

### Theory of Bijective Hypersoft Set with Application in Decision Making

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**Abstract.** Hypersoft set (an extension of soft set) is a new mathematical tool to tackle the inadequacy of soft set for attribute-valued sets. In this study, concept of bijective hypersoft set is proposed and some of its set theoretic operations like restricted-AND and relaxed-AND, are characterized. Moreover, new operations of bijective hypersoft set such as dependency, decision system, significance of decision system, reduced decision system and decision rules in decision system, are discussed with illustrated examples. A decision making algorithm and application are discussed with the support of these proposed operations.

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**Key Words:** Soft set, Bijective soft set, Hypersoft set, Bijective hypersoft set, Bijective hypersoft decision system.

#### 1. INTRODUCTION

In 1999, Molodtsov [19] developed a new structure known as soft set in literature to address the shortcoming of the existing structures (i.e. fuzzy set, intuitionistic fuzzy etc.) dealing with uncertain and vague data of information. He provided a strong basis for parameterized interpretation of hesitant information through the introduction of this structure. Later on, Maji et al. [17] investigated different elementary essentials i.e. properties, set theoretic operations, laws and results of soft set for its further implementation in certain disciplines. In 2005, Pei et al. [22] illustrated the relational concept between soft sets and information systems. They confirmed the validity of the soft set as a parameterized family of special information system. The researchers Ali et al. [2], Babitha et al. [3, 4], Sezgin et al. [39], Ge et al. [6] and Li [16] studied soft set professionally and extended the concept with the introduction of certain new features i.e. restrictedness on operations, set relation and function. Saeed et al. [29] explored the concept of soft elements and soft members under soft set environment. Kamacı et al. [9, 10, 11, 12] developed bijective soft matrix theory with multi-bijective linguistic soft decision, investigated difference operations of soft matrices with applications in decision making system, introduced  $N$ -soft algebraic structures,

and discussed bipolar  $N$ -soft set theory with applications respectively. Petchimuthu et al. [23] made valuable discussion on the row-products of inverse soft matrices with application in multicriteria decision making. Many researchers [14, 5, 15, 21, 1, 43] expanded soft set theory with the development of soft-like hybrids to have useful results as well as utilization in decision making problems. Gong et al. [7] introduced the concept of bijective soft set and investigated its some properties.

In 2018, Smarandache [40] developed a new structure hypersoft set (HSS) to adequate the soft set for multi attribute-valued functions. In 2020, Saeed et al. [30, 31] characterized the necessary basic axioms, properties, laws and set theoretic operations of HSS with the help of appropriate examples. In 2020, Rahman et al. [24, 25] enhanced the study of HSS to develop fuzzy-like structures with complex sets and also studied certain properties of convexity under HSS-environment. In 2021, Rahman et al. [26, 27, 28] studied decision making applications based on neutrosophic parameterized hypersoft Set, fuzzy parameterized hypersoft set and rough hypersoft set. Saeed et al. [32, 33, 34, 35] discussed decision making techniques for mappings on hypersoft classes, neutrosophic hypersoft mapping and complex multi-fuzzy hypersoft set. They also developed hypersoft graphs with some properties. Ihsan et al. [8] investigated hypersoft expert set with application in decision making for the best selection of product. Yolcu et al. [41, 42] conceptualized the theories of fuzzy and intuitionistic fuzzy hypersoft sets with their employment in decision making. Ozturk et al. [20] introduced neutrosophic hypersoft topological spaces and discussed their important properties. Saqlain et al. [36, 37, 38] developed single and multi-valued neutrosophic hypersoft sets along with calculation of tangent similarity measure of single valued neutrosophic hypersoft sets, characterized aggregate operators of neutrosophic hypersoft set and employed TOPSIS method for neutrosophic hypersoft sets using accuracy function with application. Martin et al. [18] investigated concentric plithogenic hypergraph based on plithogenic hypersoft sets. Kamacı [13] made very valuable research on the hybrid structures of hypersoft sets and rough sets.

In many real life situations, distinct attributes are further partitioned into disjoint attribute-valued sets. Decision makers may suffer some kind of inclination and penchant while ignoring such partitioning of attributes during the judgment. The existing soft set theory is not projected for such sets. Therefore a new structure demands its place in literature for addressing such impediment, so hypersoft set theory is conceptualized to tackle such situations. This novel structure has increased the flexibility and reliability of decision making process. It not only addresses the inadequacy of existing soft-like structures for multi-argument approximate functions but also helps the decision makers to decide the matters with deep observation. In this study, a new type of hypersoft set, bijective hypersoft set (BHSS), is characterized which assigns the disjoint approximate sets to each tuple in the cartesian product of attribute-valued sets. Moreover, certain elementary properties are investigated with supporting examples. A decision system is developed based on set theoretic operations of BHSS and is applied in decision making with illustrated example.

The rest of the paper is organized as:

In section 2, some fundamental definitions and terms are recalled from already published relevant literature, section 3 formulates the theory of bijective hypersoft set and its decision system, section 4 proposes a decision-making algorithm with application for the best

selection of an appropriate product and finally, section 5 concludes the paper with future directions.

## 2. PRELIMINARIES

In this section, we present some basic terminologies for proper understanding of the proposed work. Throughout the paper,  $\mathfrak{U}$  denotes the universe of discourse.

### Definition 1. [19]

A pair  $(\zeta_S, \Lambda)$  is called a *soft set* over  $\mathfrak{U}$ , where  $\zeta_S : \Lambda \rightarrow \mathcal{P}(\mathfrak{U})$  and  $\Lambda$  be a set of attributes of  $\mathfrak{U}$ .

### Definition 2. [17]

A soft set  $(\zeta_{S_1}, \Lambda_1)$  is a *soft subset* of another soft set  $(\zeta_{S_2}, \Lambda_2)$  if

- (i)  $\Lambda_1 \subseteq \Lambda_2$ , and
- (ii)  $\zeta_{S_1}(\omega) \subseteq \zeta_{S_2}(\omega)$  for all  $\omega \in \Lambda_1$ .

### Definition 3. [17]

Union of two soft sets  $(\zeta_{S_1}, \Lambda_1)$  and  $(\zeta_{S_2}, \Lambda_2)$  is a soft set  $(\zeta_{S_3}, \Lambda_3)$  with  $\Lambda_3 = \Lambda_1 \cup \Lambda_2$  and for  $\omega \in \Lambda_3$ ,

$$\zeta_{S_3}(\omega) = \begin{cases} \zeta_{S_1}(\omega) & \omega \in (\Lambda_1 \setminus \Lambda_2) \\ \zeta_{S_2}(\omega) & \omega \in (\Lambda_2 \setminus \Lambda_1) \\ \zeta_{S_1}(\omega) \cup \zeta_{S_2}(\omega) & \omega \in (\Lambda_1 \cap \Lambda_2) \end{cases}$$

### Definition 4. [17]

Intersection of two soft sets  $(\zeta_{S_1}, \Lambda_1)$  and  $(\zeta_{S_2}, \Lambda_2)$  is a soft set  $(\zeta_{S_3}, \Lambda_3)$  with  $\Lambda_3 = \Lambda_1 \cap \Lambda_2$  and for  $\omega \in \Lambda_3$ ,

$$\zeta_{S_3}(\omega) = \zeta_{S_1}(\omega) \cap \zeta_{S_2}(\omega)$$

### Definition 5. [17]

AND operation of two soft sets  $(\zeta_{S_1}, \Lambda_1)$  and  $(\zeta_{S_2}, \Lambda_2)$ , denoted by  $(\zeta_{S_1}, \Lambda_1) \wedge (\zeta_{S_2}, \Lambda_2)$ , is a soft set  $(\zeta_{S_3}, \Lambda_3)$  with  $\Lambda_3 = \Lambda_1 \times \Lambda_2$  and for  $\omega \in \Lambda_3$ ,

$$\zeta_{S_3}(\omega) = \zeta_{S_1}(\omega) \cap \zeta_{S_2}(\omega)$$

For more details on soft set, see [19, 17, 22, 2, 3, 4, 39, 6, 16]

### Definition 6. [7]

A soft set  $(\zeta_S, \Lambda)$  is said to be a *bijective soft set*, if

- (i)  $\cup_{\epsilon \in \Lambda} \zeta_S(\epsilon) = \mathfrak{U}$
- (ii) For any two parameters  $\epsilon_i, \epsilon_j \in \Lambda$ ,  $\epsilon_i \neq \epsilon_j$ ,  $\zeta_S(\epsilon_i) \cap \zeta_S(\epsilon_j) = \emptyset$

## 3. BIJECTIVE HYPERSOFT SET (BHS-SET)

In this section, we start with definition of hypersoft set with example then we characterize the theory of bijective hypersoft set.

### Definition 7. [40]

The pair  $(\mathfrak{F}, \mathcal{G})$  is called a *hypersoft set* over  $\mathfrak{U}$ , where  $\mathcal{G}$  is the cartesian product of  $n$  disjoint attribute-valued sets  $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \dots, \mathcal{G}_n$  corresponding to  $n$  distinct attributes  $g_1, g_2, g_3, \dots, g_n$  respectively and  $\mathfrak{F} : \mathcal{G} \rightarrow \mathcal{P}(\mathfrak{U})$ . The collection of all hypersoft sets is denoted by  $\Omega_{(\mathfrak{F}, \mathcal{G})}$ .

**Example 3.1.** Suppose that Mr. Smith wants to buy a mobile from a mobile market. There are sixteen kinds of mobiles (options) which form the set of discourse

$$\mathfrak{U} = \{m_1, m_2, m_3, m_4, m_5, \dots, m_{16}\}$$

. The best selection may be evaluated by observing the attributes i.e.  $g_1$  = Camera Resolution,  $g_2$  = storage, and  $g_3$  = Battery power. The attribute-valued sets corresponding to these attributes are:

$$\mathcal{G}_1 = \{g_{11}, g_{12}\}$$

$$\mathcal{G}_2 = \{g_{21}, g_{22}\}$$

$$\mathcal{G}_3 = \{g_{31}, g_{32}\}$$

then  $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3$

$\mathcal{G} = \{\delta_1, \delta_2, \delta_3, \delta_4, \dots, \delta_8\}$  where each  $\delta_i, i = 1, 2, \dots, 8$ , is a 3-tuple element.

The hypersoft set  $(\mathfrak{F}, \mathcal{G})$  is given as

$$(\mathfrak{F}, \mathcal{G}) = \left\{ \begin{array}{l} (\delta_1, \{m_1, m_2, m_9, m_{14}, m_{16}\}), (\delta_2, \{m_1, m_2, m_3, m_{10}, m_{12}\}), \\ (\delta_3, \{m_2, m_3, m_4, m_9, m_{13}, m_{14}\}), (\delta_4, \{m_4, m_5, m_6, m_{10}, m_{15}, m_{16}\}), \\ (\delta_5, \{m_6, m_7, m_8, m_{10}, m_{12}\}), (\delta_6, \{m_2, m_3, m_4, m_7, m_9\}), \\ (\delta_7, \{m_1, m_3, m_5, m_6, m_8, m_{10}, m_{12}, m_{14}, m_{16}\}), \\ (\delta_8, \{m_2, m_3, m_6, m_7, m_9, m_{11}, m_{13}, m_{15}\}) \end{array} \right\}$$

and tabular representation of  $(\mathfrak{F}, \mathcal{G})$  is given in TABLE 1 where if  $m_i \in \mathfrak{F}(\delta_i)$  then 1 otherwise 0.

....	$\mathfrak{F}(\delta_1)$	$\mathfrak{F}(\delta_2)$	$\mathfrak{F}(\delta_3)$	$\mathfrak{F}(\delta_4)$	$\mathfrak{F}(\delta_5)$	$\mathfrak{F}(\delta_6)$	$\mathfrak{F}(\delta_7)$	$\mathfrak{F}(\delta_8)$
$m_1$	1	1	0	0	0	0	1	0
$m_2$	1	1	1	0	0	1	0	1
$m_3$	0	1	1	0	0	1	1	1
$m_4$	0	0	1	1	0	1	0	0
$m_5$	0	0	0	1	0	0	1	0
$m_6$	0	0	0	1	1	0	1	1
$m_7$	0	0	0	0	1	1	0	1
$m_8$	0	0	0	0	1	0	1	0
$m_9$	1	0	1	0	0	1	0	1
$m_{10}$	0	1	0	1	1	0	1	0
$m_{11}$	0	0	0	0	0	0	0	1
$m_{12}$	0	1	0	0	1	0	1	0
$m_{13}$	0	0	1	0	0	0	0	1
$m_{14}$	1	0	1	0	0	0	1	0
$m_{15}$	0	0	0	1	0	0	0	1
$m_{16}$	1	0	0	1	0	0	1	0

TABLE 1. The table form of hypersoft set  $(\mathfrak{F}, \mathcal{G})$

**Definition 8.** A hypersoft set  $(\mathfrak{F}, \mathcal{G})$  is said to be bijective hypersoft set if

$$(i) \bigcup_{\delta \in \mathcal{G}} \mathfrak{F}(\delta) = \mathfrak{U}$$

(ii) For any two  $\delta_i, \delta_j \in \mathcal{G}, \delta_i \neq \delta_j, \mathfrak{F}(\delta_i) \cap \mathfrak{F}(\delta_j) = \emptyset$

Alternatively, a mapping  $\mathfrak{F} : \mathcal{G} \rightarrow \mathcal{P}(\mathcal{U})$  can be transformed to a bijective function  $\mathfrak{F} : \mathcal{G} \rightarrow \mathcal{P}_1$ , where  $\mathcal{P}_1 \subseteq \mathcal{P}(\mathcal{U})$  having disjoint sets  $\mathfrak{F}(e_i)$  as elements. The collection of all bijective hypersoft sets over  $\mathcal{U}$  is denoted by  $\Omega_{BHSS}$ .

**Example 3.2.** Consider Example 3.1, we have bijective hypersoft set

$$(\mathfrak{F}, \mathcal{G}) = \left\{ \begin{array}{l} (\delta_1, \{m_1, m_2\}), (\delta_2, \{m_3, m_{10}\}), (\delta_3, \{m_4, m_{14}\}), (\delta_4, \{m_5, m_{15}\}), \\ (\delta_5, \{m_6, m_{12}\}), (\delta_6, \{m_7, m_9\}), (\delta_7, \{m_8, m_{16}\}), (\delta_8, \{m_{11}, m_{13}\}) \end{array} \right\}$$

and its tabular representation is given in TABLE 2.

....	$\mathfrak{F}(\delta_1)$	$\mathfrak{F}(\delta_2)$	$\mathfrak{F}(\delta_3)$	$\mathfrak{F}(\delta_4)$	$\mathfrak{F}(\delta_5)$	$\mathfrak{F}(\delta_6)$	$\mathfrak{F}(\delta_7)$	$\mathfrak{F}(\delta_8)$
$m_1$	1	0	0	0	0	0	0	0
$m_2$	1	0	0	0	0	0	0	0
$m_3$	0	1	0	0	0	0	0	0
$m_4$	0	0	1	0	0	0	0	0
$m_5$	0	0	0	1	0	0	0	0
$m_6$	0	0	0	0	1	0	0	0
$m_7$	0	0	0	0	0	1	0	0
$m_8$	0	0	0	0	0	0	1	0
$m_9$	0	0	0	0	0	1	0	0
$m_{10}$	0	1	0	0	0	0	0	0
$m_{11}$	0	0	0	0	0	0	0	1
$m_{12}$	0	0	0	0	1	0	0	0
$m_{13}$	0	0	0	0	0	0	0	1
$m_{14}$	0	0	1	0	0	0	0	0
$m_{15}$	0	0	0	1	0	0	0	0
$m_{16}$	0	0	0	0	0	0	1	0

TABLE 2. The table form of bijective hypersoft set  $(\mathfrak{F}, \mathcal{G})$

**Definition 9.** AND operation of two hypersoft sets  $(\mathfrak{F}_1, \mathcal{G}_1)$  and  $(\mathfrak{F}_2, \mathcal{G}_2)$ , denoted by  $(\mathfrak{F}_1, \mathcal{G}_1) \wedge (\mathfrak{F}_2, \mathcal{G}_2)$ , is a hypersoft set  $(\mathfrak{F}_3, \mathcal{G}_3)$  with  $\mathcal{G}_3 = \mathcal{G}_1 \times \mathcal{G}_2$  and for  $\delta \in \mathcal{G}_3$ ,

$$\mathfrak{F}_3(\delta) = \mathfrak{F}_1(\delta) \cap \mathfrak{F}_2(\delta)$$

**Theorem 3.3.** If  $(\mathfrak{F}_1, \mathcal{H}_1), (\mathfrak{F}_2, \mathcal{H}_2) \in \Omega_{BHSS}$  then  $(\mathfrak{F}_1, \mathcal{H}_1) \wedge (\mathfrak{F}_2, \mathcal{H}_2) \in \Omega_{BHSS}$

*Proof.* According to Definition 9, we have

$(\mathfrak{F}_1, \mathcal{H}_1) \wedge (\mathfrak{F}_2, \mathcal{H}_2) = (\mathfrak{F}_3, \mathcal{H}_3)$ , where  $\mathcal{H}_3 = \mathcal{H}_1 \times \mathcal{H}_2$  and  $\mathfrak{F}_3(h_1, h_2) = \mathfrak{F}_1(h_1) \cap \mathfrak{F}_2(h_2), \forall (h_1, h_2) \in \mathcal{H}_3$ . Consider  $\varepsilon \in \mathcal{H}_3$  is a parameter of  $(\mathfrak{F}_3, \mathcal{H}_3)$  then

$$\mathfrak{F}_3(\varepsilon) = \mathfrak{F}_1(h_1) \cap \mathfrak{F}_2(h_2)$$

$$\begin{aligned} \therefore \bigcup_{\varepsilon \in \mathcal{H}_3} \mathfrak{F}_3(\varepsilon) &= \bigcup_{h_1 \in \mathcal{H}_1} \bigcup_{h_2 \in \mathcal{H}_2} \mathfrak{F}_1(h_1) \cap \mathfrak{F}_2(h_2) = \bigcup_{h_1 \in \mathcal{H}_1} \mathfrak{F}_1(h_1) \cap \left( \bigcup_{h_2 \in \mathcal{H}_2} \mathfrak{F}_2(h_2) \right) = \\ &= \bigcup_{h_1 \in \mathcal{H}_1} \mathfrak{F}_1(h_1) \cap \mathcal{U} = \mathcal{U}. \text{ Let } \varepsilon_i, \varepsilon_j \in \mathcal{H}_3, \varepsilon_i \neq \varepsilon_j, \varepsilon_i = \alpha_1 \times \beta_1, \alpha_1 \in \mathcal{H}_1, \beta_1 \in \mathcal{H}_2, \varepsilon_j = \end{aligned}$$

$\alpha_2 \times \beta_2, \alpha_2 \in \mathcal{H}_1, \beta_2 \in \mathcal{H}_2$ . Then

$$\mathfrak{F}_3(\varepsilon_i) \cap \mathfrak{F}_3(\varepsilon_j) = (\mathfrak{F}_1(\alpha_1) \cap \mathfrak{F}_2(\beta_1)) \cap (\mathfrak{F}_1(\alpha_2) \cap \mathfrak{F}_2(\beta_2)) = \emptyset.$$

Hence  $(\mathfrak{F}_3, \mathcal{H}_3) = (\mathfrak{F}_1, \mathcal{H}_1) \wedge (\mathfrak{F}_2, \mathcal{H}_2)$  is a bijective hypersoft set.  $\square$

**Definition 10.** A hypersoft set  $(\mathfrak{F}, \mathcal{G})$  is called a *null hypersoft set*, denoted by  $(\mathfrak{F}, \mathcal{G})_\Phi$ , if  $\mathfrak{F}(\delta) = \emptyset$ , for all  $\delta \in \mathcal{G}$ .

**Definition 11.** Union of two hypersoft sets  $(\mathfrak{F}_1, \mathcal{G}_1)$  and  $(\mathfrak{F}_2, \mathcal{G}_2)$ , denoted by  $(\mathfrak{F}_1, \mathcal{G}_1) \cup (\mathfrak{F}_2, \mathcal{G}_2)$ , is a hypersoft set  $(\mathfrak{F}_3, \mathcal{G}_3)$  with  $\mathcal{G}_3 = \mathcal{G}_1 \cup \mathcal{G}_2$  and for  $\delta \in \mathcal{G}_3$ ,

$$\mathfrak{F}_3(\delta) = \begin{cases} \mathfrak{F}_1(\delta) & \delta \in (\mathcal{G}_1 \setminus \mathcal{G}_2) \\ \mathfrak{F}_2(\delta) & \delta \in (\mathcal{G}_2 \setminus \mathcal{G}_1) \\ \mathfrak{F}_1(\delta) \cup \mathfrak{F}_2(\delta) & \delta \in (\mathcal{G}_1 \cap \mathcal{G}_2) \end{cases}$$

**Theorem 3.4.** If  $(\mathfrak{F}, \mathcal{H}) \in \Omega_{BHSS}$  then  $(\mathfrak{F}, \mathcal{H}) \cup (\mathfrak{F}, \mathcal{H})_\Phi \in \Omega_{BHSS}$ .

*Proof.* Let  $(\mathfrak{F}, \mathcal{H})_\Phi = (\mathfrak{F}_\Phi, \mathcal{H}_1)$ , then from Definition 10 and Definition 11, we have

$$\begin{aligned} (\mathfrak{F}_2, \mathcal{H}_2) &= (\mathfrak{F}, \mathcal{H}) \cup (\mathfrak{F}_\Phi, \mathcal{H}_1) \\ &= \begin{cases} \mathfrak{F}(\epsilon) & ; \epsilon \in \mathcal{H} - \mathcal{H}_1 \\ \mathfrak{F}_\Phi(\epsilon) = \emptyset & ; \epsilon \in \mathcal{H}_1 - \mathcal{H} = (\mathfrak{F}, \mathcal{H} \cup \mathcal{H}_1) \\ \mathfrak{F}(\epsilon) \cup \mathfrak{F}_\Phi(\epsilon) = \mathfrak{F}(\epsilon) \cup \emptyset & ; \epsilon \in \mathfrak{F} \cap \mathfrak{F}_1 \end{cases} \end{aligned}$$

where  $\epsilon \in \mathcal{H}_2$  and  $(\mathfrak{F}, \mathcal{H}_1) \subset (\mathfrak{F}, \mathcal{H} \cup \mathcal{H}_1)$  is a Null hypersoft set, implies

$(\mathfrak{F}_2, \mathcal{H}_2) = (\mathfrak{F}, \mathcal{H} \cup \mathcal{H}_1)$  is a bijective hypersoft set over  $\mathcal{U}$ .  $\square$

**Definition 12.** Let  $\mathcal{U}_1 \subset \mathcal{U}$  and  $(\mathfrak{F}, \mathcal{H}) \in \Omega_{BHSS}$ . The operation of  $(\mathfrak{F}, \mathcal{H})$  restricted AND  $\mathcal{U}_1$ , denoted by  $(\mathfrak{F}, \mathcal{H}) \wedge_{\mathcal{R}} \mathcal{U}_1$ , is defined as

$$\bigcup_{\delta \in \mathcal{H}} \{\mathfrak{F}(\delta) : \mathfrak{F}(\delta) \subseteq \mathcal{U}_1\}.$$

**Example 3.5.** Let  $\mathcal{U} = \{u_1, u_2, u_3, \dots, u_8\}$  and  $\mathcal{U}_1 = \{u_1, u_2, u_3, u_4\}$ . If  $(\mathfrak{F}, \mathcal{H}) \in \Omega_{BHSS}$  with

$$(\mathfrak{F}, \mathcal{H}) = \{ (\delta_1, \{u_1, u_2\}), (\delta_2, \{u_3, u_4\}), (\delta_3, \{u_5, u_6\}), (\delta_4, \{u_7, u_8\}) \}$$

then

$$(\mathfrak{F}, \mathcal{H}) \wedge_{\mathcal{R}} \mathcal{U}_1 = \{u_1, u_2\} \cup \{u_3, u_4\} = \{u_1, u_2, u_3, u_4\}$$

**Definition 13.** Let  $\mathcal{U}_1 \subset \mathcal{U}$  and  $(\mathfrak{F}, \mathcal{H}) \in \Omega_{BHSS}$ . The operation of  $(\mathfrak{F}, \mathcal{H})$  relaxed AND  $\mathcal{U}_1$ , denoted by  $(\mathfrak{F}, \mathcal{H}) \wedge_{\mathcal{Rel}} \mathcal{U}_1$ , is defined as

$$\bigcup_{\delta \in \mathcal{H}} \{\mathfrak{F}(\delta) : \mathfrak{F}(\delta) \cap \mathcal{U}_1 \neq \emptyset\}.$$

**Example 3.6.** Let  $\mathcal{U} = \{u_1, u_2, u_3, \dots, u_8\}$  and  $\mathcal{U}_1 = \{u_1, u_2, u_3, u_4\}$ . If  $(\mathfrak{F}, \mathcal{H}) \in \Omega_{BHSS}$  with

$$(\mathfrak{F}, \mathcal{H}) = \{ (\delta_1, \{u_1, u_8\}), (\delta_2, \{u_3, u_7\}), (\delta_3, \{u_2, u_6\}), (\delta_4, \{u_4, u_5\}) \}$$

then

$$(\mathfrak{F}, \mathcal{H}) \bigwedge_{\mathcal{R}el} \mathfrak{U}_1 = \{u_1, u_8\} \cup \{u_3, u_7\} \cup \{u_2, u_6\} \cup \{u_4, u_5\} = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\} = \mathfrak{U}$$

**Definition 14.** If  $(\mathfrak{F}, \mathcal{H}) \in \Omega_{BHSS}$  then its boundary region with respect to  $\mathfrak{U}_1 \subset \mathfrak{U}$ , denoted by  $(\mathfrak{F}, \mathcal{H})_\bullet$ , is defined as

$$(\mathfrak{F}, \mathcal{H})_\bullet = \left( (\mathfrak{F}, \mathcal{H}) \bigwedge_{\mathcal{R}el} \mathfrak{U}_1 \right) \setminus \left( (\mathfrak{F}, \mathcal{H}) \bigwedge_{\mathcal{R}} \mathfrak{U}_1 \right)$$

**Example 3.7.** Consider  $\mathfrak{U}$  and  $\mathfrak{U}_1$  as in Example 3.6, we have

$$(\mathfrak{F}, \mathcal{H}) = \{ (\delta_1, \{u_1\}), (\delta_2, \{u_3, u_4\}), (\delta_3, \{u_2, u_6\}), (\delta_4, \{u_7, u_5\}) \}$$

. Now

$$(\mathfrak{F}, \mathcal{H}) \bigwedge_{\mathcal{R}el} \mathfrak{U}_1 = \{u_1\} \cup \{u_3, u_4\} \cup \{u_2, u_6\} = \{u_1, u_2, u_3, u_4, u_6\}$$

and

$$(\mathfrak{F}, \mathcal{H}) \bigwedge_{\mathcal{R}} \mathfrak{U}_1 = \{u_1\} \cup \{u_3, u_4\} = \{u_1, u_3, u_4\}$$

therefore

$$(\mathfrak{F}, \mathcal{H})_\bullet = \{u_2, u_6\}$$

**Definition 15.** If  $(\mathfrak{F}_1, \mathcal{H}_1), (\mathfrak{F}_2, \mathcal{H}_2) \in \Omega_{BHSS}$  with  $\mathcal{H}_1 \cap \mathcal{H}_2 = \emptyset$  then  $(\mathfrak{F}_1, \mathcal{H}_1)$  is said to depend on  $(\mathfrak{F}_2, \mathcal{H}_2)$  to a degree  $\kappa \in [0, 1]$ , denoted by  $(\mathfrak{F}_1, \mathcal{H}_1) \xrightarrow[\kappa]{} (\mathfrak{F}_2, \mathcal{H}_2)$ , if

$$\kappa = \Gamma((\mathfrak{F}_1, \mathcal{H}_1), (\mathfrak{F}_2, \mathcal{H}_2)) = \frac{\left| \bigcup_{\delta \in \mathcal{H}_2} \{(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge_{\mathcal{R}} \mathfrak{F}_2(\delta)\} \right|}{|\mathfrak{U}|}$$

where  $|\cdot|$  = cardinality of a set.

Note:

- (i) If  $\kappa = 1$  then  $(\mathfrak{F}_1, \mathcal{H}_1)$  is full depended on  $(\mathfrak{F}_2, \mathcal{H}_2)$ .
- (ii) If  $\kappa = 0$  then  $(\mathfrak{F}_1, \mathcal{H}_1)$  is not depended on  $(\mathfrak{F}_2, \mathcal{H}_2)$ .

**Example 3.8.** Consider  $\mathfrak{U}$  as in Example 3.6, we have

$$(\mathfrak{F}_1, \mathcal{H}_1) = \left\{ (\delta_1, \{u_1\}), (\delta_2, \{u_3\}), (\delta_3, \{u_6\}), (\delta_4, \{u_7\}), \right. \\ \left. (\delta_5, \{u_5\}), (\delta_6, \{u_8\}), (\delta_7, \{u_2\}), (\delta_8, \{u_4\}) \right\}$$

and

$$(\mathfrak{F}_2, \mathcal{H}_2) = \{ (\delta_9, \{u_1, u_2\}), (\delta_{10}, \{u_3, u_4\}), (\delta_{11}, \{u_5\}), (\delta_{12}, \{u_7, u_8\}) \}$$

Now

$$(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge_{\mathcal{R}} \mathfrak{F}_2(\delta_9) = \{u_1\} \cup \{u_2\} = \{u_1, u_2\}$$

$$(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge_{\mathcal{R}} \mathfrak{F}_2(\delta_{10}) = \{u_3\} \cup \{u_4\} = \{u_3, u_4\}$$

$$(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge_{\mathcal{R}} \mathfrak{F}_2(\delta_{11}) = \{u_5\}$$

$$(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge_{\mathcal{R}} \mathfrak{F}_2(\delta_{12}) = \{u_7\} \cup \{u_8\} = \{u_7, u_8\}$$

therefore

$$\bigcup_{\delta \in \mathcal{H}_2} \{(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge_{\mathcal{R}} \mathfrak{F}_2(\delta)\} = \{u_1, u_2, u_3, u_4, u_5, u_7, u_8\}$$

with

$$\kappa = \frac{7}{8} = 0.875$$

**Definition 16.** Let  $(\mathfrak{F}, \mathcal{H}), (\mathcal{L}, \mathcal{G}) \in \Omega_{BHSS}$ . The triple  $((\mathfrak{F}, \mathcal{H}), (\mathcal{L}, \mathcal{G}), \mathfrak{U})$  is said to form a bijective hypersoft decision system over  $\mathfrak{U}$ , denoted by  $\mathbb{D}_{BH}$ , if

- (i) there exists a condition hypersoft set  $(\mathfrak{F}, \mathcal{H}) = \bigcup_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i) \in \Omega_{BHSS}$  with  $\mathcal{H}_i \cap \mathcal{H}_j = \emptyset, i \neq j$
- (ii) there exists a decision hypersoft set  $(\mathcal{L}, \mathcal{G})$  for which  $\mathcal{G} \cap \mathcal{H}_i = \emptyset$ .

**Example 3.9.** Consider  $\mathfrak{U}$  as in Example 3.6, we have

$$(\mathfrak{F}_1, \mathcal{H}_1) = \{ (\delta_1, \{u_1\}), (\delta_2, \{u_2\}), (\delta_3, \{u_3\}) \}$$

$$(\mathfrak{F}_2, \mathcal{H}_2) = \{ (\delta_4, \{u_1, u_2\}), (\delta_5, \{u_3, u_4\}), (\delta_6, \{u_5, u_6\}) \}$$

$$(\mathfrak{F}_3, \mathcal{H}_3) = \{ (\delta_7, \{u_1, u_2, u_3\}), (\delta_8, \{u_4, u_5, u_6\}), (\delta_9, \{u_7, u_8\}) \}$$

and

$$(\mathcal{L}, \mathcal{G}) = \{ (\delta_{10}, \{u_1, u_3, u_5\}), (\delta_{11}, \{u_2, u_4, u_8\}), (\delta_{12}, \{u_6, u_7\}) \}$$

therefore

$$\mathbb{D}_{BH} = \left( \bigcup_{i=1}^3 (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}), \mathfrak{U} \right)$$

**Definition 17.** The bijective hypersoft dependency between  $(\mathfrak{F}_1, \mathcal{H}_1) \wedge (\mathfrak{F}_2, \mathcal{H}_2) \wedge \dots \wedge (\mathfrak{F}_n, \mathcal{H}_n)$  and  $(\mathcal{L}, \mathcal{G})$  is called bijective hypersoft decision system dependency of  $\mathbb{D}_{BH}$  and defined by  $\kappa = \Gamma \left( \bigwedge_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}) \right)$ .

**Example 3.10.** Considering the  $\mathfrak{U}$  from Example 3.6, Let we have

$$(\mathfrak{F}_1, \mathcal{H}_1) = \{ (\delta_1, \{u_1, u_4, u_6\}), (\delta_2, \{u_2, u_5, u_7\}) \}$$

$$(\mathfrak{F}_2, \mathcal{H}_2) = \{ (\delta_3, \{u_1, u_5, u_6\}), (\delta_4, \{u_4, u_7, u_8\}) \}$$

$$(\mathfrak{F}_3, \mathcal{H}_3) = \{ (\delta_5, \{u_1, u_2, u_4\}), (\delta_6, \{u_3, u_5, u_6\}), (\delta_7, \{u_7, u_8\}) \}$$

$$(\mathcal{L}, \mathcal{G}) = \{ (\delta_8, \{u_1, u_4, u_5, u_6\}), (\delta_9, \{u_2, u_3, u_7, u_8\}) \}$$



then

$$(\mathfrak{F}, \mathcal{H}) = (\mathfrak{F}_1, \mathcal{H}_1) \wedge (\mathfrak{F}_2, \mathcal{H}_2) \wedge (\mathfrak{F}_3, \mathcal{H}_3) = \left\{ \begin{array}{l} (\varepsilon_1 = (\delta_1, \delta_3, \delta_5), \{u_1\}), (\varepsilon_2 = (\delta_1, \delta_3, \delta_6), \{u_6\}), (\varepsilon_3 = (\delta_1, \delta_3, \delta_7), \emptyset) \\ (\varepsilon_4 = (\delta_1, \delta_4, \delta_5), \{u_4\}), (\varepsilon_5 = (\delta_1, \delta_4, \delta_6), \emptyset), (\varepsilon_6 = (\delta_1, \delta_4, \delta_7), \emptyset) \\ (\varepsilon_7 = (\delta_2, \delta_3, \delta_5), \emptyset), (\varepsilon_8 = (\delta_2, \delta_3, \delta_6), \{u_5\}), (\varepsilon_9 = (\delta_2, \delta_3, \delta_7), \emptyset) \\ (\varepsilon_{10} = (\delta_2, \delta_4, \delta_5), \emptyset), (\varepsilon_{11} = (\delta_2, \delta_4, \delta_6), \emptyset), (\varepsilon_{12} = (\delta_2, \delta_4, \delta_7), \{u_7\}) \end{array} \right\}.$$

The tabular representation of  $(\mathfrak{F}_1, \mathcal{H}_1) \wedge (\mathfrak{F}_2, \mathcal{H}_2) \wedge (\mathfrak{F}_3, \mathcal{H}_3)$  is depicted in TABLE 3.

Now

$$\left( \bigwedge_{i=1}^3 (\mathfrak{F}_i, \mathcal{H}_i) \wedge_{\mathcal{R}} (\mathcal{L}, \mathcal{G}) \right) = \{u_1, u_4, u_5, u_6, u_7\}$$

therefore

$$\kappa = \Gamma \left( \bigwedge_{i=1}^3 (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}) \right) = \frac{5}{8} = 0.625$$

....	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	$u_8$
$\mathfrak{F}(\varepsilon_1)$	1	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_2)$	0	0	0	0	0	1	0	0
$\mathfrak{F}(\varepsilon_3)$	0	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_4)$	0	0	0	1	0	0	0	0
$\mathfrak{F}(\varepsilon_5)$	0	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_6)$	0	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_7)$	0	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_8)$	0	0	0	0	1	0	0	0
$\mathfrak{F}(\varepsilon_9)$	0	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_{10})$	0	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_{11})$	0	0	0	0	0	0	0	0
$\mathfrak{F}(\varepsilon_{12})$	0	0	0	0	0	0	1	0

TABLE 3. The table form of  $(\mathfrak{F}_1, \mathcal{H}_1) \wedge (\mathfrak{F}_2, \mathcal{H}_2) \wedge (\mathfrak{F}_3, \mathcal{H}_3)$

**Theorem 3.11.** Let  $\mathbb{D}_{BH} = ((\mathfrak{F}, \mathcal{H}), (\mathcal{L}, \mathcal{G}), \mathfrak{U})$ , where  $(\mathfrak{F}, \mathcal{H}) = \bigcup_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i)$  and  $(\mathfrak{F}_i, \mathcal{H}_i) \in \Omega_{BHSS}$ . If  $\kappa = \gamma \left( \bigwedge_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}) \right)$  and  $\kappa_1 = \gamma \left( \bigwedge_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}) \right)$  with  $m \leq n$  then  $\kappa_1 \leq \kappa$ .

*Proof.* Suppose that  $(\mathfrak{P}, \mathcal{C}) = \bigwedge_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i)$ ,  $(\mathfrak{J}, \mathcal{K}) = \bigwedge_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i)$  then we have

$$\kappa = \gamma \left( \bigwedge_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}) \right) = \frac{\left| \bigcup_{\varepsilon \in \mathcal{G}} (\mathfrak{P}, \mathcal{C}) \wedge_{\mathcal{R}} \mathcal{L}(\varepsilon) \right|}{|\mathfrak{U}|} = \frac{\left| \bigcup_{\varepsilon \in \mathcal{G}} \bigcup_{\alpha \in \mathcal{C}} \{\mathfrak{P}(\alpha) : \mathfrak{P}(\alpha) \subseteq \mathcal{L}(\varepsilon)\} \right|}{|\mathfrak{U}|}$$

$$\kappa_1 = \gamma \left( \bigwedge_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}) \right) = \frac{\left| \bigcup_{\varepsilon \in \mathcal{G}} (\mathfrak{J}, \mathcal{K}) \wedge_{\mathcal{R}} \mathcal{L}(\varepsilon) \right|}{|\mathfrak{U}|} = \frac{\left| \bigcup_{\varepsilon \in \mathcal{G}} \bigcup_{\alpha \in \mathcal{K}} \{\mathfrak{J}(\alpha) : \mathfrak{J}(\alpha) \subseteq \mathcal{L}(\varepsilon)\} \right|}{|\mathfrak{U}|}.$$

From Definition 2.6,

$$\mathfrak{P}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) = \mathfrak{F}_1(\varepsilon_1) \cap \mathfrak{F}_2(\varepsilon_2) \cap \dots \cap \mathfrak{F}_m(\varepsilon_m) \cap \dots \cap \mathfrak{F}_n(\varepsilon_n), \forall (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) \in \mathcal{H}_1 \times \mathcal{H}_2 \times \dots \times \mathcal{H}_n$$

$$\mathfrak{J}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m) = \mathfrak{F}_1(\varepsilon_1) \cap \mathfrak{F}_2(\varepsilon_2) \cap \dots \cap \mathfrak{F}_m(\varepsilon_m), \forall (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m) \in \mathcal{H}_1 \times \mathcal{H}_2 \times \dots \times \mathcal{H}_m$$

$n > m$

$$\mathfrak{P}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) \supseteq \mathfrak{J}(\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m)$$

and

$$\bigcup_{\varepsilon \in \mathcal{C}} \mathfrak{P}(\varepsilon) = \mathfrak{U}, \quad \bigcup_{\varepsilon \in \mathcal{K}} \mathfrak{J}(\varepsilon) = \mathfrak{U}.$$

Therefore,

$$\left| \bigcup_{\varepsilon \in \mathcal{C}} \{ \mathfrak{P}(\varepsilon) : \mathfrak{P}(\varepsilon) \subseteq \mathcal{G}(\varepsilon) \} \right| \geq \left| \bigcup_{\varepsilon \in \mathcal{K}} \{ \mathfrak{J}(\varepsilon) : \mathfrak{J}(\varepsilon) \subseteq \mathcal{G}(\varepsilon) \} \right|.$$

$$\gamma \left( \bigwedge_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i), (\mathfrak{L}, \mathcal{G}) \right) \leq \kappa.$$

□

**Definition 18.** Let  $\mathbb{D}_{BH} = ((\mathfrak{F}, \mathcal{H}), (\mathfrak{L}, \mathcal{G}), \mathfrak{U})$ , where  $(\mathfrak{F}, \mathcal{H}) = \bigcup_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i)$  and  $\bigcup_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i) \subset (\mathfrak{F}, \mathcal{H})$ . If  $\gamma \left( \bigwedge_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i), (\mathfrak{L}, \mathcal{G}) \right) = \gamma \left( \bigwedge_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i), (\mathfrak{L}, \mathcal{G}) \right) = \kappa$  then  $\bigcup_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i)$  is called a reduct of  $\mathbb{D}_{BH}$ .

**Example 3.12.** Considering the  $\mathfrak{U}$  from Example 3.6 and sets from Example 3.10, let we have

$$(\mathfrak{F}_1, \mathcal{H}_1) = \{ (\delta_1, \{u_1, u_4, u_6\}), (\delta_2, \{u_2, u_5, u_7\}) \}$$

$$(\mathfrak{F}_2, \mathcal{H}_2) = \{ (\delta_3, \{u_1, u_5, u_6\}), (\delta_4, \{u_4, u_7, u_8\}) \}$$

$$(\mathfrak{F}_3, \mathcal{H}_3) = \{ (\delta_5, \{u_1, u_2, u_4\}), (\delta_6, \{u_3, u_5, u_6\}), (\delta_7, \{u_7, u_8\}) \}$$

$$(\mathfrak{L}, \mathcal{G}) = \{ (\delta_8, \{u_1, u_4, u_5, u_6\}), (\delta_9, \{u_2, u_3, u_7, u_8\}) \}$$

then

$$(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge (\mathfrak{F}_2, \mathcal{H}_2) = \left\{ \begin{array}{l} (\varepsilon_1 = (\delta_1, \delta_3), \{u_1, u_6\}), (\varepsilon_2 = (\delta_1, \delta_4), \{u_4\}), \\ (\varepsilon_3 = (\delta_2, \delta_3), \{u_5\}), (\varepsilon_4 = (\delta_2, \delta_4), \{u_7\}) \end{array} \right\}.$$

Now

$$\left( \bigwedge_{i=1}^2 (\mathfrak{F}_i, \mathcal{H}_i) \bigwedge_{\mathcal{R}} (\mathfrak{L}, \mathcal{G}) \right) = \{u_1, u_4, u_5, u_6, u_7\}$$

therefore

$$\kappa = \Gamma \left( \bigwedge_{i=1}^2 (\mathfrak{F}_i, \mathcal{H}_i), (\mathfrak{L}, \mathcal{G}) \right) = \frac{5}{8} = 0.625 \text{ which is same as of } \Gamma \left( \bigwedge_{i=1}^3 (\mathfrak{F}_i, \mathcal{H}_i), (\mathfrak{L}, \mathcal{G}) \right)$$

calculated in Example 3.10. Hence  $(\mathfrak{F}_1, \mathcal{H}_1) \cup (\mathfrak{F}_2, \mathcal{H}_2)$  is a reduct of  $\mathbb{D}_{BH}$ .

**Definition 19.** Let  $\mathbb{D}_{BH} = (\bigcup_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}), \mathfrak{U})$ . The significance of BHSS to decision hypersoft set, denoted  $\Delta((\mathfrak{F}_j, \mathcal{H}_j), \bigcup_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G}))$ , is defined as

$$\Delta((\mathfrak{F}_j, \mathcal{H}_j), \bigcup_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G})) = \kappa - \Gamma((\mathfrak{P}, \mathcal{C}), (\mathcal{L}, \mathcal{G})),$$

where  $(\mathfrak{P}, \mathcal{C}) = \bigwedge_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i) (i \neq j)$ .

**Example 3.13.** Since from Example 3.10, we have

$$\kappa = \Gamma\left(\bigwedge_{i=1}^3 (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G})\right) = \frac{5}{8} = 0.625 \text{ and}$$

$$(\mathfrak{F}_2, \mathcal{H}_2) \bigwedge (\mathfrak{F}_3, \mathcal{H}_3) = \left\{ \begin{array}{l} (\varepsilon_1 = (\delta_3, \delta_5), \{u_1\}), (\varepsilon_2 = (\delta_3, \delta_6), \{u_5, u_6\}), (\varepsilon_3 = (\delta_3, \delta_7), \{\}) \\ (\varepsilon_4 = (\delta_4, \delta_5), \{u_4\}), (\varepsilon_5 = (\delta_4, \delta_6), \{\}), (\varepsilon_6 = (\delta_4, \delta_7), \{u_7, u_8\}) \end{array} \right\}.$$

Now

$$\left(\bigwedge_{i=2}^3 (\mathfrak{F}_i, \mathcal{H}_i) \bigwedge_{\mathcal{R}} (\mathcal{L}, \mathcal{G})\right) = \{u_1, u_4, u_5, u_6, u_7, u_8\}$$

therefore

$$\Gamma\left(\bigwedge_{i=2}^3 (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G})\right) = \frac{6}{8} = 0.75 \text{ hence}$$

$$\Delta((\mathfrak{F}_1, \mathcal{H}_1), \bigcup_{i=1}^3 (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G})) = \kappa - \Gamma\left(\bigwedge_{i=2}^3 (\mathfrak{F}_i, \mathcal{H}_i), (\mathcal{L}, \mathcal{G})\right) = 0.625 - 0.75 = -0.125$$

**Definition 20.** A BHSS  $(\mathfrak{P}, \mathcal{C})$  is said to be a core BHSS of  $\mathbb{D}_{BH}$  if it belongs to every reduct of  $\mathbb{D}_{BH}$ .

**Definition 21.** Let  $\mathbb{D}_{BH} = ((\mathfrak{F}, \mathcal{H}), (\mathcal{L}, \mathcal{G}), \mathfrak{U})$ , where  $(\mathfrak{F}, \mathcal{H}) = \bigcup_{i=1}^n (\mathfrak{F}_i, \mathcal{H}_i)$  and  $\bigcup_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i) \subset (\mathfrak{F}, \mathcal{H})$  is a reduct of  $\mathbb{D}_{BH}$ . Let  $(\mathfrak{P}, \mathcal{C}) = \bigwedge_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i)$ . We say

$$\text{if } e_i \text{ then } e_j \left( \frac{|\mathfrak{P}(e_i)|}{|\mathcal{L}(e_j)|} \right)$$

a decision rule induced by  $\bigcup_{i=1}^m (\mathfrak{F}_i, \mathcal{H}_i)$  where  $e_i \in \mathcal{C}$ ,  $\mathcal{L}(e_j) \supseteq \mathfrak{P}(e_i)$ ,  $e_j \in \mathcal{G}$  and  $\frac{|\mathfrak{P}(e_i)|}{|\mathcal{L}(e_j)|}$  denotes the coverage proportion of rule.

**Example 3.14.** Since from Example 3.12, we have

$(\mathfrak{F}_1, \mathcal{H}_1) \cup (\mathfrak{F}_2, \mathcal{H}_2)$  is a reduct of  $\mathbb{D}_{BH}$  and

$$(\mathfrak{F}_1, \mathcal{H}_1) \bigwedge (\mathfrak{F}_2, \mathcal{H}_2) = \left\{ \begin{array}{l} (\varepsilon_1 = (\delta_1, \delta_3), \{u_1, u_6\}), (\varepsilon_2 = (\delta_1, \delta_4), \{u_4\}), \\ (\varepsilon_3 = (\delta_2, \delta_3), \{u_5\}), (\varepsilon_4 = (\delta_2, \delta_4), \{u_7\}) \end{array} \right\}.$$

Now

- (i) If  $\varepsilon_1$  then  $\delta_8(2/4)$
- (ii) If  $\varepsilon_2$  then  $\delta_8(1/4)$
- (iii) If  $\varepsilon_3$  then  $\delta_8(1/4)$
- (iv) If  $\varepsilon_4$  then  $\delta_9(1/4)$

## 4. AN APPLICATION OF BIJECTIVE HYPERSOFT SET

In this section, we present an application of bijective hypersoft set to establish decision rules which further help to have right decision regarding best selection of certain material/product. Suppose we have 16 kinds of laptops that form the universe of discourse  $\mathfrak{U} = \{l_1, l_2, \dots, l_{16}\}$ . The most feasible laptop is expected to be selected by considering the attributes  $a_1 = \text{Size}$ ,  $a_2 = \text{Colour}$ ,  $a_3 = \text{Processor}$ ,  $a_4 = \text{RAM}$  and  $a_5 = \text{Price}$ . The attribute-valued sets corresponding to these attributes are:

$$\mathcal{A} = \{\text{small} = \alpha_1, \text{medium} = \alpha_2, \text{large} = \alpha_3\}$$

$$\mathcal{B} = \{\text{Silver} = \beta_1, \text{Black} = \beta_2\}$$

$$\mathcal{C} = \{2.60 \text{ GHz} = \gamma_1\}$$

$$\mathcal{D} = \{4.0 \text{ GB} = \delta_1, 5.0 \text{ GB} = \delta_2\}$$

$$\mathcal{E} = \{\text{normal} = \varepsilon_1\}.$$

Now we construct hypersoft set  $(\Psi, \mathbb{J})$  where

$$\mathbb{J} = \mathcal{A} \times \mathcal{B} \times \mathcal{C} \times \mathcal{D} \times \mathcal{E}$$

$$\mathbb{J} = \left\{ \begin{array}{l} j_1 = (\alpha_1, \beta_1, \gamma_1, \delta_1, \varepsilon_1), j_2 = (\alpha_1, \beta_1, \gamma_1, \delta_2, \varepsilon_1), j_3 = (\alpha_1, \beta_2, \gamma_1, \delta_1, \varepsilon_1), \\ j_4 = (\alpha_1, \beta_2, \gamma_1, \delta_2, \varepsilon_1), j_5 = (\alpha_2, \beta_1, \gamma_1, \delta_1, \varepsilon_1), j_6 = (\alpha_2, \beta_1, \gamma_1, \delta_2, \varepsilon_1), \\ j_7 = (\alpha_2, \beta_2, \gamma_1, \delta_1, \varepsilon_1), j_8 = (\alpha_2, \beta_2, \gamma_1, \delta_2, \varepsilon_1), j_9 = (\alpha_3, \beta_1, \gamma_1, \delta_1, \varepsilon_1), \\ j_{10} = (\alpha_3, \beta_1, \gamma_1, \delta_2, \varepsilon_1), j_{11} = (\alpha_3, \beta_2, \gamma_1, \delta_1, \varepsilon_1), j_{12} = (\alpha_3, \beta_2, \gamma_1, \delta_2, \varepsilon_1) \end{array} \right\}$$

and

$$(\Psi, \mathbb{J}) = \left\{ \begin{array}{l} (j_1, \{l_1, l_2, l_9, l_{14}, l_{16}\}), (j_2, \{l_1, l_2, l_3, l_{10}, l_{12}\}), (j_3, \{l_2, l_3, l_4, l_9, l_{13}, l_{14}\}), \\ (j_4, \{l_4, l_5, l_6, l_{10}, l_{15}, l_{16}\}), (j_5, \{l_6, l_7, l_8, l_{10}, l_{12}\}), (j_6, \{l_2, l_3, l_4, l_7, l_9\}), \\ (j_7, \{l_1, l_3, l_5, l_6, l_8, l_{10}, l_{12}, l_{14}, l_{16}\}), (j_8, \{l_2, l_3, l_6, l_7, l_9, l_{11}, l_{13}, l_{15}\}), \\ (j_9, \{l_5, l_7, l_8, l_9, l_{11}, l_{13}, l_{16}\}), (j_{10}, \{l_2, l_4, l_6, l_8, l_{10}, l_{12}, l_{14}\}), \\ (j_{11}, \{l_3, l_5, l_7, l_9, l_{11}\}), (j_{12}, \{l_1, l_4, l_7, l_{10}, l_{13}, l_{16}\}) \end{array} \right\}.$$

Now we propose an algorithm of bijective hypersoft sets to establish decision rules.

**ALGORITHM**

Step 1 Construct  $\mathbb{D}_{BH} = (\bigcup_{i=1}^n (\mathcal{W}_i, \mathcal{V}_i), (\mathcal{L}, \mathcal{J}), \mathfrak{U})$

Step 2 Calculate  $\kappa_i = \Gamma((\mathcal{W}_i, \mathcal{V}_i), (\mathcal{L}, \mathcal{J}))$  for  $i = 1, 2, \dots, n$

Step 3 Calculate  $\kappa = \Gamma\left(\bigwedge_{j=1}^n (\mathcal{W}_j, \mathcal{V}_j), (\mathcal{L}, \mathcal{J})\right)$

Step 4 Find reduct bijective hypersoft sets w.r.t.  $\mathbb{D}_{BH}$

Step 5 Determine decision rules

[descriptive diagram] Step 1, Construct  $\mathbb{D}_{BH} = (\bigcup_{i=1}^n (\mathcal{W}_i, \mathcal{V}_i), (\mathcal{L}, \mathcal{J}), \mathfrak{U})$ , Step 2,

Calculate  $\kappa_i = \Gamma((\mathcal{W}_i, \mathcal{V}_i), (\mathcal{L}, \mathcal{J}))$ , Step 3, Calculate  $\kappa = \Gamma\left(\bigwedge_{j=1}^n (\mathcal{W}_j, \mathcal{V}_j), (\mathcal{L}, \mathcal{J})\right)$ ,

Step 4, Find reduct bijective hypersoft sets w.r.t.  $\mathbb{D}_{BH}$ , Step 5, Determine decision rules

**Step 1**

Let we have bijective hypersoft sets

$$(\mathcal{W}_1, \mathcal{V}_1) = \{ (j_1, \{l_1, l_4, l_6\}), (j_2, \{l_2, l_5, l_7\}) \}$$

$$\begin{aligned}(\mathcal{W}_2, \mathcal{V}_2) &= \{ (j_3, \{l_1, l_5, l_6\}), (j_4, \{l_4, l_7, l_8\}) \} \\(\mathcal{W}_3, \mathcal{V}_3) &= \{ (j_5, \{l_1, l_2, l_4\}), (j_6, \{l_3, l_5, l_6\}), (j_7, \{l_7, l_8\}) \} \\(\mathcal{L}, \mathcal{J}) &= \{ (j_8, \{l_1, l_4, l_5, l_6\}), (j_9, \{l_2, l_3, l_7, l_8\}) \}\end{aligned}$$

which form  $\mathbb{D}_{BH} = (\bigcup_{i=1}^3 (\mathcal{W}_i, \mathcal{V}_i), (\mathcal{L}, \mathcal{J}), \mathfrak{U})$  and  $\mathcal{V}_i, \mathcal{J} \subseteq \mathbb{J}$ .

### Step 2

Since

$$(\mathcal{W}_1, \mathcal{V}_1) \wedge (\mathcal{W}_2, \mathcal{V}_2) = \left\{ \begin{array}{l} (\theta_1 = (j_1, j_3), \{l_1, l_6\}), (\theta_2 = (j_1, j_4), \{l_4\}), \\ (\theta_3 = (j_2, j_3), \{l_5\}), (\theta_4 = (j_2, j_4), \{l_7\}) \end{array} \right\}$$

and

$$(\mathcal{W}_2, \mathcal{V}_2) \wedge (\mathcal{W}_3, \mathcal{V}_3) = \left\{ \begin{array}{l} (\theta_5 = (j_3, j_5), \{l_1\}), (\theta_6 = (j_3, j_6), \{l_5, l_6\}), (\theta_7 = (j_3, j_7), \emptyset) \\ (\theta_8 = (j_4, j_5), \{l_4\}), (\theta_9 = (j_4, j_6), \emptyset), (\theta_{10} = (j_4, j_7), \{l_7, l_8\}) \end{array} \right\}$$

and

$$(\mathcal{W}_1, \mathcal{V}_1) \wedge (\mathcal{W}_3, \mathcal{V}_3) = \left\{ \begin{array}{l} (\theta_{11} = (j_1, j_5), \{l_1, l_4\}), (\theta_{12} = (j_1, j_6), \{l_6\}), (\theta_{13} = (j_1, j_7), \emptyset) \\ (\theta_{14} = (j_2, j_5), \{l_2\}), (\theta_{15} = (j_2, j_6), \{l_5\}), (\theta_{16} = (j_2, j_7), \{l_7\}) \end{array} \right\}$$

Now

$$\kappa_1 = \Gamma((\mathcal{W}_1, \mathcal{V}_1), (\mathcal{L}, \mathcal{J})) = \frac{3}{16} = 0.1875$$

$$\kappa_2 = \Gamma((\mathcal{W}_2, \mathcal{V}_2), (\mathcal{L}, \mathcal{J})) = \frac{3}{16} = 0.1875$$

$$\kappa_3 = \Gamma((\mathcal{W}_3, \mathcal{V}_3), (\mathcal{L}, \mathcal{J})) = \frac{2}{16} = 0.125$$

$$\kappa_4 = \Gamma((\mathcal{W}_1, \mathcal{V}_1) \wedge (\mathcal{W}_2, \mathcal{V}_2), (\mathcal{L}, \mathcal{J})) = \frac{5}{16} = 0.3125$$

$$\kappa_5 = \Gamma((\mathcal{W}_2, \mathcal{V}_2) \wedge (\mathcal{W}_3, \mathcal{V}_3), (\mathcal{L}, \mathcal{J})) = \frac{6}{16} = 0.375$$

$$\kappa_6 = \Gamma((\mathcal{W}_1, \mathcal{V}_1) \wedge (\mathcal{W}_3, \mathcal{V}_3), (\mathcal{L}, \mathcal{J})) = \frac{6}{16} = 0.375$$

### Step 3

$$(\mathcal{W}_1, \mathcal{V}_1) \wedge (\mathcal{W}_2, \mathcal{V}_2) \wedge (\mathcal{W}_3, \mathcal{V}_3) =$$

$$\left\{ \begin{array}{l} (\eta_1 = (j_1, j_3, j_5), \{l_1\}), (\eta_2 = (j_1, j_3, j_6), \{l_6\}), (\eta_3 = (j_1, j_3, j_7), \emptyset) \\ (\eta_4 = (j_1, j_4, j_5), \{l_4\}), (\eta_5 = (j_1, j_4, j_6), \emptyset), (\eta_6 = (j_1, j_4, j_7), \emptyset) \\ (\eta_7 = (j_2, j_3, j_5), \emptyset), (\eta_8 = (j_2, j_3, j_6), \{l_5\}), (\eta_9 = (j_2, j_3, j_7), \emptyset) \\ (\eta_{10} = (j_2, j_4, j_5), \emptyset), (\eta_{11} = (j_2, j_4, j_6), \emptyset), (\eta_{12} = (j_2, j_4, j_7), \{l_7\}) \end{array} \right\}.$$

therefore

$$\kappa = \Gamma\left(\bigwedge_{i=1}^3 (\mathcal{W}_i, \mathcal{V}_i), (\mathcal{L}, \mathcal{J})\right) = \frac{5}{16} = 0.3125$$

### Step 4

As

$$\Gamma\left((\mathcal{W}_1, \mathcal{V}_1) \wedge (\mathcal{W}_2, \mathcal{V}_2), (\mathcal{L}, \mathcal{J})\right) = 0.3125 = \Gamma\left(\bigwedge_{i=1}^3 (\mathcal{W}_i, \mathcal{V}_i), (\mathcal{L}, \mathcal{J})\right)$$

therefore  $(\mathcal{W}_1, \mathcal{V}_1) \cup (\mathcal{W}_2, \mathcal{V}_2)$  is a reduct of  $\mathbb{D}_{BH}$ .

### Step 5

Since  $(\mathcal{W}_1, \mathcal{V}_1) \cup (\mathcal{W}_2, \mathcal{V}_2)$  is a reduct of  $\mathbb{D}_{BH}$  therefore we have the following decision rules w.r.t.  $\mathbb{D}_{BH}$

- (i) If  $\theta_1$  then  $j_8(2/4)$
- (ii) If  $\theta_2$  then  $j_8(1/4)$

(iii) If  $\theta_3$  then  $j_8(1/4)$

(iv) If  $\theta_4$  then  $j_9(1/4)$

Hence we have two elements  $j_8$  and  $j_9$  in our decision set and  $j_8$  is more preferable than  $j_9$  in this set for further evaluation.

**4.1. Comparative study.** In this subsection, we compare our propose structure with the existing studies.

Authors	Structures	Remarks
H. Kamacı et al. [9]	Bijjective soft matrix theory	<ul style="list-style-type: none"> <li>• Single set of attribute is employed to develop decision system via bijjection on matrix theory</li> <li>• Multi-bijjective linguistic soft decision system is established</li> </ul>
Gong et al. [7]	The bijjective soft set	<ul style="list-style-type: none"> <li>• Single set of attribute is employed to develop decision system</li> </ul>
Proposed structure	Bijjective hypersoft set	<ul style="list-style-type: none"> <li>• Attributes are further classified into disjoint attribute-valued sets</li> <li>• Decision system is developed via employment of multi-argument approximate functions.</li> </ul>

## 5. CONCLUSION

In this study, the concept of bijjective hypersoft set is developed and its some set aggregation operations such as restricted-AND and relaxed-AND, are discussed. Decision making algorithm and its application are discussed with the support of new proposed operations of bijjective hypersoft set like dependency, decision system, significance of decision system, reduced decision system and decision rules in decision system. In order to adequate the existing soft-like structures for multi-argument functions, future work may include the hybridized study of proposed work with the following structures:

- Fuzzy Set
- Intuitionistic Fuzzy Set
- Interval-Valued Fuzzy Set
- Pythagorean Fuzzy Set
- q-rung Orthopair Fuzzy Set
- Rough Set

- Complex Set
- Neutrosophic Set etc.

and their applications in decision making.

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