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A Consequential Computation of Degree Based Topological Indices of Grasmere Geometric Graph

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Abstract.: In this paper, we compute many degree-based topological indices like First general Zagreb index, Randic index, ABC-index, Reciprocal Randic index, Reduced Reciprocal index, Inverse Sum index, Symmetric Division index,ABC_4, GA_5 , Multiplicative Randic index and Hyper-Zagreb index for Grasmere Geometric Graph. The graph comprises of a combination of different sized squares and triangles used for classic Victorian floor tile designs. Such designs are perfect for hallways, exterior paths, and porches as they are fully crystallized and frost resistant.

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1. Introduction and preliminary results

Many real world situations can conveniently be described by diagrams, consisting of a set of points together with lines joining certain pairs of these points.In mathematics, graph theory concerns with networks of points connected by lines. These mathematical structures are used to explore or model pairwise relations between objects [1, 20]. In this context, a graph consists of vertices connected by edges. The number of edges that enter or exit from a vertex is called degree. Thus,the nature of a graph is characterized by its number of vertices.The inception of graph theory can be traced back in recreational mathematics. However, it has become an important area of mathematics with applications in chemistry, computer sciences, operation research and social sciences[2]. In the study of molecular models,graph theory is an interdisciplinary field which is known as molecular topology and graph theory related to chemicals mathematically model molecules to gain insight into the physical properties of theses chemical compounds. Bioactivity of the chemical compounds speculate by the exploration of different topological indices like geometric-arithmetic (GA) index, Wiener index, Zagreb index, Szeged index, Randic index and ABC index,by the

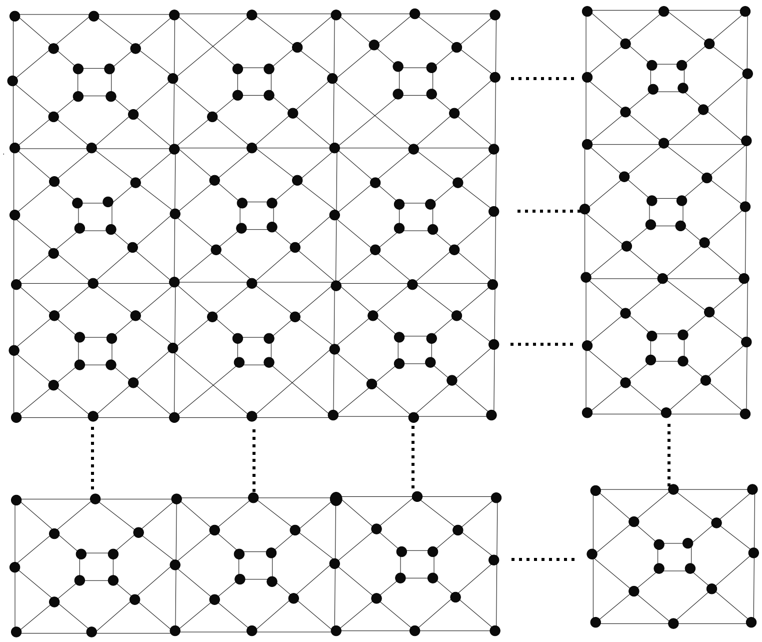

FIGURE 1. Grasmere Geometric Graph

learning of(QSAR), physic chemical properties and (QAPR)[6]. A molecular descriptor is an essential information which is obtained from the graph of molecules which shows a capable root to eloquent the analytical form, the size of molecules, branching, shape and cyclicity [3, 6].Some important kinds of molecular descriptors which are named as distance-based, degree-based, and spectrum-based. Degree-based molecular descriptor has significant importance between these classes and play an essential part in graph theory of chemicals and especially in chemistry. The idea of molecular descriptors was introduced by Wiener as he worked on paraffin's boiling point and the index was given a name as path number that was further renamed as Wiener index [5, 12, 13, 14]. One of the several and varied applications of graph theory is found in architecture and design[7, 19]. Graphs can be used, at least, in two different stages of the design. On one hand, at the initial process of design, to draw in outline or to make a sketch of the project, that establishes the relations and connections between the different parts of it. On the other hand, as an instrument of analysis of the finish or performed project, that allows a classification of different styles. This study considers the Grasmere Geometric Graph. The graph comprises of a combination of different sized squares and triangles used for classic Victorian floor tile designs. Such designs are perfect for hallways, exterior paths, and porches as they are fully crystallized and frost resistant. Thus, we determined 1stgeneral zagreb index, general Randic connectivity index , ABC in- dex, RR(G) index, RRR(G) index, inverse sum index ISI(G), symmetric-division index SD(G), 4th atom-bond connectivity index , 5th GA index , multiplicative Randic index for the Grasmere Geometric Graph.

• 1^{st} **General Zagreb Index**:
Li and Zhao offered the 1^{st} general Zagreb index in [16]:

$$M_\alpha(G) = \sum_{q \in V(G)} (d_q)^\alpha. \quad (1.\ 1)$$

• **Randić Connectivity Index**:
The Randić connectivity index was defined in [11]. The $R_\alpha(G)$ index is described as:

$$R_\alpha(G) = \sum_{qr \in E(G)} (d_q d_r)^\alpha. \quad (1.\ 2)$$

Where α is a real number.
• **Atom-Bond Connectivity Index**:
The ABC index was introduced by Estrada *et al.* [4] :

$$ABC(G) = \sum_{qr \in E(G)} \sqrt{\tfrac{d_q + d_r - 2}{d_q \times d_r}}. \quad (1.\ 3)$$

• **Reciprocal Randić Index**:
The RR(G) Index [11] is described as follows:

$$RR(G) = \sum_{qr \in E(G)} \sqrt{d_q d_r}. \quad (1.\ 4)$$

• **Reduced Reciprocal Randić Index**:
The RRR(G) Index [15] is defined as follows:

$$RRR(G) = \sum_{qr \in E(G)} \sqrt{(d_q - 1)(d_r - 1)}. \quad (1.\ 5)$$

• **Inverse Sum Index**:
The Inverse Sum Index [17] is described as follows:

$$ISI(G) = \sum_{qr \in E(G)} \frac{d_q d_r}{d_q + d_r}. \quad (1.\ 6)$$

• **Symmetric Division Index SD(G)**:
The SD(G) Index [10] is described as follows:

$$SD(G) = \sum_{qr \in E(G)} \frac{d_q^2 + d_r^2}{d_q d_r}. \quad (1.\ 7)$$

• **Fourth Atom-Bond Connectivity Index**:
In 2010, the 4th ABC index was initiated by Ghorbani and Hosseinzadeh [9] and described as:

$$ABC_4(G) = \sum_{qr \in E(G)} \sqrt{\tfrac{S_q + S_r - 2}{S_q \times S_r}}. \quad (1.\ 8)$$

• **Fifth (GA) index**:
Recently 5th class of (GA) index $(GA5)$ is suggested by Graovac [8] in 2011 and described

as:

$$GA_5(G) = \sum_{qr \in E(G)} \frac{2\sqrt{S_q \times S_r}}{S_q + S_r}. \tag{1. 9}$$

• **Multiplicative Randić Index**:
The Multiplicative Randić Index [15] has been introduced as:

$$MR(G) = \prod_{qr \in E(G)} \sqrt{\frac{1}{d_q d_r}}. \tag{1. 10}$$

• **Hyper-Zagreb Index**:
In 2013, Hyper-Zagreb index [18] has been introduced as:

$$HM(G) = \sum_{qr \in E(G)} \left(d_q + d_r\right)^2. \tag{1. 11}$$

2. RESULTS AND DISCUSSION

Classic Victorian designs are very popular and it is used as the combination of triangles and squares of different sized. These tiles look perfect in the hallways, exterior pathways and porches of the house. Any combination of colors can be choosen as all the twenty five colors are available. The popular combinations of color are shown here. The geometric tiles which are English old style and these are impenetrable, against the frost and they are suitable for interior and exterior use. They are not just utilize as floor tiles but also utilizes in variety of applications and they are combined as multitude of designs which are colorful.
Product Overview:
• Appropriate for interior and exterior use
• Without glass, fully impenetrable and frost resistant
• High strength and very low absorbent
• Thickness of 9-10mm
• 18 shapes and 25 colors accessible
• Priced the designs per square meter
Multiple topological indices have been on different graphs. For this purpose, we discuss , $M_\alpha(G)$ index [3], $R_\alpha(G)$ index, ABC index, $RR(G)$ index, $RRR(G)$ index, inverse sum index, symmetric division index, ABC_4 index, GA_5 index [8], multiplicative randić index and hyper-zagreb index of Grasmere geometric graph.

TABLE 1. The point division of graph G based on valency of points

Degree of point	No. of points
3	$4mn + 4n + 4m + 8$
4	$4mn + 6n + 6m + 8$
5	$2n + 2m$
6	$2mn + 2n + 2m$
8	mn
Total	$24mn + 26n + 26m + 28$

TABLE 2. The edge division of graph G based on degree of end points of every edge.

(d_q, d_r), where $qr \in E(G)$	No. of edges
$(3,3)$	$4mn + 4n + 4m + 4$
$(3,4)$	$4mn + 4n + 4m + 16$
$(4,4)$	$4n + 4m + 8$
$(4,5)$	$8n + 8m$
$(4,6)$	$8mn + 4n + 4m$
$(4,8)$	$4mn$
$(5,6)$	$2n + 2m$
$(6,8)$	$4mn$
Total	$24mn + 26n + 26m + 28$

Theorem 2.1. *Let G be a Grasmere geometric graph with $m > 1$ and $n > 1$, Then*

$$\begin{aligned} M_\alpha(G) &= 4(mn + n + m + 2) \cdot 3^\alpha + 2(2mn + 3n + 3m + 4) \cdot 4^\alpha \\ &+ 2(n + m) \cdot 5^\alpha + (2mn + n + m) \cdot 6^\alpha + (mn) \cdot 8^\alpha. \\ R_\alpha(G) &= (4\,mn + 4\,n + 4\,m + 4)\,9^\alpha + (4\,mn + 4\,n + 4\,m + 16)\,12^\alpha + (4\,n + 4\,m + 8)\,16^\alpha \\ &+ (8\,n + 8\,m)\,20^\alpha + (8\,mn + 4\,n + 4\,m)\,24^\alpha + 4\,mn\,32^\alpha + (2\,n + 2\,m)\,30^\alpha \\ &+ 4\,mn\,48^\alpha. \end{aligned}$$

where α is a real number.

Proof. The Grasmere geometric graph is shown in Figure 1. Let $e_{q,r}$ represents the number of edges connecting the points of degree d_q and d_r. In this graph G there are total number of vertices are $24mn + 26n + 26m + 28$. The number of points of degree three four five six and eight are $4mn + 4n + 4m + 8$, $4mn + 6n + 6m + 8$, $2n + 2m$, $2mn + n + m$ and mn respectively. By using the above values in the formula of $M_\alpha(G)$ index, we obtain the result. The total number of edges of the Grasmere geometric graph are $24mn + 26n + 26m + 28$. The division of edge established on the valency of the end points of every edge as manifest in Table 2. The general Randić index is defined as:

$$R_\alpha(G) = \sum_{qr \in E(G)} (d_q d_r)^\alpha.$$

This indicate that

$$\begin{aligned} R_\alpha(G) &= e_{3,3}\,(3 \times 3)^\alpha + e_{3,4}\,(3 \times 4)^\alpha + e_{4,4}\,(4 \times 4)^\alpha \\ &+ e_{4,5}\,(4 \times 5)^\alpha + e_{4,6}\,(4 \times 6)^\alpha + e_{4,8}\,(4 \times 8)^\alpha \\ &+ e_{5,6}\,(5 \times 6)^\alpha + e_{6,8}\,(6 \times 8)^\alpha. \\ R_\alpha(G) &= (4\,mn + 4\,n + 4\,m + 4)\,9^\alpha + (4\,mn + 4\,n + 4\,m + 16)\,12^\alpha \end{aligned}$$

$$
\begin{aligned}
&+ \quad (4\,n+4\,m+8)\,16^{\alpha}+(8\,n+8\,m)\,20^{\alpha}\\
&+ \quad (8\,mn+4\,n+4\,m)\,24^{\alpha}+4\,mn\,32^{\alpha}\\
&+ \quad (2\,n+2\,m)\,30^{\alpha}+4\,mn\,48^{\alpha}.
\end{aligned}
$$

Which consummates the proof. □

Theorem 2.2. *The ABC index of the structures of Grasmere geometric graph with $m > 1$ and $n > 1$ is given by:*

$$
\begin{aligned}
ABC(G) &= \frac{14}{3}\,mn+\frac{8}{3}\,n+\frac{8}{3}\,m+\frac{8}{3}+\frac{1}{6}\,(4\,mn+4\,n+4\,m+16)\,\sqrt{15}\\
&+ \frac{1}{4}\,(4\,n+4\,m+8)\,\sqrt{6}+\frac{1}{10}\,(8\,n+8\,m)\,\sqrt{35}\\
&+ \frac{1}{3}\,(8\,mn+4\,n+4\,m)\,\sqrt{3}+mn\,\sqrt{5}+\frac{1}{10}\,(2\,n+2\,m)\,\sqrt{30}.
\end{aligned}
$$

Proof. Consider $G(v,e)$ be a Grasmere geometric graph. Suppose $e_{q,r}$ indicates the number of edges associated with the points of degree d_q and d_r. Two-dimensional formation of the given graph contains only $e_{(3,3)}, e_{(3,4)}, e_{(4,4)}, e_{(4,5)}, e_{(4,6)}, e_{(4,8)}, e_{(5,6)}, e_{(6,8)}$ edges. The number of $e_{(3,3)}, e_{(3,4)}, e_{(4,4)}, e_{(4,5)}, e_{(4,6)}, e_{(4,8)}, e_{(5,6)}, e_{(6,8)}$ lines are introduced in Table 2. Since, the ABC index has been introduced as:

$$
ABC(G)=\sum_{qr\in E(G)}\sqrt{\tfrac{d_q+d_r-2}{d_q\times d_r}}.
$$

This indicates that

$$
\begin{aligned}
ABC(G) &= e_{3,3}\sqrt{\tfrac{3+3-2}{3\times 3}}+e_{3,4}\sqrt{\tfrac{3+4-2}{3\times 4}}+e_{4,4}\sqrt{\tfrac{4+4-2}{4\times 4}}\\
&+ e_{4,5}\sqrt{\tfrac{4+5-2}{4\times 5}}+e_{4,6}\sqrt{\tfrac{4+6-2}{4\times 6}}+e_{4,8}\sqrt{\tfrac{4+8-2}{4\times 8}}\\
&+ e_{5,6}\sqrt{\tfrac{5+6-2}{5\times 6}}+e_{6,8}\sqrt{\tfrac{6+8-2}{6\times 8}}.
\end{aligned}
$$

By using the Table 2, we get

$$
\begin{aligned}
ABC(G) &= (4mn+4n+4m+4)\sqrt{\tfrac{3+3-2}{3\times 3}}+(4mn+4n+4m+16)\sqrt{\tfrac{3+4-2}{3\times 4}}\\
&+ (8n+8m)\sqrt{\tfrac{4+5-2}{4\times 5}}+(8mn+4n+4m)\sqrt{\tfrac{4+6-2}{4\times 6}}+(4mn)\sqrt{\tfrac{4+8-2}{4\times 8}}\\
&+ (2n+2m)\sqrt{\tfrac{5+6-2}{5\times 6}}+(4mn)\sqrt{\tfrac{6+8-2}{6\times 8}}+(4n+4m+8)\sqrt{\tfrac{4+4-2}{4\times 4}}.
\end{aligned}
$$

After easy simplification we obtain

$$
\begin{aligned}
ABC(G) &= \frac{14}{3}\,mn+\frac{8}{3}\,n+\frac{8}{3}\,m+\frac{8}{3}+\frac{1}{6}\,(4\,mn+4\,n+4\,m+16)\,\sqrt{15}\\
&+ \frac{1}{4}\,(4\,n+4\,m+8)\,\sqrt{6}+\frac{1}{10}\,(8\,n+8\,m)\,\sqrt{35}
\end{aligned}
$$

$$+ \quad \frac{1}{3}\,(8\,mn + 4\,n + 4\,m)\,\sqrt{3} + mn\,\sqrt{5} + \frac{1}{10}\,(2\,n + 2\,m)\,\sqrt{30}.$$

Which consummates the proof. □

Theorem 2.3. *The Reciprocal Randić index RR of the structure of Grasmere geometric graph with $m > 1$ and $n > 1$ is given by:*

$$\begin{aligned} RR(G) &= 12\,mn + 28\,n + 28\,m + 44 + 2\,(4\,mn + 4\,n + 4\,m + 16)\,\sqrt{3} \\ &+ 2\,(8\,n + 8\,m)\,\sqrt{5} + 2\,(8\,mn + 4\,n + 4\,m)\,\sqrt{6} + 16\,mn\,\sqrt{2} \\ &+ (2\,n + 2\,m)\,\sqrt{30} + 16\,mn\,\sqrt{3}. \end{aligned}$$

Proof. The number of $e_{(3,3)}, e_{(3,4)}, e_{(4,4)}, e_{(4,5)}, e_{(4,6)}, e_{(4,8)}, e_{(5,6)}, e_{(6,8)}$ edges are mentioned in Table 2. Since, the $RR(G)$ index is given as:

$$RR(G) = \sum_{qr \in E(G)} \sqrt{d_q \times d_r}.$$

This indicates that

$$\begin{aligned} RR(G) &= e_{3,3}\,\sqrt{3 \times 3} + e_{3,4}\,\sqrt{3 \times 4} + e_{4,4}\,\sqrt{4 \times 4} + e_{4,5}\,\sqrt{4 \times 5} \\ &+ e_{4,6}\,\sqrt{4 \times 6} + e_{4,8}\,\sqrt{4 \times 8} + e_{5,6}\,\sqrt{5 \times 6} + e_{6,8}\,\sqrt{6 \times 8}. \end{aligned}$$

By using the Table 2, we get

$$\begin{aligned} RR(G) &= (4mn + 4n + 4m + 4)\,\sqrt{3 \times 3} + (4mn + 4n + 4m + 16)\,\sqrt{3 \times 4} \\ &+ (4n + 4m + 8)\,\sqrt{4 \times 4} + (8n + 8m)\,\sqrt{4 \times 5} + (8mn + 4n + 4m)\,\sqrt{4 \times 6} \\ &+ (4mn)\,\sqrt{4 \times 8} + (2n + 2m)\,\sqrt{5 \times 6} + (4mn)\,\sqrt{6 \times 8}. \end{aligned}$$

After easy simplification we obtain

$$\begin{aligned} RR(G) &= 12\,mn + 28\,n + 28\,m + 44 + 2\,(4\,mn + 4\,n + 4\,m + 16)\,\sqrt{3} \\ &+ 2\,(8\,n + 8\,m)\,\sqrt{5} + 2\,(8\,mn + 4\,n + 4\,m)\,\sqrt{6} + 16\,mn\,\sqrt{2} \\ &+ (2\,n + 2\,m)\,\sqrt{30} + 16\,mn\,\sqrt{3}. \end{aligned}$$

Which consummates the proof. □

Theorem 2.4. *The Reduced Reciprocal Randić index RRR of the structure of Grasmere geometric graph with $m > 1$ and $n > 1$ is given by:*

$$\begin{aligned} RRR(G) &= 8\,mn + 20\,n + 20\,m + 32 + (4\,mn + 4\,n + 4\,m + 16)\,\sqrt{6} \\ &+ 2\,(8\,n + 8\,m)\,\sqrt{3} + (8\,mn + 4\,n + 4\,m)\,\sqrt{15} + 4\,mn\,\sqrt{21} \\ &+ 2\,(2\,n + 2\,m)\,\sqrt{5} + 4\,mn\,\sqrt{35}. \end{aligned}$$

Proof. The number of $e_{(3,3)}, e_{(3,4)}, e_{(4,4)}, e_{(4,5)}, e_{(4,6)}, e_{(4,8)}, e_{(5,6)}, e_{(6,8)}$ edges are mentioned in Table 2. Now, the $RRR(G)$ index is described as:

$$RRR(G) = \sum_{qr \in E(G)} \sqrt{(d_q - 1) \times (d_r - 1)}.$$

This implies that

$$\begin{aligned} RRR(G) &= e_{3,3}\sqrt{(3-1)\times(3-1)} + e_{3,4}\sqrt{(3-1)\times(4-1)} + e_{4,4} \\ &\times \sqrt{(4-1)\times(4-1)} + e_{4,5}\sqrt{(4-1)\times(5-1)} + e_{4,6}\sqrt{(4-1)\times(6-1)} \\ &+ e_{4,8}\sqrt{(4-1)\times(8-1)} + e_{5,6}\sqrt{(5-1)\times(6-1)} + e_{6,8}\sqrt{(6-1)\times(8-1)}. \end{aligned}$$

By using the Table 2, we get

$$\begin{aligned} RRR(G) &= (4mn+4n+4m+4)\sqrt{(3-1)\times(3-1)} + (4mn+4n+4m+16) \\ &\times \sqrt{(3-1)\times(4-1)} + (4n+4m+8)\sqrt{(4-1)\times(4-1)} + (8n+8m) \\ &\times \sqrt{(4-1)\times(5-1)} + (8mn+4n+4m)\sqrt{(4-1)\times(6-1)} + (4mn) \\ &\times \sqrt{(4-1)\times(8-1)} + (2n+2m)\sqrt{(5-1)\times(6-1)} + (4mn)\sqrt{(6-1)\times(8-1)}. \end{aligned}$$

After easy simplification we obtain

$$\begin{aligned} RRR(G) &= 8\,mn + 20\,n + 20\,m + 32 + (4\,mn + 4\,n + 4\,m + 16)\sqrt{6} \\ &+ 2\,(8\,n + 8\,m)\sqrt{3} + (8\,mn + 4\,n + 4\,m)\sqrt{15} + 4\,mn\sqrt{21} \\ &+ 2\,(2\,n + 2\,m)\sqrt{5} + 4\,mn\sqrt{3}. \end{aligned}$$

Which consummates the proof. □

Theorem 2.5. *The inverse sum index ISI of the structure of Grasmere geometric graph with $m > 1$ and $n > 1$ is given by:*

$$ISI(G) = \frac{1407861}{2000}\,mn + \frac{10101405137}{18522000}\,n + \frac{10101405137}{18522000}\,m + \frac{22596311}{54000}.$$

Proof. The number of $e_{(3,3)}, e_{(3,4)}, e_{(4,4)}, e_{(4,5)}, e_{(4,6)}, e_{(4,8)}, e_{(5,6)}, e_{(6,8)}$ edges are introduced in Table 2. Since, the inverse sum index is described as:

$$ISI(G) = \sum_{qr \in E(G)} \frac{d_q \times d_r}{d_q + d_r}.$$

This implies that

$$\begin{aligned} ISI(G) &= e_{3,3}\frac{3\times 3}{3+3} + e_{3,4}\frac{3\times 4}{3+4} + e_{4,4}\frac{4\times 4}{4+4} + e_{4,5}\frac{4\times 5}{4+5} \\ &+ e_{4,6}\frac{4\times 6}{4+6} + e_{4,8}\frac{4\times 8}{4+8} + e_{5,6}\frac{5\times 6}{5+6} + e_{6,8}\frac{6\times 8}{6+8}. \end{aligned}$$

By using the Table 2, we get

$$\begin{aligned} ISI(G) &= (4mn+4n+4m+4)\frac{3\times 3}{3+3}+(4mn+4n+4m+16)\frac{3\times 4}{3+4} \\ &+ (8n+8m)\frac{4\times 5}{4+5}+(8mn+4n+4m)\frac{4\times 6}{4+6}+(4mn)\frac{4\times 8}{4+8} \\ &+ (4mn)\frac{6\times 8}{6+8}++(4n+4m+8)\frac{4\times 4}{4+4}+(2n+2m)\frac{5\times 6}{5+6}. \end{aligned}$$

After easy simplification we obtain

$$ISI(G) = \frac{1407861}{2000}mn+\frac{10101405137}{18522000}n+\frac{10101405137}{18522000}m+\frac{22596311}{54000}.$$

Which completes the proof. □

Theorem 2.6. *The symmetric division index SD of the structure of Grasmere geometric graph with $m > 1$ and $n > 1$ is given by:*

$$SD(G) = 52\,mn+\frac{802}{15}n+\frac{802}{15}m+\frac{172}{3}.$$

Proof. The number of $e_{(3,3)}, e_{(3,4)}, e_{(4,4)}, e_{(4,5)}, e_{(4,6)}, e_{(4,8)}, e_{(5,6)}, e_{(6,8)}$ edges are mentioned in Table 2. Since, the symmetric division index is described as:

$$SD(G) = \sum_{qr\in E(G)} \frac{d_q^2+d_r^2}{d_q\times d_r}.$$

This implies that

$$\begin{aligned} SD(G) &= e_{3,3}\frac{3^2+3^2}{3\times 3}+e_{3,4}\frac{3^2+4^2}{3\times 4}+e_{4,4}\frac{4^2+4^2}{4\times 4}+e_{4,5}\frac{4^2+5^2}{4\times 5} \\ &+ e_{4,6}\frac{4^2+6^2}{4\times 6}+e_{4,8}\frac{4^2+8^2}{4\times 8}+e_{5,6}\frac{5^2+6^2}{5\times 6}+e_{6,8}\frac{6^2+8^2}{6\times 8}. \end{aligned}$$

By using the Table 2, we get

$$\begin{aligned} SD(G) &= (4mn+4n+4m+4)\frac{3^2+3^2}{3\times 3}+(4mn+4n+4m+16)\frac{3^2+4^2}{3\times 4} \\ &+ (8n+8m)\frac{4^2+5^2}{4\times 5}+(8mn+4n+4m)\frac{4^2+6^2}{4\times 6}+(4mn)\frac{4^2+8^2}{4\times 8} \\ &+ (4mn)\frac{6^2+8^2}{6\times 8}+(4n+4m+8)\frac{4^2+4^2}{4\times 4}+(2n+2m)\frac{5^2+6^2}{5\times 6}. \end{aligned}$$

After easy simplification we obtain

$$SD(G) = 52\,mn+\frac{802}{15}n+\frac{802}{15}m+\frac{172}{3}.$$

Which completes the proof. □

In the following two theorems we deliberated the ABC_4 index and the GA_5 index. There are twenty three variety of edges on degree based sum of neighbors points of every edge in the Grasmere geometric graph, We utilize this division of edges to compute ABC_4 and GA_5 indices. Table 3 offers such variety of edges of the Grasmere geometric graph.

TABLE 3. The edge division of graph G based on degree sum of neighbor points of end points of each edge.

(S_q, S_r), where $qr \in E(G)$	No. of edges	(S_q, S_r), where $qr \in E(G)$	No. of edges
$(10,10)$	$4mn+4n+4m+4$	$(18,18)$	$4n+4m-8$
$(10,14)$	4	$(18,22)$	$8n+8m-8$
$(10,18)$	$4n+4m$	$(18,26)$	0
$(10,23)$	$4mn$	$(18,29)$	$4n+4m$
$(12,14)$	4	$(22,26)$	0
$(12,16)$	8	$(22,29)$	$2n+2m$
$(14,14)$	0	$(23,29)$	$4n+4m$
$(14,16)$	8	$(23,32)$	$8mn-4n-4m$
$(14,18)$	0	$(23,40)$	$4mn$
$(14,22)$	0	$(29,40)$	$2n+2m$
$(16,18)$	8	$(32,40)$	$4mn-2n-2m$
$(16,22)$	8		
Total	$24mn+26n+26m+28$		

Theorem 2.7. *The ABC_4 index of Grasmere geometric graph with $m>1$ and $n>1$ is given by*

$$\begin{aligned} ABC_4(G) &= \frac{3}{10}\,(4\,mn+4\,n+4\,m+4)\,\sqrt{2}+\frac{2}{35}\,\sqrt{770}+\frac{1}{30}\,(4\,n+4\,m)\,\sqrt{130}\\ &+ \frac{4}{7}\,\sqrt{7}+\frac{1}{3}\,\sqrt{78}+\frac{8}{3}+2\,\sqrt{2}+\frac{6}{11}\,\sqrt{22}+\frac{1}{18}\,(4\,n+4\,m-8)\,\sqrt{34}\\ &+ \frac{1}{66}\,(8\,n+8\,m-8)\,\sqrt{418}+\frac{1}{58}\,(4\,n+4\,m)\,\sqrt{290}+\frac{7}{638}\,(2\,n+2\,m)\,\sqrt{638}\\ &+ \frac{5}{667}\,(4\,n+4\,m)\,\sqrt{1334}+\frac{1}{184}\,(8\,mn-4\,n-4\,m)\,\sqrt{2438}+\frac{1}{115}\,mn\,\sqrt{14030}\\ &+ \frac{1}{580}\,(2\,n+2\,m)\,\sqrt{19430}+\frac{1}{16}\,(4\,mn-2\,n-2\,m)\,\sqrt{14}++\frac{2}{115}\,mn\,\sqrt{7130}. \end{aligned}$$

Proof. Suppose $e_{i,j}$ indicates the number of edges of the Grasmere geometric graph along $g=s_q$ and $h=s_r$. It is simple to perceive that the abstract of valency of line endpoints of particular graph has 23 edge kinds $e_{10,10}, e_{10,14}, e_{10,18}...e_{32,40}$ that are manifest in Table 3. The 4th atom-bound connectivity index ABC_4 is defined as:

$$ABC_4(G)=\sum_{qr\in E(G)}\sqrt{\frac{S_q+S_r-2}{S_q\times S_r}}.$$

This indicates that

$$\begin{aligned} ABC_4(G) &= e_{10,10}\sqrt{\frac{10+10-2}{10\times10}}+e_{10,14}\sqrt{\frac{10+14-2}{10\times14}}+e_{10,18}\sqrt{\frac{10+18-2}{10\times18}}\\ &+ e_{10,23}\sqrt{\frac{10+23-2}{10\times23}}+e_{12,14}\sqrt{\frac{12+14-2}{12\times14}}+e_{12,16}\sqrt{\frac{12+16-2}{12\times16}} \end{aligned}$$

$$
\begin{aligned}
&+ \quad e_{14,14}\sqrt{\tfrac{14+14-2}{14\times 14}} + e_{14,16}\sqrt{\tfrac{14+16-2}{14\times 16}} + e_{14,18}\sqrt{\tfrac{14+18-2}{14\times 18}}\\
&+ \quad e_{14,22}\sqrt{\tfrac{14+22-2}{14\times 22}} + e_{16,18}\sqrt{\tfrac{16+18-2}{16\times 18}} + e_{16,22}\sqrt{\tfrac{16+22-2}{16\times 22}}\\
&+ \quad e_{18,18}\sqrt{\tfrac{18+18-2}{18\times 18}} + e_{18,22}\sqrt{\tfrac{18+22-2}{18\times 22}} + e_{18,26}\sqrt{\tfrac{18+26-2}{18\times 26}}\\
&+ \quad e_{18,29}\sqrt{\tfrac{18+29-2}{18\times 29}} + e_{22,26}\sqrt{\tfrac{22+26-2}{22\times 26}} + e_{22,29}\sqrt{\tfrac{22+29-2}{22\times 29}}\\
&+ \quad e_{23,29}\sqrt{\tfrac{23+29-2}{23\times 29}} + e_{23,32}\sqrt{\tfrac{23+32-2}{23\times 32}} + e_{23,40}\sqrt{\tfrac{23+40-2}{23\times 40}}\\
&+ \quad e_{29,40}\sqrt{\tfrac{29+40-2}{29\times 40}} + e_{32,40}\sqrt{\tfrac{32+40-2}{32\times 40}}.
\end{aligned}
$$

$$
\begin{aligned}
ABC_4(G) &= (4mn+4n+4m+4)\sqrt{\tfrac{10+10-2}{10\times 10}} + (4)\sqrt{\tfrac{10+14-2}{10\times 14}} + (4n+4m)\sqrt{\tfrac{10+18-2}{10\times 18}}\\
&+ (4mn)\sqrt{\tfrac{10+23-2}{10\times 23}} + (4)\sqrt{\tfrac{12+14-2}{12\times 14}} + (8)\sqrt{\tfrac{12+16-2}{12\times 16}} + (0)\sqrt{\tfrac{14+14-2}{14\times 14}}\\
&+ (8)\sqrt{\tfrac{14+16-2}{14\times 16}} + (0)\sqrt{\tfrac{14+18-2}{14\times 18}} + (0)\sqrt{\tfrac{14+22-2}{14\times 22}} + (8)\sqrt{\tfrac{16+18-2}{16\times 18}}\\
&+ (8)\sqrt{\tfrac{16+22-2}{16\times 22}} + (4n+4m-8)\sqrt{\tfrac{18+18-2}{18\times 18}} + (8n+8m-8)\sqrt{\tfrac{18+22-2}{18\times 22}}\\
&+ (0)\sqrt{\tfrac{18+26-2}{18\times 26}} + (4n+4m)\sqrt{\tfrac{18+29-2}{18\times 29}} + (0)\sqrt{\tfrac{22+26-2}{22\times 26}} + (2n+2m)\sqrt{\tfrac{22+29-2}{22\times 29}}\\
&+ (4n+4m)\sqrt{\tfrac{23+29-2}{23\times 29}} + (8mn-4n-4m)\sqrt{\tfrac{23+32-2}{23\times 32}}\\
&+ (4mn)\sqrt{\tfrac{23+40-2}{23\times 40}} + (2n+2m)\sqrt{\tfrac{29+40-2}{29\times 40}} + (4mn-2n-2m)\sqrt{\tfrac{32+40-2}{32\times 40}}.\\
ABC_4(G) &= \frac{3}{10}\,(4\,mn+4\,n+4\,m+4)\sqrt{2} + \frac{2}{35}\sqrt{770} + \frac{1}{30}\,(4\,n+4\,m)\sqrt{130}\\
&+ \frac{4}{7}\sqrt{7} + \frac{1}{3}\sqrt{78} + \frac{8}{3} + 2\sqrt{2} + \frac{6}{11}\sqrt{22} + \frac{1}{18}\,(4\,n+4\,m-8)\sqrt{34}\\
&+ \frac{1}{66}\,(8\,n+8\,m-8)\sqrt{418} + \frac{1}{58}\,(4\,n+4\,m)\sqrt{290} + \frac{7}{638}\,(2\,n+2\,m)\sqrt{638}\\
&+ \frac{5}{667}\,(4\,n+4\,m)\sqrt{1334} + \frac{1}{184}\,(8\,mn-4\,n-4\,m)\sqrt{2438} + \frac{1}{115}\,mn\sqrt{14030}\\
&+ \frac{1}{580}\,(2\,n+2\,m)\sqrt{19430} + \frac{1}{16}\,(4\,mn-2\,n-2\,m)\sqrt{14} + \frac{2}{115}\,mn\sqrt{7130}.
\end{aligned}
$$

Which completes the proof. □

Theorem 2.8. *The 5th geometric-arithmetic index GA_5 of the Grasmere geometric graph with $m > 1$ and $n > 1$ is given by:*

$$\begin{aligned} GA_5(G) &= -4 + 8\,n + 4\,mn + 8\,m + \frac{8}{13}\sqrt{42} + \frac{32}{7}\sqrt{3} + \frac{32}{15}\sqrt{14} \\ &+ \frac{96}{17}\sqrt{2} + \frac{32}{19}\sqrt{22} + \frac{2}{3}\sqrt{35} + \frac{3}{7}\,(4\,n + 4\,m)\sqrt{5} \\ &+ \frac{344}{693}\,mn\sqrt{230} + \frac{3}{10}\,(8\,n + 8\,m - 8)\sqrt{11} + \frac{6}{47}\,(4\,n + 4\,m)\sqrt{58} \\ &+ \frac{2}{51}\,(2\,n + 2\,m)\sqrt{638} + \frac{1}{26}\,(4\,n + 4\,m)\sqrt{667} + \frac{8}{55}\,(8\,mn - 4\,n - 4\,m)\sqrt{46} \\ &+ \frac{4}{69}\,(2\,n + 2\,m)\sqrt{290} + \frac{4}{9}\,(4\,mn - 2\,n - 2\,m)\sqrt{5}. \end{aligned}$$

Proof. Suppose $e_{i,j}$ indicates the number of edges of the Grasmere geometric graph along $g = s_q$ and $h = s_r$. It is simple to perceive that the abstract of valency of line endpoints of specific graph has 23 edge kinds $e_{10,10}, e_{10,14}, e_{10,18}...e_{32,40}$ which are manifest in Table 3. The GA_5 is described as:

$$GA_5(G) = \sum_{qr \in E(G)} \frac{2\sqrt{S_q \times S_r}}{S_q + S_r}.$$

This indicates that

$$\begin{aligned} GA_5(G) &= e_{10,10}\frac{2\sqrt{10\times 10}}{10+10} + e_{10,14}\frac{2\sqrt{10\times 14}}{10+14} + e_{10,18}\frac{2\sqrt{10\times 18}}{10+18} + e_{10,23}\frac{2\sqrt{10\times 23}}{10+23} \\ &+ e_{12,14}\frac{2\sqrt{12\times 14}}{12+14} + e_{12,16}\frac{2\sqrt{12\times 16}}{12+16} + e_{14,14}\frac{2\sqrt{14\times 14}}{14+14} + e_{14,16}\frac{2\sqrt{14\times 16}}{14+16} \\ &+ e_{14,18}\frac{2\sqrt{14\times 18}}{14+18} + e_{14,22}\frac{2\sqrt{14\times 22}}{14+22} + e_{16,18}\frac{2\sqrt{16\times 18}}{16+18} + e_{16,22}\frac{2\sqrt{16\times 22}}{16+22} \\ &+ e_{18,18}\frac{2\sqrt{18\times 18}}{18+18} + e_{18,22}\frac{2\sqrt{18\times 22}}{18+22} + e_{18,26}\frac{6\sqrt{18\times 26}}{18+26} + e_{18,29}\frac{2\sqrt{18\times 29}}{18+29} \\ &+ e_{22,26}\frac{2\sqrt{22\times 26}}{22+26} + e_{22,29}\frac{2\sqrt{22\times 29}}{22+29} + e_{23,29}\frac{2\sqrt{23\times 29}}{23+29} + e_{23,32}\frac{2\sqrt{23\times 32}}{23+32} \\ &+ e_{23,40}\frac{2\sqrt{23\times 40}}{23+40} + e_{29,40}\frac{2\sqrt{29\times 40}}{29+40} + e_{32,40}\frac{2\sqrt{32\times 40}}{32+40}. \end{aligned}$$

$$GA_5(G) = (4mn + 4n + 4m + 4)\frac{2\sqrt{10\times 10}}{10+10} + (4)\frac{2\sqrt{10\times 14}}{10+14} + (4n + 4m)\frac{2\sqrt{10\times 18}}{10+18}$$

$$
\begin{aligned}
&+ \quad (4mn)\,\frac{2\sqrt{10\times 23}}{10+23} + (4)\,\frac{2\sqrt{12\times 14}}{12+14} + (8)\,\frac{2\sqrt{12\times 16}}{12+16} + (0)\,\frac{2\sqrt{14\times 14}}{14+14}\\
&+ \quad (8)\,\frac{2\sqrt{14\times 16}}{14+16} + (0)\,\frac{2\sqrt{14\times 18}}{14+18} + (0)\,\frac{2\sqrt{14\times 22}}{14+22} + (8)\,\frac{2\sqrt{16\times 18}}{16+18}\\
&+ \quad (8)\,\frac{2\sqrt{16\times 22}}{16+22} + (4n+4m-8)\,\frac{2\sqrt{18\times 18}}{18+18} + (8n+8m-8)\,\frac{2\sqrt{18\times 22}}{18+22}\\
&+ \quad (0)\,\frac{6\sqrt{18\times 26}}{18+26} + (4n+4m)\,\frac{2\sqrt{18\times 29}}{18+29} + (0)\,\frac{2\sqrt{22\times 26}}{22+26} + (2n+2m)\,\frac{2\sqrt{22\times 29}}{22+29}\\
&+ \quad (4n+4m)\,\frac{2\sqrt{23\times 29}}{23+29} + (8mn-4n-4m)\,\frac{2\sqrt{23\times 32}}{23+32} + (4mn)\,\frac{2\sqrt{23\times 40}}{23+40}\\
&+ \quad (2n+2m)\,\frac{2\sqrt{29\times 40}}{29+40} + (4mn-2n-2m)\,\frac{2\sqrt{32\times 40}}{32+40}.
\end{aligned}
$$

$$
\begin{aligned}
GA_5(G) \quad &= \quad -4+8\,n+4\,mn+8\,m+\frac{8}{13}\sqrt{42}+\frac{32}{7}\sqrt{3}+\frac{32}{15}\sqrt{14}\\
&+ \quad \frac{96}{17}\sqrt{2}+\frac{32}{19}\sqrt{22}+\frac{2}{3}\sqrt{35}+\frac{3}{7}\,(4\,n+4\,m)\sqrt{5}\\
&+ \quad \frac{344}{693}\,mn\sqrt{230}+\frac{3}{10}\,(8\,n+8\,m-8)\sqrt{11}+\frac{6}{47}\,(4\,n+4\,m)\sqrt{58}\\
&+ \quad \frac{2}{51}\,(2\,n+2\,m)\sqrt{638}+\frac{1}{26}\,(4\,n+4\,m)\sqrt{667}+\frac{8}{55}\,(8\,mn-4\,n-4\,m)\sqrt{46}\\
&+ \quad \frac{4}{69}\,(2\,n+2\,m)\sqrt{290}+\frac{4}{9}\,(4\,mn-2\,n-2\,m)\sqrt{5}.
\end{aligned}
$$

Which completes the proof. □

Theorem 2.9. *The Multiplicative Randić index of Grasmere geometric graph with* $m > 1$ *and* $n > 1$ *is given as:*

$$
\begin{aligned}
MR(G) \quad &= \quad \left(\frac{1}{3}\right)^{4\,mn+4\,n+4\,m+4} \times \left(\frac{1}{6}\sqrt{3}\right)^{4\,mn+4\,n+4\,m+16} \times \left(\frac{1}{4}\right)^{4\,n+4\,m+8}\\
&\times \quad \left(\frac{1}{10}\sqrt{5}\right)^{8\,n+8\,m} \times \left(\frac{1}{12}\sqrt{6}\right)^{8\,mn+4\,n+4\,m} \times \left(\frac{1}{8}\sqrt{2}\right)^{4\,mn}\\
&\times \quad \left(\frac{1}{30}\sqrt{30}\right)^{2\,n+2\,m} \times \left(\frac{1}{12}\sqrt{3}\right)^{4\,mn}.
\end{aligned}
$$

Proof. Let G be a Grasmere geometric graph. The edge set $E(G)$ splitted into eight edge divisions based on degree of end points. The $1st$ partition of $E_1(G)$ contains $4mn +$

$4n + 4m + 4$ edges qr, where $d_q = 3$, $d_r = 3$. The $2nd$ partition of $E_2(G)$ contains $4mn+4n+4m+16$ lines qr, where $d_q = 3$, $d_r = 4$. The $3rd$ partition of $E_3(G)$ contains $4n + 4m + 8$ edges qr, where $d_q = 4$, $d_r = 4$. and so on. It is simple to perceive that $|E_1(G)| = e_{3,3}$, $|E_2(G)| = e_{3,4}$ and $|E_3(G)| = e_{4,4}$... Since,

$$MR(G) = \prod_{qr\in E(G)} \sqrt{\frac{1}{d_q d_r}}.$$

This indicates that

$$\begin{aligned} MR(G) &= \prod_{qr\in E_1(G)} \sqrt{\frac{1}{d_q d_r}} \times \prod_{qr\in E_2(G)} \sqrt{\frac{1}{d_q d_r}} \times \prod_{qr\in E_3(G)} \sqrt{\frac{1}{d_q d_r}} \\ &\times \prod_{qr\in E_4(G)} \sqrt{\frac{1}{d_q d_r}} \times \prod_{qr\in E_5(G)} \sqrt{\frac{1}{d_q d_r}} \times \prod_{qr\in E_6(G)} \sqrt{\frac{1}{d_q d_r}} \\ &\times \prod_{qr\in E_7(G)} \sqrt{\frac{1}{d_q d_r}} \times \prod_{qr\in E_8(G)} \sqrt{\frac{1}{d_q d_r}}. \end{aligned}$$

By using the Table 2, we get

$$\begin{aligned} MR(G) &= 9^{|E_1(G)|} \times 12^{|E_2(G)|} \times 16^{|E_3(G)|} \times 20^{|E_4(G)|} \\ &\times 24^{|E_5(G)|} \times 32^{|E_6(G)|} \times 30^{|E_7(G)|} \times 48^{|E_8(G)|}. \end{aligned}$$

$$\begin{aligned} MR(G) &= 9^{(4mn+4n+4m+4)} \times 12^{(4mn+4n+4m+16)} \times 16^{(4n+4m+8)} \\ &\times 20^{(8n+8m)} \times 24^{(8mn+4n+4m)} \times 32^{(4mn)} \times 30^{(2n+2m)} \times 48^{(4mn)}. \end{aligned}$$

After easy simplification we obtain

$$\begin{aligned} MR(G) &= \left(\frac{1}{3}\right)^{4\,mn+4\,n+4\,m+4} \times \left(\frac{1}{6}\sqrt{3}\right)^{4\,mn+4\,n+4\,m+16} \times \left(\frac{1}{4}\right)^{4\,n+4\,m+8} \\ &\times \left(\frac{1}{10}\sqrt{5}\right)^{8\,n+8\,m} \times \left(\frac{1}{12}\sqrt{6}\right)^{8\,mn+4\,n+4\,m} \times \left(\frac{1}{8}\sqrt{2}\right)^{4\,mn} \\ &\times \left(\frac{1}{30}\sqrt{30}\right)^{2\,n+2\,m} \times \left(\frac{1}{12}\sqrt{3}\right)^{4\,mn}. \end{aligned}$$

Which completes the proof. □

We calculate hyper-Zagreb index $HM(G)$ for structures of Grasmere geometric graph in the next thm.

Theorem 2.10. *Let G be a Grasmere geometric graph with $m > 1$ and $n > 1$, then*

$$HM(G) = 2500\,mn + 1886\,n + 1886\,m + 1440.$$

Proof. Suppose G be a Grasmere geometric graph. The edge set $E(G)$ divided into eight edge partitions based on degree of end points. The division of 1st $E_1(G)$ contains $4mn + 4n + 4m + 4$ lines qr, where $d_q = 3$, $d_r = 3$. The division of 2nd $E_2(G)$ contains $4mn+4n+4m+16$ edges qr, where $d_q = 3$, $d_r = 4$. The division of 3rd $E_3(G)$ contains $4n + 4m + 8$ edges qr, where $d_q = 4$, $d_r = 4$. and so on. It is simple to perceive that $|E_1(G)| = e_{3,3}$, $|E_2(G)| = e_{3,4}$ and $|E_3(G)| = e_{4,4}$... Since,

$$HM(G) = \sum_{qr \in E(G)} \left(d_q + d_r\right)^2.$$

This indicates that

$$\begin{aligned} HM(G) &= \sum_{qr\in E_1(G)} \left[d_q + d_r\right]^2 + \sum_{qr\in E_2(G)} \left[d_q + d_r\right]^2 + \sum_{qr\in E_3(G)} \left[d_q + d_r\right]^2 \\ &+ \sum_{qr\in E_4(G)} \left[d_q + d_r\right]^2 + \sum_{qr\in E_5(G)} \left[d_q + d_r\right]^2 + \sum_{qr\in E_6(G)} \left[d_q + d_r\right]^2 \\ &+ \sum_{qr\in E_7(G)} \left[d_q + d_r\right]^2 + \sum_{qr\in E_8(G)} \left[d_q + d_r\right]^2. \end{aligned}$$

By using the Table 2, we get

$$\begin{aligned} HM(G) &= 36|E_1(G)| + 49|E_2(G)| + 64|E_3(G)| + 81|E_4(G)| \\ &+ 100|E_5(G)| + 144|E_6(G)| + 121|E_7(G)| + 196|E_8(G)|. \end{aligned}$$

$$\begin{aligned} HM(G) &= 36(4mn + 4n + 4m + 4) + 49(4mn + 4n + 4m + 16) + 64(4n + 4m + 8) \\ &+ 81(8n + 8m) + 100(8mn + 4n + 4m) + 144(4mn) \\ &+ 121(2n + 2m) + 196(4mn). \end{aligned}$$

After easy simplification we obtain

$$HM(G) = 2500\, mn + 1886\, n + 1886\, m + 1440.$$

Which completes the proof. □

3. Conclusion:

In this paper, different degree based molecular descriptors, namely, $M_\alpha(G)$ index, Randić connectivity index, ABC-index, $RR(G)$ index, $RRR(G)$ index, $ISI(G)$ index, $SD(G)$ index, ABC_4 index, GA_5 index, $MR(G)$ index, $HM(G)$ index of Grasmere Geometric graph and Rhombic type graph are deliberated. These results provide remarkable contribution in graph theory and also in network science. They provide virtuous basis to perceive the networks and the topology of these graphs.

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