Punjab University Journal of Mathematics (2022),54(5),315-332
https://doi.org/10.52280/pujm.2022.540503

An Inclusive Study on Fundamentals of Hypersoft Expert Set with Application

Muhammad Ihsan [1], Muhammad Saeed [2,*], Atiqe Ur Rahman [2], Florentin Smarandache [3]
[1,2] Department of Mathematics,
University of Management and Technology, Lahore, Pakistan,
Email: mihkhb@gmail.com [1], muhammad.saeed@umt.edu.pk [2,*], aurkhb@gmail.com [2]
[3] Department of Mathematics,
University of New Mexico, Gallup, NM 87301, USA,
Email: smarand@unm.edu

Received: 04 February,2022 / Accepted: 19 May, 2022 / Published online: 27 May, 2022

Abstract.: Soft set deals with single set of attributes whereas its extension hypersoft set deals with multi attribute-valued disjoint sets corresponding to distinct attributes. Many researchers have created some models based on soft set to solve problems in decision-making, but most of these models deal with only one expert. This causes a problem with the users, especially with those who use questionnaires in their work. Therefore we present a novel model hypersoft expert set which not only addresses this limitation of soft-like models with the emphasis on the opinion of all experts but also resolves the inadequacy of soft set for attribute-valued disjoint sets corresponding to distinct attributes. In this study, the existing concept of hypersoft expert set is modified and some fundamental properties i.e. subset, not set and equal set, whole set, absolute set, relative null set, relative absolute set; results i.e. commutative, associative, distributive and De' Morgan's Laws and set-theoretic operations i.e. complement, union intersection, restricted union, extended intersection, AND, and OR are developed. An algorithm is proposed to solve decision-making problem and applied to recruitment process for hiring "right person for the right job".

AMS (MOS) Subject Classification Codes: 91B06; 93E25
Key Words: Soft Set, Soft Expert Set, Hypersoft Set, Hypersoft Expert Set, Recruitment Process.

1. INTRODUCTION

Molodtsov [**?**] conceptualized soft set (s-set) as it deals with the single approximate functions. The s-set also regarded as a new parameterized family of subsets of the universe of discourse as it transforms the single attribute-valued set into subsets of the universe of discourse. Chen et al. [**?**] introduced the parameter reduction of s-set and applied in application of different areas. Maji et al. [**?**] worked on s-set and initiated its different characteristics like equality, union and intersection of two or more s-sets, null and absolute s-sets and some generalized operations especially AND and OR. They also verified certain results as well. After introducing fundamentals of s-sets, Maji [**?**] applied successfully this theory in decision-making problems (DMPs) by giving its an application using rough mathematics. Ali et al. [**?**] developed characteristics like restricted union, intersection, difference and extended intersection. Babitha et al. [**?**] introduced some relations and functions on s-set. Fatimah et al. [**?**] developed N-soft sets and discussed their decision-making algorithms with applications. Akram et al. [**?**] have made great contributions by introducing group like methods using hesitant N-soft sets with numerical cases in DMPs. Deli [**?**] introduced the concept of convexity using structures of s-set and fuzzy soft set (fs-set). He proved some important results by using operations like union, intersection and complement. Later on Majeed [**?**] introduced the concept of convex hull and cone for s-set to meet the demand of computational geometry with uncertain and vague information. Rahman et al. [**?**, **?**] introduced the concept of (m, n)-convexity cum concavity by defining first and second senses on s-set. They discussed the various properties of convexity cum concavity under fs-set and s-set. The s-set has been constructed for the opinion of single expert in a single model. But certain circumstances demand opinions of more than one experts using single model. To address this scarcity, soft expert set (se-set) has been constructed. Alkhazaleh et al. [**?**] converted successfully the structure of s-set to se-set by combining s-set and expert set. They characterized its necessary characteristics i.e. complement, intersection, AND, OR etc., and successfully applied the concept in DMPs. Alkhazaleh et al. [**?**, **?**, **?**] extended the work of se-set and developed the theories of soft multi sets and possibility fuzzy s-set to adequate their already proposed structures for other scenarios. Ihsan et al. [**?**, **?**] extended the work of convexity cum concavity on se-set, fuzzy se-set and discussed its several properties with numerical cases.

1.1. **Research Gap and Motivation.** Following points are provided to explain the research gap and motivation behind the choice of proposed structure:

(1) The s-set is usually useful for single argument approximate functions but it fails when functions are of multi-argument nature. To solve this kind of issue, Smarandache took initiative and brought about a new type of model hypersoft set (hs-set). Smarandache [**?**] made extension of s-set by introducing hs-set. He made use of multi-attribute valued functions in replace of single attribute-valued functions. Saeed et al. [**?**] introduced several fundamentals of hs-set for its applicability in various other fields of study. Abbas et al. [**?**] introduced basic notions of of hs-set points and discussed its certain properties in topological structures. They also verified certain results with the help of examples. Rahman et al. [**?**] developed the hybrids of hs-set with different structures and discussed its theoretic operations with generalized results. Rahman et al. [**?**] introduced decision-making application based on neutrosophic parameterized hypersoft set theory. Rahman et al. [**?**] conceptualized possibility neutrosophic hypersoft sets with application in diagnosis of heart diseases. Rahman et al. [**?**] introduced new structure of bijective hypersoft set with application in decision-making. Rahman et al. [**?**] applied

decision-making application based on aggregations of complex fuzzy hypersoft set and developed interval-valued complex fuzzy hypersoft set. Rahman et al. [**?**] also presented decision-making algorithmic approaches based on parameterization of neutrosophic set under hypersoft set environment with fuzzy, intuitionistic fuzzy and neutrosophic settings. Rahman et al. [**?**] worked on decision-making algorithmic techniques based on aggregation operations and similarity measures of possibility intuitionistic fuzzy hypersoft sets. Rahman et al. [**?**] made use of theoretical and analytical approach to the conceptual framework of convexity cum concavity on fuzzy hypersoft sets with some generalized properties. Rahman et al. [**?**] introduced the multi-attribute decision-support system based on aggregations of interval-valued complex neutrosophic hypersoft set. Saqlain et al. [**?**] gave the idea of aggregation operators for neutrosophic hypersoft set. Yolcu & Öztürk [**?**] introduced the concept of fuzzy hypersoft set with its fundamental operators and applied them in decision-making. Yolcu et al. [**?**] also conceptualized intuitionistic fuzzy hypersoft set and discussed its applications in decision-making problems. Öztürk & Yolcu [**?**] redefined the operations of neutrosophic hypersoft topological spaces and discussed its basic properties.

(2) It can be viewed that the s-set like models deal with opinion of only single expert. But in real life, there are certain situations where we need different opinions of different experts in one model. To tackle this situation, se-set has been developed. However, there are also certain situations when features are farther classified into their relevant numerical-characteristics disjoint sets. Ihsan et al. [**?**] made extension of hs-set and introduced a new structure called hypersoft expert set (hse-set) and then the researchers [**?**, **?**, **?**, **?**] made contributions by developing fuzzy hse-set, single valued neutrosophic hse-set and bijective hse-set respectively with applications in DMPs.

(3) Having motivation from [**?**, **?**], fundamentals of hse-set are developed and a new method is adopted to explain an application in DMPs.

The paper is written in this order: section 2 has definitions of s-sets, se-set and hs-set. Section 3 contains the basic notions of hse-set with properties. Section 4 contains a numerical case of of main structure in DMPs. In section 5 conclusion has been described.

2. Preliminaries

In first part of the paper, some necessary definitions are described from the literature to support the main study. Now some important symbols are mentioned that will be used throughout the paper: $P(\widetilde{\Omega})$ for the power set of $\widetilde{\Omega}$ (universe of discourse), $\daleth$ for the collection of parameters, Ⓢ for the collection of experts and $\eth$ for the set of conclusions, $\mathrm{T} = \daleth \times Ⓢ \times \eth$ with $\mathcal{S} \subseteq \mathrm{T}$.

Definition 1. [**?**]
A soft set is a collection of pairs $(\Upsilon_M, \daleth)$ with Υ_M is a mapping defined by $\Upsilon_M : \daleth \to P(\widetilde{\Omega})$ where $\daleth$ is a set of parameters.

Definition 2. [**?**]
The union of two s-sets $(\Gamma_1, Æ_1)$ and $(\Gamma_2, Æ_2)$ over $\widetilde{\Omega}$ is a s-set $(\Gamma_3, Æ_3)$ with $Æ_3 \doteq$

$\text{Æ}_1 \cup \text{Æ}_2$, and $\forall\, o \in \text{Æ}_3$,

$$\Gamma_3(o) = \begin{cases} \Gamma_1(o) & ; o \in \text{Æ}_1 \setminus \text{Æ}_2 \\ \Gamma_2(o) & ; o \in \text{Æ}_2 \setminus \text{Æ}_1 \\ \Gamma_1(o) \cup \Gamma_2(o) & ; o \in \text{Æ}_1 \cap \text{Æ}_2. \end{cases}$$

Definition 3. [**?**]
The intersection of two s-sets $(\vec{\Theta}_1, \text{Æ}_1)$ and $(\vec{\Theta}_2, \text{Æ}_2)$ is a s-set $(\vec{\Theta}_3, \text{Æ}_3)$ with $\text{Æ}_3 \doteq \text{Æ}_1 \cup \text{Æ}_2$, for all $o \in \text{Æ}_3$,

$$\vec{\Theta}_3(o) = \begin{cases} \vec{\Theta}_1(o) & ; o \in \text{Æ}_1 \setminus \text{Æ}_2 \\ \vec{\Theta}_2(o) & ; o \in \text{Æ}_2 \setminus \text{Æ}_1 \\ \vec{\Theta}_1(o) \cup \vec{\Theta}_2(o) & ; o \in \text{Æ}_1 \cap \text{Æ}_2. \end{cases}$$

Definition 4. [**?**]
A collection of pairs $(\hbar_A, \mathcal{S})$ is called a soft expert set over $\widetilde{\Omega}$ with $\hbar_A$ is a mapping given by $\hbar_A : \mathcal{S} \to P(\widetilde{\Omega})$ where $\mathcal{S} \subseteq \mathrm{T} = \daleth \times \circledS \times \eth$ and $\daleth$ stands for set of parameters, $\circledS$ is the set of experts and $\eth$ is the set of conclusions. For simplicity, $\{0 = agree, 1 = disagree\}$ is being used as of set of conclusion.

Definition 5. [**?**]
A soft expert set $(\text{œ}_1, \breve{\text{ø}})$ will be subset of $(\text{œ}_2, \breve{\text{æ}})$ over $\widetilde{\Omega}$, if $\text{œ}_1 \subseteq \text{œ}_2$, $\forall\, o \in \breve{\text{ø}}, \text{œ}_1(o) \subseteq \text{œ}_2(o)$. Moreover $(\text{œ}_2, \breve{\text{æ}})$ is a superset of $(\text{œ}_1, \breve{\text{ø}})$.

Definition 6. [**?**]
Suppose $\mho_1, \mho_2, \mho_3, ..., \mho_\alpha$, for $\alpha \geq 1$, be α disjoint attributes, while the sets $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, ..., \mathcal{L}_\alpha$, are corresponding attribute valued sets with $\mathcal{L}_m \cap \mathcal{L}_n = \emptyset$ for $m \neq n$ and $m, n \in \{1, 2, 3, ..., \alpha\}$. Then the pair $(\eta, \mathfrak{G})$ while $\mathfrak{G} = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times ... \times \mathcal{L}_\alpha$ and $\eta : \mathfrak{G} \to P(\widetilde{\Omega})$ is called a hypersoft set over $\widetilde{\Omega}$.

3. Fundamentals of Hypersoft Expert Set

In this section, the definition of hypersoft expert set and its fundamental properties (subset, equal set, not set, complement of a set, relative complement,relative null set, relative whole set, agree and disagree set etc.) are presented with examples.

Definition 7. [**?**] *Hypersoft Expert set* (hse-set): A pair$(\underbrace{\Psi}, \mathcal{S})$ is named as a hypersoft expert set over $\widetilde{\Omega}$ with $\underbrace{\Psi} : \mathcal{S} \to P(\widetilde{\Omega})$ where $\mathcal{S} \subseteq \mathrm{T} = \Im \times \circledS \times \eth$; $\Im = \Im_1 \times \Im_2 \times \Im_3 \times .... \times \Im_n$ with $\Im_1, \Im_2, \Im_3, ..., \Im_n$ are disjoint attributes sets corresponding to n disjoint attributes $\partial_1, \partial_2, \partial_3, ..., \partial_n$; $\circledS$ represents the set of experts and $\eth$ represents the set of conclusion.

Example 3.1. Assume that a multi-national manufacturing company plans to assess its manufactured items through external evaluators. Let $\widetilde{\Omega} = \{v_1, v_2, v_3, v_4\}$ be a set of items and $\Lambda_1 = \{g_{11}, g_{12}\}$, $\Lambda_2 = \{g_{21}, g_{22}\}$, $\Lambda_3 = \{g_{31}, g_{32}\}$, be disjoint parametric valued sets for distinct attributes g_1= simple to utilize, g_2= nature, g_3= modest. Now $\Lambda = \Lambda_1 \times \Lambda_2 \times \Lambda_3$

$$\Lambda = \left\{ \begin{array}{l} \eta_1 = (g_{11}, g_{21}, g_{31}), \eta_2 = (g_{11}, g_{21}, g_{32}), \eta_3 = (g_{11}, g_{22}, g_{31}), \eta_4 = (g_{11}, g_{22}, g_{32}), \\ \eta_5 = (g_{12}, g_{21}, g_{31}), \eta_6 = (g_{12}, g_{21}, g_{32}), \eta_7 = (g_{12}, g_{22}, g_{31}), \eta_8 = (g_{12}, g_{22}, g_{32}) \end{array} \right\}.$$

Now $\Pi = \Lambda \times \Upsilon \times \Gamma$

$$\Pi = \left\{ \begin{array}{l} (\eta_1, q, 0), (\eta_1, s, 1), (\eta_1, t, 0), (\eta_1, t, 1), (\eta_1, u, 0), (\eta_1, u, 1), (\eta_8, s, 0), \\ (\eta_2, s, 0), (\eta_2, s, 1), (\eta_2, t, 0), (\eta_2, t, 1), (\eta_2, u, 0), (\eta_2, u, 1), (\eta_8, s, 1), \\ (\eta_3, s, 0), (\eta_3, s, 1), (\eta_3, t, 0), (\eta_3, t, 1), (\eta_3, u, 0), (\eta_3, u, 1), (\eta_8, t, 0), \\ (\eta_4, s, 0), (\eta_4, s, 1), (\eta_4, t, 0), (\eta_4, t, 1), (\eta_4, u, 0), (\eta_4, u, 1), (\eta_8, t, 1), \\ (\eta_5, s, 0), (\eta_5, s, 1), (\eta_5, t, 0), (\eta_5, t, 1), (\eta_5, u, 0), (\eta_5, u, 1), (\eta_8, u, 0), \\ (\eta_6, s, 0), (\eta_6, s, 1), (\eta_6, t, 0), (\eta_6, t, 1), (\eta_6, u, 0), (\eta_6, u, 1), (\eta_8, u, 1) \\ (\eta_7, s, 0), (\eta_7, s, 1), (\eta_7, t, 0), (\eta_7, t, 1), (\eta_7, u, 0), (\eta_7, u, 1), \end{array} \right\}.$$

Let

$$\digamma = \left\{ \begin{array}{l} (\eta_1, q, 0), (\eta_1, s, 1), (\eta_1, t, 0), (\eta_1, t, 1), (\eta_1, u, 0), (\eta_1, u, 1), \\ (\eta_3, s, 0), (\eta_3, s, 1), (\eta_3, t, 0), (\eta_3, t, 1), (\eta_3, u, 0), (\eta_3, u, 1), \\ (\eta_5, s, 0), (\eta_5, s, 1), (\eta_5, t, 0), (\eta_5, t, 1), (\eta_5, u, 0), (\eta_5, u, 1) \end{array} \right\}$$

be a subset of Π and $\Upsilon = \{s, t, u\}$ represents a set of specialists and Γ= $\{0 = agree, 1 = disagree\}$ represents a set of conclusion. Following are the approximations of prescribed attributes with respect to selected evaluators:
$\mho_1 = \mho(\eta_1, s, 1) = \{v_1, v_2, v_4\}, \mho_2 = \mho(\eta_1, t, 1) = \{v_3, v_4\}, \mho_3 = \mho(\eta_1, u, 1) = \{v_3, v_4\}, \mho_4 = \mho(\eta_3, s, 1) = \{v_4\}, \mho_5 = \mho(\eta_3, t, 1) = \{v_1, v_3\},$
$\mho_6 = \mho(\eta_3, u, 1) = \{v_1, v_2, v_4\},$ $\mho_7 = \mho(\eta_5, s, 1) = \{v_3, v_4\}, \mho_8 = \mho(\eta_5, t, 1) = \{v_1, v_2\}, \mho_9 = \mho(\eta_5, u, 1) = \{v_4\},$
$\mho_{10} = \mho(\eta_1, s, 0) = \{v_3\}, \mho_{11} = \mho(\eta_1, t, 0) = \{v_2, v_3\}, \mho_{12} = \mho(\eta_1, u, 0) = \{v_1, v_2\},$
$\mho_{13} = \mho(\eta_3, s, 0) = \{v_1, v_2, v_3\}, \mho_{14} = \mho(\eta_3, t, 0) = \{v_2, v_4\}, \mho_{15} = \mho(\eta_3, u, 0) = \{v_3\},$ $\mho_{16} = \mho(\eta_5, s, 0) = \{v_1, v_2\}, \mho_{17} = \mho(\eta_5, t, 0) = \{v_3, v_4\}, \mho_{18} = \mho(\eta_5, u, 0) = \{v_1, v_2, v_3\}.$
The hypersoft expert set is

$$(\mho, \digamma) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), ((\eta_1, u, 1), \{v_3, v_4\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), ((\eta_3, u, 1), \{v_1, v_2, v_4\}), \\ ((\eta_5, s, 1), \{v_3, v_4\}), ((\eta_5, t, 1), \{v_1, v_2\}), ((\eta_5, u, 1), \{v_4\}), \\ ((\eta_1, s, 0), \{v_3\}), ((\eta_1, t, 0), \{v_2, v_3\}), ((\eta_1, u, 0), \{v_1, v_2\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}), ((\eta_3, u, 0), \{v_3\}) \\ ((\eta_5, s, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), ((\eta_5, u, 0), \{v_1, v_2, v_3\}) \end{array} \right\}.$$

Definition 8. Hypersoft Expert Subset
A hse-set $(\mho_1, \digamma) \subseteq (\mho_2, \urcorner)$ over $\widetilde{\Omega}$, if
(i) $\digamma \subseteq \urcorner$, (ii) $\forall\, o \in \digamma, \mho_1(o) \subseteq \mho_2(o)$ and shown by $(\mho_1, \digamma) \subseteq (\mho_2, \urcorner)$.

Example 3.2. Considering Example **??**, suppose

$$Æ_1 = \left\{ (\eta_1, s, 1), (\eta_3, s, 0), (\eta_1, t, 1), (\eta_3, t, 1), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1) \right\}$$

$$Æ_2 = \left\{ \begin{array}{l} (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1), (\eta_1, t, 1), (\eta_3, t, 1), \\ (\eta_5, t, 0), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1), (\eta_5, u, 1) \end{array} \right\}.$$

$\Rightarrow$ $Æ_1 \subset Æ_2$. Suppose $(\mho_1, Æ_1)$ and $(\mho_2, Æ_2)$ be two hse-sets

$$(\mho_1, Æ_1) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), \\ ((\eta_3, t, 1), \{v_1, v_3\}), ((\eta_3, u, 1), \{v_1, v_2\}), \\ ((\eta_1, u, 0), \{v_1\}), ((\eta_3, s, 0), \{v_1, v_2\}), \\ ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}$$

$$(\mho_2, Æ_2) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_5, u, 1), \{v_4\}), ((\eta_3, u, 1), \{v_1, v_2, v_4\}), \\ ((\eta_1, u, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}$$

$\Rightarrow (\mho_1, Æ_1) \subseteq (\mho_2, Æ_2)$.

Definition 9. Two hse-sets $(\mho_1, \delta_1)$ and $(\mho_2, \delta_2)$ will be equal if $(\mho_1, \delta_1) \subseteq (\mho_2, \delta_2)$ and $(\mho_2, \delta_2) \subseteq (\mho_1, \delta_1)$.

Definition 10. The NOT set of $\Pi = \Lambda \times \Upsilon \times \Gamma$ denoted by $\sim \Lambda$, is shown as $\sim \Lambda = \{(\sim o_i, a_j, v_k) \forall i, j, k\}$ with $\sim o_i$ is not o_i.

Definition 11. Let $(\mho, F)$ be a hse-set, then its compliment is defined by $(\mho, F)^c = (\mho^c, \sim F)$ such that $\mho^c : \sim F \to P(\widetilde{\Omega})$ is represented by $\mho^c(0) = \widetilde{\Omega} - \mho(\sim o)$, for $o \in \sim F$.

Example 3.3. We can find the compliment of hse-set in Example **??**, as

$$(\mho, F)^c = \left\{ \begin{array}{l} ((\sim \eta_1, s, 1), \{v_3\}), ((\sim \eta_1, t, 1), \{v_2, v_3\}), ((\sim \eta_1, u, 1), \{v_1, v_2\}), \\ ((\sim \eta_3, s, 1), \{v_1, v_2, v_3\}), ((\sim \eta_3, t, 1), \{v_2, v_4\}), ((\sim \eta_3, u, 1), \{v_1, v_2, v_4\}), \\ ((\sim \eta_5, s, 1), \{v_1, v_2\}), ((\sim \eta_5, t, 1), \{v_3, v_4\}), ((\sim \eta_5, u, 1), \{v_1, v_2, v_3\}), \\ ((\sim \eta_1, s, 0), \{v_1, v_2, v_4\}), ((\sim \eta_1, t, 0), \{v_1, v_4\}), ((\sim \eta_1, u, 0), \{v_1, v_2\}), \\ ((\sim \eta_3, s, 0), \{v_4\}), ((\sim \eta_3, t, 0), \{v_1, v_3\}), ((\sim (\eta_3, u, 0), \{v_3\}) \\ ((\sim \eta_5, s, 0), \{v_3, v_4\}), ((\sim \eta_5, t, 0), \{v_1, v_3\},), ((\sim \eta_5, u, 0), \{v_4\}) \end{array} \right\}.$$

Definition 12. Let $(\mho, F)$ is a hse-set, then its relative compliment is $(\mho, F)^* = (\mho^*, F)$ with $\mho^* : F \to P(\widetilde{\Omega})$, as well as $\mho^*(o) = \widetilde{\Omega} - \mho(o)$ for all o $\in F$.

Example 3.4. We can find the relative compliment of hse-set in Example **??**, as

$$(\mho, F)^* = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_3\}), ((\eta_1, t, 1), \{v_2, v_3\}), ((\eta_1, u, 1), \{v_1, v_2\}), \\ ((\eta_3, s, 1), \{v_1, v_2, v_3\}), ((\eta_3, t, 1), \{v_2, v_4\}), ((\eta_3, u, 1), \{v_1, v_2, v_4\}), \\ ((\eta_5, s, 1), \{v_1, v_2\}), ((\eta_5, t, 1), \{v_3, v_4\}), ((\eta_5, u, 1), \{v_1, v_2, v_3\}), \\ ((\eta_1, s, 0), \{v_1, v_2, v_4\}), ((\eta_1, t, 0), \{v_1, v_4\}), ((\eta_1, u, 0), \{v_1, v_2\}), \\ ((\eta_3, s, 0), \{v_4\}), ((\eta_3, t, 0), \{v_1, v_3\}), (((\eta_3, u, 0), \{v_3\}) \\ ((\eta_5, s, 0), \{v_3, v_4\}), ((\eta_5, t, 0), \{v_1, v_3\},), ((\eta_5, u, 0), \{v_4\}) \end{array} \right\}.$$

Definition 13. Suppose $(\mho, F)$ be a hse-set, then the following properties hold:

(1) $((\mho, F)^c)^c = (\mho, F)$
(2) $((\mho, F)^*)^* = (\mho, F)$
(3) $((\mho_1, F_1)_{\widetilde{\Omega}})^c = (\mho_1, F_1)_\Phi = ((\mho_1, F_1)_{\widetilde{\Omega}})^*$ with $F_1 \subseteq F$.
(4) $((\mho_1, F_1)_\Phi)^c = (\mho_1, F_1)_{\widetilde{\Omega}} = ((\mho_1, F_1)_\Phi)^*$ with $F_1 \subseteq F$.

Definition 14. A hse-set $(\mho, F_1)$ is called a relative null hse-set with respect to $F_1 \subset F$, denoted by $(\mho, F_1)_{\widetilde{\Omega}}$, if $\mho(o) = \emptyset, \forall$ o $\in F_1$.

Example 3.5. Taking Example 3.2, if
$(\mho, \digamma_1)_{\widetilde{\Omega}}$ = $\{((\eta_1, s, 1), \emptyset), ((\eta_1, t, 1), \emptyset), ((\eta_1, u, 1), \emptyset)\}$, where $\digamma_1 \subseteq \digamma$.

Definition 15. A hse-set $(\mho, \digamma_2)$ is called a relative whole hse-set with respect to $\digamma_2 \subset \digamma$, denoted by $(\mho, \digamma_2)_{\widetilde{\Omega}}$, if $\mho(o) = \widetilde{\Omega}, \forall$ o $\in \digamma_2$.

Example 3.6. Taking Example 3.2, if
$(\mho, \digamma_2)_{\widetilde{\Omega}}$ = $\{((\eta_2, s, 1), \widetilde{\Omega}), ((\eta_2, t, 1), \widetilde{\Omega}), ((\eta_2, u, 1), \widetilde{\Omega})\}$, where $\digamma_2 \subseteq \digamma$.

Definition 16. A hse-set $(\mho, \digamma)$ is called absolute whole hse-set shown by $(\mho, \digamma)_{\widetilde{\Omega}}$, if $\mho(o) = \widetilde{\Omega}, \forall$ o $\in \digamma$.

Example 3.7. Taking Example 3.2, if $(\mho, \digamma)_{\widetilde{\Omega}} =$

$$\left\{ \begin{array}{l} \left((\eta_1, s, 1), \widetilde{\Omega}\right), \left((\eta_1, t, 1), \widetilde{\Omega}\right), \left((\eta_1, u, 1), \widetilde{\Omega}\right), \left((\eta_3, s, 1), \widetilde{\Omega}\right), \left((\eta_3, t, 1), \widetilde{\Omega}\right), \left((\eta_3, u, 1), \widetilde{\Omega}\right), \\ \left((\eta_5, s, 1), \widetilde{\Omega}\right), \left((\eta_5, t, 1), \widetilde{\Omega}\right), \left((\eta_5, u, 1), \widetilde{\Omega}\right), \left((\eta_1, s, 0), \widetilde{\Omega}\right), \left((\eta_1, t, 0), \widetilde{\Omega}\right), \left((\eta_1, u, 0), \widetilde{\Omega}\right), \\ \left((\eta_3, s, 0), \widetilde{\Omega}\right), \left((\eta_3, t, 0), \widetilde{\Omega}\right), \left((\eta_3, u, 0), \widetilde{\Omega}\right) \left((\eta_5, s, 0), \widetilde{\Omega}\right), \left((\eta_5, t, 0), \widetilde{\Omega}\right), \left((\eta_5, u, 0), \widetilde{\Omega}\right) \end{array} \right\}.$$

Proposition 3.8. *Suppose $(\mho_1, \digamma_1)_{\widetilde{\Omega}}$, $(\mho_2, \digamma_2)_{\widetilde{\Omega}}$, $(\mho_3, \digamma_3)_{\widetilde{\Omega}}$, be three hse-sets over $\widetilde{\Omega}$, then following properties hold:*

(1) $(\mho_1, \digamma_1) \subset (\mho_2, \digamma_2)_{\widetilde{\Omega}}$
(2) $(\mho_1, \digamma_1)_{\Phi} \subset (\mho_1, \digamma_1)$
(3) $(\mho_1, \digamma_1) \subset (\mho_1, \digamma_1)$
(4) *If* $(\mho_1, \digamma_1) \subset (\mho_2, \digamma_2)$, *and* $(\mho_2, \digamma_2) \subset (\mho_3, \digamma_3)$, *then* $(\mho_1, \digamma_1) \subset (\mho_3, \digamma_3)$
(5) *If* $(\mho_1, \digamma_1) = (\mho_2, \digamma_2)$, *and* $(\mho_2, \digamma_2) = (\mho_3, \digamma_3)$, *then* $(\mho_1, \digamma_1) = (\mho_3, \digamma_3)$.

Definition 17. An Agree-hse-set $(\mho, \digamma)_{ag}$ is a hse-subset of $(\mho, \digamma)$ and is characterized as $(\mho, \digamma)_{ag} = \{\mho_{ag}(o) : o \in \Lambda \times \Upsilon \times \{1\}\}$.

Example 3.9. We can find Agree-hse-set using Example **??**, we get

$$(\mho, \digamma) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), ((\eta_1, u, 1), \{v_3, v_4\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), ((\eta_3, u, 1), \{v_1, v_2, v_4\}), \\ ((\eta_5, s, 1), \{v_3, v_4\}), ((\eta_5, t, 1), \{v_1, v_2\}), ((\eta_5, u, 1), \{v_4\}) \end{array} \right\}.$$

Definition 18. A Disagree-hse-set $(\mho, \digamma)_{dag}$ over $\widetilde{\Omega}$, is a hse-subset of $(\mho, \digamma)$ and is characterized as $(\mho, \digamma)_{dag} = \{\mho_{dag}(o) : o \in \Lambda \times \Upsilon \times \{0\}\}$.

Example 3.10. We can find Disagree-hse-set in Example **??**,

$$(\mho, \digamma) = \left\{ \begin{array}{l} ((\eta_1, s, 0), \{v_3\},), ((\eta_1, t, 0), \{v_2, v_3\}), ((\eta_1, u, 0), \{v_1, v_2\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}), ((\eta_3, u, 0), \{v_3\}), \\ ((\eta_5, s, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), ((\eta_5, u, 0), \{v_1, v_2, v_3\}) \end{array} \right\}.$$

Proposition 3.11. *Consider a hse-subset $(\mho, \digamma)$ on $\widetilde{\Omega}$, then following properties hold:*

(1) $((\mho, \digamma)^c)^c = (\mho, \digamma)$
(2) $(\mho, \digamma)^c_{ag} = (\mho, \digamma)_{dag}$
(3) $(\mho, \digamma)^c_{dag} = (\mho, \digamma)_{ag}$.

Definition 19. The union of $(\mho_1, \digamma)$ and $(\mho_2, \urcorner)$ over $\widetilde{\Omega}$ is $(\mho_3, \sqsupset)$ with $\sqsupset = \digamma \cup \urcorner$, defined as

$$\mho_3(o) = \begin{cases} \mho_1(o) & ; o \in \digamma - \urcorner \\ \mho_2(o) & ; o \in \urcorner - \digamma \\ \mho_1(o) \cup \mho_2(o) & ; o \in \digamma \cap \urcorner. \end{cases}$$

Example 3.12. Taking Example **??**, and two sets

$$\text{Æ}_1 = \left\{ \begin{matrix} (\eta_1, s, 1), (a_3, s, 0), (a_3, s, 1), (\eta_1, t, 1), (a_3, t, 1), \\ (a_5, t, 0), (a_3, t, 0), (\eta_1, u, 0), (a_3, u, 1), (a_5, u, 1) \end{matrix} \right\}$$

$$\text{Æ}_2 = \left\{ (\eta_1, s, 1), (a_3, s, 0), (a_3, s, 1), (\eta_1, t, 1), \ (a_3, t, 1), (a_5, t, 0), (a_3, t, 0), (\eta_1, u, 0), (a_3, u, 1) \right\}.$$

Consider two two hse-sets $(\mho_1, \text{Æ}_1)$ and $(\mho_2, \text{Æ}_2)$ on $\widetilde{\Omega}$

$$(\mho_1, \text{Æ}_1) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_3, u, 1), \{v_1, v_2\}), ((\eta_1, u, 0), \{v_1\}), ((\eta_3, s, 0), \{v_1, v_2\}), \\ ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}$$

$$(\mho_2, \text{Æ}_2) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_5, u, 1), \{v_4\}), (((\eta_3, u, 1), \{v_1, v_2, v_4\}) ((\eta_1, u, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}.$$

Then $(\mho_1, \text{Æ}_1) \cup (\mho_2, \text{Æ}_2) = (\mho_3, \text{Æ}_3)$

$$\left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_5, u, 1), \{v_4\}), ((\eta_3, u, 1), \{v_1, v_2, v_4\}), ((\eta_1, u, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}.$$

Definition 20. Restricted Union of two hse-sets $(\mho_1, \text{Æ}_1)$, $(\mho_2, \text{Æ}_2)$ over $\widetilde{\Omega}$ is $(\mho_3, \text{Œ})$ with $\text{Œ} = \text{Æ}_1 \cap \text{Æ}_2$, defined as $\mho_3(o) = \mho_1(o) \cup_R \mho_2(o)$ for all $o \in \text{Æ}_1 \cap \text{Æ}_2$.

Example 3.13. Dealing Example **??** with two sets

$$\text{Æ}_1 = \left\{ \begin{matrix} (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1), (\eta_1, t, 1), (\eta_3, t, 1), \\ (\eta_5, t, 0), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1), (\eta_5, u, 1) \end{matrix} \right\}$$

$$\text{Æ}_2 = \left\{ \begin{array}{l} (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1), (\eta_1, t, 1), (\eta_3, t, 1), \\ (\eta_5, t, 0), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1) \end{array} \right\}.$$

Consider two hse-sets

$$(\mho_1, \text{Æ}_1) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_3, u, 1), \{v_1, v_2\}), ((\eta_1, u, 0), \{v_1\}), ((\eta_3, s, 0), \{v_1, v_2\}), \\ ((\eta_3, t, 0), \{v_2, v_4\}), ((\eta_5, u, 1), \{v_4\}), ((\eta_5, t, 0), \{v_3\}) \\ ((\eta_3, s, 1), \{v_3, v_4\}) \end{array} \right\}$$

$$(\mho_2, \text{Æ}_2) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_3, u, 1), \{v_1, v_2, v_4\}) ((\eta_1, u, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}.$$

Then $(\mho_1, \AE_1) \cup_R (\mho_2, \AE_2) = (\mho_3, \OE)$

$$(\mho_3, \OE) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_3, u, 1), \{v_1, v_2, v_4\}), \\ ((\eta_1, u, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}.$$

Proposition 3.14. *Consider three hse-sets* $(\mho_1, \AE_1)$,$(\mho_2, \AE_2)$ *and* $(\mho_3, \AE_3)$ *on* $\widetilde{\Omega}$*, then*

(1) $(\mho_1, \AE_1) \cup (\mho_2, \AE_2) = (\mho_2, \AE_2) \cup (\mho_1, \AE_1)$

(2) $((\mho_1, \AE_1) \cup (\mho_2, \AE_2)) \cup (\mho_3, \AE_3) = (\mho_1, \AE_1) \cup ((\mho_2, \AE_2) \cup (\mho_3, \AE_3))$.

Definition 21. The intersection of $(\mho_1, \AE_1)$ and $(\mho_2, \AE_2)$ over $\widetilde{\Omega}$ is $(\mho_3, \OE)$ with $\OE = \AE_1 \cap \AE_2$, defined as $\mho_3(o) = \mho_1(o) \cap \mho_2(o)$ for all $o \in \AE_1 \cap \AE_2$.

Example 3.15. Dealing Example **??**, and following two sets

$$\AE_1 = \left\{ \begin{array}{l} (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1), (\eta_1, t, 1), (\eta_3, t, 1), \\ (\eta_5, t, 0), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1), (\eta_5, u, 1) \end{array} \right\}$$

$$\AE_2 = \left\{ \begin{array}{l} (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1), (\eta_1, t, 1), (\eta_3, t, 1), \\ (\eta_5, t, 0), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1) \end{array} \right\}.$$

Consider two hse-sets over $\widetilde{\Omega}$, then

$$(\mho_1, \AE_1) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_3, u, 1), \{v_1, v_2\}), ((\eta_1, u, 0), \{v_1\}), ((\eta_3, s, 0), \{v_1, v_2\}), \\ ((\eta_3, t, 0), \{v_2, v_4\}), ((\eta_5, u, 1), \{v_4\}), ((\eta_5, t, 0), \{v_3\}) \\ ((\eta_3, s, 1), \{v_3, v_4\}) \end{array} \right\}$$

$$(\mho_2, \AE_2) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1, v_4\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), (((\eta_3, u, 1), \{v_1, v_2, v_4\}) \\ ((\eta_1, u, 0), \{v_1, v_2\}), ((\eta_5, t, 0), \{v_3, v_4\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}.$$

Then $(\mho_1, \AE_1) \cap (\mho_2, \AE_2) = (\mho_3, \AE_3)$

$$\left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_3, t, 1), \{v_1, v_3\}), ((\eta_3, u, 1), \{v_1, v_2\}), \\ ((\eta_1, u, 0), \{v_1\}), ((\eta_3, s, 0), \{v_1, v_2\}), ((\eta_3, t, 0), \{v_2, v_4\}), ((\eta_3, s, 1), \{v_4\}) \\ ((\eta_5, t, 0), \{v_3\}), \end{array} \right\}.$$

Definition 22. Extended intersection of two hse-sets $(\mho_1, \digamma)$ and $(\mho_2, \daleth)$ is $(\mho_3, \beth)$ with $\beth = \digamma \cup \daleth$, and

$$\mho_3(o) = \left\{ \begin{array}{cl} \mho_1(o) & ; o \in \digamma - \daleth \\ \mho_2(o) & ; o \in \daleth - \digamma \\ \mho_1(o) \cap \mho_2(o) & ; o \in \digamma \cap \daleth. \end{array} \right.$$

Example 3.16. By utilizing Example **??**,and with two sets

$$\AE_1 = \left\{ \begin{array}{l} (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1), (\eta_1, t, 1), (\eta_3, t, 1), \\ (\eta_5, t, 0), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1), (\eta_5, u, 1) \end{array} \right\}$$

$$Æ_2 = \left\{ \begin{array}{l} (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1), (\eta_1, t, 1), (\eta_3, t, 1), \\ (\eta_5, t, 0), (\eta_3, t, 0), (\eta_1, u, 0), (\eta_3, u, 1) \end{array} \right\}$$

Suppose $(\mho_1, Æ_1)$ and $(\mho_2, Æ_2)$ are two hse-sets

$$(\mho_1, Æ_1) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_3, u, 1), \{v_1, v_2\}), ((\eta_1, u, 0), \{v_1\}), ((\eta_3, s, 0), \{v_1, v_2\}), \\ ((\eta_3, t, 0), \{v_2, v_4\}), ((\eta_5, u, 1), \{v_4\}), ((\eta_5, t, 0), \{v_3\}) \\ ((\eta_3, s, 1), \{v_3, v_4\}) \end{array} \right\}$$

$$(\mho_2, Æ_2) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_5, t, 0), \{v_3, v_4\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, t, 1), \{v_1, v_3\}), \\ ((\eta_3, u, 1), \{v_1, v_2, v_4\}) ((\eta_1, u, 0), \{v_1, v_2\}), \\ ((\eta_3, s, 0), \{v_1, v_2, v_3\}), ((\eta_3, t, 0), \{v_2, v_4\}) \end{array} \right\}$$

Then $(\mho_1, Æ_1) \cap_E (\mho_2, Æ_2) = (\mho_3, \sqsupset)$

$$\left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_3, t, 1), \{v_1, v_3\}), ((\eta_3, u, 1), \{v_1, v_2\}), \\ ((\eta_1, u, 0), \{v_1\}), ((\eta_3, s, 0), \{v_1, v_2\}), ((\eta_3, t, 0), \{v_2, v_4\}), ((\eta_5, t, 0), \{v_3\}), \\ ((\eta_5, u, 1), \{v_4\}), ((\eta_3, s, 1), \{v_4\}) \end{array} \right\}.$$

Proposition 3.17. *Consider three hse-sets* $(\mho_1, Æ_1)$,$(\mho_2, Æ_2)$ *and* $(\mho_3, Æ_3)$ *over* $\widetilde{\Omega}$*, then*

(1) $(\mho_1, Æ_1) \cap (\mho_2, Æ_2) = (\mho_2, Æ_2) \cap (\mho_1, Æ_1)$
(2) $((\mho_1, Æ_1) \cap (\mho_2, Æ_2)) \cap (\mho_3, Æ_3) = (\mho_1, Æ_1) \cap ((\mho_2, Æ_2) \cap (\mho_3, Æ_3))$.

Proposition 3.18. *Consider three hse-sets* $(\mho_1, Æ_1)$,$(\mho_2, Æ_2)$ *and* $(\mho_3, Æ_3)$ *over* $\widetilde{\Omega}$*, then*

(1) $(\mho_1, Æ_1) \cup ((\mho_2, Æ_2) \cap (\mho_3, Æ_3)) = ((\mho_1, Æ_1) \cup ((\mho_2, Æ_2)) \cap ((\mho_1, Æ_1) \cup (\mho_3, Æ_3))$
(2) $(\mho_1, Æ_1) \cap ((\mho_2, Æ_2) \cup (\mho_3, Æ_3)) = ((\mho_1, Æ_1) \cap ((\mho_2, Æ_2)) \cup ((\mho_1, Æ_1) \cap (\mho_3, Æ_3))$.

Definition 23. Let $(\mho_1, Æ_1)$ and $(\mho_2, Æ_2)$ are two hse-sets then $(\mho_1, Æ_1)$ AND $(\mho_2, Æ_2)$ shown as $(\mho_1, Æ_1) \wedge (\mho_2, Æ_2)$ and can be defined as $(\mho_1, Æ_1) \wedge (\mho_2, Æ_2) = (\mho_3, Æ_1 \times Æ_2)$, with $\mho_3(\beta, \gamma) = \mho_1(\beta) \cap \mho_2(\gamma), \forall (\beta, \gamma) \in Æ_1 \times Æ_2$.

Example 3.19. Taking Example **??**, let two sets
$Æ_1 = \{ (\eta_1, s, 1), (\eta_1, t, 1), (\eta_3, s, 1), (\eta_3, s, 0) \}$, $Æ_2 = \{ (\eta_1, s, 1), (\eta_3, s, 0), (\eta_3, s, 1) \}$.
Consider two hse-sets

$$(\mho_1, Æ_1) = \{ ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_1, t, 1), \{v_1\}), ((\eta_3, s, 1), \{v_4\}), ((\eta_3, s, 0), \{v_1, v_2\}), \}$$

and

$$(\mho_2, Æ_2) = \{ ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_3, s, 0), \{v_1, v_2, v_3\}) \}$$

Then $(\mho_1, Æ_1) \wedge (\mho_2, Æ_2) = (\mho_3, Æ_1 \times Æ_2)$

$$\left\{ \begin{array}{l} (((\eta_1, s, 1), (\eta_1, s, 1)), \{v_1, v_2\}), (((\eta_1, s, 1), (\eta_3, s, 0)), \{v_1, v_2\}), \\ (((\eta_1, t, 1), (\eta_1, s, 1)), \{v_1\}), (((\eta_1, t, 1), (\eta_3, s, 0)), \{v_1\}), \\ (((\eta_3, s, 1), (\eta_1, s, 1)), \{v_4\}), (((\eta_3, s, 1), (\eta_3, s, 0)), \phi), \\ (((\eta_3, s, 0), (\eta_1, s, 1)), \{v_1, v_2\}), (((\eta_3, s, 0), (\eta_3, s, 0)), \{v_1, v_2\}), \end{array} \right\}.$$

Definition 24. Consider two hse-sets $(\mho_1, Æ_1)$ and $(\mho_2, Æ_2)$, then $(\mho_1, Æ_1)$ OR $(\mho_2, Æ_2)$ shown as $(\mho_1, Æ_1) \vee (\mho_2, Æ_2)$ can be defined as $(\mho_1, Æ_1) \vee (\mho_2, Æ_2)$ = $(\mho_3, Æ_1 \times Æ_2)$, with $\mho_3(\beta, \gamma) = \mho_1(\beta) \cup \mho_2(\gamma), \forall (\beta, \gamma) \in Æ_1 \times Æ_2$.

Example 3.20. Dealing Example **??**, and with sets
$Æ_1 = \{ (\eta_1, s, 1), (\eta_1, t, 1), (a_3, s, 1), (a_3, s, 0) \}$, $Æ_2 = \{ (\eta_1, s, 1), (a_3, s, 0), (a_3, s, 1) \}$.
Consider two hse-sets

$$(\mho_1, Æ_1) = \{ \ ((a_1, s, 1), \{v_1, v_2\}), ((a_1, t, 1), \{v_1\}), ((a_3, s, 1), \{v_4\}), ((a_3, s, 0), \{v_1, v_2\}) \ \}$$

and

$$(\mho_2, Æ_2) = \{ \ ((a_1, s, 1), \{v_1, v_2, v_4\}), ((a_3, s, 0), \{v_1, v_2, v_3\}) \ \}$$

Then $(\mho_1, Æ_1) \vee (\mho_2, Æ_2)$ = $(\mho_3, Æ_1 \times Æ_2)$

$$\left\{ \begin{array}{l} (((a_1, s, 1), (a_1, s, 1)), \{v_1, v_2, v_4\}), (((a_1, s, 1), (a_3, s, 0)), \{v_1, v_2, v_3\}), \\ (((a_1, t, 1), (a_1, s, 1)), \{v_1, v_2, v_4\}), (((a_1, t, 1), (a_3, s, 0)), \{v_1, v_2\}), \\ (((a_3, s, 1), (a_1, s, 1)), \{v_1, v_2, v_4\}), (((a_3, s, 1), (a_3, s, 0)), \{v_1, v_2, v_3, v_4\}), \\ (((a_3, s, 0), (a_1, s, 1)), \{v_1, v_2, v_4\}), (((a_3, s, 0), (a_3, s, 0)), \{v_1, v_2, v_3\}). \end{array} \right\}.$$

Definition 25. Restricted Difference of two hse-sets $(\partial_1, Æ_1)$ and $(\partial_2, Æ_2)$ over $\widetilde{\Omega}$, shown by $(\partial_1, Æ_1) \setminus_R (\mho_2, Æ_2))$, is a hse-set $(\partial_3, Æ_3)$ with $Æ_3 = Æ_1 \cap Æ_2$
$\partial_3(o) = \partial_1(o) \setminus \partial_2(o)$ for $o \in Æ_3$.

Example 3.21. Dealing Example **??**, and with sets
$Æ_1 = \{ (\eta_1, s, 1), (\eta_1, t, 1), (a_3, s, 1), (a_3, s, 0) \}$, $Æ_2 = \{ (\eta_1, s, 1), (a_3, s, 0) \}$.
Consider two hse-sets $(\mho_1, Æ_1)$ and $(\mho_2, Æ_2)$, then

$$(\mho_1, Æ_1) = \{ \ ((a_1, s, 1), \{v_1, v_2, v_4\}), ((a_1, t, 1), \{v_1\}), ((a_3, s, 1), \{v_4\}), ((a_3, s, 0), \{v_1, v_2, v_3\}), \ \}$$

$$(\mho_2, Æ_2) = \{ \ ((a_1, s, 1), \{v_1, v_2\}), ((a_3, s, 0), \{v_1, v_2\}) \ \}.$$

$(\mho_1, Æ_1) \setminus_R (\mho_2, Æ_2))$ = $(\mho_3, Æ_3)$

$$(\mho_3, Æ_3) = \{ \ ((a_1, s, 1), \{v_4\}), ((a_3, s, 0), \{v_3\}) \ \}.$$

Definition 26. Restricted Symmetric Difference of two hse-sets $(\mho_1, Æ_1)$ and $(\mho_2, Æ_2)$ on $\widetilde{\Omega}$, shown by $(\mho_1, Æ_1) \bigstar (\mho_2, Æ_2))$, is a hse-set $(\mho_3, Æ_3)$ characterized by $(\mho_3, Æ_3)$ = $\{((\mho_1, Æ_1) \cup_R (\mho_2, Æ_2)) \setminus_R ((\mho_1, Æ_1) \cap (\mho_2, Æ_2))\}$.

Example 3.22. Dealing Example **??**, with sets
$Æ_1 = \{ (\eta_1, s, 1), (\eta_1, t, 1), (\eta_3, s, 1), (\eta_3, s, 0) \}$, $Æ_2 = \{ (\eta_1, s, 1), (\eta_3, s, 0) \}$.
Consider three hse-sets $(\mho_1, Æ_1)$ and $(\mho_2, Æ_2)$ over $\widetilde{\Omega}$, then

$$(\mho_1, Æ_1) = \left\{ \begin{array}{l} ((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_1, t, 1), \{v_1\}), \\ ((\eta_3, s, 1), \{v_4\}), ((\eta_3, s, 0), \{v_1, v_2, v_3\}), \end{array} \right\}$$

$$(\mho_2, Æ_2) = \{ \ ((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_3, s, 0), \{v_1, v_2\}) \ \}$$

$(\mho_1, Æ_1) \cup_R (\mho_2, Æ_2)$ = $((\eta_1, s, 1), \{v_1, v_2, v_4\}), ((\eta_3, s, 0), \{v_1, v_2, v_3\})$, and
$\{(\mho_1, Æ_1) \cap (\mho_2, Æ_2)\}$ = $\{((\eta_1, s, 1), \{v_1, v_2\}), ((\eta_3, s, 0), \{v_1, v_2\})\}$
then $(\mho_3, Æ_3)$ = $\{((\eta_1, s, 1), \{v_4\}), ((\eta_3, s, 0), \{v_3\})\}$.

Proposition 3.23. *Consider three hse-sets* $(\mho_1, Æ_1)$,$(\mho_2, Æ_2)$ *and* $(\mho_3, Æ_3)$*, then*

(1) $((\mho_1, Æ_1) \wedge (\mho_2, Æ_2))^c$ = $((\mho_1, Æ_1))^c \vee ((\mho_2, Æ_2))^c$

(2) $((\mho_1, \AE_1) \vee (\mho_2, \AE_2))^c = ((\mho_1, \AE_1))^c \wedge ((\mho_2, \AE_2))^c$.

Proposition 3.24. *Consider three hse-sets* $(\mho_1, \AE_1)$, $(\mho_2, \AE_2)$ *and* $(\mho_3, \AE_3)$*, then*

(1) $((\mho_1, \AE_1) \wedge (\mho_2, \AE_2)) \wedge (\mho_3, \AE_3) = (\mho_1, \AE_1) \wedge ((\mho_2, \AE_2) \wedge (\mho_3, \AE_3))$
(2) $((\mho_1, \AE_1) \vee (\mho_2, \AE_2)) \vee (\mho_3, \AE_3) = (\mho_1, \AE_1) \vee ((\mho_2, \AE_2) \vee (\mho_3, \AE_3))$
(3) $(\mho_1, \AE_1) \vee ((\mho_2, \AE_2) \wedge (\mho_3, \AE_3) = ((\mho_1, \AE_1) \vee ((\mho_2, \AE_2)) \wedge ((\mho_1, \AE_1) \vee (\mho_3, \AE_3))$
(4) $(\mho_1, \AE_1) \wedge ((\mho_2, \AE_2) \vee (\mho_3, \AE_3)) = ((\mho_1, \AE_1) \wedge ((\mho_2, \AE_2)) \vee ((\mho_1, \AE_1) \wedge (\mho_3, \AE_3))$.

4. Basic Properties and Laws

In this section of the paper, some properties like exclusion, contraction and laws like idempotent, identity, domination etc. are described.

(1) Idempotent Laws
(2) $(\Xi, \partial) \cup (\Xi, \partial) = (\Xi, \partial) = (\Xi, \partial) \cup_R (\Xi, \partial)$
(3) $(\Xi, \partial) \cap (\Xi, \partial) = (\Xi, \partial) = (\Xi, \partial) \cap_\varepsilon (\Xi, \partial)$.

(1) Identity Laws
(2) $(\Xi, \partial) \cup (\Xi, \partial)_\Phi = (\Xi, \partial) = (\Xi, \partial) \cup_R (\Xi, \partial)_\Phi$
(3) $(\Xi, \partial) \cap (\Xi, \partial)_U = (\Xi, \partial) = (\Xi, \partial) \cap_\varepsilon (\Xi, \partial)_U$
(4) $(\Xi, \partial) \setminus_R (\Xi, \partial)_\Phi = (\Xi, \partial) = (\Xi, \partial) \bigstar (\Xi, \partial)_\Phi$
(5) $(\Xi, \partial) \setminus_R (\Xi, \partial) = (\Xi, \partial)_\Phi = (\Xi, \partial) \bigstar (\Xi, \partial)$.

(1) Domination Laws
(2) $(\Xi, \partial) \cup (\Xi, \partial)_U = (\Xi, \partial)_U = (\Xi, \partial) \cup_R (\Xi, \partial)_U$
(3) $(\Xi, \partial) \cap (\Xi, \partial)_\Phi = (\Xi, \partial)_\Phi = (\Xi, \partial) \cap_\varepsilon (\Xi, \partial)_\Phi$.

(1) Property of Exclusion
(2) $(\Xi, \partial) \cup (\Xi, \partial)^* = (\Xi, \partial)_U = (\Xi, \partial) \cup_R (\Xi, \partial)^*$.

(1) Property of Contradiction
(2) $(\Xi, \partial) \cap (\Xi, \partial)^* = (\Xi, \partial)_\Phi = (\Xi, \partial) \cap_\varepsilon (\Xi, \partial)^*$.

(1) Absorption Laws
(2) $(\eth, \mathcal{H}_1) \cup ((\eth, \mathcal{H}_1) \cap (\eth, \mathcal{H}_1)) = (\eth, \mathcal{H}_1)$
(3) $(\eth, \mathcal{H}_1) \cap ((\eth, \mathcal{H}_1) \cup (\eth, \mathcal{H}_1)) = (\eth, \mathcal{H}_1)$
(4) $(\eth, \mathcal{H}_1) \cup_R ((\eth, \mathcal{H}_1) \cap_\varepsilon (\eth, \mathcal{H}_1)) = (\eth, \mathcal{H}_1)$
(5) $(\eth, \mathcal{H}_1) \cap_\varepsilon ((\eth, \mathcal{H}_1) \cup_R (\eth, \mathcal{H}_1)) = (\eth, \mathcal{H}_1)$.

(1) Absorption Laws
(2) $((\eth, \mathsf{C}_1) \cup (\eth, \mathsf{C}_2)) = ((\eth, \mathsf{C}_1) \cup (\eth, \mathsf{C}_2))$
(3) $((\eth, \mathsf{C}_1) \cup_R (\eth, \mathsf{C}_2)) = ((\eth, \mathsf{C}_1) \cup_R (\eth, \mathsf{C}_2))$
(4) $((\eth, \mathsf{C}_1) \cap (\eth, \mathsf{C}_2)) = ((\eth, \mathsf{C}_1) \cap (\eth, \mathsf{C}_2))$
(5) $((\eth, \mathsf{C}_1) \cap_\varepsilon (\eth, \mathsf{C}_2)) = ((\eth, \mathsf{C}_1) \cap_\varepsilon (\eth, \mathsf{C}_2))$
(6) $((\eth, \mathsf{C}_1) \bigstar (\eth, \mathsf{C}_2)) = ((\eth, \mathsf{C}_1) \bigstar (\eth, \mathsf{C}_2))$.

(1) Associative Laws
(2) $(\Upsilon, \mathsf{C}_1) \cup ((\Upsilon, \mathsf{C}_2) \cup (\omega, \mathsf{C}_3)) = ((\Upsilon, \mathsf{C}_1) \cup (\Upsilon, \mathsf{C}_2)) \cup (\omega, \mathsf{C}_3)$
(3) $(\Upsilon, \mathsf{C}_1) \cup_R ((\Upsilon, \mathsf{C}_2) \cup_R (\omega, \mathsf{C}_3)) = ((\Upsilon, \mathsf{C}_1) \cup_R (\Upsilon, \mathsf{C}_2)) \cup_R (\omega, \mathsf{C}_3)$
(4) $(\Upsilon, \mathsf{C}_1) \cap ((\Upsilon, \mathsf{C}_2) \cap (\omega, \mathsf{C}_3)) = ((\Upsilon, \mathsf{C}_1) \cap (\Upsilon, \mathsf{C}_2)) \cap (\omega, \mathsf{C}_3)$
(5) $(\Upsilon, \mathsf{C}_1) \cap_\varepsilon ((\Upsilon, \mathsf{C}_2) \cap_\varepsilon (\omega, \mathsf{C}_3)) = ((\Upsilon, \mathsf{C}_1) \cap_\varepsilon (\Upsilon, \mathsf{C}_2)) \cap_\varepsilon (\omega, \mathsf{C}_3)$
(6) $(\Upsilon, \mathsf{C}_1) \bigvee ((\Upsilon, \mathsf{C}_2) \bigvee (\omega, \mathsf{C}_3)) = ((\Upsilon, \mathsf{C}_1) \bigvee (\Upsilon, \mathsf{C}_2)) \bigvee (\omega, \mathsf{C}_3)$

(7) $(\Upsilon, \mathtt{C}_1) \bigwedge ((\Upsilon, \mathtt{C}_2) \bigwedge (\omega, \mathtt{C}_3)) = ((\Upsilon, \mathtt{C}_1) \bigwedge (\Upsilon, \mathtt{C}_2)) \bigwedge (\omega, \mathtt{C}_3)$.

(1) De Morgan's Laws
(2) $((\Theta, \mathtt{C}_1) \cup (\Theta, \mathtt{C}_2))^c = (\Theta, \mathtt{C}_1)^c \cap_\varepsilon (\Theta, \mathtt{C}_2)^c$
(3) $((\Theta, \mathtt{C}_1) \cap_\varepsilon (\Theta, \mathtt{C}_2))^c = (\Theta, \mathtt{C}_1)^c \cup (\Theta, \mathtt{C}_2)^c$
(4) $((\Theta, \mathtt{C}_1) \cup_R (\Theta, \mathtt{C}_2))^* = (\Theta, \mathtt{C}_1)^* \cap (\Theta, \mathtt{C}_2)^*$
(5) $((\Theta, \mathtt{C}_1) \cap (\Theta, \mathtt{C}_2))^* = (\Theta, \mathtt{C}_1)^* \cup_R (\Theta, \mathtt{C}_2)^*$
(6) $((\Theta, \mathtt{C}_1) \bigvee (\Theta, \mathtt{C}_2))^c = (\Theta, \mathtt{C}_1)^c \bigwedge (\Theta, \mathtt{C}_2)^c$
(7) $((\Theta, \mathtt{C}_1) \bigwedge (\Theta, \mathtt{C}_2))^c = (\Theta, \mathtt{C}_1)^c \bigvee (\Theta, \mathtt{C}_2)^c$
(8) $((\Theta, \mathtt{C}_1) \bigvee (\Theta, \mathtt{C}_2))^* = (\Theta, \mathtt{C}_1)^* \bigwedge (\Theta, \mathtt{C}_2)^*$
(9) $((\Theta, \mathtt{C}_1) \bigwedge (\Theta, \mathtt{C}_2))^* = (\Theta, \mathtt{C}_1)^* \bigvee (\Theta, \mathtt{C}_2)^*$.

(1) Distributive Laws
(2) $(\Theta, \mathtt{C}_1) \cup ((\Theta, \mathtt{C}_2) \cap (\omega, \mathtt{C}_3)) = ((\Theta, \mathtt{C}_1) \cup (\Theta, \mathtt{C}_2)) \cap ((\Theta, \mathtt{C}_1) \cup (\omega, \mathtt{C}_3))$
(3) $(\Theta, \mathtt{C}_1) \cap ((\Theta, \mathtt{C}_2) \cup (\omega, \mathtt{C}_3)) = ((\Theta, \mathtt{C}_1) \cap (\Theta, \mathtt{C}_2)) \cup ((\Theta, \mathtt{C}_1) \cap (\omega, \mathtt{C}_3))$
(4) $(\Theta, \mathtt{C}_1) \cup_R ((\Theta, \mathtt{C}_2) \cap_\varepsilon (\omega, \mathtt{C}_3)) = ((\Theta, \mathtt{C}_1) \cup_R (\Theta, \mathtt{C}_2)) \cap_\varepsilon ((\Theta, \mathtt{C}_1) \cup_R (\omega, \mathtt{C}_3))$
(5) $(\Theta, \mathtt{C}_1) \cap_\varepsilon ((\Theta, \mathtt{C}_2) \cup_R (\omega, \mathtt{C}_3)) = ((\Theta, \mathtt{C}_1) \cap_\varepsilon (\Theta, \mathtt{C}_2)) \cup_R ((\Theta, \mathtt{C}_1) \cap_\varepsilon (\omega, \mathtt{C}_3))$
(6) $(\Theta, \mathtt{C}_1) \cup_R ((\Theta, \mathtt{C}_2) \cap (\omega, \mathtt{C}_3)) = ((\Theta, \mathtt{C}_1) \cup_R (\Theta, \mathtt{C}_2)) \cap ((\Theta, \mathtt{C}_1) \cup_R (\omega, \mathtt{C}_3))$
(7) $(\Theta, \mathtt{C}_1) \cap ((\Theta, \mathtt{C}_2) \cup_R (\omega, \mathtt{C}_3)) = ((\Theta, \mathtt{C}_1) \cap (\Theta, \mathtt{C}_2)) \cup_R ((\Theta, \mathtt{C}_1) \cap (\omega, \mathtt{C}_3))$.

5. AN APPLICATION TO HYPERSOFT EXPERT SET

In this section, the application of hse-set theory in decision-making problem is presented.

Statement of the problem

An assembling organization advertises an "open position" to fill its an empty position. Its primary trademark is "the perfect individual for the right post". Eight applications got from the appropriate applicants and the experts need to finish this employing system through the choice leading group of certain specialists for certain recommended ascribes..

Proposed Algorithm

The following is the algorithm which is adopted for the solution of the problem.

Step-1

FIGURE 1. Algorithm for recruitment process

Let universe of discourse $\widetilde{\Omega} = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8\}$ consists of eight candidates and X = $\{E_1, E_2, E_3\}$ is representing a set of experts. Attributes with corresponding attribute-valued sets are given as:

$\mathcal{O}_1 = Qualification = \{\triangle_1 = M.phil, \triangle_2 = Ph.D\}$
$\mathcal{O}_2 = Experience = \{\triangle_3 = 5years, \triangle_4 = 10years\}$
$\mathcal{O}_3 = ComputerKnowledge = \{\triangle_5 = Yes, \triangle_6 = No\}$
$\mathcal{O}_4 = Confidence = \{\triangle_7 = Low, \triangle_8 = High\}$
$\mathcal{O}_5 = Skills = \{\triangle_9 = Good, \triangle_{10} = Excellent\}$
and then $H = \mathcal{O}_1 \times \mathcal{O}_2 \times \mathcal{O}_3 \times \mathcal{O}_4 \times \mathcal{O}_5$, $H =$

$$\left\{\begin{array}{l}
(\triangle_1,\triangle_3,\triangle_5,\triangle_7,\triangle_9),(\triangle_1,\triangle_3,\triangle_5,\triangle_7,\triangle_{10}),(\triangle_1,\triangle_3,\triangle_5,\triangle_8,\triangle_9),(\triangle_1,\triangle_3,\triangle_5,\triangle_8,\triangle_{10}),\\
(\triangle_1,\triangle_3,\triangle_6,\triangle_7,\triangle_9),(\triangle_1,\triangle_3,\triangle_6,\triangle_7,\triangle_{10}),(\triangle_1,\triangle_3,\triangle_6,\triangle_8,\triangle_9),(\triangle_1,\triangle_3,\triangle_6,\triangle_8,\triangle_{10}),\\
(\triangle_1,\triangle_4,\triangle_5,\triangle_7,\triangle_9),(\triangle_1,\triangle_4,\triangle_5,\triangle_7,\triangle_{10}),(\triangle_1,\triangle_4,\triangle_5,\triangle_8,\triangle_9),(\triangle_1,\triangle_4,\triangle_5,\triangle_8,\triangle_{10}),\\
(\triangle_1,\triangle_4,\triangle_6,\triangle_7,\triangle_9),(\triangle_1,\triangle_4,\triangle_6,\triangle_7,\triangle_{10}),(\triangle_1,\triangle_4,\triangle_6,\triangle_8,\triangle_9),(\triangle_1,\triangle_4,\triangle_6,\triangle_8,\triangle_{10}),\\
(\triangle_2,\triangle_3,\triangle_5,\triangle_7,\triangle_9),(\triangle_2,\triangle_3,\triangle_5,\triangle_7,\triangle_{10}),(\triangle_2,\triangle_3,\triangle_5,\triangle_8,\triangle_9),(\triangle_2,\triangle_3,\triangle_5,\triangle_8,\triangle_{10}),\\
(\triangle_2,\triangle_3,\triangle_6,\triangle_7,\triangle_9),(\triangle_2,\triangle_3,\triangle_6,\triangle_7,\triangle_{10}),(\triangle_2,\triangle_3,\triangle_6,\triangle_8,\triangle_9),(\triangle_2,\triangle_3,\triangle_6,\triangle_8,\triangle_{10}),\\
(\triangle_2,\triangle_4,\triangle_5,\triangle_7,\triangle_9),(\triangle_2,\triangle_4,\triangle_5,\triangle_7,\triangle_{10}),(\triangle_2,\triangle_4,\triangle_5,\triangle_8,\triangle_9),(\triangle_2,\triangle_4,\triangle_5,\triangle_8,\triangle_{10}),\\
(\triangle_2,\triangle_4,\triangle_6,\triangle_7,\triangle_9),(\triangle_2,\triangle_4,\triangle_6,\triangle_7,\triangle_{10}),(\triangle_2,\triangle_4,\triangle_6,\triangle_8,\triangle_9),(\triangle_2,\triangle_4,\triangle_6,\triangle_8,\triangle_{10})
\end{array}\right\}$$

and now take $\Omega \subseteq H$ as $\Omega = \{S_1 = (\triangle_1,\triangle_3,\triangle_5,\triangle_7,\triangle_9), S_2 = (\triangle_1,\triangle_3,\triangle_6,\triangle_7,\triangle_{10}), S_3 = (\triangle_1,\triangle_4,\triangle_6,\triangle_8,\triangle_9), S_4 = (\triangle_2,\triangle_3,\triangle_6,\triangle_8,\triangle_9), S_5 = (\triangle_2,\triangle_4,\triangle_6,\triangle_7,\triangle_{10})\}$ and $(\mho,\Omega) =$

$$\left\{\begin{array}{l}
((S_1,E_1,1)=\{C_1,C_2,C_4,C_7,C_8\}),((S_1,E_2,1)=\{C_1,C_4,C_5,C_8\}),\\
((S_1,E_3,1)=\{C_1,C_3,C_4,C_5,C_6,C_7,C_8\}),((S_5,E_3,0)=\{C_2,C_4,C_6\})\\
((S_2,E_1,1)=\{C_3,C_5,C_8\}),((S_2,E_2,1)=\{C_1,C_3,C_4,C_5,C_6,C_8\}),\\
((S_2,E_3,1)=\{C_1,C_2,C_4,C_7,C_8\}),((S_3,E_3,1)=\{C_1,C_7,C_8\}),\\
((S_3,E_1,1)=\{C_3,C_4,C_5,C_7\}),((S_3,E_2,1)=\{C_1,C_2,C_5,C_8\}),\\
((S_4,E_1,1)=\{C_1,C_7,C_8\}),((S_4,E_2,1)=\{C_5,C_1,C_4,C_8\}),\\
((S_4,E_3,1)=\{C_1,C_6,C_7,C_8\}),((S_5,E_3,1)=\{C_1,C_3,C_4,C_5,C_7,C_8\}),\\
((S_5,E_1,1)=\{C_1,C_3,C_4,C_5,C_7,C_8\}),((S_5,E_2,1)=\{C_1,C_4,C_5,C_8\}),\\
((S_1,E_1,0)=\{C_3,C_5,C_6\}),((S_1,E_2,0)=\{C_2,C_3,C_6,C_7\}),\\
((S_1,E_3,0)=\{C_2,C_5\}),((S_2,E_3,0)=\{C_2,C_3,C_4,C_5,C_6\}),\\
((S_2,E_1,0)=\{C_1,C_2,C_4,C_5,C_6,C_7\}),((S_2,E_2,0)=\{C_2,C_7\}),\\
((S_3,E_1,0)=\{C_1,C_2,C_6,C_8\}),((S_3,E_2,0)=\{C_3,C_4,C_6,C_7\}),\\
((S_3,E_3,0)=\{C_2,C_3,C_4,C_5,C_7\}),((S_4,E_3,0)=\{C_2,C_3,C_4,C_5\}),\\
((S_4,E_1,0)=\{C_2,C_3,C_3,C_4,C_5,C_7\}),((S_4,E_2,0)=\{C_2,C_3,C_6,C_7\}),\\
((S_5,E_1,0)=\{C_4,C_6,C_7\}),((S_5,E_2,0)=\{C_2,C_3,C_6,C_7\}),
\end{array}\right\}$$

is a hypersoft expert set.

Step-2 Agree-hse-set and Disagree-hse-set have been presented in Tables 1 and 2 respectively in such way when $C_i \in F_1(o)$ then C_{ij} = ↑ = 1 otherwise C_{ij} = ↓ = 0, and when $C_i \in F_0(o)$ then C_{ij} = ↑ = 1 otherwise C_{ij} = ↓ = 0.

TABLE 1. Agree-hse-set

v	C_1	C_2	C_3	C_4	C_5	C_6	C_8	C_7
(S_1,E_1)	↑	↑	↓	↑	↓	↓	↑	↑
(S_2,E_1)	↓	↓	↑	↓	↑	↓	↓	↑
(S_3,E_1)	↓	↓	↑	↑	↑	↓	↑	↓
(S_4,E_1)	↑	↓	↓	↓	↑	↓	↑	↑
(S_5,E_1)	↑	↑	↑	↓	↑	↓	↓	↑
(S_1,E_2)	↑	↓	↓	↑	↓	↓	↓	↑
(S_2,E_2)	↑	↓	↑	↑	↑	↑	↓	↑
(S_3,E_2)	↓	↓	↑	↑	↑	↓	↑	↓
(S_4,E_2)	↑	↓	↓	↑	↑	↓	↓	↑
(S_5,E_2)	↑	↓	↓	↑	↑	↓	↓	↑
(S_1,E_3)	↑	↓	↑	↑	↓	↑	↑	↑
(S_2,E_3)	↑	↑	↓	↑	↓	↓	↑	↑
(S_3,E_3)	↑	↓	↓	↓	↓	↓	↑	↑
(S_4,E_3)	↑	↓	↓	↓	↓	↑	↑	↑
(S_5,E_3)	↑	↓	↑	↓	↑	↓	↑	↑
$\otimes_j = \sum_i C_{ij}$	$\otimes_1 = 12$	$\otimes_2 = 3$	$\otimes_3 = 7$	$\otimes_4 = 9$	$\otimes_5 = 9$	$\otimes_6 = 3$	$\otimes_7 = 9$	$\otimes_8 = 13$

TABLE 2. Disagree-hse-set

V	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
$(\wp_1, E_1)$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
(S_2, E_1)	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
(S_3, E_1)	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
(S_4, E_1)	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
(S_5, E_1)	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
(S_1, E_2)	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
(S_2, E_2)	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$
(S_3, E_2)	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$
(S_4, E_2)	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
(S_5, E_2)	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$
(S_1, E_3)	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
(S_2, E_3)	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
(S_3, E_3)	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
(S_4, E_3)	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
(S_5, E_3)	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\oslash_i = \sum_i C_{ij}$	$\oslash_1 = 3$	$\oslash_2 = 12$	$\oslash_3 = 8$	$\oslash_4 = 6$	$\oslash_5 = 5$	$\oslash_6 = 12$	$\oslash_7 = 6$	$\oslash_8 = 2$

Step-(3-6)
The $\otimes_i = \sum_i C_{ij}$ for Agree-hse-set and $\oslash_i = \sum_i C_{ij}$ for Disagree-hse-set have been shown in Table 3, then $\uplus_j = \otimes_j - \oslash_j$ is calculated so that decision can be made.

TABLE 3. Optimal

$\otimes_i = \sum_i C_{ij}$	$\oslash_i = \sum_i C_{ij}$	$\uplus_j = \otimes_j - \oslash_j$
$\otimes_1 = 12$	$\oslash_1 = 3$	$\uplus_1 = 9$
$\otimes_2 = 3$	$\oslash_2 = 12$	$\uplus_2 = -9$
$\otimes_3 = 7$	$\oslash_3 = 8$	$\uplus_3 = -1$
$\otimes_4 = 9$	$\oslash_4 = 6$	$\uplus_4 = 3$
$\otimes_5 = 9$	$\oslash_5 = 5$	$\uplus_5 = 4$
$\otimes_6 = 3$	$\oslash_6 = 12$	$\uplus_6 = -9$
$\otimes_7 = 9$	$\oslash_7 = 6$	$\uplus_7 = 3$
$\otimes_8 = 13$	$\oslash_8 = 2$	$\uplus_8 = 11$

Decision
As $\uplus_8$ is getting best position in table, so candidate C_8 will be selected. Then max $\uplus_8$, so the committee will decide to select applicant C_8 for the job.

6. COMPARATIVE ANALYSIS

The performance of hypersoft expert model outperforms all other existing models. Such a model is popular in decision-making problems. This can be seen by comparing hypersoft expert set with the others models like soft set, soft expert set and hypersoft set This proposed model is more useful to others as it contains the multi argument approximate function, which is highly effective in decision-making problems. Comparison analysis has been shown in Table **??**. From the Table **??**, it is clear

TABLE 4. Comparison Analysis

Features	soft set	soft expert set	hypersoft set	Pro. Structure
Multi Decisive Opinion	No	Yes	No	Yes
Multi Argument Apro.Function	No	No	Yes	Yes
Single Ar. Apro.Function	Yes	Yes	Yes	Yes
Ranking	No	Yes	No	Yes

that our proposed model is more generalized than the above described models.

7. CONCLUSIONS

In this study, fundamental properties, aggregation operations, and basic set laws are characterized under hypersoft expert set environment. Moreover, an algorithm is proposed for the solution of a decision-making problem. Future work may include the development of hybrids of hypersoft expert set with fuzzy set, rough set, cubic set etc. and algebraic structures like hypersoft expert topological spaces, hypersoft expert functional spaces, hypersoft expert groups, hypersoft expert vector spaces, hypersoft expert ring, hypersoft expert measure etc.

REFERENCES

[1] M. Abbas, G. Murtaza and F. Smarandache, *Basic operations on hypersoft sets and hypersoft point,* Neutrosophic Sets System, **35**(2020) 407-421.

[2] M. Akram, A. Adeel and J. C. R. Alcantud, *Group decision-making methods based on hesitant N-soft sets,* Expert Systems with Applications, **115**(2019) 95-105.

[3]

[4] S. Alkhazaleh, A. R. Salleh and N. Hassan, *Possibility fuzzy soft set,* Advances in Decision Sciences, **2011** (2011) 1-18, Article ID 479756.

[5] S. Alkhazaleh, and A. R. Salleh, Soft Expert Sets, *Advances in Decision Sciences,* **2011**(2011) 757868-1.

[6] S. Alkhazaleh, A. R. Salleh and N. Hassan, *Soft multi sets theory,* Applied Mathematical Sciences, **5,** No.72(2011) 3561-3573.

[7] S. Alkhazaleh, A. R. Salleh and N. Hassan, *Fuzzy parameterized interval-valued fuzzy soft set,* Applied Mathematical Sciences, **5**, No. 67(2011) 3335-3346.

[8] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, *On some new operations in soft set theory,* Computers and Mathematics with Applications, **57,** No.9 (2009) 1547-1553.

[9] K. V. Babitha and J. Sunil, *Soft set relations and functions,* Computers & Mathematics with Applications, **60,** No.7 (2010) 1840-1849.

[10] D. Chen, E. C. C. Tsang, D. S. Yeung and X. Wang, *The parameterization reduction of soft sets and its applications,* Computers and Mathematics with Applications, **49,**No.5-6 (2005) 757-763.

[11] I. Deli, *Convex and Concave Sets Based on Soft Sets and Fuzzy Soft Sets,* Journal of New Theory, **29**(2019) 101-110.

[12]

[13] F. Fatimah, D. Rosadi, R. F. Hakim and J. C. R. Alcantud, *N-soft sets and their decision making algorithms,* Soft Computing, **22,** No.12 (2018) 3829-3842.

[14] M. Ihsan, A. U. Rahman, M. Saeed and H. A. E. W. Khalifa, *Convexity-cum-concavity on fuzzy soft expert set with certain properties,* International Journal of Fuzzy Logic and Intelligent Systems,**21,** No.3 (2021) 233-242.

[15] M. Ihsan, M. Saeed and A. U. Rahman, A rudimentary approach to develop context for convexity cum concavity on soft expert set with some generalized results, Punjab Univ. j. maths. **53,** No. 9(2021) 621-629.

[16] M. Ihsan, M. Saeed and A. U. Rahman, *Multi-attribute decision support model based on bijective hypersoft expert set*, Punjab Univ. j. maths.**54,** No.1(2021) 55-73.

[17] M. Ihsan, A. U. Rahman and M. Saeed, *Hypersoft expert set with application in decision making forrecruitment process,* Neutrosophic Sets and Systems, **42**(2021) 191-207.

[18] M. Ihsan, M. Saeed and A. U. Rahman, *Fuzzy hypersoft expert set with application in decision making for the best selection of product*, Neutrosophic Sets and Systems, **46**(2021) 318-335.

[19] M. Ihsan, M. Saeed and A. U. Rahman, *Single valued neutrosophic hypersoft expert set with application in decision making,* Neutrosophic Sets and Systems, **47**(2021) 451-471.

[20] M. Ihsan, M. Saeed, A. Alburaikan and H. A. E. W. Khalifa, *Product evaluation through multi-criteria decision making based on fuzzy parameterized Pythagorean fuzzy hypersoft expert set,* AIMS Mathematics, **7,** No.6(2022) 11024-11052.

[21] T. Y. Öztürk and A. Yolcu, *On neutrosophic hypersoft topological spaces,* Theory and Application of Hypersoft Set (2021) 215-234.

[22] S. N. Majeed, *Some notions on convex soft sets,* Annals of Fuzzy Mathematics and Informatics, **12,** No.4(2016) 517-526.

[23] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory,* Computers and Mathematics with Applications, **45,** No.4-5 (2005) 555-562.

[24] P. K. Maji, A. R. Roy and R. Biswas, *An application of soft sets in a decision making problem,* Computers and Mathematics with Applications, **44,** No.8-9(2002) 1077-1083.

[25] D. Molodtsov, *Soft set theoryfirst results,* Computers and Mathematics with Applications, **37** No.4-5(1999) 19-31.

[26] A. U. Rahman, M. Saeed, M. Ihsan, M. Arshad and S. Ayaz, *A conceptual framework of m-convex and m-concave sets under soft set environmentwith properties,* Transactions in Mathematical and Computational Sciences, **1,** No. 1(2021) 50-60.

[27] A. U. Rahman, M. Saeed, M. Arshad, M. Ihsan and M. R. Ahmad, *(m, n)-convexity-cum-concavity on fuzzy soft set with applications in first and second sense,* Punjab Univ. j. maths. **53,** No.1(2021) 19-33.

[28] A. U. Rahman, M. Saeed, F. Smarandache and M. R. Ahmad, *Development of hybrids of hypersoft set with complex fuzzy set,complex intuitionistic fuzzy set and complex neutrosophic set,* Neutrosophic Sets and Syst,**38**(2020) 335-354.

[29] A. U. Rahman, M. Saeed and F. Smarandache, *Convex and concave hypersoft sets with some properties,* Neutrosophic Sets Syst, **38**(2020) 497-508.

[30] A. U. Rahman, M. Saeed and A. Dhital, *Decision making application based on neutrosophic parameterized hypersoft set theory,* Neutrosophic Sets and Systems, **41,**(2021) 1-14.

[31] A. U. Rahman, M. Saeed, M. A. Mohammed, S. Krishnamoorthy, S. Kadry and F. Eid, *An integrated algorithmic MADM approach for heart diseases diagnosis based on neutrosophic hypersoft set with possibility degree-based setting,* Life, **12,** No. 5(2022) 729.

[32] A. U. Rahman, M. Saeed and A. Hafeez, *Theory of bijective hypersoft set with application in decision making,* Punjab Univ. j.maths. **53,** No.7(2021)511-526..

[33] A. U. Rahman, M. Saeed, A. Khalid, M. R., Ahmad and S. Ayaz, *Decision-making application based on aggregations of complex fuzzy hypersoft set and development of interval-valued complex fuzzy hypersoft set,* Neutrosophic Sets and Systems, **46,** No.1 (2021) 300-317.

[34] A. U. Rahman, M. Saeed, S. S., Alodhaibi and H. Abd, *Decision making algorithmic approaches based on parameterization of neutrosophic set under hypersoft set environment with fuzzy, intuitionistic fuzzy and neutrosophic settings,* CMES-Computer Modeling in Engineering & Sciences, **128,** No.2(2021) 743-777.

[35] A. U. Rahman, M. Saeed, H. A. E. W., Khalifa and W. A. Afifi, *Decision making algorithmic techniques based on aggregation operations and similarity measures of possibility intuitionistic fuzzy hypersoft sets,* AIMS Mathwmatics, **7,** No.3 (2022) 3866-3895.

[36] A. U. Rahman, M. Saeed and F. Smarandache, *A theoretical and analytical approach to the conceptual framework of convexity cum concavity on fuzzy hypersoft sets with some generalized properties,* Soft Computing, **26,** No. 9(2022) 4123-4139.

[37] A. U. Rahman, M. Saeed, M. Arshad and S. El-Morsy, *Multi-Attribute Decision-Support System Based on Aggregations of Interval-Valued Complex Neutrosophic Hypersoft Set,* Applied Computational Intelligence and Soft Computing, **2021** (2021).

[38] M. Saeed, A. U. Rahman, M. Ahsan and F. Smarandache, *An inclusive study on fundamentals of hypersoft set*, Theory and Application of Hypersoft Set, Pons Publishing House, Brussels, (2021) 1-23.

[39] M. Saqlain, S. Moin, M. N. Jafar, M. Saeed and F. Smarandache, *Aggregate operators of neutrosophic hypersoft set*, Neutrosophic Sets and Systems, **32**(2020) 294-306.

[40] F. Smarandache, *Extension of soft set to hypersoft set, and then to plithogenic hypersoft set*, Neutrosophic Sets System, **22**(2018) 168-170.

[41] A. Yolcu, F. Smarandache and T. Y. Öztürk, *Intuitionistic fuzzy hypersoft sets,* Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, **70,**(No.1 (2021) 443-455.

[42] A. Yolcu and T. Y. Öztürk, *Fuzzy hypersoft sets and its application to decision-making* Theory and Application of Hypersoft Set (2021) 50-64.