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Application of Some Analytical Techniques to Solve Boundary layer Flow over Exponentially Stretching Sheet Problems

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Abstract.: In this research paper, we apply OHAM and HAM to establish and solve the problem concerning two-dimensional exponential stretching sheets. The governing nonlinear differential equations are modeled with the aid of suitable transformation. A concrete graphical analysis is carried to investigate the behavior of similarity between the analytical and numerical solutions. The techniques OHAM and HAM are capable of producing resemblance between the two graphs. These methods have high accuracy, controlling convergence and generally trustworthy for the higher-order problems solutions.

AMS (MOS) Subject Classification Codes: 35S29; 40S70; 25U09
Key Words: OHAM, HAM, Exponential stretching sheet and Boundary layer flow.

1. Introduction

The stretching surface predicament was modified by Crane and McCormack in 1973 [1]. The enlarging sheet problems for time-independent flow have been used in several manufacturing methods and engineering, resembling non-Newtonian fluid, Magneto-hydrodynamics fluid flow, permeable medium and temperature transference analysis. Magneto-hydrodynamics is the learn of the interface of conducting fluid flow with electro-magnetic process. The electrically conducting fluid flow in the presence of a magnetic area is significant in several fields of engineering as Magneto-hydrodynamics power generation, Magneto-hydrodynamics fluid flow meters, and MHD pump. It is of interest to investigate the behavior of flow of incompressible fluid flow over an exponential extending surface; recently all the researchers take different manufacturing processes over the boundary layer idea [32-34]. The processes are aerodynamic extrusion of hot rolling, plastic surface, gluing of labels on warm bodies, paper manufacture and relevance in polymer industries. A large number of numerical and analytical techniques are used in the study of these systematic models. The mathematical structure of a large number of physical systems clues to non-linear partial or ordinary differential equations in several areas such as physics, chemistry, and engineering technologies. A useful technique is vital to investigate the mathematical model, illustrating techniques conforming physical truth. Common analytic measures, liberalizing the systems or assuming that non-linearity?s are comparatively irrelevant. Such postulates from time to time powerfully, influence the solutions with respect to physics of the phenomena. Thus in search of the exact solutions non-linear PDEs or ODEs are of vast significance. Several controlling mathematical techniques such as (HPM) [2-6] ADM [7-11] Laplace Decomposition Method (LDM) [12-16] Variational Iteration Method (VIM) [17-21]. We used OHAM [22-28] and HAM [29-31].

2. FUNDAMENTALS OF OHAM:

To exhibit the basic technique of OHAM [22- 28] we take Differential Equation

$$T(v(r)) + l(r) = 0 \tag{2. 1}$$

$$D(v) = 0 \tag{2. 2}$$

With T is wide-ranging differential operator D an operator on the boundary and $l(r)$ is a known function where

$$T = L + N \tag{2. 3}$$

In equation (2.3) N and L are non-linear and linear parts. By this method equations of deformation are given as:

$$(1 - s)\left[L(\Psi(t, s)) + l(t)\right] - h(s)\left[L(\Psi(t, s)) + l(t) + N(\Psi(t, s))\right] \tag{2. 4}$$

$$D(\Psi(t, s)) = 0 \tag{2. 5}$$

In equation (2.4), $\Psi(t, s)$ is unknown function, s∈[0,1] is embedding parameter and $h(s)$ is a non- zero assisting function for s≠0 and $h(0) = 0$ as s increases from 0 to 1 the solution $\Psi(t, s)$ is changing between $v_0(t)$ the preliminary guess and the solution $v(t)$. Obviously, when $s = 0$ and $s = 1$ it gives the following

$$\Psi(t, 0) = v_0(t), \Psi(t, 1) = v(t) \tag{2. 6}$$

We select $h(s)$ in the form

$$h(s) = s\beta_1 + s^2\beta_2 + s^3\beta_3 + ... \tag{2. 7}$$

When β_1,β_2,β_3... are convergence control parameters that are to be dogged later. Expressing $\Psi(t, s)$ in terms of perturbation series dependent on ,

$$\Psi(t, s, \beta_i) = v_0(t) + \sum_{k=1}^{\infty} v_k(t, \beta_i)s^k \tag{2. 8}$$

$i = 1, 2, 3...$ putting equation (2.8) in equation (2.4) and equating the coefficients of $s^0, s^1, s^2, ...$

$$L(v_0(t)) + l(t) = 0, D(v_0) = 0 \tag{2. 9}$$

$$L(v_1(t)) = \beta_1 N_0(v_0(t)), D(v_1) = 0 \tag{2. 10}$$

And we have

$$L(v_k(t) - v_{k-1}(t)) = \beta_k N_0(v_0(t)) + \sum_{i=1}^{k-1} \beta_i \left[L(v_{k-i}(t)) + N_{k-i}(v_0(t), v_1(t)...v_{k-i}(t)) \right] \tag{2. 11}$$

For $k = 2, 3, 4...$
$D(v_k) = 0$ Wherever N_i, $i \geq 0$ signifies the coefficients of s^i in N

$$N(v(t)) = N_0(v_0(t)) + sN_1(v_0(t), v_1(t)) + s^2 N_2(v_0(t), v_1(t), v_2(t)) + ... \tag{2. 12}$$

We should emphasize that v_n for $n \geq 0$ are dominated by the equations from (2.9) to (2.11) with boundary condition that we get from an innovative problem is solved. The merging of progression (2.8) relies on the convergence control parameters $\beta_1, \beta_2, \beta_3...$ if it is convergent at $s = 1$ we obtain

$$v(t, \beta_i) = v_0(t) + \sum_{k=1}^{\infty} v_k(t, \beta_i) \tag{2. 13}$$

In general equation (2.1) can be solved approximately in the form

$$v^q = v_0(t) + \sum_{k=1}^{q} v_k(t, \beta_i) \tag{2. 14}$$

We admit that the ending coefficients β_q have a dependence on putting equation (2.14) in (2.1) we get the residual

$$S(t, \beta_i) = L(v)(t, \beta_i) + l(t) + N(v(t, \beta_i)), i = 1, 2, 3, ... \tag{2. 15}$$

If $S(t, \beta_i) = 0$ we have $v(t, \beta_i)$ is an analytical solution. In general, this type of assumption cannot occur for the problems which are nonlinear, but the following functional can be reduced

$$P(\beta_1, \beta_2, \beta_3..., \beta_n) = \int_a^b S^2(t, \beta_1, \beta_2, \beta_3..., \beta_n) dt \tag{2. 16}$$

For $i = 1, 2, 3, ...$
Now we apply Least square technique the values of $\beta_1, \beta_2, \beta_3...$ are determined using conditions of calculus.

$$\frac{\partial P}{\partial \beta_1} = 0, \frac{\partial P}{\partial \beta_2} = 0, ..., \frac{\partial P}{\partial \beta_n} = 0. \tag{2. 17}$$

Wherever a, b lie in the dominion of the given problem, we can also find the values of $\beta_1, \beta_2, \beta_3...$ by using Galerkin's method that is

$$\int_a^b S \frac{\partial P}{\partial \beta_1} = 0, \int_a^b S \frac{\partial P}{\partial \beta_2} = 0, ..., \int_a^b S \frac{\partial P}{\partial \beta_n} = 0. \tag{2. 18}$$

As for obtaining solutions of the problems of higher-order we use optimization techniques such as OHAM, HAM we can also use other employing optimization techniques such as OHPM and OAFM [31,32].

3. BASICS OF HAM

Initiating from

$$N[u(x)] = 0 \tag{3. 19}$$

If N is a tool which is nonlinear, $u(x)$ is function which is to be determined and x is not a dependent variable. The 0^{th}-order functional is given as:

$$(1-s)L[\phi(x,s) - u_0(x)] = shH(x)N[\phi(x,s)] \tag{3. 20}$$

For $0 \leq s \leq 1$ is an inserting parameter,$H(x)$ is an auxiliary function L represents the operator, which is a linear, $h \neq 0$ is a non-zero assisting parameter,$u_0(x)$ shows the starting value of $u(x)$, $\phi(x,s)$ is a function which is to be determined, from equation (3.20) $s = 0$ and $s = 1$ we get

$$\phi(x,0) = u_0(x), \phi(x,1) = u(x) \tag{3. 21}$$

As s increases in the unit interval, its answers $\phi(x,s)$ change from the $u_0(x)$ to $u(x)$. Escalating $\phi(x,s)$ in Taylor series which is given as:

$$\phi(x,s) = u_0(x) + \sum_{j=1}^{\infty} u_j(x)s^j \tag{3. 22}$$

Where

$$u_j = \frac{1}{j!}\frac{\partial^j \phi(x,s)}{\partial s^j} \tag{3. 23}$$

at $s = 0$

The initial guess, linear operator, the auxiliary parameter and assisting function are so correctly selected that Taylor series (3.22) converge at $s = 1$

$$u(x) = u_0(x) + \sum_{j=1}^{\infty} u_j(x) \tag{3. 24}$$

That is the first answer of the unique equation which is nonlinear as showed by Liao [28] selecting $h = -1$ and $H(x) = 1$ Equation (3.20) takes the form

$$(1-s)L[\phi(x,s) - u_0(x)] + sN[\phi(x,s)] = 0 \tag{3. 25}$$

That is we use typically the HPM technique for getting a straight solution, ignoring Taylor expansions and evaluation between HAM and HPM can be seen in [28].As $H(x) = 1$ eq. (3.20) takes the form

$$(1-s)L[\phi(x,s) - u_0(x)] + shV[\phi(x,s)] = 0 \tag{3. 26}$$

Which can be evaluated in the HAM technique. For such a situation $H(x)$ is ignored as the fundamental operators. Differentiating eq. (3.20) j times w.r.t s and then $s = 0$ and lastly dividing them by $j!$ we have j^{th}-order deformation eq.

$$L[u_j(x) - \chi_j u_{j-1}(x)] = hH(x)R_j(\overline{u}_{j-1}, x) \tag{3. 27}$$

$$R_j(\overline{u}_{j-1}, x) = \frac{1}{(j-1)!}\frac{\partial^{j-1} N[\phi(x,s)]}{\partial s^{j-1}} \tag{3. 28}$$

at $s = 0$

$$\overline{u}_j = \left\{u_1(x), u_2(x), ..., u_j(x)\right\} \tag{3. 29}$$

$$\chi_i = \begin{cases} 1, & e > 1 \\ 0, & e \leqslant 1 \end{cases} \tag{3. 30}$$

putting equation (3.22) into (3.28) we have

$$R_j(\overline{u}_{j-1}, x) = \frac{1}{(j-1)!} \frac{\partial^{j-1}}{\partial s^{j-1}} N \left[\sum_{n=0}^{\infty} u_n(x) s^n \right] \tag{3. 31}$$

at $s = 0$

It should be reserved that $u_j(x)$ for $j \geq 1$ is dominated by the linear eq. (3.27) with the linear boundary condition that is found from the innovative problem which is solved by Matlab.

4. FORMULATION:

We deliberate flow of in-compressible several fluids in excess of exponential extending at $y = 0$. The suppositions boundary layer hypothesis, the momentum and continuity equations as:

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} = 0, \tag{4. 32}$$

$$\overline{u} \frac{\partial \overline{u}}{\partial x} + \overline{v} \frac{\partial \overline{u}}{\partial y} = \overline{v} \frac{\partial^2 \overline{u}}{\partial x^2} \tag{4. 33}$$

Here the velocity components along horizontal and vertical directions are $\overline{u}$ and $\overline{v}$, v signifies Kinematic viscosity which is $v = \dfrac{\mu}{\rho}$.

The boundary conditions matching to the exponential stretching surface are

$$u = U_0 e^{\frac{x}{\Upsilon}}, v = 0 \, at \, y = 0, u \longrightarrow 0 \, as \, y \longrightarrow \infty \tag{4. 34}$$

U_0 is orientation rapidity and Υ shows constant. Depending on the given resemblance alterations in equations (4.32) to (4.34) we get

$$\xi = \sqrt{\frac{U_0}{2v\Upsilon}} e^{\frac{x}{\Upsilon}} y, u = U_0 e^{\frac{x}{\Upsilon}} f(\xi), v = -\sqrt{\frac{vU_0}{2\Upsilon}} e^{\dfrac{x}{2\Upsilon}} [f(\xi) - \xi f(\xi)] \tag{4. 35}$$

After the transformations are given above the reduced nonlinear differential equations with the related boundary conditions as:

$$f''' - 2f'^2 + ff'' = 0, f(0) = 0, f'(0) = 1, f(\infty) = 0 \tag{4. 36}$$

We are interested in solving (4.36) by using OHAM and HAM techniques given in sections (2) and (3)

Applying the OHAM technique, on the problem in equation (4.36) we have the OHAM pattern as:

$$G'''(y) - 2G'^2(y) + G(y)G''(y) = 0, G(0) = 0, G'(0) = 0, G'(\infty) = 0 \tag{4. 37}$$

For simplicity and justification of the problem, the boundary conditions of equation (4.37) get the custom which will be verified at the end of the graphs.

$$G(0) = 0, G'(0) = 1, G'(2) = 0 \tag{4. 38}$$

The exact solution is

$$G(y) = y - 0.25y^2 \tag{4. 39}$$

Applying the technique of section 2, 0^{th}-order problem is

$$G_0^{(3)}(y) = 0, G_0(0) = 0, G_0'(0) = 1, G_0'(2) = 0 \tag{4. 40}$$

Its solution is

$$G_0(y) = \frac{1}{4}(4y - y^2) \tag{4. 41}$$

The problem of 1^{st} order is

$$\begin{aligned} G_1^{(3)}(y, C_1) &= -2C_1(G_0'(y))^2 + C_1(G_0(y))G_0^{(2)}(y) + (1 + C_1)G_0^{(3)}(y), \\ &G_1(0) = 0, G_1'(0) = 0, G_1'(2) = 0 \end{aligned} \tag{4. 42}$$

Its solution is

$$G_1(y, C_1) = \frac{C_1}{480}[300y^2 - 160y^3 + 30y^4 - 3yy^5] \tag{4. 43}$$

The problem of 2^{nd} order is

$$\begin{aligned} G_2^{(3)}(y, C_1, C_2) &= -2C_2(G_0'(y))^2 - 4C_1G_0'(y)G_1'(y) + C_2G_0(y)G_0''(y) \\ &+ C_1G_1'(y)G_0''(y) + C_1G_0(y)G_1'(y) + C_2G_0''(y) + (1 + C_1)G_1''(y), \\ &G_2(0) = 0, G_2'(0) = 0, G_2'(2) = 0. \end{aligned} \tag{4. 44}$$

Its solution is

$$G_2^{(3)}(y, C_1, C_2) = \frac{1}{322560}\begin{bmatrix} C_1\{201600y^2 - 107520y^3 + 20160y^4 - 2016y^5\} \\ +C_1^2\{350976y^2 - 107520y^3 - 30240y^4 + 18876y^5 \\ -4256y^6 + 432y^7 - 27y^8\} + C_2\{201600y^2 \\ -107520y^3 + 20160y^4 - 2016y^5\} \end{bmatrix} \tag{4. 45}$$

Now consuming equations (4.41), (4.43), (4.45) we have

$$\overline{G}(y, C_1, C_2) = G_0(y) + G_1(y, C_1) + G_2(y, C_1, C_2) \tag{4. 46}$$

Using the method of the second section and using in the interval [0,2] we acquire the residual

$$S = \overline{G}'''(y) - 2(\overline{G}'(y))^2 + \overline{G}(y)\overline{G}'(y) \tag{4. 47}$$

Now using the least square or Galerkin, s methods and minimizing using calculus we get the values $C_1 = 0.0000000$, $C_2 = -0.18299306687060843$
We acquire the OHAM solution as:

$$\overline{G}(y) = y - 0.364371y^2 + 0.0609977y^3 - 0.0114371y^4 + 0.00114371y^5 + O(y^6) \tag{4. 48}$$

The HAM solution is

$$\overline{G}(y) = y - 0.568159y^2 + 0.206473y^3 - 0.611674y^4 + 0.013211y^5 - 0.00191192y^6 + O(y^7) \tag{4. 49}$$

5. TABLE.1

Y	Exact Solution	OHAM Solution	HAM Solution	OHAM Error	HAM Error
0.0	0.0	0.0	0.0	0.0	0.0
0.1	0.0975	0.0964162	0.0945171	1.08384 E-3	2.9821 E-3
0.2	0.19	0.185895	0.178824	4.10478 E-3	1.11763 E-2
0.3	0.2775	0.268764	0.253949	8.73628 E-3	2.35512 E-2
0.4	0.36	0.345323	0.320806	1.46765 E-2	3.91935 E-2
0.5	0.4375	0.415853	0.380205	2.1647 E-2	5.72955 E-2
0.6	0.51	0.480609	0.432856	2.93912 E-2	7.7144 E-2
0.7	0.5775	0.539827	0.479389	3.76732 E-2	9.81112 E-2
0.8	0.64	0.593724	0.520355	4.62763 E-2	1.19645 E-1
0.9	0.6975	0.642499	0.556235	5.50014 E-2	1.41265 E-1
1.0	0.75	0.686334	0.587446	6.36663 E-2	1.62554 E-1
1.1	0.7975	0.725396	0.614347	7.21036 E-2	1.83153 E-1
1.2	0.84	0.75984	0.637237	8.01597 E-2	2.02763 E-1
1.3	0.8775	0.789807	0.65636	8.76934 E-2	2.2114 E-1
1.4	0.91	0.815426	0.671903	9.45744 E-2	2.38097 E-1
1.5	0.9375	0.836818	0.683999	1.00682 E-1	2.53501 E-1
1.6	0.96	0.854096	0.692717	1.05904 E-1	2.67283 E-1
1.7	0.9775	0.867366	0.698064	1.10134 E-1	2.79436 E-1
1.8	0.99	0.876727	0.699978	1.13273 E-1	2.90022 E-1
1.9	0.9975	0.882275	0.698319	1.15225 E-1	2.99181 E-1
2.0	1.00	0.884104	0.692863	1.15896 E-1	3.07137 E-1

The graphs are given in Fig.1 and Fig.2 exhibit coincidence between the OHAM, HAM and their exact solutions.

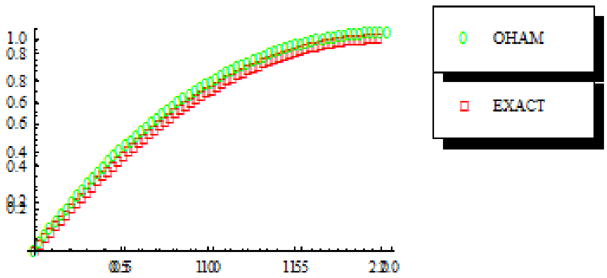

FIGURE 1

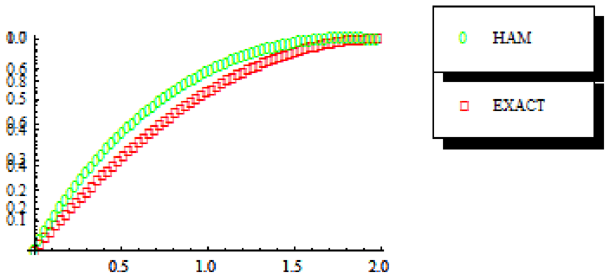

FIGURE 2

6. GENERAL CONCLUSION:

The main aims and objectives of this task are offering the chain elucidation of the boundary layers? equation of two-dimensional fluid flow on an exponential permeable by considering OHAM and HAM techniques. These techniques are controlling tools to pursue the solutions of different non-linear boundary value problems. These methods eliminate the complexity as compared to other techniques because these are resourceful. We get consequently fast convergent results by using OHAM and HAM. We can compare the outcomes achieved using these techniques with the analytical solution results. We found coincidence between them. The graphs of OHAM and HAM were found overlapping with their exact solution graphs. These methods overcome the difficulty in other methods because it is efficient, we derived fast convergent results by using these techniques. We, the authors tried the formulated problem by modified optimal homotopy asymptotic method (MOHAM) by splitting $l(t)$ into $l_1(t)$ and $l_2(t)$.

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