mixed view.

λ	$X=(x_1,x_2,x_3)$	$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$	
0.25	(5.9518, 0.4642, 0)		31.8887	260.2596	15.1118	7.9992
		μ	0.7929	0.8701	0.7763	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.5	(5.9518, 0.4642, 0)		31.8887	260.2596	15.1118	7.9992
		μ	0.7929	0.8701	0.7763	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.75	(5.9623, 0.44834, 0)		31.9042	261.3676	15.1301	8.0468
		μ	0.7512	0.8018	0.7224	1.0000
		v	0.2487	0.1982	0.2776	0.0000
0.25	(5.9593, 0.4572, 0)		31.9096	261.0544	15.1268	8.0245
		μ	0.7956	0.8879	0.7493	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.50	(5.9586, 0.4570, 0)		31.9056	260.9752	15.1250	8.0238
		μ	0.7938	0.8854	0.7530	1.0000
		v	0.0000	0.0000	0.0000	0.0000
0.75	(5.9715, 0.4294, 0)		31.9055	262.3344	15.1436	8.0986
		μ	0.7365	0.7948	0.7162	1.0000
		v	0.2634	0.2052	0.2838	0.0000

6. COMPARATIVE STUDY

Two problems are considered from the existing study and solved using the proposed approach. The obtained solutions to the problems are compared with the solutions using the existing methods.

Example 6.1. [20] *A manufacturing factory produces three types of products A, B and C during one month. Three types of resources R_1, R_2 and R_3 are required to produce these products. One unit of type A product needs around 3 units of R_1, 2 units of R_2 and 3 units of R_3; one unit of type B product needs around 4 units of R_1, 3 units of R_2 and 2 units of R_3 and one unit of type C product needs around 2 units of R_1, 3 units of R_2 and 3 units of R_3. The planned available resource of R_1 and R_2 are around 320 and 350 units respectively, with the additional amount of around 25 and 20 units in safety stock for emergency purposes which is administrated by the general manager. For better quality of the products at least 360 units of resource R_3 must be utilized. To reach the goals, let x_1, x_2 and x_3 units be the planned production quantities of A, B and C. The profit of selling each unit of products A, B and C are around 7, 10 and 8 rupees respectively, and the estimated time requirements in producing each unit of products A, B and C are around 3, 4 and 5 hours respectively. The general manager wants to maximize the total profit and minimize the total time required.*

The problem is solved considering the optimistic view using model (4.34) taking equal weights for the objectives and constraints and also using equal fuzzifcation parameter $\Lambda = 0.5$ for both membership and non-membership functions, when $\lambda = 0.25$, $\lambda = 0.5$ and $\lambda = 0.75$ for $t = 1$ and $t = 2$. The solutions are presented in Table 6. The upper and lower parts of the table represent the solutions when $t = 1$ and $t = 2$ respectively.

The solutions of the problem for the optimistic approach using Singh and Yadav approach [20] is presented in the upper and lower parts of Table 7, for the first and second reference functions respectively. Comparing the obtained solutions using the proposed approach in Table 6 and the solutions with Singh and Yadav approach [20] in Table 7, it is observed that the proposed approach significantly improved their solutions. Furthermore, the proposed approach provides detailed information and several options to the decision-maker compared to the existing approach. Similarly, the solutions for pessimistic and mixed views are also significantly improved by employing the proposed approach.

TABLE 6. Solutions for optimistic view.

λ	$X=(x_1,x_2,x_3)$		$f_1(X)$	$f_2(X)$	$g_1(X)$	$g_2(X)$	$g_3(X)$
0.25	(0, 118.6554, 7.3179)		1259.9290	497.2939	266.2231	363.6434	372.5209
		μ	0.7399	0.7725	1.0000	0.5990	-
		v	0.2080	0.1820	0.0000	0.3207	-
0.50	(0, 118.8592, 7.07182)		1260.0220	496.8217	265.9202	363.7340	369.9218
		μ	0.7419	0.7762	1.0000	0.5945	-
		v	0.1721	0.1492	0.0000	0.2703	-
0.75	(0, 118.9338, 6.9813)		1260.0550	496.6476	265.8085	363.7670	369.8756
		μ	0.7425	0.7775	1.0000	0.5929	-
		v	0.1471	0.1271	0.0000	0.2326	-
0.25	(0, 116.4652, 10.35042)		1262.014	504.3486	270.6138	363.4643	372.5209
		μ	0.7790	0.7414	1.0000	0.6145	-
		v	0.1776	0.2008	0.0000	0.3062	-
0.50	(0, 116.3774,10.51189)		1262.416	504.8359	270.9071	363.5394	372.7373
		μ	0.7874	0.7377	1.0000	0.6108	-
		v	0.1431	0.1649	0.0000	0.2559	-
0.75	(0, 116.2757, 10.66920)		1262.645	505.2480	271.1594	363.5651	372.9006
		μ	0.7921	0.7347	1.0000	0.6095	-
		v	0.1206	0.1391	0.0000	0.2186	-

TABLE 7. Solutions for optimistic view using Singh and Yadav's approach [20].

k	$X=(x_1,x_2,x_3)$	$f_1(X)$	$f_2(X)$	η
1	(0.00, 103.987, 24.321)	1247.44	527.595	0.86
2	(0.00, 104.007, 24.294)	1247.42	527.534	0.70
3	(0.00, 104.284, 23.914)	1247.19	526.661	0.70
4	(0.00, 104.699, 23,332)	1246.73	525.283	0.70
5	(0.00, 106.149, 21.269)	1244.91	520.331	0.70
1	(0.00, 103.073, 25.622)	1248.59	530.723	0.87
2	(0.00, 103.064, 25.631)	1248.57	530.730	0.84
3	(0.00, 103.061, 25.640)	1248.62	530.769	0.81
4	(0.00, 103.058, 25,645)	1248.62	530.781	0.78
5	(0.00, 103.044, 25.660)	1248.60	530.803	0.75

Example 6.2. [9] *Solve*

$$\begin{aligned}
&\min\ 400x_1^{-2}x_2^{-1}x_3^{-1}+400x_1^{-2}x_2^{-2}x_3^{-1},\\
&\min\ 400x_1^{-1}x_2^{-3}x_3^{-1}+400x_1^{-1}x_2^{-2}x_3^{-2},\\
&\min\ 400x_1^{-1}x_2^{-1}x_3^{-2}+200x_1^{-2}x_2^{-2}x_3^{-1}\\
&\qquad\textit{subject to}\\
&x_1x_2+x_1x_3+x_2x_3\leq 6,\\
&x_j\geq 1,\ j=1,2,3\\
&x_j>0,\ j=1,2,3.
\end{aligned} \tag{6. 44}$$

Let $\ell = 1.5$, $\Lambda = 0.5$ for both membership and non-membership functions and equal weights for the objectives and constraint be assumed. Then the problem is solved considering the optimistic view employing model (4.34) for $\lambda = 0.25$, $\lambda = 0.5$, $\lambda = 0.75$, $t = 1$ and $t = 2$ and the solutions are summarized in the upper and lower parts of Table 8 for $t = 1$ and $t = 2$, respectively.
From the solution in Table 8, a relatively better solution is obtained when $\lambda = 0.75$ and $t = 1$. Using the two-phase method, the same solution is obtained for this particular case. Hence, the solution $X^* = (1.5764, 1.6340, 1.2288)$ satisfies both the intuitionistic fuzzy efficiency and Pareto-optimality conditions. Similarly, the problem can be solved for the remaining cases until the decision-maker is satisfied with the results.

TABLE 8. Solutions for optimistic view.

λ	$X=(x_1,x_2,x_3)$		$f_1(X)$	$f_2(X)$	$f_3(X)$	$g_1(X)$
0.25	(1.5726, 1.6312, 1.2323)		129.8034	110.5160	127.3606	6.5129
		μ	0.9757	0.8874	1.0000	0.6580
		v	0.0194	0.0898	0.0000	0.2735
0.50	(1.5752, 1.6332, 1.2298)		129.3955	110.2935	127.3605	6.5188
		μ	0.9800	0.8883	1.0000	0.6542
		v	0.0132	0.0741	0.0000	0.2305
0.75	(1.5764, 1.6340, 1.2288)		129.2114	110.2560	127.3606	6.5211
		μ	0.9819	0.8887	1.0000	0.6525
		v	0.0103	0.0632	0.0000	0.1985
0.25	(1.5818, 1.5389, 1.2760)		134.3040	119.9413	127.3607	6.4165
		μ	0.9466	0.9074	1.0000	0.7446
		v	0.0399	0.0654	0.0000	0.1988
0.50	(1.5834, 1.5362, 1.2766)		134.2906	120.2508	127.3607	6.4154
		μ	0.9467	0.9063	1.0000	0.7453
		v	0.0312	0.0494	0.0000	0.1608
0.75	(1.6165, 1.5889, 1.2311)		127.4883	114.7575	127.3607	6.5150
		μ	1.0000	0.9249	1.0000	0.6816
		v	0.0000	0.0307	0.0000	0.0000

Problem (6.44) is suggested by Jafarian et al. [9] and by their method, the decision maker needs to choose three violation parameters for each constraint and each objective is solved four times independently with the help of these subjectively chosen violation parameters.

TABLE 9. Solution obtained using Jafarian et al. approach [9].

$X = (x_1, x_2, x_3)$		$f_1(X)$	$f_2(X)$	$f_3(X)$	$g_1(X)$
(1.55, 1.58, 1.41)		123.06	99.44	106.56	6.84
	μ	0.55	0.70	0.60	0.44
	v	0.33	0.22	0.26	0.22

Then, based on the extreme solutions obtained for the four cases, the violation parameters for objectives are also determined and finally, the problem is solved by incorporating these violation parameters using model (3.8). Although their approach gives relatively better values for the objectives of this problem, it has computational burden and needs a judicious choice of values of violations and more iterations to achieve the solution compared to the proposed approach. Whereas the proposed method has reduced the number of subjectively chosen violations to just one for each constraint and the violation for each objective is determined from the extreme solutions of the problem. The remaining optional values of violations for constraints and objectives are generated using $\lambda \in (0, 1)$ as shown in Section 4 and then the problem is solved depending on the views of the decision-maker using model (4.34) or (4.35) or (4.36). To search for the best solution, as per the interest of the decision-maker, the problem can be solved again for different values of $\lambda \in (0, 1)$, $t = 1$ or $t = 2$ to generate other solutions and from which the decision-maker can choose the optimum solution.

In general, it is relatively challenging to apply Jafarian et al. [9] approach for solving IF-MOOP with many constraints. As it increases the subjectively chosen violation parameters and the number of stages required to solve the problem. The proposed approach overcomes such difficulties and efficiently solves this type of problems.

All problems in this study are solved with the help of LINGO 17.0 [12]. One can solve the problems using other mathematical software as well.

The main advantages of the proposed technique relative to the existing methods stated in this work are summarized in Table 10.

TABLE 10. Advantages of the proposed technique.

The existing methods	The proposed method
1. Yager [21], Dubey et al. [7], Rani et al. [16], Singh and Yadev [20] have worked to optimize the worst character in the problem to find the optimal solution to the problem. The obtained solution may not be POS.	1. Every objective and constraint of the problem are optimized using a compensatory aggregation operator to get an optimal compromise solution. It uses an interactive two-phase approach to find a POS.
2. In Razimi et al. [17] and Jafarian et al. [9] decision-maker's viewpoint is considered from a single perspective.	2. The three main viewpoints of decision-maker are considered independently in the solution methodology in order to further justify the subjective nature of decision-maker.
3. Razimi et al. [17] and Jafarian et al. [9] have used three subjectively chosen violation parameters to describe the membership and non-membership functions of a constraint. Similarly, to describe the membership and non-membership functions of an objective function, three violation parameters have to be chosen in the identified interval after solving each objective function with respect to the constraints four times involving the previously assigned violation parameters of the constraints.	3. Only one subjectively chosen violation parameter is required to describe the membership and non-membership functions of a constraint and the remaining tolerance values can be generated using the designed method. To describe the membership and non-membership functions of an objective function, each objective function is solved once concerning the constraints to identify the extreme values and based on these values the required violation parameters and tolerance values can be generated.
4. The Jafarian et al. [9] approach involves multiple stages and iterations to solve an IFMOOP with many objectives and constraints. It is a relatively taxing approach to apply it to real-life problems.	4. The proposed approach needs a few stages and iterations to solve an IFMOOP which involves several objectives and constraints. It is an effortless and simple approach to handle complex practical problems.

7. CONCLUSION

In many MOOPs, one of the challenging task is determining appropriate values of violations and tolerances to describe the degrees of acceptance and rejection in the solution process. Since almost all practical problems are solved under uncertainty, the choice of violation parameters varies from expertise to each decision-maker. Hence it is obvious to have several options for the values of violation parameters and tolerances for a specific problem. On the other hand, the these values have a direct impact on the solutions of the problems, the best choice of values of violation parameters and tolerances would result a better solution while the worst choice would give an inappropriate or wrong solution. As a result of this, it is imperative to develop a technique that can avoid such difficulties in solving IFMOOPs. The main contribution of this research is that it designs a method for generating alternative violation parameters and tolerance values and using post opitmality analysis one can choose reasonable values of violations and tolerances of constraints and objectives in order to achieve POS to the problem using a relatively simple algorithm.

In this article, we have modified the approach proposed by Jafarian et al. [9] and presented a computationally efficient and effective methodology for dealing with IFMOOPs. The Jafarian et al. approach has many significant advantages for solving IFMOOP but it is difficult to determine appropriate violation parameters and tolerance values for constraints and has an additional computational burden to choose suitable violation parameters for objective functions. In this study, efforts have been made to reduce the number of violation parameters for constraints and to minimize the computational stages required to choose suitable violation parameters for objectives. Furthermore, the proposed approach considers the solution methodology from the decision-makers perspective to handle real problems more efficiently.

In many real-life situations, several decision-makers maybe involved in solving a specific problem in the hierarchical structure depending on the nature of the problem. In such a case, a multi-level optimization approach effectively solves the problem. However, the proposed method is limited to single level problems, in which the decision-makers equally involve at all levels of the problem. The other limitation of this work is that only the parameters involved in the problem are assumed to be intuitionistic fuzzy numbers neglecting the intuitionistic fuzziness of the decision variables. Further research can be contemplated to avoid the stated difficulties by incorporating the proposed method with other IFO tools.

Besides the present study, the authors of this research are also working on the application of the proposed method in the determination of optimal cropping pattern and optimization of balanced diet problems.

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