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Bielecki–Ulam–Hyers stability of non–linear Volterra impulsive integro–delay dynamic systems on time scales

Syed Omar Shah*[1], Akbar Zada[2], Cemil Tunç[3] and Asad Ali[4]

[1,4]Department of Physical and Numerical Sciences, Qurtuba University of Science and Information Technology, Peshawar 25000, Pakistan

[2]Department of Mathematics, University of Peshawar, Peshawar 25000, Pakistan

[3]Department of Mathematics, Faculty of Sciences, Van Yuzuncu Yil University 65080, Van–Turkey

Email: [1]omarshah89@yahoo.com, [2]zadababo@yahoo.com, [3]cemtunc@yahoo.com, [4]asadali16102@gmail.com

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Abstract.: In this manuscript, the stability in terms of Bielecki–Ulam–Hyers and stability in terms of Bielecki–Ulam–Hyers–Rassias of non–linear Volterra impulsive integro–delay dynamic systems on time scales are obtained using the fixed point approach along with Grönwall inequality and Lipschitz condition.

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1. Introduction

The theory of dynamic equations with impulses are utilized for modeling the mathematical problems. These problems are subjected to sudden changes of state at certain instant. These dynamic equations have got appreciable consideration from the researchers due to their various applications in different fields, including population dynamics, electrodynamics, viscoelasticity, blood flows, mathematical economy, pharmacokinetics etc. [5,6,17,32].

In mathematical analysis, stability analysis is grown to be one of the most significant areas. In the literature, different types of stability can be found including exponential stability but the interesting and important type of stability is Ulam–Hyers stability. This stability problem was identified by Ulam [30, 31] in 1940 at Wisconsin university and it was solved by Hyers [12] partially for the case of Banach spaces which was generalized for the case of linear mapping by Rassias [21] in 1978. For more details, see [2, 13–15, 18, 19, 22–29, 32–37, 40–42, 44].

The idea of time scale was given by Hilger in 1988 [11]. The importance of time scales theory is due to the fact that it is utilized not only in differential equations but also in difference equations. For details, see [1, 3, 4, 7–10, 16, 20, 22, 23, 26–29, 38–40, 42, 43]. Nowadays, many researchers are working on the existence, uniqueness and stability results of non–linear impulsive integro–delay dynamic systems on time scales. Recently, Zada *et al.* [43] studied the existence, uniqueness and stability results of non–linear impulsive Volterra integro–delay dynamic systems on time scales by using fixed point theory.

As we studied, the Bielecki–Ulam–Hyers stability and Bielecki–Ulam–Hyers–Rassias stability of non–linear Volterra impulsive integro–delay dynamic systems on time scales are not yet investigated. So getting motivation from the results proved in [43], in this paper, we obtain existence, uniqueness, Bielecki–Ulam–Hyers and Bielecki–Ulam–Hyers–Rassias stability of solution of the following non–linear impulsive Volterra integro–delay dynamic system,

$$\begin{cases} \Theta^{\Delta}(\upsilon) = M(\upsilon)\Theta(\upsilon) + \int_{\upsilon_0}^{\upsilon} \mathrm{K}(\upsilon,\nu,\Theta(\nu),\Theta(h(\nu)))\Delta\nu, \\ \upsilon \in \mathrm{T}_{\mathfrak{S}}{}' = \mathrm{T}_{\mathfrak{S}}{}^0 \backslash \{\upsilon_1,\upsilon_2,\ldots,\upsilon_m\}, \\ \Delta\Theta(\upsilon_k) = \Theta(\upsilon_k^+) - \Theta(\upsilon_k^-) = \Upsilon_k(\Theta(\upsilon_k^-)),\ k = 1,2,\ldots,m, \\ \Theta(\upsilon) = \alpha(\upsilon),\ \upsilon \in [\upsilon_0-\tau,\upsilon_0], \\ \Theta(\upsilon_0) = \alpha(\upsilon_0) = \Theta_0, \end{cases} \tag{1. 1}$$

where $\tau > 0$, $M(\upsilon)$ is piecewise continuous and a regressive square matrix of order m on $\mathrm{T}_{\mathfrak{S}}{}^0 := [\upsilon_0,\upsilon_f]^{z}_{\mathrm{T}_{\mathfrak{S}}}$, $\upsilon_f > \upsilon_0 \geq 0$ and $\mathrm{K}(\upsilon,\nu,\Theta(\nu),\Theta(h(\nu)))$ is piecewise continuous operator on $\Gamma = \{(\upsilon,\nu,\Theta) : \upsilon_0 \leq \nu \leq \upsilon < \upsilon_f,\ \Theta \in \mathbb{R}^m\}$. Also $\Upsilon_k : \mathbb{R} \to \mathbb{R}$, $\alpha : [\upsilon_0-\tau,\upsilon_0] \to \mathbb{R}$ are continuous functions, $\Theta(\upsilon_k^+) = \lim_{\tau\to 0^+}\Theta(\upsilon_k+\tau)$ and $\Theta(\upsilon_k^-) = \lim_{\tau\to 0^-}\Theta(\upsilon_k-\tau)$ are respectively the right and left side limits of $\Theta(\upsilon)$ at υ_k, where υ_k are not isolated points and satisfies $\upsilon_0 < \upsilon_1 < \upsilon_2 < \cdots < \upsilon_m < \upsilon_{m+1} = \upsilon_f < +\infty$. Moreover, $h : \mathrm{T}_{\mathfrak{S}}{}^0 \to \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0-\tau,\upsilon_0]$ is a continuous delay function such that $h(\upsilon) \leq \upsilon$. It should be noted that throughout this paper, we assume that the time scale $\mathrm{T}_{\mathfrak{S}}$ is not the subset of integers and the impulses $\Theta(\upsilon_k^+) - \Theta(\upsilon_k^-)$ are considered to be zero on isolated points.

2. Preliminaries

The time scale, denoted by $\mathrm{T}_{\mathfrak{S}}$, is defined to be an arbitrary closed subset of real numbers. The forward jump operator $\omega : \mathrm{T}_{\mathfrak{S}} \to \mathrm{T}_{\mathfrak{S}}$, backward jump operator $\rho : \mathrm{T}_{\mathfrak{S}} \to \mathrm{T}_{\mathfrak{S}}$ and graininess function $\mu : \mathrm{T}_{\mathfrak{S}} \to [0,\infty)$ are respectively defined as:

$$\omega(\nu) = \inf\{\upsilon \in \mathrm{T}_{\mathfrak{S}} : \upsilon > s\},\ \rho(\nu) = \sup\{\upsilon \in \mathrm{T}_{\mathfrak{S}} : \upsilon < s\},\ \mu(\nu) = \omega(\nu) - \nu.$$

An arbitrary $\upsilon \in \mathrm{T}_{\mathfrak{S}}$ is called left scattered (respectively left dense) when $\upsilon > \rho(\upsilon)$ (respectively $\upsilon = \rho(\upsilon)$). While, in case of $\upsilon < \omega(\upsilon)$ (respectively $\omega(\upsilon) = \upsilon$), we call υ as right scattered (respectively right dense (rd)). The set ${\mathrm{T}_{\mathfrak{S}}}^z$ known as derived form of $\mathrm{T}_{\mathfrak{S}}$ is:

$$ {\mathrm{T}_{\mathfrak{S}}}^z = \begin{cases} \mathrm{T}_{\mathfrak{S}} \backslash (\rho(\sup \mathrm{T}_{\mathfrak{S}}), \sup \mathrm{T}_{\mathfrak{S}}], & \text{if } \sup \mathrm{T}_{\mathfrak{S}} < \infty, \\ \mathrm{T}_{\mathfrak{S}}, & \text{if } \sup \mathrm{T}_{\mathfrak{S}} = \infty. \end{cases} $$

The function $\mathcal{W} : \mathrm{T}_{\mathfrak{S}} \to \mathbb{R}$ is said to be rd-continuous if its continuity and left–sided limit existence hold at every rd and left–dense point on $\mathrm{T}_{\mathfrak{S}}$, respectively. The function $\mathcal{W} : \mathrm{T}_{\mathfrak{S}} \to \mathbb{R}$ is called regressive, denoted by $\mathcal{R}_{\mathcal{G}}(\mathrm{T}_{\mathfrak{S}})$, (respectively positively regressive, denoted by $\mathcal{R}_{\mathcal{G}}(\mathrm{T}_{\mathfrak{S}})^+$) if $1 + \mu(\upsilon)\mathcal{W}(\upsilon) \neq 0$, (respectively $1 + \mu(\upsilon)\mathcal{W}(\upsilon) > 0$) $\forall\, \upsilon \in {\mathrm{T}_{\mathfrak{S}}}^z$. The delta derivative and $\Delta-$integral of $\mathcal{W} : \mathrm{T}_{\mathfrak{S}} \to \mathbb{R}$ are respectively defined as

$$ \mathcal{W}^{\Delta}(\upsilon) = \lim_{s \to \upsilon,\; s \neq \omega(\upsilon)} \frac{\mathcal{W}(\omega(\upsilon)) - \mathcal{W}(s)}{\omega(\upsilon) - s}, \; \upsilon \in {\mathrm{T}_{\mathfrak{S}}}^z, $$

$$ \int_{\mathrm{a}}^{\mathrm{b}} \mathcal{W}(\upsilon)\Delta\upsilon = w(\mathrm{b}) - w(\mathrm{a}), \; \forall\, \mathrm{a}, \mathrm{b} \in \mathrm{T}_{\mathfrak{S}}, $$

where $w^{\Delta} = \mathcal{W}$ on ${\mathrm{T}_{\mathfrak{S}}}^z$.
The generalized exponential function $e_W(a, b)$ for $W \in \mathcal{R}_{\mathcal{G}}(\mathrm{T}_{\mathfrak{S}})$ on $\mathrm{T}_{\mathfrak{S}}$ is

$$ e_W(a, b) = \exp\left(\int_a^b \Theta_{\mu(\nu)} W(\nu)\Delta\nu \right) \; \forall\, a, b \in \mathrm{T}_{\mathfrak{S}}, $$

where

$$ \Theta_{\mu(\upsilon)} W(\upsilon) = \begin{cases} \dfrac{Log(1 + \mu(\upsilon)W(\upsilon))}{\mu(\upsilon)}, & \text{if } \mu(\upsilon) \neq 0, \\ W(\upsilon), & \text{if } \mu(\upsilon) = 0. \end{cases} $$

The general solution of $\Theta^{\Delta}(\upsilon) = M(\upsilon)\Theta(\upsilon),\; \Theta(\upsilon_0) = \Theta_0,\; \upsilon \in {\mathrm{T}_{\mathfrak{S}}}^0$ is known as fundamental matrix denoted by $\zeta_M(\upsilon, \upsilon_0)$.

3. Some Basic Concepts

Let $P_{\mathrm{C}}({\mathrm{T}_{\mathfrak{S}}}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}, \mathbb{R}^m)$ be the Banach space of piecewise continuous functions with
$\|\Theta\| = \sup_{\upsilon \in {\mathrm{T}_{\mathfrak{S}}}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}} \|\Theta(\upsilon)\|$ and Bielecki norm
$\|\Theta\|_B = \sup_{\upsilon \in {\mathrm{T}_{\mathfrak{S}}}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}} \|\Theta(\upsilon)\| e_{-\theta}(\upsilon, \upsilon_0)$
$= \sup_{\upsilon \in {\mathrm{T}_{\mathfrak{S}}}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}} \|\Theta(\upsilon)\| e_{-\theta}(\omega(\upsilon), \upsilon_0)$, keep in mind that $-\theta$ is a positively regressive constant function. Finally, we denote by $P^1_{\mathrm{C}}({\mathrm{T}_{\mathfrak{S}}}^0, \mathbb{R}^m) = \{\Theta \in P_{\mathrm{C}}({\mathrm{T}_{\mathfrak{S}}}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}, \mathbb{R}^m) : \Theta^{\Delta} \in P_{\mathrm{C}}({\mathrm{T}_{\mathfrak{S}}}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}, \mathbb{R}^m)\}$, the Banach space with $\|\Theta\|_1 = \max\{\|\Theta\|, \|\Theta^{\Delta}\|\}$. Consider the following inequalities,

$$ \begin{cases} \left\| \chi^{\Delta}(\upsilon) - M(\upsilon)\chi(\upsilon) - \displaystyle\int_{\upsilon_0}^{\upsilon} \mathrm{K}(\upsilon, \nu, \chi(\nu), \chi(h(\nu)))\Delta\nu \right\| \leq \epsilon; \; \upsilon \in {\mathrm{T}_{\mathfrak{S}}}', \\ \left\| \Delta\chi(\upsilon_k) - \Upsilon_k(\chi(\upsilon_k^-)) \right\| \leq \epsilon, \; k = 1, 2, \ldots, m, \end{cases} \tag{3. 2} $$

$$\begin{cases} \left\|\chi^{\Delta}(\upsilon) - M(\upsilon)\chi(\upsilon) - \int_{\upsilon_0}^{\upsilon} \mathrm{K}(\upsilon,\nu,\chi(\nu),\chi(h(\nu)))\Delta\nu\right\| \leq \varphi(\upsilon);\ \upsilon \in \mathrm{T}_{\mathfrak{S}}{}',\\ \left\|\Delta\chi(\upsilon_k) - \Upsilon_k(\chi(\upsilon_k^-))\right\| \leq \kappa,\ k = 1,2,\ldots,m, \end{cases} \tag{3. 3}$$

where $\psi : \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}} \to \mathbb{R}^+$ is continuous and increasing function.

Definition 3.1. *Eq.* (1. 1) *is stable in terms of Bielecki–Ulam–Hyers on* $\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]$ *if for every* $\Theta_0 \in P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0], \mathbb{R}^m)$ *satisfying* (3. 2), $\exists$ *a solution* $\Theta \in P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0], \mathbb{R}^m)$ *of* (1. 1) *with* $\|\Theta(\upsilon) - \Theta_0(\upsilon)\| e_{-\theta}(\upsilon, \upsilon_0) \leq C\epsilon,\ C > 0,\ \forall\ \upsilon \in \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}$.

Definition 3.2. *Eq.* (1. 1) *is stable in terms of Bielecki–Ulam–Hyers–Rassias on* $\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]$ *if for every* $\Theta_0 \in P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0], \mathbb{R}^m)$ *satisfying* (3. 3), $\exists$ *a solution* $\Theta \in P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0], \mathbb{R}^m)$ *of* (1. 1) *with* $\|\Theta(\upsilon) - \Theta_0(\upsilon)\| e_{-\theta}(\upsilon, \upsilon_0) \leq C\psi(\upsilon)$, $C > 0, \forall\ \upsilon \in \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}$.

Lemma 3.3. *[16] Let* $\tau \in \mathrm{T}_{\mathfrak{S}}{}^+,\ y,\ b \in \mathcal{R}_{\mathcal{G}}(\mathrm{T}_{\mathfrak{S}}{}^+),\ p \in \mathcal{R}_{\mathcal{G}}(\mathrm{T}_{\mathfrak{S}}{}^+)^+$ *and* $c,\ b_k \in \mathbb{R}^+,\ k = 1, 2, \ldots$, *then*

$$\chi(\upsilon) \leq c + \int_{\tau}^{\upsilon} p(\nu)\chi(\nu)\Delta\nu + \sum_{\tau<\upsilon_k<\upsilon} b_k \chi(\upsilon_k),$$

implies

$$\chi(\upsilon) \leq c \prod_{\tau<\upsilon_k<\upsilon} (1 + b_k) e_p(\upsilon, \tau),\ \upsilon \geq \tau.$$

Remark 3.4. *A function* $\chi \in P_{\mathrm{C}}^1(\mathrm{T}_{\mathfrak{S}}{}^0, \mathbb{R}^m)$ *satisfies* (3. 2) *if and only if there exists* $f \in P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0], \mathbb{R}^m)$ *and a sequence* f_k *bounded by* ϵ *such that*

$$\begin{cases} \chi^{\Delta}(\upsilon) = M(\upsilon)\chi(\upsilon) + \int_{\upsilon_0}^{\upsilon} \mathrm{K}(\upsilon,\nu,\chi(\nu),\chi(h(\nu)))\Delta\nu + f(\upsilon),\ \chi(\upsilon_0) = \chi_0,\ \upsilon \in \mathrm{T}_{\mathfrak{S}}{}',\\ \Delta\chi(\upsilon_k) = \Upsilon_k(\chi(\upsilon_k^-)) + f_k,\ k = 1,2,\ldots,m. \end{cases}$$

We do similar remark for (3. 3).

Lemma 3.5. *[43] Every* $\chi \in P_{\mathrm{C}}^1(\mathrm{T}_{\mathfrak{S}}{}^0, \mathbb{R}^m)$ *that satisfies* (3. 2) *also satisfies*

$$\Bigg\|\chi(\upsilon) - \zeta_M(\upsilon,\upsilon_0)\chi_0 - \sum_{j=1}^{k} \Upsilon(\chi(\upsilon_j^-)) - \int_{\upsilon_0}^{\upsilon} \zeta_M(\upsilon,\omega(\nu)) \int_{\upsilon_0}^{\nu} \mathrm{K}(\nu,\vartheta,\chi(\vartheta),\chi(h(\vartheta)))\Delta\vartheta\Delta\nu\Bigg\| \leq C(m + \upsilon_f - \upsilon_0)\epsilon,$$

for $\upsilon \in (\upsilon_k, \upsilon_{k+1}] \subset \mathrm{T}_{\mathfrak{S}}{}^0$, *where* $\|\zeta_M(\upsilon, \omega(\nu))\| \leq C$.

4. Main Results

This section is comprised of existence, uniqueness, Bielecki–Ulam–Hyers and Bielecki–Ulam–Hyers–Rassias stablity of solution of Eq. (1. 1). Let

$(\mathbf{A_1})$ The function K satisfies the Lipschitz condition $\|\mathrm{K}(\upsilon,\nu,x_1,x_2)-\mathrm{K}(\upsilon,\nu,\chi_1,\chi_2)\| \leq \sum_{i=1}^{2} L\|x_i-\chi_i\|$, $L>0$ for $\upsilon_0 \leq \nu \leq \upsilon < \upsilon_f$ and for all $x_i, \chi_i \in \mathbb{R}^m$, $i \in \{1,2\}$;

$(\mathbf{A_2})$ $\Upsilon_k : \mathbb{R} \to \mathbb{R}$ satisfies $\|\Upsilon_k(\upsilon_1) - \Upsilon_k(\upsilon_2)\| \leq M_k\|\upsilon_1-\upsilon_2\|$, $M_k > 0$, $\forall\, k \in \{1,2,\ldots,m\}$ and $\upsilon_1, \upsilon_2 \in \mathbb{R}$;

$(\mathbf{A_3})$ $\left(\sum_{j=1}^{m} M_j + \frac{2CL(\upsilon_f-\upsilon_0)}{\theta} e^{-\theta(\upsilon_f-\upsilon_0)}\right) < 1$;

$(\mathbf{A_4})$ $\psi : \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0-\tau,\upsilon_0]_{\mathrm{T}_{\mathfrak{S}}} \to \mathbb{R}^+$ satisfies $\int_{\upsilon_0}^{\upsilon} \psi(\nu)\Delta\nu \leq \rho\psi(\upsilon),\ \rho>0$.

Theorem 4.1. *If assumptions* $(\mathbf{A_1})-(\mathbf{A_3})$ *hold, then Eq.* (1. 1) *has only one solution in* $P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0-\tau,\upsilon_0],\mathbb{R}^m)$.

Proof. Define $\Lambda : P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0-\tau,\upsilon_0],\mathbb{R}^m) \to P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0-\tau,\upsilon_0],\mathbb{R}^m)$ by

$$(\Lambda z)(\upsilon) = \begin{cases} \alpha(\upsilon),\ \upsilon \in [\upsilon_0-\tau,\upsilon_0], \\ \alpha(\upsilon_0) + \zeta_M(\upsilon,\upsilon_0)\Theta_0 \\ \quad + \displaystyle\int_{\upsilon_0}^{\upsilon} \zeta_M(\upsilon,\omega(\nu)) \int_{\upsilon_0}^{\nu} \mathrm{K}(\nu,\vartheta,\Theta(\vartheta),\Theta(h(\vartheta)))\Delta\vartheta\Delta\nu,\ \upsilon \in (\upsilon_0,\upsilon_1], \\ \alpha(\upsilon_0) + \Upsilon_1(\Theta(\upsilon_1^-)) + \zeta_M(\upsilon,\upsilon_0)\Theta_0 \\ \quad + \displaystyle\int_{\upsilon_0}^{\upsilon} \zeta_M(\upsilon,\omega(\nu)) \int_{\upsilon_0}^{\nu} \mathrm{K}(\nu,\vartheta,\Theta(\vartheta),\Theta(h(\vartheta)))\Delta\vartheta\Delta\nu,\ \upsilon \in (\upsilon_1,\upsilon_2], \\ . \\ . \\ . \\ \alpha(\upsilon_0) + \displaystyle\sum_{j=1}^{m} \Upsilon_j(\Theta(\upsilon_j^-)) + \zeta_M(\upsilon,\upsilon_0)\Theta_0 \\ \quad + \displaystyle\int_{\upsilon_0}^{\upsilon} \zeta_M(\upsilon,\omega(\nu)) \int_{\upsilon_0}^{\nu} \mathrm{K}(\nu,\vartheta,\Theta(\vartheta),\Theta(h(\vartheta)))\Delta\vartheta\Delta\nu,\ \upsilon \in (\upsilon_m,\upsilon_{m+1}]. \end{cases} \tag{4. 4}$$

We see that for any $\Theta_1, \Theta_2 \in P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0-\tau,\upsilon_0],\mathbb{R}^m)$ and $\forall\, \upsilon \in [\upsilon_0-\tau,\upsilon_0]$, we have $\|(\Lambda\Theta_1)(\upsilon)-(\Lambda\Theta_2)(\upsilon)\| = 0$. For $\upsilon \in (\upsilon_m,\upsilon_{m+1}]$ consider,

$$\Big\|(\Lambda\Theta_1)(\upsilon)-(\Lambda\Theta_2)(\upsilon)\Big\| = \sum_{j=1}^{m} \Big\|\Upsilon_j(\Theta_1(\upsilon_j^-)) - \Upsilon_j(\Theta_2(\upsilon_j^-))\Big\|$$
$$+\Big\| \int_{\upsilon_0}^{\upsilon} \zeta_M(\upsilon,\omega(\nu)) \int_{\upsilon_0}^{\nu} \Big(\mathrm{K}(\nu,\vartheta,\Theta_1(\vartheta),\Theta_1(h(\vartheta)))$$

$$
\begin{aligned}
&-\mathrm{K}(\nu,\vartheta,\Theta_2(\vartheta),\Theta_2(h(\vartheta)))\Big)\Delta\vartheta\Delta\nu\Big\|\\
\le\ & \sum_{j=1}^{m} M_j\Big\|\Theta_1(\upsilon_j^-)-\Theta_2(\upsilon_j^-)\Big\|\\
&+\int_{\upsilon_0}^{\upsilon}\|\zeta_M(\upsilon,\omega(\nu))\|\int_{\upsilon_0}^{\nu}\Big\|\Big(\mathrm{K}(\nu,\vartheta,\Theta_1(\vartheta),\Theta_1(h(\vartheta)))\\
&-\mathrm{K}(\nu,\vartheta,\Theta_2(\vartheta),\Theta_2(h(\vartheta)))\Big)\Big\|\Delta\vartheta\Delta\nu
\end{aligned}
$$

$$
\begin{aligned}
\le\ & \sum_{j=1}^{m} M_j \sup_{\upsilon\in\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]}\Big\|\Theta_1(\upsilon_j^-)-\Theta_2(\upsilon_j^-)\Big\|\\
&+\int_{\upsilon_0}^{\upsilon}C\int_{\upsilon_0}^{\nu}e_{-\theta}(\vartheta_0,\omega(\vartheta))L\sup_{\upsilon\in\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]}\|\Theta_1(\vartheta)\\
&-\Theta_2(\vartheta)\|e_{-\theta}(\omega(\vartheta),\vartheta_0)\Delta\vartheta\Delta\nu\\
&+\int_{\upsilon_0}^{\upsilon}C\int_{\upsilon_0}^{\nu}e_{-\theta}(\vartheta_0,\omega(\vartheta))L\sup_{\upsilon\in\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]}\|\Theta_1(h(\vartheta))\\
&-\Theta_2(h(\vartheta))\|e_{-\theta}(\omega(\vartheta),\vartheta_0)\Delta\vartheta\Delta\nu\\
\le\ & \sum_{j=1}^{m} M_j\|\Theta_1-\Theta_2\|+\frac{2L}{-\theta}\|\Theta_1-\Theta_2\|_B\int_{\upsilon_0}^{\upsilon}C\int_{\upsilon_0}^{\nu}-\theta e_{-\theta}(\vartheta_0,\omega(\vartheta))\Delta\vartheta\Delta\nu\\
\le\ & \sum_{j=1}^{m} M_j\|\Theta_1-\Theta_2\|+\frac{2L}{-\theta}\|\Theta_1-\Theta_2\|_B\int_{\upsilon_0}^{\upsilon}C\big(e_{-\theta}(\vartheta_0,\nu)-1\big)\Delta\nu\\
\le\ & \sum_{j=1}^{m} M_j\|\Theta_1-\Theta_2\|+\frac{2CL}{\theta}\|\Theta_1-\Theta_2\|_B\int_{\upsilon_0}^{\upsilon}\Delta\nu\\
\le\ & \sum_{j=1}^{m} M_j\|\Theta_1-\Theta_2\|+\frac{2CL(\upsilon_f-\upsilon_0)}{\theta}\|\Theta_1-\Theta_2\|_B.
\end{aligned}
$$

Thus

$$
\begin{aligned}
&\|(\Lambda\Theta_1)(\upsilon)-(\Lambda\Theta_2)(\upsilon)\|e_{-\theta}(\upsilon,\upsilon_0)\le\sup_{\upsilon\in\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]}\|(\Lambda\Theta_1)(\upsilon)\\
&-(\Lambda\Theta_2)(\upsilon)\|e_{-\theta}(\upsilon,\upsilon_0)\\
\le\ & \sum_{j=1}^{m} M_j\|\Theta_1-\Theta_2\|e_{-\theta}(\upsilon,\upsilon_0)+\frac{2CL(\upsilon_f-\upsilon_0)}{\theta}e_{-\theta}(\upsilon,\upsilon_0)\|\Theta_1-\Theta_2\|_B\\
\Rightarrow\ & \sup_{\upsilon\in\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]}\|(\Lambda\Theta_1)(\upsilon)-(\Lambda\Theta_2)(\upsilon)\|e_{-\theta}(\upsilon,\upsilon_0)\le\sup_{\upsilon\in\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]}\sum_{j=1}^{m}M_j\|\Theta_1\\
-\ & \Theta_2\|e_{-\theta}(\upsilon,\upsilon_0)+\frac{2CL(\upsilon_f-\upsilon_0)}{\theta}e^{-\theta(\upsilon-\upsilon_0)}\|\Theta_1-\Theta_2\|_B
\end{aligned}
$$

$$\Rightarrow \sup_{\upsilon\in \mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]} \|(\Lambda\Theta_1)(\upsilon)-(\Lambda\Theta_2)(\upsilon)\|e_{-\theta}(\upsilon,\upsilon_0) \leq \sum_{j=1}^{m} M_j\|\Theta_1-\Theta_2\|_B$$

$$+ \quad \frac{2CL(\upsilon_f-\upsilon_0)}{\theta}e^{-\theta(\upsilon-\upsilon_0)}\|\Theta_1-\Theta_2\|_B$$

$$\Rightarrow \|(\Lambda\Theta_1)(\upsilon)-(\Lambda\Theta_2)(\upsilon)\|_B \leq \left(\sum_{j=1}^{m} M_j\right.$$

$$\left.+\frac{2CL(\upsilon_f-\upsilon_0)}{\theta}e^{-\theta(\upsilon_f-\upsilon_0)}\right)\|\Theta_1-\Theta_2\|_B.$$

From $(\mathbf{A_3})$, Λ is a Picard operator on $P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0],\mathbb{R}^m)$. Therefore, the only one fixed point of Λ is actually the only one solution of (1. 1) in $P_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0],\mathbb{R}^m)$. □

Theorem 4.2. *If assumptions* $(\mathbf{A_1})-(\mathbf{A_3})$ *hold, then Eq.* (1. 1) *is Bielecki–Ulam–Hyers stable on* $\mathrm{T}_{\mathfrak{S}}{}^0\cup[\upsilon_0-\tau,\upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}$.

Proof. Let $\chi\in P^1_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0,\mathbb{R}^m)$ satisfies (3. 2). The only one solution $\Theta\in P^1_{\mathrm{C}}(\mathrm{T}_{\mathfrak{S}}{}^0,\mathbb{R}^m)$ of the dynamic equation

$$\begin{cases} \Theta^{\Delta}(\upsilon)=M(\upsilon)\Theta(\upsilon)+\displaystyle\int_{\upsilon_0}^{\upsilon}\mathrm{K}(\upsilon,\nu,\Theta(\nu),\Theta(h(\nu)))\Delta\nu,\\ \upsilon\in\mathrm{T}_{\mathfrak{S}}{}'=\mathrm{T}_{\mathfrak{S}}{}^0\backslash\{\upsilon_1,\upsilon_2,\ldots,\upsilon_m\},\\ \Delta\Theta(\upsilon_k)=\Theta(\upsilon_k^+)-\Theta(\upsilon_k^-)=\Upsilon_k(\Theta(\upsilon_k^-)),\ k=1,2,\ldots,m,\\ \Theta(\upsilon)=\chi(\upsilon),\ \upsilon\in[\upsilon_0-\tau,\upsilon_0],\\ \Theta(\upsilon_0)=\chi(\upsilon_0)=\Theta_0, \end{cases}$$

is given by

$$\Theta(\upsilon)=\begin{cases} \chi(\upsilon),\ \upsilon\in[\upsilon_0-\tau,\upsilon_0],\\ \chi(\upsilon_0)+\zeta_M(\upsilon,\upsilon_0)\Theta_0\\ \ +\displaystyle\int_{\upsilon_0}^{\upsilon}\zeta_M(\upsilon,\omega(\nu))\int_{\upsilon_0}^{\nu}\mathrm{K}(\nu,\vartheta,\Theta(\vartheta),\Theta(h(\vartheta)))\Delta\vartheta\Delta\nu,\ \upsilon\in(\upsilon_0,\upsilon_1],\\ \chi(\upsilon_0)+\Upsilon_1(\Theta(\upsilon_1^-))+\zeta_M(\upsilon,\upsilon_0)\Theta_0\\ \ +\displaystyle\int_{\upsilon_0}^{\upsilon}\zeta_M(\upsilon,\omega(\nu))\int_{\upsilon_0}^{\nu}\mathrm{K}(\nu,\vartheta,\Theta(\vartheta),\Theta(h(\vartheta)))\Delta\vartheta\Delta\nu,\ \upsilon\in(\upsilon_1,\upsilon_2],\\ .\\ .\\ .\\ \chi(\upsilon_0)+\displaystyle\sum_{j=1}^{m}\Upsilon_j(\Theta(\upsilon_j^-))+\zeta_M(\upsilon,\upsilon_0)\Theta_0\\ \ +\displaystyle\int_{\upsilon_0}^{\upsilon}\zeta_M(\upsilon,\omega(\nu))\int_{\upsilon_0}^{\nu}\mathrm{K}(\nu,\vartheta,\Theta(\vartheta),\Theta(h(\vartheta)))\Delta\vartheta\Delta\nu,\ \upsilon\in(\upsilon_m,\upsilon_{m+1}]. \end{cases}$$

We observe that $\forall\ \upsilon \in [\upsilon_0 - \tau, \upsilon_0]$, we have $\|\chi(\upsilon) - \Theta(\upsilon)\| = 0$. For $\upsilon \in (\upsilon_m, \upsilon_{m+1}]$, using Lemma 3.5, we have

$$
\begin{aligned}
& \|\chi(\upsilon) - \Theta(\upsilon)\| e_{-\theta}(\upsilon, \upsilon_0) \leq \|\chi(\upsilon) - \Theta(\upsilon)\|_B \leq \Bigg\|\chi(\upsilon) - \zeta_M(\upsilon, \upsilon_0)\chi_0 - \sum_{j=1}^{m} \Upsilon(\chi(\upsilon_j^-)) \\
& - \int_{\upsilon_0}^{\upsilon} \zeta_M(\upsilon, \omega(\nu)) \int_{\upsilon_0}^{\nu} \mathrm{K}(\nu, \vartheta, \chi(\vartheta), \chi(h(\vartheta)))\Delta\vartheta\Delta\nu \Bigg\| e_{-\theta}(\upsilon, \upsilon_0) \\
& + \sum_{j=1}^{m} \Bigg\| \Upsilon_j(\chi(\upsilon_j^-)) - \Upsilon_j(\Theta(\upsilon_j^-)) \Bigg\| e_{-\theta}(\upsilon, \upsilon_0) \\
& + \Bigg\| \int_{\upsilon_0}^{\upsilon} \zeta_M(\upsilon, \omega(\nu)) \int_{\upsilon_0}^{\nu} \Big(\mathrm{K}(\nu, \vartheta, \chi(\vartheta), \chi(h(\vartheta))) \\
& - \mathrm{K}(\nu, \vartheta, \Theta(\vartheta), \Theta(h(\vartheta))) \Big) \Delta\vartheta\Delta\nu \Bigg\| e_{-\theta}(\upsilon, \upsilon_0) \\
\leq\ & (m + \upsilon_f - \upsilon_0)\epsilon e_{-\theta}(\upsilon, \upsilon_0) + \sum_{j=1}^{m} M_j \Bigg\| \chi(\upsilon_j^-) - \Theta(\upsilon_j^-) \Bigg\| e_{-\theta}(\upsilon, \upsilon_0) \\
& + \int_{\upsilon_0}^{\upsilon} C \int_{\upsilon_0}^{\nu} L \|\chi(\vartheta) - \Theta(\vartheta)\| e_{-\theta}(\upsilon, \upsilon_0)\Delta\vartheta\Delta\nu \\
& + \int_{\upsilon_0}^{\upsilon} C \int_{\upsilon_0}^{\nu} L \|\chi(h(\vartheta)) - \Theta(h(\vartheta))\| e_{-\theta}(\upsilon, \upsilon_0)\Delta\vartheta\Delta\nu \\
\leq\ & (m + \upsilon_f - \upsilon_0)\epsilon e^{-\theta(\upsilon - \upsilon_0)} + \sum_{j=1}^{m} M_j \sup_{\upsilon \in \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]} \Bigg\| \chi(\upsilon_j^-) - \Theta(\upsilon_j^-) \Bigg\| e_{-\theta}(\upsilon, \upsilon_0) \\
& + \int_{\upsilon_0}^{\upsilon} C \int_{\upsilon_0}^{\nu} L \sup_{\upsilon \in \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]} \|\chi(\vartheta) - \Theta(\vartheta)\| e_{-\theta}(\upsilon, \upsilon_0)\Delta\vartheta\Delta\nu \\
& + \int_{\upsilon_0}^{\upsilon} C \int_{\upsilon_0}^{\nu} L \sup_{\upsilon \in \mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]} \|\chi(h(\vartheta)) - \Theta(h(\vartheta))\| e_{-\theta}(\upsilon, \upsilon_0)\Delta\vartheta\Delta\nu \\
\leq\ & (m + \upsilon_f - \upsilon_0)\epsilon e^{-\theta(\upsilon_f - \upsilon_0)} + \sum_{j=1}^{m} M_j \|\chi(\upsilon) - \Theta(\upsilon)\|_B \\
& + 2\|\chi(\upsilon) - \Theta(\upsilon)\|_B \int_{\upsilon_0}^{\upsilon} C \int_{\upsilon_0}^{\nu} L \Delta\vartheta\Delta\nu
\end{aligned}
$$

Thus by Lemma 3.3, we get

$$
\|\chi(\upsilon) - \Theta(\upsilon)\|_B \leq (m + \upsilon_f - \upsilon_0)\epsilon e^{-\theta(\upsilon_f - \upsilon_0)} \prod_{\upsilon_0 < \upsilon_j < \upsilon} (1 + M_j) e_P(\upsilon, \upsilon_0),
$$

where $P = \int_{\upsilon_0}^{\nu} 2CL\Delta\vartheta$ is a positively regressive function. On further calculations, we get

$$
\|\chi(\upsilon) - \Theta(\upsilon)\|_B \leq (m + \upsilon_f - \upsilon_0)\epsilon e^{-\theta(\upsilon_f - \upsilon_0)} \prod_{\upsilon_0 < \upsilon_j < \upsilon} (1 + M_j) e^{P(\upsilon_f - \upsilon_0)}.
$$

By choosing $K = (m + \upsilon_f - \upsilon_0)e^{-\theta(\upsilon_f - \upsilon_0)} \prod_{\upsilon_0<\upsilon_j<\upsilon}(1 + M_j)e^{P(\upsilon_f - \upsilon_0)}$, Eq. (1. 1) is Bielecki–Ulam–Hyers stable on $\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}$.

□

Theorem 4.3. *If assumptions* $(\mathbf{A_1}) - (\mathbf{A_4})$ *hold, then Eq.* (1. 1) *is Bielecki–Ulam–Hyers–Rassias stable on* $\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}$.

Proof. Using Lemma 3.5 (with the help of Remark 3.4) in the similar manner for the case of increasing function $\psi(\upsilon)$, it can be easily proved that the Eq. (1. 1) is Bielecki–Ulam–Hyers–Rassias stable on $\mathrm{T}_{\mathfrak{S}}{}^0 \cup [\upsilon_0 - \tau, \upsilon_0]_{\mathrm{T}_{\mathfrak{S}}}$. So the proof is Trivial. □

5. CONCLUSION

This paper is based on the existence, uniqueness, Bielecki–Ulam–Hyers and Bielecki–Ulam–Hyers–Rassias stability of solution of Eq. (1. 1). The fixed point theory is used to establish the main results. Our work assures the existence of an exact solution of (1. 1) near to approximate solution. We are confident that the achieved results will be valuable to the present literature. In fact, our results are significant when finding exact solution is quite difficult and hence are important in approximation theory etc. Moreover, it is clear that the stable systems are of high importance while unstable systems are useless. So the stability of Eq. (1. 1) will be of great interest for applied mathematicians in mathematical modeling, image segmentation, numerical coding etc.

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