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An Analytic Solution of MHD Viscous Flow With Thermal Radiation Over a Permeable Flat Plate

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Abstract. The problem of thermal radiation on MHD viscous flow over a stationary permeable wall is investigated. The governing partial differential equations are transformed to a system of ordinary differential equations with the help of similarity transformations. The resulting nonlinear differential equations are then solved analytically by a purely analytic technique, namely, homotopy analysis method. The effect of wall suction/injection on velocity field and temperature distribution is discussed in detail with the help of graphs.

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1. Introduction

The MHD flow with continuous heat transfer has many practical applications in industrial manufacturing processes. However, of late, the radiation effect on MHD flow and heat transfer problems has become more important industrially. At high operating temperatures, the radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and the knowledge of radiation heat transfer becomes very much important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. When the difference between the surface temperature and the ambient temperature is large, the radiation effect becomes important. In the aspect of convection radiation, Viskanta and Grosh [1] considered the effect of thermal radiation on the temperature distribution and the heat transfer in an absorbing and emitting media flowing over a wedge by using the Rosseland diffusion approximation. This approximation leads to a considerable simplification in the expression for radiant flux. In [1] the temperature differences within the flow were assumed to be sufficiently small such that

T^4 may be expressed as a linear function of temperature, i.e., $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$. The thermal radiation of gray fluid, which is emitting and absorbing radiation in non-scattering medium, has been examined by Ali et al. [2], Ibrahim [3] ,Mansour [4], Hossain et al. [5]-[6], Elbashbeshy [7] and Elbashbeshy and Dimian [8] .The thermal radiation effect on a micropolar fluid was studied by Raptis [9] Recently, Raptis et al. [10] investigated the effect of thermal radiation on a flow of an electrically conducting viscous fluid. In [10] the authors considered the rigid stationary plate and computed a numerical solution of the problem. In the present study we extend the work of Raptis et al. [10] for the case of permeable stationary wall. The objective of this investigation is two fold; firstly, to investigate the effect of suction/injection on velocity and temperature profiles, secondly, to present complete analytic solution to the governing nonlinear equations. A newly developed analytic technique, namely, homotopy analysis method [11] is used to get the explicit analytic solution.

Currently, Liao [11] introduced an analytic technique for highly nonlinear problems in science and engineering. Liao named it as ”homotopy analysis method”. The homotopy analysis method is very useful to fluid mechanics problems and Liao himself proved the validity of the method by solving number of nonlinear problems in fluid mechanics (see for instance [12]-[20]). In recent years, the popularity of the method has grown considerably and number of researchers have successfully applied it to many nonlinear problems (see [21]-[29]).

The paper is organized into five sections. Section 2 contains the mathematical description of the problem. Analytic solution of the governing equations and the issue of convergence of the solution series is discussed in section 3. Graphical representation of results and their discussion is given in section 4 and finally some concluding remarks are given in section 5.

2. Mathematical description of the problem

We consider an incompressible, viscous and electrically conducting fluid bounded by a permeable semi-infinite flat plate situated at $y = 0$. The fluid is assumed to be flowing with uniform free stream velocity U(x) at infinity. A uniform magnetic field of strength B_0 is applied perpendicular to the plate. After neglecting the induced magnetic field and the radiative heat flux in the $x-$direction we get the continuity, momentum and the energy equations [10]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + U\frac{dU}{dx} + \frac{\sigma B_0^2}{\rho}\left(U - u\right) \tag{2.2}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 \theta}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{2.3}$$

subject to the boundary conditions

$$\begin{aligned} &u = 0,\ v = -V_0^*,\ \theta = \theta_0,\ \text{at}\ y = 0,\\ &u \to U\left(x\right),\ \theta = \theta_\infty,\ \text{as}\ y \to \infty. \end{aligned} \tag{2.4}$$

where V_0^* is constant suction/injection velocity (positive values of V_0^* correspond to suction and the negative values correspond to the constant injection at the plate); ρ is the density, ν is the kinematic viscosity, σ is the electric conductivity of the fluid, θ is the temperature, k is the thermal conductivity, c_p is the specific heat of the fluid under constant

pressure and q_r is the radiation heat flux.
We assume the velocity of the free stream of the form

$$U(x) = ax + bx^2 \tag{2.5}$$

and where a and b are constants.
By using the Rosseland approximation for radiation for an optically thick layer([1]-[2]) and the following transformations

$$\eta = \sqrt{\frac{a}{\nu}}y,\ u = axf'(\eta) + bx^2 g'(\eta),\ v = -\sqrt{a\nu}f(\eta) - \frac{2bx}{\sqrt{\frac{a}{\nu}}}g(\eta),$$

$$\theta = \theta_0 + (\theta_\infty - \theta_0)\left[T(\eta) + \frac{2bx}{a}\tau(\eta)\right], \tag{2.6}$$

the system(2.1) - (2.4) reduces to

$$f''' + ff'' - f'^2 + M(1 - f') + 1 = 0, \tag{2.7}$$

$$g''' + fg'' - 3f'g' + 2f''g + M(1 - g') + 3 = 0, \tag{2.8}$$

$$(3K + 4)T'' + 3K\,\mathrm{Pr}\,fT' = 0, \tag{2.9}$$

$$(3K + 4)\tau'' + 3K\,\mathrm{Pr}\,(-f'\tau + gT' + f\tau') = 0, \tag{2.10}$$

with boundary conditions

$$f = w,\ f' = 0,\ g = 0,\ g' = 0,\ T = 0,\ \tau = 0, \text{at } \eta = 0,$$

$$f' \to 1,\ g' \to 1,\ T \to 1,\ \tau \to 0, \text{as } \eta \to \infty. \tag{2.11}$$

3. Analytic solution

We use homotopy analysis method to solve the system (2.7) - (2.11) analytically. Due to the boundary conditions (2.11) , one can express the solution series of $f(\eta)$,$g(\eta)$, $T(\eta)$ and $\tau(\eta)$ in the following form:

$$f(\eta) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} A_{i,j}\eta^j e^{-i\eta}, \tag{3.1}$$

$$g(\eta) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} B_{i,j}\eta^j e^{-i\eta} \tag{3.2}$$

or

$$T(\eta) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} C_{i,j}\eta^j e^{-i\eta}, \tag{3.3}$$

$$\tau(\eta) = \sum_{i=1}^{\infty}\sum_{j=0}^{\infty} D_{i,j}\eta^j e^{-i\eta}, \tag{3.4}$$

respectively, where $A_{i,j}$, $B_{i,j}$, $C_{i,j}$, and $D_{i,j}$ are constant coefficients. They provided us with the so-called rule of solution expression, which plays an important role in the frame work of homotopy analysis method. According to the boundary conditions (2.11) and the foregoing rule of solution expression defined in (3.1)- (3.4) , we choose the initial approximations for $f(\eta)$, $g(\eta)$, $T(\eta)$ and $\tau(\eta)$ of the form

$$f_0(\eta) = w - 1 + \eta + e^{-\eta} \tag{3.5}$$

$$g_0(\eta) = -1 + \eta + e^{-\eta}, \tag{3.6}$$

$$T_0(\eta) = 1 - e^{-\eta}, \tag{3.7}$$

$$\tau_0(\eta) = e^{-\eta} - e^{-2\eta}, \tag{3.8}$$

respectively and the auxiliary linear operators

$$\mathcal{L}_1 \equiv \frac{\partial^3}{\partial\eta^3} - \frac{\partial}{\partial\eta}, \tag{3.9}$$

$$\mathcal{L}_2 \equiv \frac{\partial^2}{\partial\eta^2} + \frac{\partial}{\partial\eta}, \tag{3.10}$$

which satisfy the properties

$$\mathcal{L}_1\left[C_1 + C_2 e^{\eta} + C_3 e^{-\eta}\right] = 0, \tag{3.11}$$

and

$$\mathcal{L}_2\left[C_4 - C_5 e^{-\eta}\right] = 0, \tag{3.12}$$

where C_i $(i = 1, 2, \cdots, 5)$ are arbitrary constants, the subscripts 1 and 2 indicate that the operator corresponds to velocity and temperature functions respectively.

We construct the so-called zeroth-order deformation equations

$$(1-p)\,\mathcal{L}_1\left[F(\eta;p) - f_0(\eta)\right] = p\hbar_1 N_1\left[F(\eta;p)\right], \tag{3.13}$$

$$(1-p)\,\mathcal{L}_1\left[G(\eta;p) - g_0(\eta)\right] = p\hbar_1 N_2\left[F(\eta;p), G(\eta;p)\right], \tag{3.14}$$

$$(1-p)\,\mathcal{L}_2\left[\Gamma(\eta;p) - T_0(\eta)\right] = p\hbar_2 N_3\left[\Gamma(\eta;p), F(\eta;p)\right], \tag{3.15}$$

$$(1-p)\,\mathcal{L}_2\left[\Lambda(\eta;p) - \tau_0(\eta)\right] = p\hbar_2 N_4\left[F(\eta;p), G(\eta;p), \Gamma(\eta;p), \Lambda(\eta;p)\right], \tag{3.16}$$

subject to the boundary conditions

$$\begin{aligned} &F(0,p) = V_0,\ G(0,p) = 0,\ \frac{\partial F(\eta,p)}{\partial\eta}\Big|_{\eta=0} = 0,\ \frac{\partial G(\eta,p)}{\partial\eta}\Big|_{\eta=0} = 0,\\ &\Gamma(0,p) = 0, \Lambda(0,p) = 0,\ \frac{\partial F(\eta,p)}{\partial\eta}\Big|_{\eta=\infty} = 1,\ \frac{\partial G(\eta,p)}{\partial\eta}\Big|_{\eta=\infty} = 1,\\ &\Gamma(\infty,p) = 1,\ \Lambda(\infty,p) = 0, \end{aligned} \tag{3.17}$$

where $p \in [0,1]$ is the embedding parameter, $\hbar_1$ and $\hbar_2$ are the non-zero auxiliary parameters corresponding to the velocity and temperature profiles respectively and the nonlinear operators $N_1[F(\eta;p)]$, $N_2[F(\eta;p), G(\eta;p)]$, $N_3[\Gamma(\eta;p), F(\eta;p)]$, and $N_4[F(\eta;p), G(\eta;p), \Gamma(\eta;p), \Lambda(\eta;p)]$ are defined through

$$N_1[F(\eta;p)] = \frac{\partial^3 F}{\partial\eta^3} + F\frac{\partial^2 F}{\partial\eta^2} - \left(\frac{\partial F}{\partial\eta}\right)^2 + M\left(1 - \frac{\partial F}{\partial\eta}\right) + 1, \tag{3.18}$$

$$N_2[F(\eta;p), G(\eta;p)] = \frac{\partial^3 G}{\partial\eta^3} + F\frac{\partial^2 G}{\partial\eta^2} - 3\frac{\partial F}{\partial\eta}\frac{\partial G}{\partial\eta} + 2\frac{\partial^2 F}{\partial\eta^2}G + M\left(1 - \frac{\partial G}{\partial\eta}\right) + 3, \tag{3.19}$$

$$N_3[\Gamma(\eta;p), F(\eta;p)] = (3K+4)\frac{\partial^2\Gamma}{\partial\eta^2} + 3K\,\mathrm{Pr}\,F\frac{\partial\Gamma}{\partial\eta}, \tag{3.20}$$

$$\begin{aligned} N_4[F(\eta;p), G(\eta;p), \Gamma(\eta;p), \Lambda(\eta;p)] &= (3K+4)\frac{\partial^2\Lambda}{\partial\eta^2}\\ &+3K\,\mathrm{Pr}\left(-\frac{\partial F}{\partial\eta}\Lambda + G\frac{\partial\Gamma}{\partial\eta} + F\frac{\partial\Lambda}{\partial\eta}\right). \end{aligned} \tag{3.21}$$

When $p = 0$ we have the initial guess approximations

$$F(\eta;0) = f_0(\eta),\ G(\eta;0) = g_0(\eta),\ \Gamma(\eta;0) = T_0(\eta),\quad \Lambda(\eta;0) = \tau_0(\eta). \tag{3.22}$$

When $p = 1$ equations (3.13 -3.17) are the same as (2.7 -2.11) respectively, therefore at $p = 1$ we get the final solutions

$$F(\eta;1) = f(\eta), \; G(\eta;1) = g(\eta), \; \Gamma(\eta;1) = T(\eta), \; \Lambda(\eta;1) = \tau(\eta). \tag{3.23}$$

The initial guess approximations $f_0(\eta), g_0(\eta), T_0(\eta)$, and $\tau_0(\eta)$, the linear operators $\mathcal{L}_1, \mathcal{L}_2$ and the auxiliary parameters $\hbar_1$ and $\hbar_2$ are assumed to be selected such that equations $(3.13 - 3.17)$ have solution at each point $p \in [0,1]$, and also with the help of Maclaurin's series and due to eq. (3.22), $F(\eta;p), G(\eta;p), T(\eta;p)$, and $\tau(\eta;p)$ can be expressed as

$$F(\eta;p) = f_0(\eta) + \sum_{k=1}^{\infty} f_k(\eta)\, p^k, \tag{3.24}$$

$$G(\eta;p) = g_0(\eta) + \sum_{k=1}^{\infty} g_k(\eta)\, p^k, \tag{3.25}$$

$$\Gamma(\eta;p) = T_0(\eta) + \sum_{k=1}^{\infty} T_k(\eta)\, p^k, \tag{3.26}$$

$$\Lambda(\eta;p) = \tau_0(\eta) + \sum_{k=1}^{\infty} \tau_k(\eta)\, p^k, \tag{3.27}$$

where

$$f_k(\eta) = \frac{1}{k!}\frac{\partial^k F(\eta;p)}{\partial p^k}\,|_{p=0}, \; g_k(\eta) = \frac{1}{k!}\frac{\partial^k G(\eta;p)}{\partial p^k}\,|_{p=0},$$
$$T_k(\eta) = \frac{1}{k!}\frac{\partial^k \Gamma(\eta;p)}{\partial p^k}\,|_{p=0}, \; \tau_k(\eta) = \frac{1}{k!}\frac{\partial^k \Lambda(\eta;p)}{\partial p^k}\,|_{p=0}\,. \tag{3.28}$$

Assume that the auxiliary parameters $\hbar_1$ and $\hbar_2$ are so properly chosen that the series (3.24) -(3.27) are convergent at $p = 1$, then due to (3.27) we have

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{3.29}$$

$$g(\eta) = g_0(\eta) + \sum_{m=1}^{\infty} g_m(\eta), \tag{3.30}$$

$$T(\eta) = T_0(\eta) + \sum_{m=1}^{\infty} T_m(\eta), \tag{3.31}$$

$$\tau(\eta) = \tau_0(\eta) + \sum_{m=1}^{\infty} \tau_m(\eta). \tag{3.32}$$

Equations (3.29) -(3.32) provide us with a relationship between the initial guess approximations $f_0(\eta), g_0(\eta), T_0(\eta)$, and $\tau_0(\eta)$ and the unknown solutions $f(\eta), g(\eta), T(\eta)$, and $\tau(\eta)$ respectively. In order to get the governing equations for $f_m(\eta), g_m(\eta), T_m(\eta)$, and $\tau_m(\eta)$ $(m \geq 1)$, we first differentiate m times the two sides of eqs. (3.13) -(3.17) with respect to the embedding parameter p at $p = 0$ and then divide them by $m!$. In this way we get

$$\mathcal{L}_1[f_m - \chi_m f_{m-1}] = \hbar_1 P_m(\eta), \tag{3.33}$$
$$\mathcal{L}_1[g_m - \chi_m g_{m-1}] = \hbar_1 Q_m(\eta), \tag{3.34}$$
$$\mathcal{L}_2[T_m - \chi_m T_{m-1}] = \hbar_2 R_m(\eta), \tag{3.35}$$

$$\mathcal{L}_2\left[\tau_m - \chi_m \tau_{m-1}\right] = \hbar_2 W_m\left(\eta\right), \tag{3.36}$$

with the boundary conditions

$$\begin{aligned} f_m(0) = 0, \quad f_m'(0) = 0, \quad g_m'(0) = 0, \quad T_m(0) = 0, \quad \tau_m(0) = 0, \\ f_m'(\infty) = 0, \quad g_m'(\infty) = 0, \quad T_m(\infty) = 0, \quad \tau_m(\infty) = 0, \end{aligned} \tag{3.37}$$

where

$$P_m(\eta) = f_{m-1}''' - M f_{m-1}' + (1-\chi_m)(M+1) + \sum_{k=0}^{m-1}\left[f_{m-1-k} f_k'' - f_{m-1-k}' f_k'\right], \tag{3.38}$$

$$\begin{aligned} Q_m(\eta) &= g_{m-1}''' - M g_{m-1}' + (1-\chi_m)(M+3) \\ &+ \sum_{k=0}^{m-1}\left[f_{m-1-k} g_k'' - 3 f_{m-1-k}' g_k' + 2 f_{m-1-k}'' g_k\right], \end{aligned} \tag{3.39}$$

$$R_m(\eta) = (3K+4)\, T_{m-1}'' + 3K \,\mathrm{Pr} \sum_{k=0}^{m-1} f_{m-1-k} T_k', \tag{3.40}$$

$$W_m(\eta) = (3K+4)\, \tau_{m-1}'' + 3K \,\mathrm{Pr} \sum_{k=0}^{m-1}\left[f_{m-1-k}\tau_k' + g_{m-1-k} T_k' - f_{m-1-k}' \tau_k\right], \tag{3.41}$$

and, for k being any integer

$$\begin{aligned} \chi_k &= 0 \quad \text{if} \quad k \leq 1, \\ &= 1 \quad \text{if} \quad k > 1. \end{aligned} \tag{3.42}$$

We emphasize here that eqs. (3.33) -(3.36) are linear for all $m \geq 1$. Also, the left-hand sides of all (3.33) -(3.36) are governed by the same linear operators $\mathcal{L}_1$ and $\mathcal{L}_2$, for all $m \geq 1$. These linear equations can be easily solved by means of symbolic computation software such as MATHEMATICA, MATLAB, MAPLE and so on. We solve the system ((3.33) -(3.37) for first few values of m and find that the solution expressions for $f_m(\eta)$, $g_m(\eta)$, $T_m(\eta)$, and $\tau_m(\eta)$ can be written as

$$f_m(\eta) = \sum_{n=0}^{2m+2}\sum_{q=0}^{2m+2-n} a_{m,n}^q \eta^q e^{-n\eta}, \tag{3.43}$$

$$g_m(\eta) = \sum_{n=0}^{2m+2}\sum_{q=0}^{2m+2-n} b_{m,n}^q \eta^q e^{-n\eta}, \tag{3.44}$$

$$T_m(\eta) = \sum_{n=0}^{2m+2}\sum_{q=0}^{2m+2-n} c_{m,n}^q \eta^q e^{-n\eta}, \tag{3.45}$$

$$\tau_m(\eta) = \sum_{n=1}^{2m+2}\sum_{q=0}^{2m+2-n} d_{m,n}^q \eta^q e^{-n\eta}. \tag{3.46}$$

In this way we get the explicit analytic solution

$$f(\eta) = \lim_{M\to\infty}\sum_{m=0}^{M}\sum_{n=0}^{2m+2}\sum_{q=0}^{2m+2-n} a_{m,n}^q \eta^q e^{-n\eta}, \tag{3.47}$$

$$g(\eta) = \lim_{M\to\infty} \sum_{m=0}^{M} \sum_{n=0}^{2m+2} \sum_{q=0}^{2m+2-n} b_{m,n}^{q} \eta^{q} e^{-n\eta}, \tag{3.48}$$

$$T(\eta) = \lim_{M\to\infty} \sum_{m=0}^{M} \sum_{n=0}^{2m+2} \sum_{q=0}^{2m+2-n} c_{m,n}^{q} \eta^{q} e^{-n\eta}, \tag{3.49}$$

$$\tau(\eta) = \lim_{M\to\infty} \sum_{m=0}^{M} \sum_{n=1}^{2m+2} \sum_{q=0}^{2m+2-n} d_{m,n}^{q} \eta^{q} e^{-n\eta}, \tag{3.50}$$

of the original eqs. (2.7) -(2.10).

Therefore, at the Mth-order approximation, the solution can be expressed as follows

$$f(\eta) \approx \sum_{m=0}^{M} \sum_{n=0}^{2m+2} \sum_{q=0}^{2m+2-n} a_{m,n}^{q} \eta^{q} e^{-n\eta}, \tag{3.51}$$

$$g(\eta) \approx \sum_{m=0}^{M} \sum_{n=0}^{2m+2} \sum_{q=0}^{2m+2-n} b_{m,n}^{q} \eta^{q} e^{-n\eta}, \tag{3.52}$$

$$T(\eta) \approx \sum_{m=0}^{M} \sum_{n=0}^{2m+2} \sum_{q=0}^{2m+2-n} c_{m,n}^{q} \eta^{q} e^{-n\eta}, \tag{3.53}$$

$$\tau(\eta) \approx \sum_{m=0}^{M} \sum_{n=1}^{2m+2} \sum_{q=0}^{2m+2-n} d_{m,n}^{q} \eta^{q} e^{-n\eta} \tag{3.54}$$

As mentioned by Liao [11] that whenever the solution series obtained by homotopy analysis method converges it will be one of the solution of the original equations. The convergence of the solution series depends upon the choice of initial approximations, the auxiliary linear operators and the nonzero auxiliary parameters. Once if the initial guess approximations and the auxiliary linear operators have been selected then the convergence of the solution series will strictly depend upon the auxiliary parameters only. Therefore, the convergence of the solution series is determined by the values of such kind of parameters. The admissible values of the parameters $\hbar_1$ and $\hbar_2$ are determined by the so-called $\hbar-$curves. In order to find the allowed values of $\hbar_1$ and $\hbar_2$ to make the series (3.47) -(3.50) convergent we have plotted the $\hbar-$curves corresponding to $f''(0)$, $g''(0)$, $T'(0)$, and $\tau'(0)$ in figs. 1 and 2.

In fig. 1, the $\hbar-$curves have been plotted for $f(\eta)$ and $g(\eta)$. Notice that for $\hbar_1 \in (-0.16, -0.12)$ both curves have their line segments parallel to the $\hbar_1-$axis. If $\hbar_1$ is chosen from this interval, then the series (3.47) and (3.48) will converge, also our analysis shows that these series are convergent for $\hbar_1 = -0.14$. Similarly, the series (3.49) and (3.50) are found to be convergent at a same value of $\hbar_2 = -0.10$. To show that the series (3.47) -(3.50) are uniformly convergent series, we have calculated the differences between their successive terms at particular orders of approximation (see table 1). We have define the successive absolute differences at $\eta = 0$ in the following way

$$\Delta_i f'' = \left| f_i'' - f_{i-1}'' \right|,$$

$$\Delta_i g'' = \left| g_i'' - g_{i-1}'' \right|,$$

$$\Delta_i T' = \left| T_i' - T_{i-1}' \right|,$$

$$\Delta_i \tau' = \left| \tau_i' - \tau_{i-1}' \right|.$$

From table 1 it is clear that by increasing the order of approximation the contribution of the higher order terms is decreasing and after a certain order of approximations the contribution of the next terms becomes negligible which confirms the convergence of the solutions series.

4. Graphical results and discussion

To see the effect of wall suction/injection on velocity and temperature we have plotted the graphs. In figs. 3 and 4 the velocity f' is plotted against η for different values of suction and injection parameter w. It is observed that the boundary layer thickness decreases by increasing the suction velocity and the effect is totally reversed in the case of injection, increasing injection at the wall causes to increase the layer thickness. In fig. 5, it is shown that there is a direct effect of suction on the velocity $f(\eta)$. By increasing suction at the wall the velocity $f(\eta)$ also increases at the plate. Similar effect of suction/injection is observed on the velocities $g(\eta)$ and $g'(\eta)$ as shown in figs. $(6)-(8)$. However, it is noted that the suction/injection effects are strong in $f(\eta)$ and $f'(\eta)$ as compared with $g(\eta)$ and $g'(\eta)$.

In figs. 9 and 10 the temperature $T(\eta)$ is plotted against η for different values of the suction and injection velocity respectively. Clearly, the thermal boundary layer thickness decreases by increasing w and increases by decreasing w as shown in figs. 9 and 10. Similar effects of suction are shown in fig. 11 for the temperature $\tau(\eta)$.

5. Concluding remarks

In this communication we investigated the effect of suction/injection on MHD viscous flow with thermal radiation. Explicit purely analytic solution for velocity and temperature distribution are obtained by homotopy analysis method. The solution is explicit and totally analytic valid for all values of the dimensionless parameters involved in the problem. Convergence of the solution has been shown through a table of absolute differences of the successive terms of the solution series at different orders. The effect of suction is to decrease the boundary layer thickness and the thermal boundary layer thickness whereas the effect of injection is reverse to it which is in accordance with the results present in literature. We also remark here that the homotopy analysis method is a very useful analytic technique to solve highly nonlinear problems.

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Table 1: Successive differences at different orders.

Table 1				
	$\hbar_1 = -0.14$, $\hbar_2 = -0.1$, $M = 0.5$, $w = 0.5$, $K = 0.5$, $\Pr = 0.7$, $\eta = 0$			
i	$\Delta_i f''$	$\Delta_i g''$	$\Delta_i T'$	$\Delta_i \tau'$
5	0.05623560	0.13780800	0.014116700	0.023131500
10	0.02579640	0.05093700	0.005753070	0.002770360
15	0.01129380	0.02010250	0.000430896	0.000286050
20	0.00510152	0.00835626	0.000204585	0.000440658
25	0.00232260	0.00359375	0.000024761	0.000014915

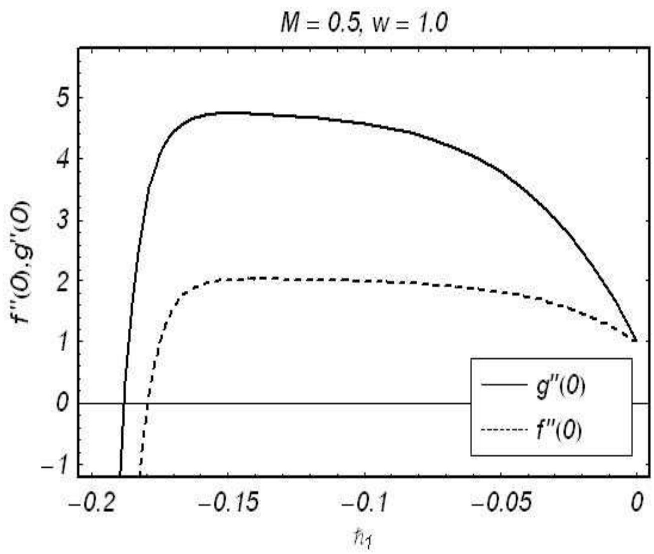

FIGURE 1. $\hbar-$curves corresponding to the velocity components.

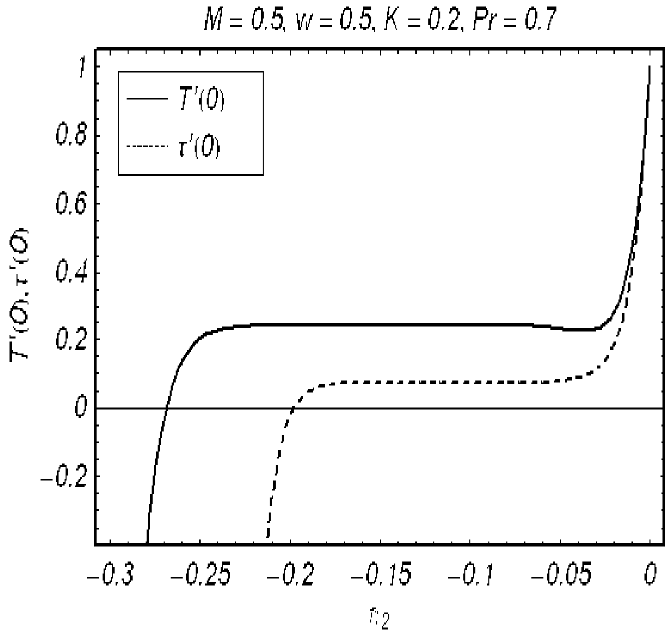

FIGURE 2. $\hbar-$curves for temperature.

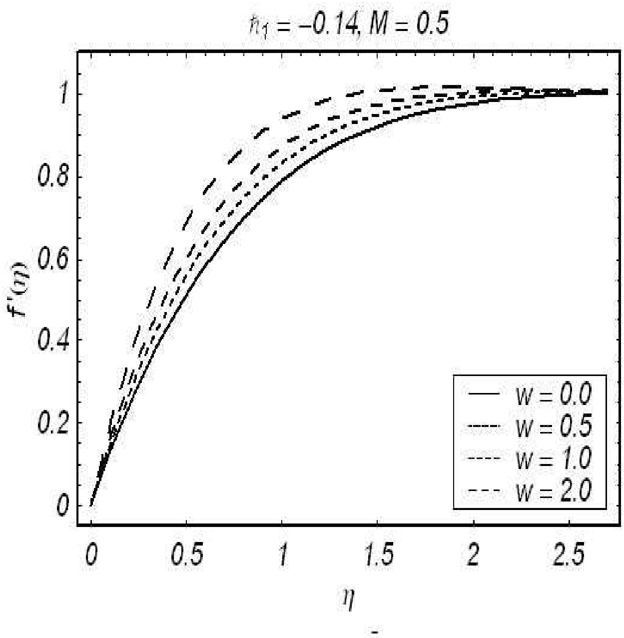

FIGURE 3. Effect of constant suction on the velocity component $f'(\eta)$

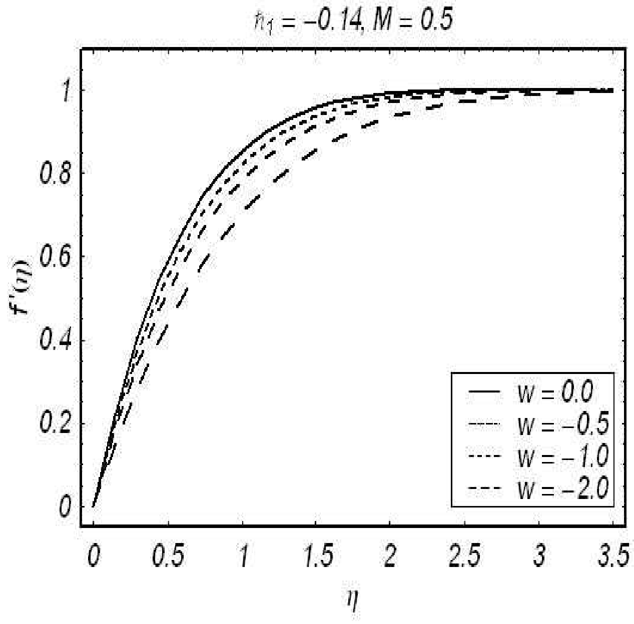

FIGURE 4. Velocity component $f'(\eta)$ in the case of constant injection.

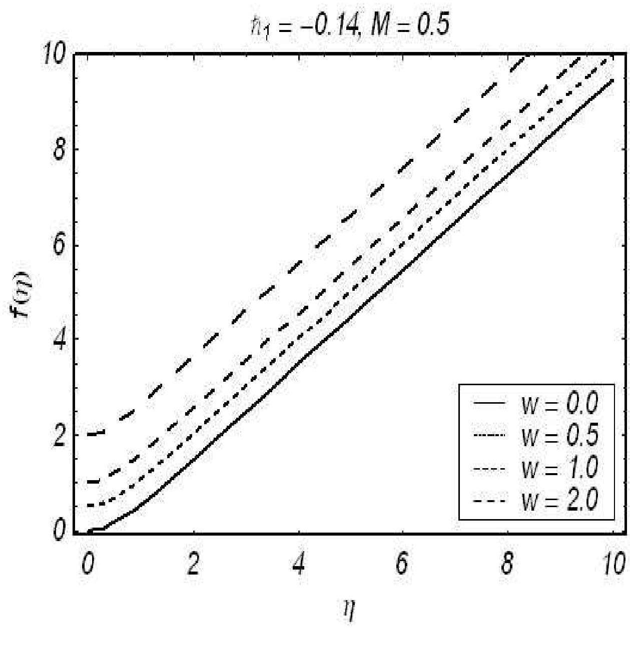

FIGURE 5. $f(\eta)$ for different values of the suction parameter w.

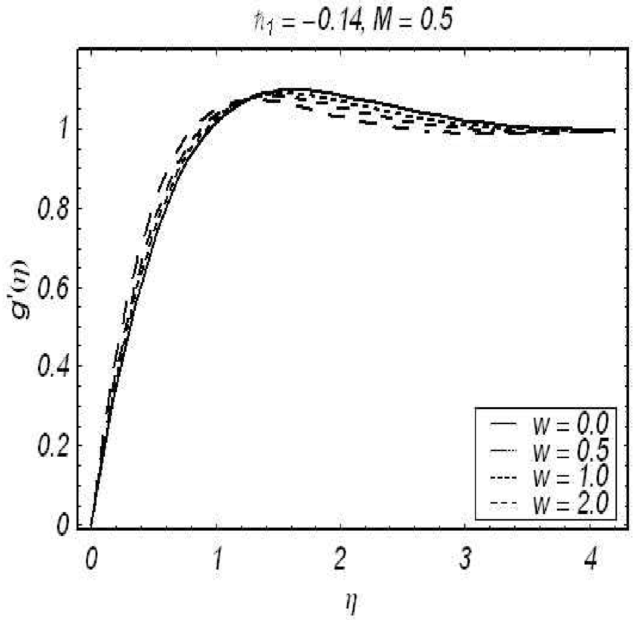

FIGURE 6. Variation of the velocity component $g'(\eta)$ for different values of the suction velocity w.

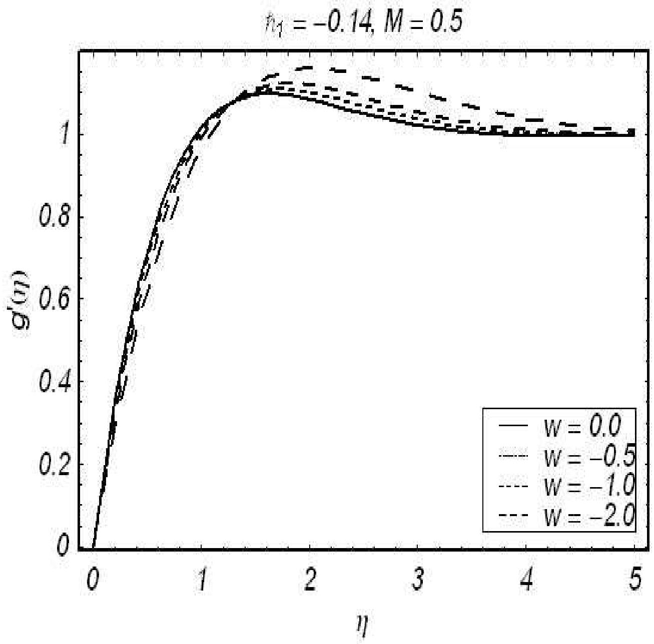

FIGURE 7. Injection effects on the velocity $g'(\eta)$.

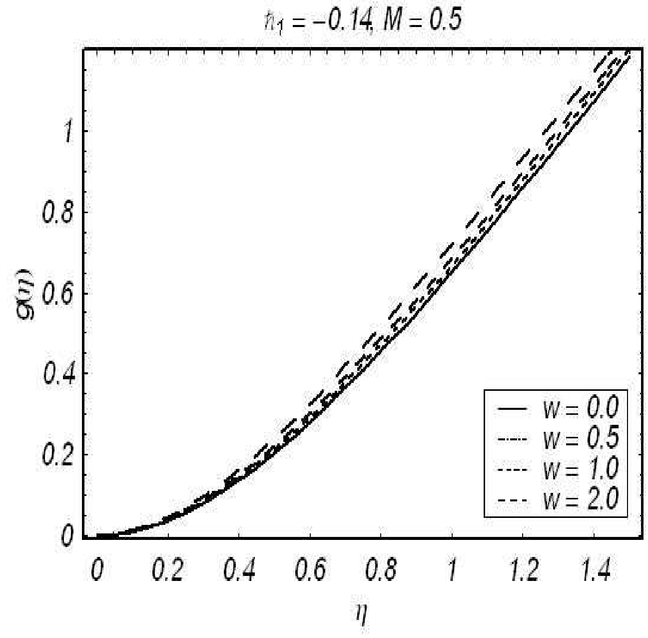

FIGURE 8. $g(\eta)$ for different values of the suction parameter w.

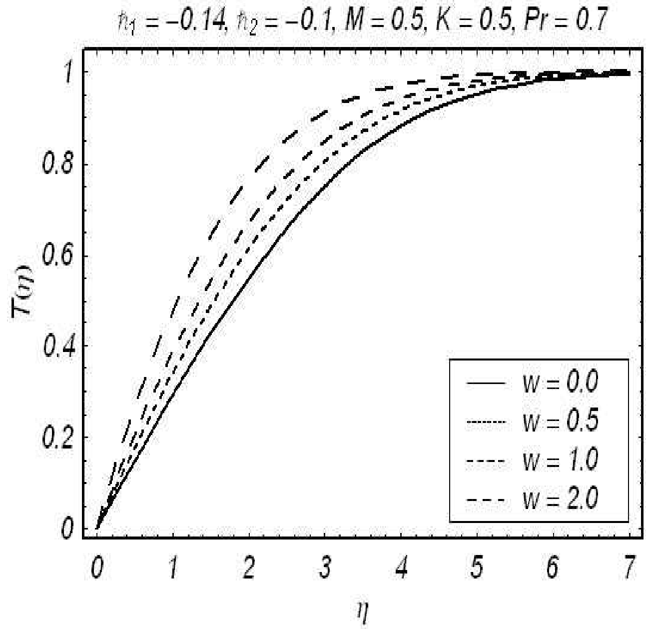

FIGURE 9. Temperature profiles at different w.

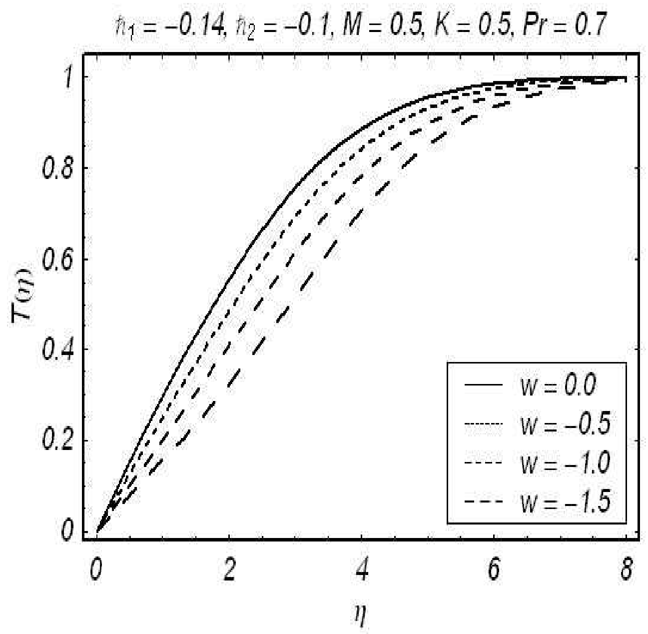

FIGURE 10. Injection effects on the temperature distribution.

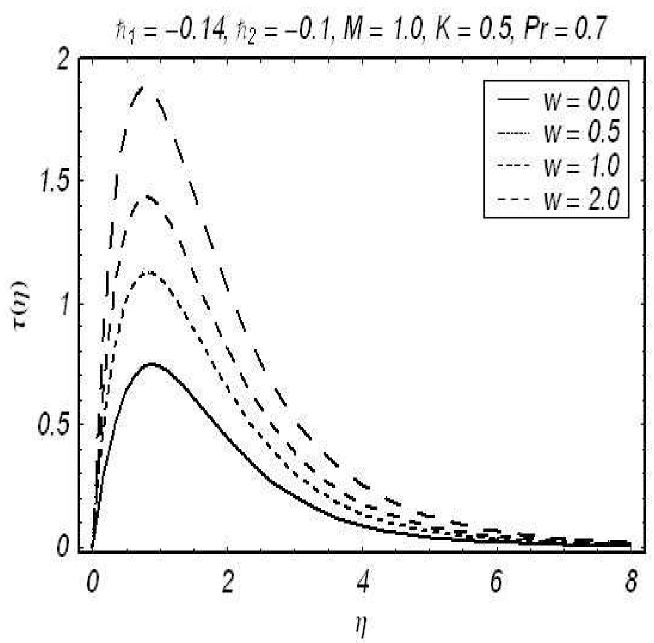

FIGURE 11. Effect of constant suction on $\tau(\eta)$.