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Magnetohydrodynamic Flow and Heat Transfer for A Peristaltic Motion of Carreau Fluid Through A Porous Medium

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Abstract. The two-dimensional peristaltic motion of magnetohydrodynamic flow and heat transfer for incompressible non-Newtonian fluid through a porous medium in uniform channel with a sinusoidal wave are studied. The system is influenced by uniform magnetic field. The problem is formulated and analyzed using a perturbation expansion in terms of a variant of the Weicsenberg number. Carreau flow is considered in this study to investigate the effect of porous medium. An analytic forms for axial velocity,pressure gradient and heat transfer have been obtained. The results were studied for various values of the physical parameters of the problem and illustrated graphically.

1. Introduction

Our purpose is to investigate the mechanism by which a fluid is transported through a duct when contraction waves propagate progressively along its wall. This valveless-pumping principle, which is called peristalsis [1], plays a role in many physiological processes with fluid transport and is also exploited in technology, e.g. in so-called ”roller pumps”.There are many investigations on peristaltic flow of Newtonian fluids have been carried out . Rath [2] has given a survey of this subject, with a probably complete summary of the bibliography unit. Studying peristaltic flows, especially with a view to applications in biomechanics and physiology, one should consider real material properties of the fluid being transported and determine the essential departures from the results of the theories for Newtonian fluids. These investigations are, also, interesting for technological applications, e.g. in the field of polymer processing. In this regard there are only few contributions in the literature.

The analysis of the mechanisms responsible for peristaltic transport have been studied by many authors. Latham's investigation[3], may be the first study in this field and since that time several theoretical and experimental investigations have been made to understand peristaltic action in both mechanical and physiological situations.

Some of these studies were made by Burns and Parkes [4], Barton and Raynor [5], Shapiro et al. [6], Lykoudis and Roos [7], Roos and Lykoudis [8], Shuka et al. [9], Elshehawey and Mekheimer [10].

Since most of physiological fluids in the human body behave like non-Newtonian fluids, some researches on non-Newtonian fluids were recently published.

Bohme and Friedrich [1] have investigated peristaltic flow of second-order viscoelastic liquid assuming that the reliant Reynolds number is small enough to neglect inertia forces, and that the ratio of the wave length and the channel height is large so that the pressure is constant over the cross-section. ELmisery, Elshehawey and Hakeem [11] have studied the peristaltic motion of an incompressible generalized Newtonian fluid in a planar channel of uniform geometry in case of long-wave approximation. The same problem for non-uniform channel has been studied by Elshehawey, EL Misery [12].

The effect of porous medium on the motion of the fluid have been studied by many authors, Elshehawey et al. [13] studied the effect of porous medium on peristaltic motion of a Newtonian fluid. Eldabe [14] studied magnetohydrodynamic flow through a porous medium fluid at a rear stagnation point. Eldabe et al. [15] studied MHD flow and heat transfer in a viscoelastic incompressible fluid confined between a horizontal stretching sheet and a parallel porous wall. Elshehawey et al. [16] studied the peristaltic motion of a Generalized Newtonian fluid through a porous medium.

Elshehawey et al. [17] studied the peristaltic motion of a Generalized Newtonian fluid under the affect of transverse magnetic field. This problem studied the effect of porous boundaries on peristaltic transport through a porous medium. Elshehawey and Sobh [18] studied the peristaltic viscoelastic fluid motion in a tube.

The main aim of this work is to study the effect a magnetic field and heat transfer on a peristaltic motion of Carreau fluid through a porous medium in uniform channel with a sinusoidal wave. The system is expressed by uniform magnetic field and heat transfer. By using Weicsenberg perturbation technique, in fact we have to choose the parameters for the Carreau fluid such that the Weicsenberg number $W \prec 1$, the wave number δ is neglected and the Reynold's number R_e is very small [17]. The velocity, pressure gradient and heat transfer have been obtained in explicit forms. The effects of the parameters of the problem on these solutions (the magnetic number, the Prandtl number, the Eckert number and the Weicsenberg number) are discussed and shown graphically.

2. Basic equations

The basic equations of MHD motion neglecting displacement and free charges are

2.1. The continuity equation.

$$\nabla \cdot \underline{V} = 0 \tag{2. 1}$$

where $\underline{V}$ the velocity vector

2.2. The momentum equation.

$$\rho[\frac{\partial \underline{V}}{\partial t} + (\underline{V} \cdot \nabla)\underline{V}\,] = -\nabla P + \nabla \cdot \underline{\tau} - \frac{\mu}{\epsilon_0}\underline{V} + \underline{J} \times \underline{B} \tag{2. 2}$$

where ρ is the density of the fluid, t is the time, P is the pressure of fluid, $\underline{\tau}$ is the extra stress tensor, μ is the viscosity coefficient, ϵ_0 is the permeability fluid, $\underline{J}$ is the current density and $\underline{B}$ the magnetic flux density.

2.3. Energy equation.

$$\rho c_p[\frac{\partial T}{\partial t} + (\underline{V} \cdot \nabla)T\,] = k\nabla^2 T + \tau \cdot (\nabla \underline{V}) \tag{2. 3}$$

where c_p, k and T are capacity, thermal coductivity and temperature of the fluid.

2.4. **Maxwell's equations.**

$$\nabla \times \underline{B} = \mu_e \underline{J} \tag{2. 4}$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t} \tag{2. 5}$$

$$\nabla \cdot \underline{B} = 0 \tag{2. 6}$$

where $\underline{E}$ denotes the electric field and μ_e is the magnetic permeability.

2.5. **Ohm's equation.**

$$\underline{J} = \sigma(\underline{E} + \underline{V} \times \underline{B}) \tag{2. 7}$$

where σ is the electric conductivity.

2.6. **Constitutive equation for Carreau fluid** [16] **is given by.**

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = [1 + (\Gamma\gamma\cdot)^2]^{(\frac{n-1}{2})} \tag{2. 8}$$

where η_0 is the zero-shear-rate viscosity, η_∞ is the infinite-shear-rate viscosity, Γ is a time constant, n is the dimensionless power law index and $\gamma\cdot$ is defined by:

$$\gamma\cdot = \sqrt{\frac{1}{2}\sum_i \sum_j \gamma\cdot_{ij}\gamma\cdot_{ij}} = \sqrt{\frac{1}{2}\Pi_{\gamma\cdot_{ij}}} \tag{2. 9}$$

where $\Pi_{\gamma\cdot_{ij}}$ is second invariant of strain -rate tensor $\gamma\cdot_{ij}$.

In equation (2. 8) we shall consider the case for which $\eta_\infty = 0$ and $\Gamma\gamma\cdot < 1$, so τ_{ij} can be written as [16]:

$$\tau_{ij} = -\eta_0[1 + \frac{n-1}{2}(\Gamma\gamma\cdot)^2]\gamma\cdot_{ij} \tag{2. 10}$$

τ_{ij} are the components of the extra stress tensor.

3. Formulation of the problem

We shall consider a two-dimensional channel of uniform thickness $2a$, filled with an incompressible Carreau fluid through a porous medium. A uniform magnetic flux density B_0 fixed relative to the fluid is imposed along Y-axis.

The walls of the channel are flexible and non-conducting, on which are imposed travelling sinusoidal waves of moderate amplitude.The geometry of the wall surface is defined as in fig. (1)

$$H(X, t) = a + b\,\sin\frac{2\pi}{\lambda}(X - ct), \tag{3. 1}$$

where b is the wave amplitude, λ is the wave length, c is the speed of the wave and X is the same direction of the wave propagation .

We choose moving coordinates (x, y), wave frame, which travel in the X-direction with the same speed as the wave, the unsteady flow in the laboratory frame (X, Y) can be treated as steady [6].The coordinates frame is related by :

$$x = X - ct,\ y = Y \tag{3. 2}$$

$$u = \overline{U} - c,\ v = \overline{V} \tag{3. 3}$$

where $\overline{U}$, $\overline{V}$and u, v are the velocity components in the corresponding coordinate systems.

Equation of continuity, Navier-Stoke's equation and heat equation, respectively, take the following forms [12], [14]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3. 4}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{1}{\rho}(\frac{\partial \tau_{11}}{\partial x}+\frac{\partial \tau_{21}}{\partial y})-(\frac{\nu}{\epsilon_0}+\frac{\sigma}{\rho}B_0^2)u \tag{3. 5}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}-\frac{1}{\rho}(\frac{\partial \tau_{21}}{\partial x}+\frac{\partial \tau_{22}}{\partial y})-\frac{\nu}{\epsilon_0}v \tag{3. 6}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho c_p}(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+\frac{1}{\rho c_p}[\tau_{11}\frac{\partial u}{\partial x}+\tau_{22}\frac{\partial v}{\partial y}+\tau_{21}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})] \tag{3. 7}$$

where ν is the kinematic viscosity. From equation (2. 10) for $(i, j = 1, 2)$ we get:

$$\tau_{11}=-\eta_0[1+\frac{n-1}{2}(\Gamma\gamma\cdot)^2]\gamma\cdot_{11}$$

$$\tau_{12}=-\eta_0[1+\frac{n-1}{2}(\Gamma\gamma\cdot)^2]\gamma\cdot_{12}$$

$$\tau_{22}=-\eta_0[1+\frac{n-1}{2}(\Gamma\gamma\cdot)^2]\gamma\cdot_{22} \tag{3. 8}$$

where:

$$\gamma\cdot_{11}=2\frac{\partial u}{\partial x},\ \gamma\cdot_{12}=\gamma\cdot_{21}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},\ \gamma\cdot_{22}=2\frac{\partial v}{\partial y} \tag{3. 9}$$

The appropriate boundary conditions are:

$$u=-c,\ v=-c\frac{dH}{dx},\ T=T_s\ at\ y=H(x)$$

$$\frac{\partial u}{\partial y}=0\ ,\ v=0,\ T=T_0\quad at\ y=0 \tag{3. 10}$$

Let us introduce the following non-dimensional variables

$$x^*=\frac{x}{\lambda},\ X^*=\frac{X}{\lambda},\ y^*=\frac{y}{a},\ Y^*=\frac{Y}{a},\ t^*=\frac{c}{\lambda}t,\ P^*=\frac{a^2}{c\lambda\eta_0}P$$

$$u^*=\frac{u}{c},\ \overline{U}^*=\frac{\overline{U}}{c},\ v^*=\frac{\lambda}{ca}v,\overline{V}^*=\frac{\lambda}{ca}\overline{V},\ T^*=\frac{T-T_s}{T_w-T_s}$$

$$\tau^*_{ij}=\frac{\lambda}{c\eta_0}\tau_{ij}\ i=j,\ \tau^*_{ij}=\frac{a}{c\eta_0}\tau_{ij}\ i\neq j,\ \gamma\cdot^*_{ij}=\frac{\lambda}{c}\gamma\cdot_{ij}\ i=j$$

$$\gamma\cdot^*_{ij}=\frac{a}{c}\gamma\cdot_{ij}\ i\neq j,\ \gamma\cdot^*=\frac{a}{c}\gamma\cdot,\ \epsilon^*_0=\frac{\epsilon_0}{a^2} \tag{3. 11}$$

Using (3. 11) in equations (3. 1) and (3. 4 - 3. 9) after dropping star, we obtain the following equations:

$$H(x)=1+\phi\ \sin 2\pi x \tag{3. 12}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3. 13}$$

$$R_e\delta(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial P}{\partial x}-(\delta^2\frac{\partial \tau_{11}}{\partial x}+\frac{\partial \tau_{21}}{\partial y})-(\frac{1}{\epsilon_0}+M)u \tag{3. 14}$$

$$R_e\delta^3(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial P}{\partial y}-\delta^2(\frac{\partial \tau_{21}}{\partial x}+\frac{\partial \tau_{22}}{\partial y})-\frac{\delta^2}{\epsilon_0}v \tag{3. 15}$$

$$\begin{aligned} R_e\delta(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}) &= \frac{1}{P_r}(\delta^2\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+E_c[\delta^2\tau_{11}\frac{\partial u}{\partial x}+\delta^2\tau_{22}\frac{\partial v}{\partial y} \\ &\quad +\tau_{21}(\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x})], \end{aligned} \tag{3. 16}$$

$$\begin{aligned} \tau_{11} &= -[1+\frac{n-1}{2}W^2\gamma^{\cdot 2}]\gamma^{\cdot}{}_{11}, \\ \tau_{12} &= -[1+\frac{n-1}{2}W^2\gamma^{\cdot 2}]\gamma^{\cdot}{}_{12}, \\ \tau_{22} &= -[1+\frac{n-1}{2}W^2\gamma^{\cdot 2}]\gamma^{\cdot}{}_{22} \end{aligned} \tag{3. 17}$$

$$\gamma^{\cdot}{}_{11}=2\frac{\partial u}{\partial x},\ \gamma^{\cdot}{}_{12}=\gamma^{\cdot}{}_{21}=\frac{\partial u}{\partial y}+\delta^2\frac{\partial v}{\partial x},\ \gamma^{\cdot}{}_{22}=2\frac{\partial v}{\partial y} \tag{3. 18}$$

where:

$$\begin{aligned} \phi &= \frac{a}{b}, \\ \delta &= \frac{a}{\lambda} \text{ is the wave number,} \\ R_e &= \frac{\rho a c}{\eta_0} \text{ is the Reynold number,} \\ M &= \frac{\sigma a^2 B_0^2}{\eta_0} \text{ is the magnetic number,} \\ P_r &= \frac{\eta_0}{K}c_p \text{ is the Prandtl number,} \\ E_c &= \frac{c}{c_p(T_w-T_s)} \text{ is the Eckert number,} \\ W &= \frac{c\Gamma}{a} \text{ is the Weicsenberg number.} \end{aligned}$$

The dimensionless boundary conditions are:

$$\begin{aligned} &u=-1,\ v=-\frac{dH}{dx},\ T=0 \qquad at\ y=H(x) \\ &\frac{\partial u}{\partial y}=0,\ v=0,\ T=1 \quad at\ y=0 \end{aligned} \tag{3. 19}$$

Using long wavelength approximation ($\delta=\frac{a}{\lambda}=0$), equations (3. 14 - 3. 18) become:

$$\frac{\partial P}{\partial x}=-\frac{\partial \tau_{21}}{\partial y}-(\frac{1}{\epsilon_0}+M)u \tag{3. 20}$$

$$\frac{\partial P}{\partial y}=0 \tag{3. 21}$$

$$\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}=-E_c\tau_{21}(\frac{\partial u}{\partial y}) \tag{3. 22}$$

$$\tau_{12} = \tau_{21} = -[1 + \frac{n-1}{2} W^2 \gamma \cdot] \gamma \cdot_{12}, \tag{3. 23}$$

$$\gamma \cdot_{12} = \gamma \cdot_{21} = \frac{\partial u}{\partial y} \tag{3. 24}$$

Eliminating the pressure from (3. 20)and (3. 21), we get:

$$-\frac{\partial^2 \tau_{21}}{\partial y^2} - (\frac{1}{\epsilon_0} + M) \frac{\partial u}{\partial y} = 0 \tag{3. 25}$$

From equations (3. 23) and (3. 24), we have:

$$\tau_{12} = \tau_{21} = -[1 + \frac{n-1}{2} W^2 \left(\frac{\partial u}{\partial y} \right)^2] \frac{\partial u}{\partial y} \tag{3. 26}$$

4. Rate of volume flow

The rate of volume flow in the fixed frame is given by:

$$Q(X,t) = \int_0^{H(X,t)} \overline{U}(X,Y,t) dY \tag{4. 1}$$

The rate of volume flow in the moving frame (wave frame) is given by:

$$q(x) = \int_0^{H(x)} u(x,y) dy \tag{4. 2}$$

With the help of equations (3. 2) and (3. 3), one can show that these two rates of volume flow are related by:

$$Q = q + cH(x) \tag{4. 3}$$

The time-mean flow over a period $\overline{t} = \frac{\lambda}{c}$ at a fixed position X is defined as:

$$\overline{Q} = \frac{1}{\overline{t}} \int_0^{\overline{t}} Q dt \tag{4. 4}$$

By using (2. 10),(4. 2) in (4. 3) we get :

$$\overline{Q} = q + ca \tag{4. 5}$$

Defining the dimensionless time-mean flows θ and F in the fixed and wave frame, respectively as:

$$\theta = \frac{\overline{Q}}{ac} and F = \frac{q}{ac} \tag{4. 6}$$

Equation (4. 4) can be rewritten as:

$$\theta = 1 + F \tag{4. 7}$$

where:

$$F = \int_0^{H(x)} u(x,y) dy \tag{4. 8}$$

5. METHOD OF SOLUTION

We expand the following quantities as power series in the small parameter W as follows:

$$u = W^0 u_o + W^2 u_1 + O(W^4)$$

$$\frac{\partial P}{\partial x} = W^0 \frac{\partial P_0}{\partial x} + W^2 \frac{\partial P_1}{\partial x} + O(W^4)$$
$$\tau_{12} = W^0 \tau_{12(0)} + W^2 \tau_{12(1)} + O(W^4)$$
$$T = W^0 T_o + W^2 T_1 + O(W^4)$$

$$F = W^0 F_o + W^2 F_1 + O(W^4) \tag{5. 1}$$

The use of expansion (5. 1) with equations(3. 19),(3. 20),(3. 22),(3. 26)and (4. 4) gives the systems of equations and after comparing the coefficient of $W^0$and W^2, we get:

$$\frac{\partial^2 u_0}{\partial y^2} - (\frac{1}{\epsilon_0} + M)u_0 = \frac{\partial P_0}{\partial x},\ F_0 = \int_0^{H(x)} u_0(x,y)dy \tag{5. 2}$$

$$\frac{\partial^2 u_1}{\partial y^2} - (\frac{1}{\epsilon_0} + M)u_1 = \frac{\partial P1}{\partial x} - \frac{3}{2}(n-1)(\frac{\partial^2 u_0}{\partial y^2})(\frac{\partial u_0}{\partial y})^2,\ F_1 = \int_0^{H(x)} u_1(x,y)dy \tag{5. 3}$$

$$\frac{\partial^2 T_0}{\partial y^2} = p_r E_c(\frac{\partial u_0}{\partial y})^2 \tag{5. 4}$$

$$\frac{\partial^2 T_1}{\partial y^2} = p_r E_c[2(\frac{\partial u_0}{\partial y})(\frac{\partial u_1}{\partial y}) + \frac{n-1}{2}(\frac{\partial u_0}{\partial y})^4] \tag{5. 5}$$

With corresponding boundary conditions:

$$u_0 = u_1 = -1,\ v_0 = v_1 = -\frac{dH}{dx},\ T_0 = T_1 = 0 \qquad at\ y = H(x)$$

$$\frac{\partial u_0}{\partial y} = \frac{\partial u_1}{\partial y} = 0,\ v_0 = v_1 = 0,\ T_0 = 1,\ T_1 = 0 \quad at\ y = 0 \tag{5. 6}$$

The solutions of equations (5. 2 - 5. 5) subject to the boundary conditions (5. 6) give the axial velocity component u, the pressure gradient $\frac{\partial P}{\partial x}$ and the temperature distribution T as:

$$u = b_0 + b_1 \cosh\frac{y}{\sqrt{N}} + W^2[b_2 + b_3 \cosh\frac{y}{\sqrt{N}} + b_4 \cosh\frac{3y}{\sqrt{N}} + b_5 y \sinh\frac{y}{\sqrt{N}}], \tag{5. 7}$$

$$\begin{aligned}\frac{\partial P}{\partial x} &= (F\cosh\frac{H}{\sqrt{N}} + \sqrt{N}\sinh\frac{H}{\sqrt{N}})/(N^{\frac{3}{2}}\sinh\frac{H}{\sqrt{N}} \\ &\quad -NH\cosh\frac{H}{\sqrt{N}}) - W^2\frac{3(n-1)}{16N^3}\beta f(H,N),\end{aligned} \tag{5. 8}$$

$$\begin{aligned}T &= \frac{b_6}{8}\cosh\frac{2y}{\sqrt{N}} - \frac{b_6}{2N}y^2 + b_7 y + b_8 + W^2[b_9\cosh\frac{4y}{\sqrt{N}} \\ &\quad + b_{10}\cosh\frac{2y}{\sqrt{N}} + b_{11} y \sinh\frac{y}{\sqrt{N}} + b_{12} y^2 + b_{13} y + b_{14}],\end{aligned} \tag{5. 9}$$

where N, β and $(b_0, ..., \; b_{14})$ are defined in the appendix.

6. Results and discussion

The momentum and energy equations of magnetohydrodynamic flow and heat transfer in a peristaltic motion of Generalized Newtonian fluid through a porous medium are solved analytically by using Weicsenberg perturbation technique, in fact we have to choose the parameters for the carreau fluid such that the Weicsenberg number $W < 1$, δ is neglected and the Reynold number R_e is very small [16]. Our system of linear partial differential equations are solved and the effects of the parameters of the problem on these solutions are shown graphically.

Fig. (2) illustrates that the velocity distribution u increases with increasing the permeability fluid ϵ_0, but at $y < 0.57$ the vise verse occurs. It is found that the velocity distribution u decreases when the magnetic number M increases, but after that at $y = 0.57$, u starts to increase with increasing M in fig. (3). The effect of the parameter $\phi = \frac{a}{b}$ on the velocity distribution u is shown in fig. (4), where u increases as ϕ increases. From fig. (5) we have seen that the pressure gradient $\frac{\partial P}{\partial x}$ decreases as the permeability fluid ϵ_0 increases. In fig. (6) we note that the pressure gradient $\frac{\partial P}{\partial x}$ increases with increasing the magnetic number M. It is clear from fig (7) that the pressure gradient $\frac{\partial P}{\partial x}$ decreases as the parameter $\phi = \frac{a}{b}$ increases and the inverse effect occurs at $\theta = 0.5$. It seems from figs. (9) and (10) that the temperature T increases with increasing the magnetic number M and the parameter $\phi = \frac{a}{b}$. Figs.(8), (11) and (12) clear that the temperature T decreases when the permeability fluid ϵ_0, the Prandtl number P_r and the Eckert number E_c increase.

7. Conclusion and Applications

In this work, we study of magnetohydrodynamics flow and heat transfer of the two-dimensional peristaltic motion for incompressible non-Newtonian fluid through a porous medium analytically. The governing partial differential equation of this problem, subject to the boundary conditions are solved by using Weicsenberg perturbation technique. The analytical forms for the velocity distribution u, the pressure gradient $\frac{\partial P}{\partial x}$ and the temperature T are obtained. The effects of the various physical parameters of the problem are discussed and have been shown graphically. It is seen that the velocity distribution u decreases or increases as ϵ_0 and M, but the pressure gradient $\frac{\partial P}{\partial x}$ and the temperature T increase with increasing the magnetic number M, the temperature T decreases when the permeability fluid ϵ_0, the Prandtl number P_r and the Eckert number E_c increase and the velocity distribution u and the temperature T decrease with increasing ϕ.

The study of this phenomena is very important, because the study of flow through porous medium have many applications. It has an important role in agricultural, engineering, science and petroleum industry. For example, ground water hydrology, extracting pure petrol from crude oil and chemical engineering. There are examples of natural porous media such as wood, filter paper, cotton, leather and plastics. As a good biological examples on the porous medium the human lung gall bladder and the walls of vessels. The peristaltic motion has been found to involved in many biological organs such as esophagus, small and large intestine, stomach, the human ureter, lymphatic vessels and small blood vessels. Also, peristaltic transport occurs in many practical applications involving biomechanical systems such as finger pumps [16].

8. Appendix

$$
\begin{aligned}
N &= \frac{1}{(\frac{1}{\epsilon_0}+M)},\\
b_0 &= -1-\frac{(F+H)\cosh\frac{H}{\sqrt{N}}}{(\sqrt{N}\sinh\frac{H}{\sqrt{N}}-H\cosh\frac{H}{\sqrt{N}})},\\
b_1 &= \frac{(F+H)}{(\sqrt{N}\sinh\frac{H}{\sqrt{N}}-H\cosh\frac{H}{\sqrt{N}})},\\
\beta &= \frac{(F^3+H^3+3H^2F+3HF^2)}{(\sqrt{N}\sinh\frac{H}{\sqrt{N}}-H\cosh\frac{H}{\sqrt{N}})^3},\\
f(H,N) &= (H+\frac{\sqrt{N}}{4}\cosh\frac{3H}{\sqrt{N}}\sinh\frac{H}{\sqrt{N}}-\frac{\sqrt{N}}{12}\cosh\frac{H}{\sqrt{N}}\sinh\frac{3H}{\sqrt{N}}\\
&\quad -\sqrt{N}\cosh\frac{H}{\sqrt{N}}\sinh\frac{H}{\sqrt{N}})/(\sqrt{N}\sinh\frac{H}{\sqrt{N}}-H\cosh\frac{H}{\sqrt{N}}),\\
b_2 &= \frac{3(n-1)}{16N}\beta f(H,N),\\
b_3 &= \frac{3(n-1)}{16N^{\frac{3}{2}}}\beta(\frac{\sqrt{N}}{4}\cosh\frac{3H}{\sqrt{N}}-H\sinh\frac{H}{\sqrt{N}})/\\
&\quad (\cosh\frac{H}{\sqrt{N}})-\frac{3(n-1)}{16N^{\frac{3}{2}}}\beta f(H,N),\\
b_4 &= \frac{-3(n-1)}{64N}\beta,\\
b_5 &= \frac{3(n-1)}{16N^{\frac{3}{2}}}\beta,\\
b_6 &= p_rE_cb_1^2,\\
b_7 &= -b_6(\frac{1}{8H}\cosh\frac{2H}{\sqrt{N}}-\frac{H}{4N}-\frac{1}{8H}+\frac{1}{Hb_6}),\\
b_8 &= b_6(\frac{1}{b_6}-\frac{1}{8}),\\
b_9 &= \frac{b_6}{16b_1}(\frac{(n-1)}{16N^3}b^3+3b_4),
\end{aligned}
$$

$$
\begin{aligned}
b_{10} &= \frac{b_6}{8b_1}(2b_3-6b_4-3\sqrt{N}b_5-\frac{(n-1)}{64N^{\frac{5}{2}}}b^3),\\
b_{11} &= \frac{b_6b_5}{2b_1},\\
b_{12} &= \frac{b_6}{8Nb_1}(\frac{3(n-1)}{4N^3}b^3-4b_3-12\sqrt{N}b_5),\\
b_{13} &= \frac{1}{H}(b_9(1-\cosh\frac{4H}{\sqrt{N}})+b_{10}(1-\cosh\frac{2H}{\sqrt{N}})-b_{11}H\sinh\frac{H}{\sqrt{N}}\\
&\quad -b_{12}H^2),\\
b_{14} &= -b_9-b_{10}.
\end{aligned}
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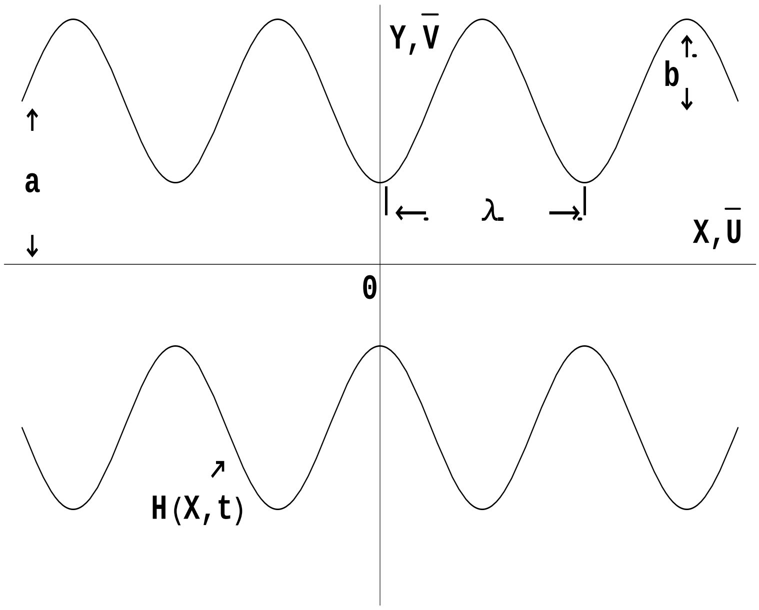


FIGURE 1

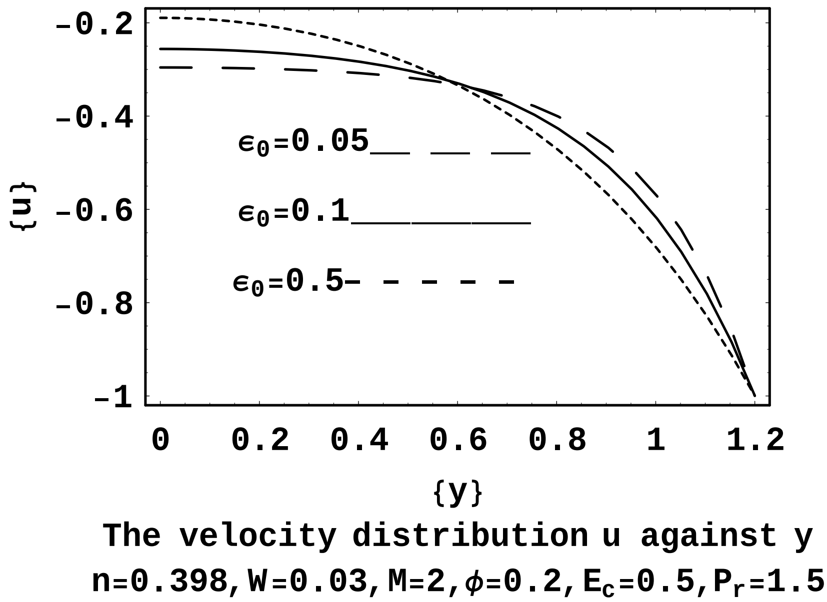
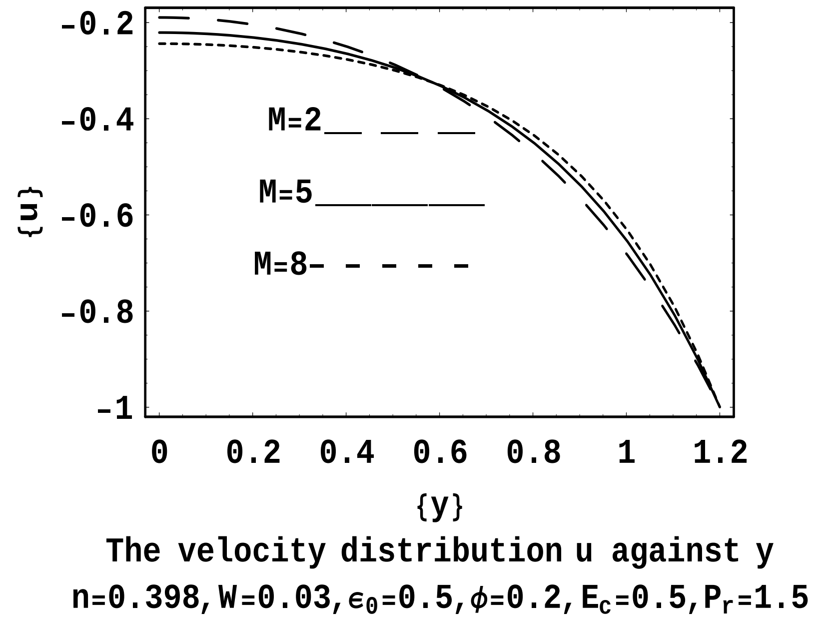


FIGURE 2



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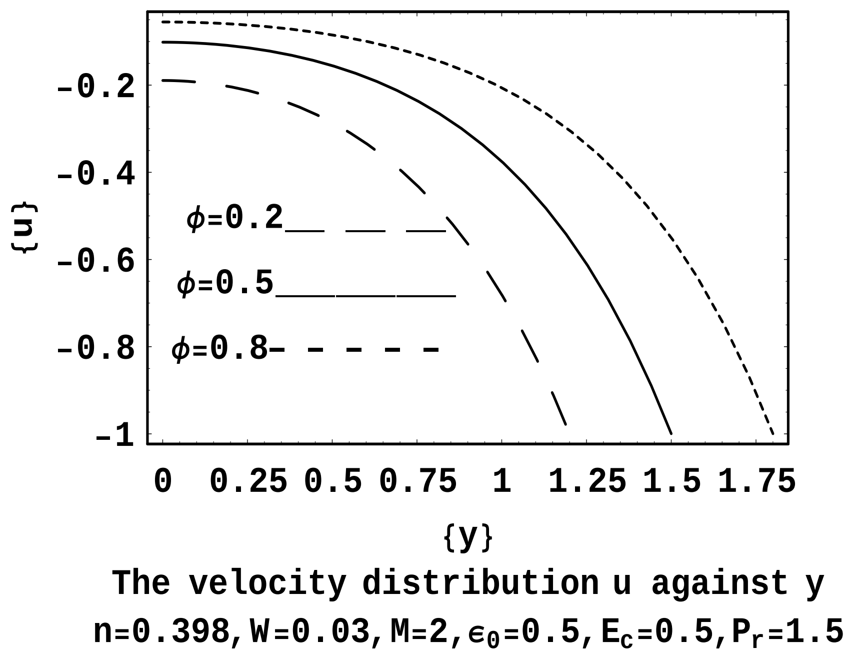

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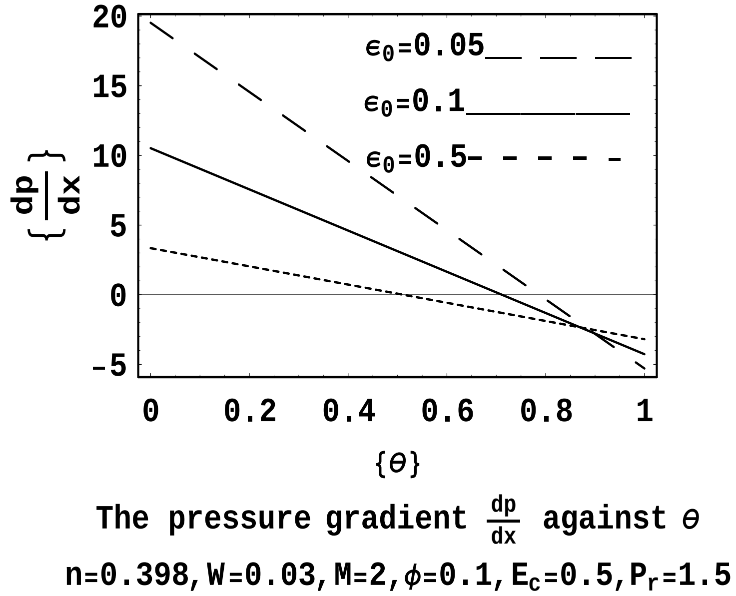

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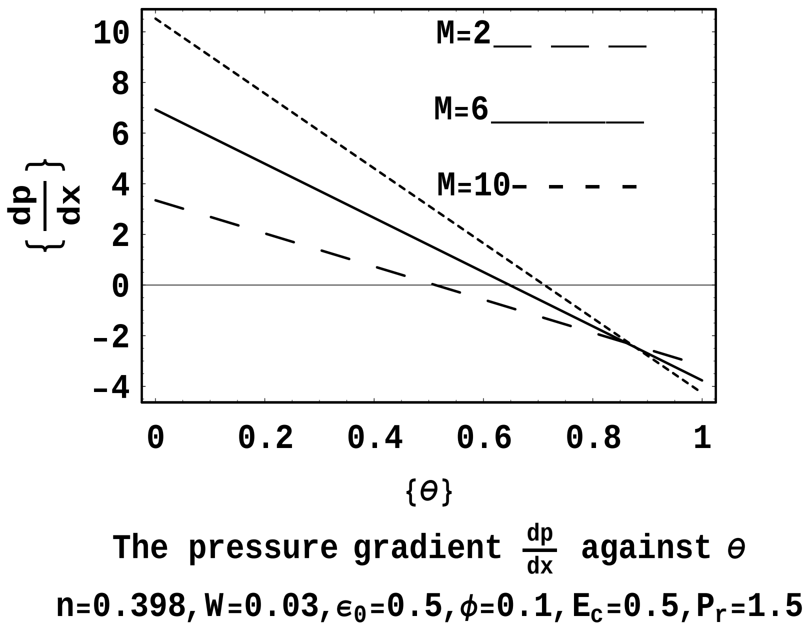

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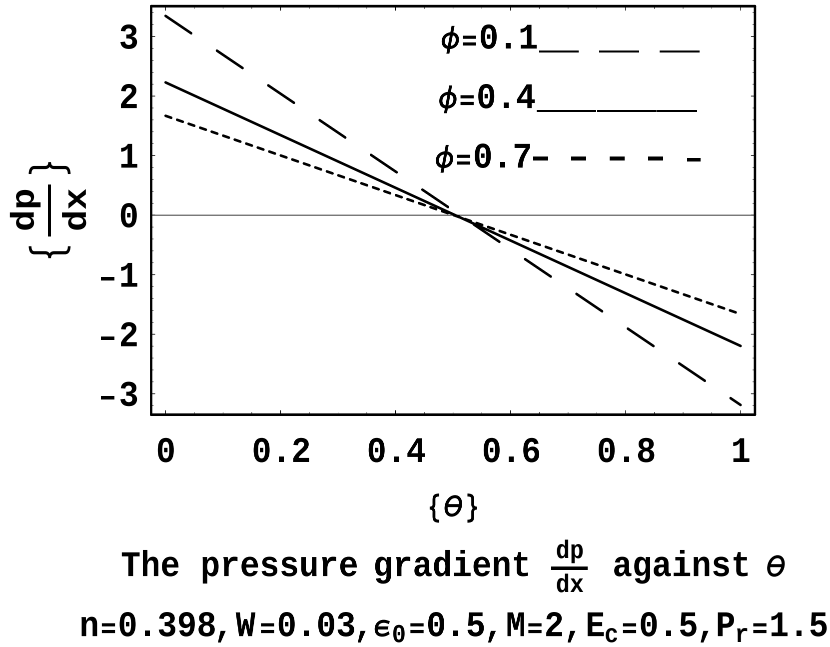

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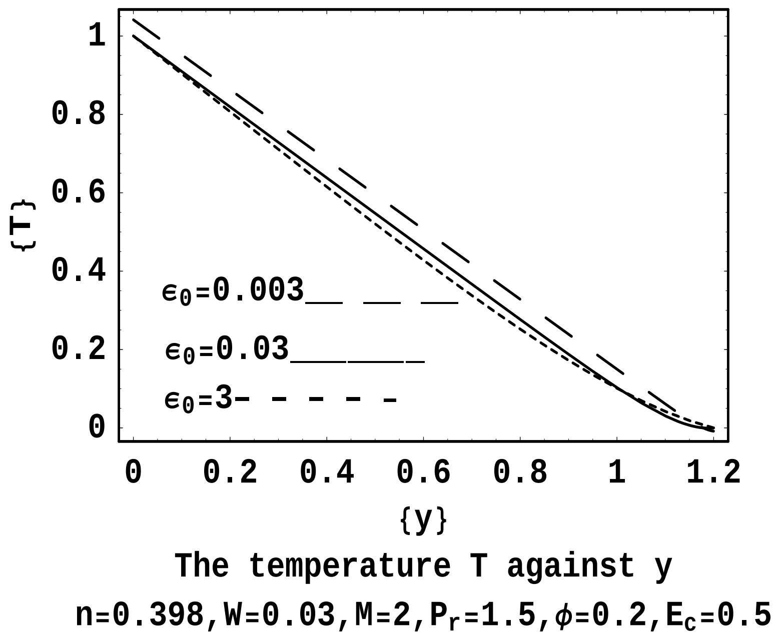

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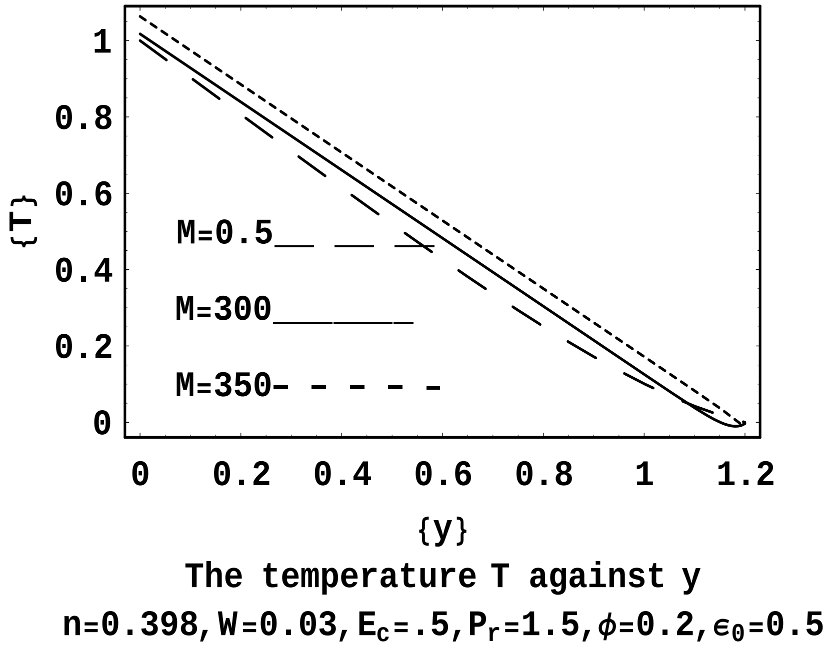

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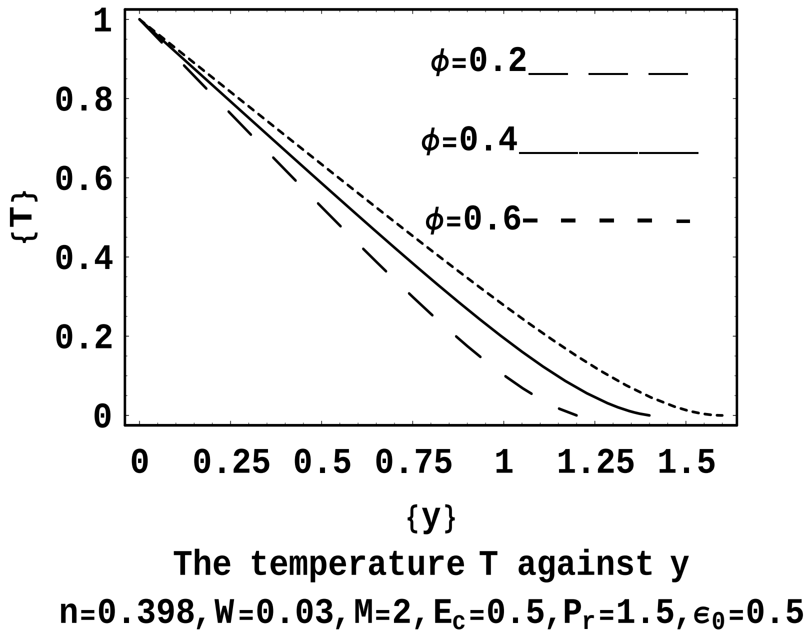

FIGURE 10

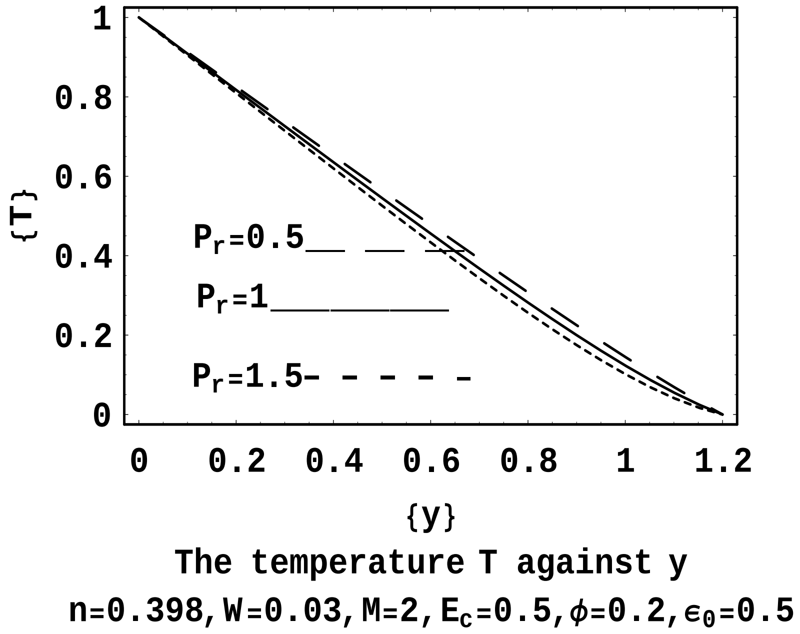

FIGURE 11

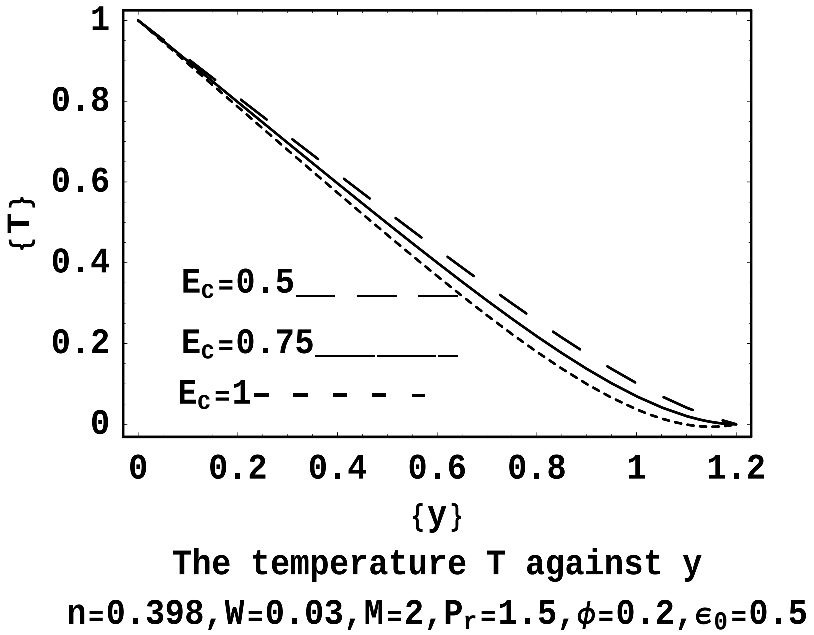

FIGURE 12