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NONLINEARITY IN INFLATION
A Case of Pakistan

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Abstract. Recent research work has shown that inflation rate is asymmetric and it is also well known that asymmetry is a non-linear phenomenon. In order to better understand this non-linearity in inflation of Pakistan, we investigate the possible presence of Smooth Transition Autoregressive (STAR) non-linearity in inflation series. The study finds that month on month inflation series for Pakistan possesses both logistic and exponential STAR type non-linearity. Exponential Smooth transition function was proven to be more relevant on the basis of Dijk *et al.* (2000). Therefore, we develop ESTAR model in this paper which outperforms its linear rivals in forecasting.

I. INTRODUCTION

We usually develop models for forecasting purposes which we use in setting monetary and fiscal policies. Unfortunately, if we look into the history, the forecasting record of economic variables is poor. To some extent this could be owing to random human behaviour or availability of virgin data however rigid structural assumptions of the model may also be responsible for the weak forecasting performance (Moshiri, 1997). For instance if we try to estimate a model with a linear regression whose underlying data generating process (DGP) has a non-linear pattern will generate poor results. In this scenario only non-linear models will likely give better results. Stock and Watson's (1999) prove that simple auto-regressive models, AR (p), out-perform other models, however, AR (p) models have low forecasting power if DGP is nonlinear (Dijk *et al.*, 2000).

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Recent empirical literature shows that the dynamic generating mechanism of inflation rate is asymmetric, *i.e.* its behaviour is different during different phases of business cycle. This means there is a possibility that inflation has a nonlinear data generating process. For example, Shyh-Wei Chen (2010) provides evidence of non-linearity of inflation rate in OECD countries. Similarly Yildirim (2004) in his thesis provides the evidence for non-linearity in Turkish inflation rate and estimates Logistic smooth transition auto-regressive model (LSTAR). Testing and modeling non-linearity in inflation rate has attracted substantial interest because they outperform their linear rivals in forecasting and also proven presence of non-linearity questions many different theories. In the presence of non-linearity in inflation, different theories will have to be re-evaluated for example fisher effect or quantity theory of money etc. If inflation rate is non-linear then traditional unit root tests for stationarity will not work. This implies need to re-test unit root in inflation to estimate co-integration relationship with other macro-economic variables.

Despite the abundance of studies on the behavior of inflation rates in Pakistan, non-linearity has not been considered yet by the existing literature. This study is an attempt to bridge this gap. In order to test the hypothesis of non-linearity, we split annual real GDP growth ranges from 1950 to 2011 into two groups- above and below average growth. Then we try to explore the corresponding inflation rates responses to one standard deviation (SD) of GDP growth in both groups. We observe that one SD above the average brings a change of only 2 basis points while the change is 36 basis points for the below average group, which to some extent support our concept of asymmetry (non-linearity). After establishing this preliminary evidence of non-linearity in inflation we formally test and model the non-linearity phenomenon using the STAR model developed specially to address this issue.

In recent times a number of nonlinear models have been proposed to capture observed asymmetries. Comprehensive surveys are given by Granger and Teräsvirta (1993), Potter (1999) and Dijk *et al.* (2000). The most common nonlinear models are Threshold autoregressive (TAR) models smooth transition autoregressive (STAR) models and Markov-switching regime models. These models are actually set of different linear AR models. In TAR model, AR models change in different regimes which are built via fixed threshold(s). Pining down the threshold (s) is a difficult task. In STAR models we replace this threshold with continuous smooth transition function. In Markov-switching regime models it is assumed that the thresholds are stochastic.

We will focus on STAR models because they are more flexible and have power to allow the possible different dynamics of inflation rate.

II. REPRESENTATION OF STAR MODELS

Generally the two regime smooth transition autoregressive (STAR) model of order p for y_t can be written as

$$y_t = \theta_1' x_t \left(1 - F(y_{t-d} : \gamma, c)\right) + \theta_2' x_t F\left(y_{t-d} : \gamma, c\right) + \varepsilon_t$$

$$\text{Or} \quad y_t = \theta_1' x_t + \theta_3' x_t F(y_{t-d} : \gamma, c) + \varepsilon_t, \quad \varepsilon_t \sim i.i.d(0, \sigma^2) \tag{1}$$

Where $x_t = (1, y_{t-1}, \ldots\ldots, y_{t-p})'$, $\theta_1 = (\theta_{11}, \ldots\ldots, \theta_{1p})'$, $\theta_2 = (\theta_{21}, \ldots\ldots, \theta_{2p})'$, and $\theta_3 = (\theta_2 - \theta_1)$.

$F(s_t : \gamma, c)$ is known as transition function which allows the model to switch between different regimes smoothly. It is bounded between zero and one, *i.e.*

$$0 \le F(y_{t-d} : \gamma, c) \le 1$$

Exogenous variable y_{t-d} is known as transition variable and d is delay parameter. $\gamma > 0$, is smoothness parameter for transition function $F(s_t : \gamma, c)$. c is a location or threshold parameter.

Transition Function, $F(s_t : \gamma, c)$, can have different functional choices. For each choice of transition function, we get different regime switching behaviour. The most common choices are

$$F\left(y_{t-d} : \gamma, c\right) = \left(1 + \exp\left(-\gamma\left(y_{t-d} - c\right)\right)\right)^{-1} \tag{2}$$

$$\text{Or} \quad F\left(y_{t-d} : \gamma, c\right) = \left[1 - \exp\left(-\gamma(y_{t-d} - c)\right)^2\right] \tag{3}$$

The transition functions in equation (2), is a logistic function and in equation (3) is exponential function. The STAR model with logistic transition function is known as Logistic STAR (LSTAR) model and for the exponential functional, it is known as Exponential STAR (ESTAR) model.

The STAR model presented in equation (2) can be estimated if the null hypothesis of “parameters constancy” is rejected. The estimated STAR model might give information about where the parameters change and also that how this change happens. From logistic transition function we can easily see that if $\gamma = 0$, $F(y_{t-d} : 1, c) = 0$ and we will get simple AR (p) model. If $\gamma \to \infty$, we will get AR (p) model with one structural change. The intermediate values of transition function give us combination of two AR (p) models. Therefore, LSTAR modeling is appropriate for asymmetric data.

III. DATA

The economic series we consider in this section represents the month on month (MOM) inflation rate of consumer price index (CPI) of Pakistan, at the monthly frequency covering the period July 1992 until February 2011 (224 observations). The CPI series is obtained from the State Bank of Pakistan.

We use the series up to July 2009 (204 observations) for estimation and reserve 20 observations from August 2009 to February 2011 for forecasting purpose.

IV. METHODOLOGY FOR ESTIMATING CYCLES OF STAR MODELS

In estimation we will follow Teräsvirta (1994) who developed a data-based five steps method for specification, estimation and evaluation of cycles of STAR models. These steps are

1. Specification of Linear AR (p) model
2. Testing Linearity against STAR model
3. Selection of the form of transition function
4. Estimation of the parameters of STAR
5. Evaluation of the model

These step are discussed in detail below:

Specification of Linear AR (p) model

Teräsvirta (1994) recommends to construct AR (p) model for the given time series. This gives basis for estimation and evaluation of non-linear model.

Following Teräsvirta (1994), we first specified the lag length of AR (p) model for month on month CPI inflation rate. Since it is a monthly data we first allow for max of lag 18. From Akaike information criterion (AIC), Schwarz Baysian criterion (SBC), Hannan-Quinn information criterion (HQ), sequential modified LR test statistic (LR) and Final prediction error (FPE) tests statistic show that AR (3) is appropriate model.

We try to estimate AR (3) with the assumption of deterministic seasonality in the data. We assume that the monthly dummy variables can effectively capture systematic component of seasonality.

Let $d_{i,t} = 1$ if $t = i$, otherwise $d_{i,t} = 0$, here $i = 1, 2, \ldots, 11$ represents the months.

After applying various tests we find the most parsimonious model given below:

$$y_t = 0.19y_{t-1} + 0.16y_{t-2} + 0.33y_{t-3} + 0.66d_2 + 0.63d_3 + 0.77d_6 + 0.49d_7 - 0.4d_6 + \varepsilon_t$$

Skewness = –0.03, E.Kurt = 0.53, JB test = 2.4(0.3), SE of regression = 0.59, LM(2) = (0.94), LM(4) = (0.77), LM(8) = (0.73), LM(12) = (0.44), ARCH(1) = (0.24), ARCH(3) = (0.00).

Numbers in parenthesis show the p-value of the corresponding test statistics. LM test is Breusch-Godfrey test for no residual auto-correlation. LM tests for different lags indicate that there is no evidence for auto correlation in residuals. There is no skewness and E- kurtosis problem which is a good news. JB test shows that residuals are normal. All coefficients are significant. LM for no ARCH effect up to lag 3 indicates heteroskedasticity problem which might be due to abrupt changes in MOM inflation after 2008. Over all model looks adequate and ready for the further analysis.

Testing Linearity against STAR Model

In processes of developing STAR model Teräsvirta (1994) recommends to test null hypothesis of linearity in the residuals of the chosen AR (p) model against the alternative of STAR non-linearity.

Consider

$$y_t = \theta_1' x_t + \theta_3' x_t F(y_{t-d} : \gamma, c) + \varepsilon_t, \quad \varepsilon_t \sim i.i.d(0, \sigma^2) \tag{4}$$

To test linearity our null hypothesis is $H_0 : \theta_3 = 0$, against the alternative of $H_1 : \theta_3 \neq 0$.

In other words, we need to test the equivalence of two regimes. Testing procedure face the problem of parameter identification. Teräsvirta (1994) devises an intelligent way to solve the problem. He proposes to replace the transition function with its Taylor approximation. This technique solves the problem of identification of parameters.

Linearity Test in LSTAR Model

The null hypothesis of linearity can be illustrated in different ways. For example,

$$H_0 : \gamma = 0$$

If we approximate $F(y_{t-d} : \gamma, c) = (1 + \exp(-\gamma(y_{t-d} - c)))^{-1}$ by Taylor series around $\gamma = 0$, the resultant third order auxiliary equation can be written as:

$$y_t = \alpha_1' x_t + \alpha_2' x_t y_{t-d} + \alpha_3' x_t y_{t-d}^2 + \alpha_4' x_t y_{t-d}^3 + \varepsilon_t \quad (5)$$

Where $x_t = (1, y_{t-1}, \ldots\ldots, y_{t-p})'$, $\alpha_{i1} = (\alpha_{i1}, \ldots\ldots, \alpha_{ip})'$, $i = 1, \ldots, 4$.

Null hypothesis of linearity can be written as:

$$H_0 : \alpha_{i1} = (\alpha_{i1}, \ldots\ldots, \alpha_{ip})' = (0, \ldots\ldots, 0)', \quad i = 2, \ldots, 4.$$

This is Simple LM test which has χ^2 distribution with 3 $(p + 1)$ degree of freedom.

The LM test statistic can be computed, for different values of delay parameter ranges from 1 to 12, as:

$$LM = T * \frac{(RSS - USS)}{RSS}$$

RSS = Restricted Sum of Squares residuals, *USS* = Unrestricted Sum of Squares of residuals.

We can get *RSS* by simply regressing y_t on x_t and *USS* can be estimated by equation (5).

Linearity Test in ESTAR Model

Saikkonen and Luukkonen (1988) suggests a linearity test against an ESTAR model by approximating equation (1) via first order Taylor series with respect to equation (3) around $\gamma = 0$. The auxiliary regression is

$$y_t = \alpha_1' x_t + \alpha_2' x_t y_{t-d} + \alpha_3' x_t y_{t-d}^2 + \varepsilon_t \quad (6)$$

Null hypothesis of linearity can be written as:

$$H_0 : \alpha_{i1} = (\alpha_{i1}, \ldots\ldots, \alpha_{ip})' = (0, \ldots\ldots, 0)', \quad i = 2, \ldots, 3.$$

This is again Simple LM test which has χ^2 distribution with 2 $(p + 1)$ degree of freedom.

The *p*-values of LM tests to check linearity against LSTAR and ESTAR models are given in Table 1.

Results clearly indicate that linearity can be rejected at 5% significance level at $d = 8$ against LSTAR and ESTAR models. Hence STAR type non-linearity exits. Therefore we can say that inflation rate adjust non-linearly and can be characterized by STAR model.

TABLE 1

Linearity Test Results against LSTAR and ESTAR Model of Inflation

Chi-test (*p*-value)		
d	LSTAR	ELSTAR
1	0.35	0.46
2	0.40	0.39
3	0.56	0.56
4	0.35	0.43
5	0.11	0.21
6	0.12	0.11
7	0.14	0.27
8	0.02	0.03
9	0.32	0.31

Selection of the Form of Transition Function

In building STAR model next step is to choose appropriate type of smooth transition function $F(y_{t-d} : \gamma, c)$. From Table 1 it can be observed that LSTAR and ESTAR type of non-linearity exits in inflation rate series. For the selection of appropriate type of smooth transition function we will follow the Dijk *et al.* (2000) and run the following sequences of null hypothesis in regression presented in equation (5)

1. $H_{01} = \alpha_4 = 0$
2. $H_{02} = \alpha_3 = \frac{0}{\alpha_4} = 0$
3. $H_{03} = \alpha_2 = \frac{0}{\alpha_4} = \alpha_3 = 0$

All null can be tested by LM tests. Dijk *et al.* (2000) shows that (*i*) H_{01} is rejected only if the model is an LSTAR model and (*ii*) H_{01} is accepted but H_{02} is rejected then the model is ESTAR model. Again if H_{01} and H_{02} is accepted but H_{03} is rejected then again it is LSTAR model.

In our case H_{01} is accepted — *p*-value of LM test (0.24) but H_{02} is rejected — *p*-value of LM test (0.0059) which means that ESTAR model is appropriate for inflation data.

To develop STAR model we need stationary series. Since Dickey Fuller unit root test has lower power against ESTAR model. Kapetanios, Shin and Snell (2003) — *KSS* — has developed a testing procedure to detect the presence of non-stationarity for ESTAR models. Under the unit root null hypothesis, the OLS test regression is given below:

$$dy_t = \delta y_{t-1}^2 + \varepsilon_t \tag{7}$$

The test statistic is simply

$$t_{KSS} = \frac{\hat{\delta}}{\sigma_\delta}$$

Where σ_δ is SE of $\hat{\delta}$. Using 5000 stochastic simulations with 1000 observations, *KSS* obtained 1, 5 and 10% asymptotic null critical values of the t statistic as –3.48, –2.93 and –2.66 respectively.

In our case $t_{KSS} = -6.73$ which means that *KSS* test reject unit root for ESTAR model in inflation series. Now we can proceed further.

Estimation of the Parameters of STAR

After the selection of transition function, $F(y_{t-d} : \gamma, c)$, next step in modeling cycle is to estimate the parameters for STAR model.

STAR model can be estimated by non-linear least square (NLS) and if we assume that errors ε_t are normally distributed then NLS is equivalent to maximum likelihood estimate (quasi-maximum Likelihood estimate). High dimensionality in estimating NLS causes computational problems. Leybourne, Newbold and Vougas (1998) suggest a grid search technique to cope with these problems. In this technique, we first fix the γ and c in transition function. When these parameters are known and fixed, the STAR model becomes linear in θ_1 and θ_2 so this can be estimated by simple OLS. This technique, which is conditional on γ and c, reduces dimensionality problem considerably. Adjusting the values of γ and c we try to minimize the sum of square of residuals. Teräsvirta (1994) suggests standardizing the transition function to make γ almost scale free. He also suggests to select c as some percentile of y_{t-d} and γ can be varied between 1 and 200.

The results our model are presented below:

$$\begin{aligned} y_i t = {} & 0.531 D_i 1t + 0.52 D_i 2t - 0.285 D_i 3t + 0.703 D_i 5t + 0.48 D_i 6t - \\ & 0.425 D_i 10t + [0.889 + 0.447 y_i(t-3)]\,[1 - F(y_i(t-d) : \gamma, c)] + \\ & [0.187 y_i(t-1) + 0.17 y_i(t-2) + 0.408 y_i(t-3)]\, F(y_i(t-d) : \gamma, c) \end{aligned}$$

$$F(y_{t-d} : \gamma, c) = 1 + \exp(-1\,(y_{t-8} - 0.3058)^2)$$

$$\text{JB} = 0.11,\ \hat{\sigma} = 0.58,\ \frac{\hat{\sigma}_{STAR}}{\hat{\sigma}_{AR}} = 0.94,\ \text{ARCH}(1) = 0.53.$$

Evaluation of the Model

Next step is the evaluation of the model. We evaluate the model by applying battery of the tests.

For example model is estimated under the assumption of constant parameters and constant variance. To check the constant variance we use the Lagrange Multiplier test of Engle (1982). The probability of LM test is (0.53), which shows that there is no ARCH effect till first lag. To check constant parameters we use stability test proposed by Hansen (1990) which shows that parameters are constants. To test the forecast performance of ESTAR model, we use Meese and Rogoff (1983) – MR – criterion for forecast performance evaluation.

$$MR = \frac{\overline{S}_{uv}}{\sqrt{\frac{1}{n^2}\sum_{i=1}^{n}\left(u_i^2 - v_i^2\right)}} \sim N(0,1)$$

Here $u_i = e_{1i} - e_{2i}$, $v_i = e_{1i} - e_{2i}$, where e_{1i} is the i^{th} forecast residual from bench mark model which is AR (p) model in our case and e_{2i} is i^{th} forecast residual from ESTAR model. $\overline{S}_{uv}$ is covariance of u and v. Our null hypothesis is that there is no improvement in the forecast. Results are –2.002 (0.022) which reject the null hypothesis of no forecast improvement. The forecast improvement of ESTAR model over simple AR (p) model was 62%. The test $\frac{\hat{\sigma}_{STAR}}{\hat{\sigma}_{AR}} = 0.94$ also demonstrated significant improvement in standard deviation of ESTAR model than that of bench mark model. JB test showed that residuals are normal.

V. CONCLUSION

Inflation is a very important indicator of an economy owing to its serious implications for economic growth and income distribution. Therefore, it is a matter of serious concern for the researchers and policy makers. Different models have been developed to analyze the dynamics of inflation rate. For this purpose, usually they use naive a-theoretical time series models. These models perform well only when the series under investigation is linear. So linearity of time series is a crucial assumption in developing a model.

However, empirical evidences show that underlining data generating mechanism of inflation is non-linear. Keeping this in view, the present study uses a class of nonlinear models (STAR models) to investigate possible non-linearity in month on month CPI inflation rate in Pakistan. Analysis shows that we cannot reject LSTAR and ESTAR type nonlinearities in the data. To develop a model, we need to choose one between these two different types of model. Further empirical investigation shows that ESTAR model is more appropriate in our case. We develop ESTAR model and compare it with AR (P) models. We find that model has lower residual variance and has better forecast performance than its linear counter-part – AR (p).

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