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DO EXCHANGE RATES FOLLOW RANDOM WALKS?
An Application of Variance-Ratio Test

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Abstract. This paper tests the random walk hypothesis for foreign exchange market in Pakistan. To proceed with this, Lo and Mackinlay's (1988) variance-ratio tests are used. The study uses weekly five pairs of nominal exchange rate series over the span about 10 years. The analysis indicates that nominal exchange rates follow random walks. These findings are robust to the two sub-sample periods. The presence of a random walk implies that today's exchange rate is the best predictor of the future spot exchange rate. It suggests that economic models of exchange rate determination must be used with caution for predicting exchange rate dynamics.

I. INTRODUCTION

Forex market is the largest financial market in terms of size. The role of forex market in economic development is always remaining debatable and controversial in the development literature. With the passage of time, the importance of the forex market is increasing due to the financial reforms and trade liberalization alike. Fluctuations in exchange rates are of great concern to households, businesses and policymakers. In the 1990s, markets for goods and finance are global. When individuals, business firms, and governments in one country want to trade, borrow, or lend in another country, they have to conduct their transactions in different currencies. Therefore, the abrupt exchange rate movements have a significant impact on individuals'

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economic decisions.[1] Greater exchange rate fluctuations (uncertainty) may increase the value of waiting and hence diminish investment and job creation. A less volatility of exchange rate has positive impact on the demand side of the labor market and makes domestic industries relatively less competitive.

In addition to the above, forex markets have an immediate and direct impact on economic growth. The balance of payments, budget deficit and government policies directly affected by changes in exchange rates. Especially, those economies that rely strongly on remittances of overseas contract workers or tourism are quite sensitive to exchange rate fluctuations. Hence, knowledge of the dynamics of foreign exchange markets is important not only for policy makers and macroeconomic managers in government and banking sector (including leasing companies and other financial institutes) but also for individual businessmen and consumers. Therefore, they would be interested to know whether fluctuations in exchange rates can be predicted by using information about past exchange rates fluctuations or/and other variables like economic growth, inflation, and interest rate.[2] Therefore, the core purpose of this document is to investigate whether the Pakistani foreign exchange market behaves like a random walk vis-à-vis the markets have a tendency to slip back/mean revert.

Prior to observe the behavior of the forex exchange market, it would be useful to consider the recent trend in Pak rupee value relative to foreign currency (particularly to US Dollar). Overtime the value of Pak rupee has declined consistently by 14.13 percent in 1993 through 1995, by 43.28 percent in 1998, by 17.49 percent in 2000, and by 1.31 percent in 2003. Overall, the currency value has declined almost 70.55 percent over eight years (from 1995 to 2002).[3] Based on this historical piece of evidence, can we predict the next year change in the value of currency?

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[1]For example, when the rupee appreciates significantly, Pakistani goods become more expensive abroad and Pakistan exports decline, hurting Pakistan industries. At the same time, Pakistani consumers get benefits from the appreciating rupee because foreign consumer products sold in the Pakistan become more affordable.

[2]According to the Purchasing Power Parity (PPP) that exchange rate can be predicted by using the inflation rate differential between two countries. On the other side, Uncovered Interest Parity (UIP) implies that the interest rate differential between two countries should equal the expected change in the nominal 'spot' exchange rate.

[3]Source: *Global Stock Market Fact Book* 2003.

Of course, in practice, it is very difficult task to foresee future behaviour of exchange rates because at a time a large number of rational profit maximizing traders have attempt to acquire even the smallest informational advantages, and in doing so, they incorporate their information into market value of currency and quickly the gain opportunities that gave rise to their effort. Under this scenario, the market is very unpredictable; exchange rates must fully reflect all available information and no gain can be garnered by using information about past movements. A market said to be "efficient" in which the expected change is independent of all available information. The weak form only considers past information (*i.e.* historical prices). It implies that prices follow a random walk in which successive price increments have zero correlation (*see* for details Trippi and Lee, 1996). This subset of the efficient market hypothesis (EMH) is known as the random walk hypothesis (RWH) (*see* for details Peirson *et al.*, 1995).

The issue whether exchange rates follow random walk or not has been hotly debated between finance practitioner and finance academic for a long time. The random walk hypothesis also has been occupied an important place in modern finance. It is therefore not surprising that it has been widely tested resulting in a large empirical literature with a few studies for developing countries.

In the existing literature, the random walk is usually reported either because a unit root component is detected in the exchange rate series or because the increment in the exchange rates is found to be serially uncorrelated. Since a "unit root" and "uncorrelated increments" are both required for a random walk process (*see* for details, Meese and Singleton, 1982 and Hsieh, 1988; respectively).

In this paper, I examined the random walk hypothesis for the nominal exchange rate series by applying variance-ratio tests developed in Lo and Mackinlay (1988) to five pairs of weekly nominal exchange rate series over the span about 10 years. The present study focuses on the uncorrelated increments aspect. This is not only because there are some important departures from the random walk that unit root test cannot detect, but also because the autocorrelation aspect may yield interesting implications for alternative exchange rates models. In particular, the correlated increment suggests the possibility of either exchange rate overshooting (Dornbusch, 1976) or undershooting (Frenkle and Rodriguez, 1982). However, an uncorrelated increment implies that the series follows random walk.

Despite the exchange rate overshooting or undershooting phenomenon, the random walk in exchange rate has a number of useful implications. The presence of random walk implies that we cannot foresee the behavior of the forex market using past exchange rate movements. Since, the direction and size of changes are independently and randomly chosen from normal distribution (*see* for details Hull, 1995). Therefore, the best prediction is no change; this is known as the naive forecast. In contrast, if exchange rates do not follow random walks, there are sufficient gain opportunities to compensate traders for the cost of information gathering.

An other very important implication of the random walk is that a researcher be interested in testing whether the rate of growth of the exchange rate, call it R_t, is random walk or whether it can be explained by the lagged values of the rates of growth some fundamentals, U_{t-i}. Thus, we can use the following two models to forecast exchange rates:

Model 1: $R_t = \varepsilon_t$

Model 2: $R_t = \beta U_{t-i} + \varepsilon_t$

where ε_t is unpredictable, "Model 1" is the random walk and "Model 2" is the economic model.[4] The presence of the random walks in exchange rates suggests that economic models of exchange rate determination cannot be used to foresee the exchange rate fluctuations. In contrast, if exchange rates do not follow random walk then the economic models do better than the random walk in terms of point forecasts. This analysis has also very useful implications for both hedgers and speculators. Most hedgers do not employ continuous hedges to reduce the currency risk. Instead, they select the times at which they want protection against adverse changes in exchange rates. Definitely, knowledge of market fluctuations can be used for effective hedging. Therefore, it is very important for business firms, households, financial service providers, policy makers, and finance academics to know the behavior of foreign exchange market.

The remainder of the paper is organized as follows: Section II presents the literature survey. Section III demonstrates the data sources. The foreign currency returns estimation, the random walk hypothesis and empirical methodology are also discussed in this section. Section IV presents the empirical results and provides a comparison of different tests results. Section

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[4]Meese and Rogoff (1988) estimated the Model 2 in first-difference form. In their analysis, thus, R_t is the rate of growth of the exchange rate and U_{t-1} is the rate of growth of the interest rate differential.

V discusses the policy implications derived from these results and final section summarizes and concludes.

II. THE CURRENT STATE OF THE ART

Goodhart and Charles (1988) explored some anomalies in the foreign exchange markets. Their results stated that longer-term speculation based on fundamentals is strictly limited and interplay between speculations based on fundamentals, and on a random walk influences the outcomes.

Christina and Jia He (1991) tested the random walk hypothesis by adopting variance-ratio tests under homoscedasticity and heteroscedasticity. They use five pairs of weekly nominal exchange rate series (the Canadian Dollar, British Pound, Japanese Yen, French Franc, and Deutsche Mark, all relative to the US Dollar) over the period from August 7, 1974 to March 29, 1989. Their findings are quite contrary to previous evidence in the literature. They rejected the random walk hypothesis for four out of five (apart from Canadian Dollar) nominal exchange rate series at the five percent significance level.

In addition to this, their evidence is consistent for the two sub-periods from August 7, 1974 to October 10, 1979 and from October 17, 1979 to March 29, 1989. However, the most of the rejections under homoscedasticity are robust to heteroscedasticity; this implies that the variance-ratio is different from one due to autocorrelation, rather than to heteroscedasticity. Furthermore, the results based on the Box-Pierce Q-statistic rejected the random walk only under the null of homoscedasticity.

Richard and David (1996) used the variance ratio tests proposed by Cochrane (1988) to examine the random walk hypothesis for the currencies of eight economies of the Pacific Basin. Their results show that the random walk model is not consistent with the dynamics of daily or weekly exchange rate innovations in the majority of the examined markets.

Swarna and Ghosh (1999) examined the weak and strong forms of the foreign exchange markets efficiency hypothesis for seven major currencies of the European Economic Community (US Dollar, Belgian franc, Danish krone, German mark, French franc, Italian lira, and Netherlandish guilder). They use daily data on the spot and forward exchange rates over the period from January 1973 to April 1994. The results, based on Harris-Inder (1994) methodology, indicated that both forms of the market efficiency hypothesis are soundly rejected for all the major currencies of the European Economic Community (EEC).

Jose and Peter (2004) employed a nested purchasing power parity (PPP), random walk (RW), and uncovered interest parity (UIP) specification to investigate the impact of interest rate liberalization on exchange rate expectations in the Dominican Republic. The analysis employs monthly data on prices and commercial banks' interest rates for Dominican Republic and the USA, and exchange rate data for the Dominican Republic covering the period ranging from January 1992 to December 2002. The findings naked that the Dominican Republic financial market does not follow random walk and exchange rate fluctuations can be predicted by the interest rate differential between the Dominican Republic and the United States.

III. RANDOM WALK MODEL (RWM)

Let consider EX_t, $t = 1, 2, \ldots, T$ denote the closing price of a currency on successive trading days. The random walk model treats a permanent arbitrary drift in the price change from one period to another with a component on increments that are independent and identically distributed. Thus, the random walk model states:

$$EX_{t+i} = EX_t + u + \psi_{t-i} \tag{1}$$

$$EX_{t-i} = u + \psi_{t+i}$$

therefore, the increment in exchange rate in i period as:

$$EX_{t+i} = EX_{t+i} - EX_t$$

where $\psi_{t-i} \approx NID\left(\sigma_\psi^2\right)$, and u is an arbitrary float parameter. The ψ_{t-i} is referred to in time series literature as Gaussian White Noise. The equation (1) indicates that the exchange rate in next period will be equal to the current exchange rate plus some arbitrary drift and stochastic term. We know the stochastic term naturally is unpredictable, therefore, the future exchange rate movements cannot foresee using historical exchange rate movements.[5]

THE RANDOM WALK HYPOTHESIS (RWH)

There are two fundamental implications of the random walk model:

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[5]Random walk model implies no investor can find over or under valued currency using any investment strategy. However, some special causes of abnormal returns are luck, insider trading, and forecast formula but only when proven better than market average.

1. Expected future exchange rate increments are unpredictable in both short and long spans.

2. The variance of a sample is proportional to the sampling interval.

The testing of first hypothesis implies the successive values of a time series are uncorrelated. It means the series has a unit root. Thus, the information about historical changes of a time series is ineffective for prediction of future changes. A policy maker also cannot design an effective policy based on past information to reduce the probability of collapse. Second hypothesis deals with testing the variance of a time series' return is linear in the observation interval. It means the increments are uncorrelated. This hypothesis has several important implications for traders and researchers. It is very important, for a trader, to explore the risk of trading in foreign currency. A trader has interest to know the possibility of profits and losses. Furthermore, it provides information about the pattern of returns.

Our study focuses on testing second hypothesis for Pakistani foreign exchange market. This is not only because there are some important departure from the random walk that unit root test cannot detect, also because the autocorrelation aspect may yield interesting implications for alternative models of exchange rates. For this purpose, the study uses variance-ratio test that was discussed by Lo and Mackinlay (1988).

TESTING METHODOLOGY

1. Box and Pierce Q-Statistic

Box and Pierce (1970) gave Q-statistic as an alternative to various hypothesis of autocorrelation with different time lags. It is defined as:

$$Q_{im} = T\sum_{k=1}^{m} \hat{\rho}_i^2(k) \qquad i = 1, 2, \ldots, N \tag{2}$$

where, $\rho_i\,(k)$ is autocorrelation with k lags and T is the sample size.[6]

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[6] $\hat{\rho}_k = \dfrac{Cov(ER_t, ER_{t-1})}{ER_t}$, where ER_t is the currency return in time t. The standard error of $\rho\,(k)$ is defined as: $SE(\hat{\rho}_k) = (\dfrac{1}{T-k})^{1/2}$.

To test the validity of market efficiency the Q-statistic is tested for various values of m, for the following null hypothesis:

H_0 : All the ρ (k) autocorrelation coefficients are simultaneously equal to zero.[7]

Q-statistic can capture departure from no autocorrelation in either direction and at all time lags (governed by m). It approximately follows (*i.e.*, in large samples) a Chi square (χ^2) distribution with m degrees of freedom. Here m is the number of autocorrelation included in the Q-statistic. For small samples, Ljung and Box (1978) provide a finite-sample correction that yields a better fit to the chi-square distributions,

$$LB_{im} = T(T+2)\sum_{k=1}^{m}\frac{\hat{\rho}_i^2(k)}{T-K} \qquad i = 1, 2, \ldots, N \qquad (3)$$

2. The Variance-Ratio Tests

We employ variance-ratio tests, developed by Lo and Mackinlay (1988), to test the both homoscedastic and heteroscedastic random walk hypothesizes for Pakistani foreign exchange market.[8] The random walk null hypothesis suggests that the variance of a sample is linearly associated with sampling interval. Hence, the variance of the q-period return is must be equal to the q times the variance of the one-period return.

$$Var(EX_t^q) = q \times Var(EX_t) \qquad (4)$$

or $$\frac{Var(EX_t^q)}{q \times Var(EX_t)} = 1$$

where q is any integer greater than one. The alternative hypothesis will be the ratio of the variance of the q-period return to the variance of the 1-period

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[7]It implies that the return distribution is the identical independently distribution (IID), in our context, the series follows random walk.

[8]This is because the random walk model is a proper subset of the unit root null hypothesis. Moreover, even though the variance-ratio test and unit root test are direct competitors, it is shown by Lo and Mackinlay (1989) that the variance-ratio test is more reliable than Dickey-Fuller test, which is developed in Dickey and Fuller (1979) for detecting the unit root component and adopted in Meese and Singleton (1982) for examining the exchange rate series.

return divided by q is not equal to 1.[9] To explain the variance-ratio test, let EX_t be the natural log of nominal exchange rate at time t (*i.e.*, $EX_t \equiv \ln(P_t)$, where P_t is a exchange rate). A simple recursive relation as:

$$EX_t = \alpha + EX_{t-1} + \xi_t \tag{5}$$

where α is an arbitrary drift parameter and ξ_t is the random disturbance term. We assume that, $E\{\xi_t\} = 0$. Suppose that we obtain $nq + 1$ observation EX_0, EX_1, …, EX_{nq}, where q is any integral greater than 1 and consider the following estimators for the unknown parameters u and δ_0^2:

$$\hat{U} = \frac{1}{nq}\sum_{k=1}^{nq}(EX_k - EX_{k-1}) = \frac{1}{nq}(EX_{nq} - EX_0) \tag{6}$$

$$\hat{\delta}_a^2 \equiv \frac{1}{nq}\sum_{k=1}^{nq}(EX_k - EX_{k-1} - \hat{U})^2 \tag{7}$$

$$\hat{\delta}_b^2(q) \equiv \frac{1}{nq}\sum_{k=1}^{n}(EX_{qk} - EX_{qk-q} - q\hat{U})^2 \tag{8}$$

Now,

$$J_d(q) = \frac{\hat{\delta}_b^2(q)}{\hat{\delta}_a^2} - 1$$

Under the finite-sample properties, the $J_d(q)$ test will convert in more powerful test:

$$M_r(q) = \frac{\hat{\delta}_c^2(q)}{\hat{\delta}_a^2} - 1$$

where

$$\hat{\delta}_c^2(q) \equiv \frac{1}{nq^2}\sum_{k=q}^{nq}(EX_k - EX_{k-q} - q\hat{U})^2 \tag{9}$$

[9]While this variance-ratio would be exactly equal to one only under homoscedasticity, it still approaches one under the specification of the heteroscedasticity in Lo and Mackinlay (1988).

This differs from the estimator $\hat{\delta}_c^2(q)$ since this sum contains $nq - q + 1$ term, whereas the estimator $\hat{\delta}_n^2(q)$ contains on n terms. Finally, by using the unbiased variance estimators, the M-statistic is defined as:[10]

$$\overline{M}_r(q) = \frac{\overline{\delta}_c^2(q)}{\overline{\delta}_n^2} - 1$$

where

$$\overline{\delta}_a^2 = \frac{1}{nq-1}\sum_{k=1}^{nq}\left(EX_k - EX_{k-1} - \hat{U}\right)^2 \tag{10}$$

$$\overline{\delta}_c^2(q) = \frac{1}{m}\sum_{k=q}^{nq}\left(EX_k - EX_{k-q} - q\hat{U}\right)^2 \tag{11}$$

$$m = q(nq - q + 1)\left(1 - \frac{q}{nq}\right) \tag{12}$$

For an aggregate value q of 2, the $\overline{M}_r(q)$ can expend as:

$$\overline{M}_r(q) = \hat{\rho}(1) - \frac{1}{4n\overline{\delta}_c^2}\left\{\left(EX_1 - EX_0 - \hat{U}\right)^2 + \left(EX_{2n} - EX_{2n-1} - \hat{U}\right)^2\right\} \cong \hat{\rho}(1) \tag{13}$$

Hence, for $q = 2$ the $\overline{M}_r(q)$ statistic is approximately the first-order autocorrelation coefficient estimator $\hat{\rho}(1)$ of the differences. More generally, it may be shown that

$$\overline{M}_r(q) \cong \frac{2(q-1)}{q}\hat{\rho}(1) + \frac{2(q-2)}{q}\hat{\rho}(2) + \ldots + \frac{2}{q}\hat{\rho}(q-1)$$

$$\overline{M}_r(q) \cong 2\sum_{k=1}^{q-1}\left(1 - \frac{k}{q}\right)\hat{\rho}(k)$$

where $\hat{\rho}(k)$ is the K^{th} order autocorrelation coefficient estimator of the first

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[10] $\overline{M}_r(q) + 1 = \frac{\overline{\delta}_c^2(q)}{\overline{\delta}_a^2}$ is called variance-ratio and generally denoted by VR(q).

differences of EX_t.[11] Hence, the variance-ratio can be written in terms of the autocorrelation function (ACF) for the returns – it is simply a declining weighted sum of the first $q-1$ autocorrelation coefficient estimators of the first differences (returns).

3. Testing the Random Walk Hypothesis

The null and alternative hypotheses define as:

$$\text{H}_0\text{:}\quad \frac{\overline{\delta}_c^2(q)}{\overline{\delta}_a^2} = 1 \text{ (series follow random walk)}$$

$$\text{H}_a\text{:}\quad \frac{\overline{\delta}_c^2(q)}{\overline{\delta}_a^2} \neq 1 \text{ (series does not follow random walk[12])}$$

After deriving an asymptotic distribution of the variance ratios, two alternative statistics are derived to test the null hypothesis for different specifications of error term behavior.

1. The Homoscedastic Standard Normal Test-Statistic, Z (q)

This test statistic considers an independent and identical distributed normal error term. Therefore, the standard normal test statistic for homoscedastic increments is computed as follows:

$$Z(q) \doteq \frac{\overline{M}_r(q)}{\{\eta(q)\}^{1/2}} \overset{a}{\approx} N(0,1)$$

where $\eta(q)$ is the asymptotic variance of variance-ratio under homoscedasticity, defined as:

$$\eta(q) = \frac{2(2q-1)(q-1)}{3q(nq)}$$

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[11] However, the Box-Pierce Q-statistic is a linear combination of squared autocorrelations with all the weights set identically equal to unity.

[12] i.e., $\frac{\overline{\delta}_c^2(q)}{\overline{\delta}_a^2} < 1$ or $\frac{\overline{\delta}_c^2(q)}{\overline{\delta}_a^2} > 1$ is mean reversion.

2. The Heteroscedastic Standard Normal Test-Statistic, Z* (q)

A rejection of the Random Walk Hypothesis because of heteroscedasticity would not be of much interest. Hence, to avoid this, the heteroscedasticity-consistent standard normal test statistic is employed, which relaxed the assumption of normality. The heteroscedasticity-robust test statistic is defined as follows:

$$Z^*(q) = M_r(q)/\sqrt{\hat{\theta}(q)} \overset{a}{\approx} N(0,1)$$

where $\hat{\theta}(q)$ is the heteroscedasticity-consistent asymptotic variance of the variance ratio:

$$\hat{\theta}(q) = \sum_{j=1}^{q-1}\left[\frac{2(q-j)}{q}\right]^2 \hat{\vartheta}(j)$$

or $$\hat{\theta}(q) = 4\sum_{j=1}^{q-1}\left[1-\frac{j}{q}\right]^2 \hat{\vartheta}(j)$$

where

$$\hat{\vartheta}(j) = \frac{\sum_{k=j+1}^{nq}\left(EX_k - EX_{k-1} - \hat{U}\right)^2\left(EX_{k-j} - EX_{k-j-1} - \hat{U}\right)^2}{\left[\sum_{k=1}^{nq}\left(EX_k - EX_{k-1} - \hat{U}\right)^2\right]^2}$$

In this study, both homoscedasticity test statistic, $Z(q)$ and heteroscedasticity-robust test statistic, $Z^*(q)$[13] are employed.

4. The Data

To test the random walk in exchange rate series, we cover the 383-week time span from January 1, 1995 to May 31, 2004 for five pairs of nominal exchange rate series (Pak rupee/US dollar, Pak rupee/Japanese yen, Pak

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[13]If the random walk hypothesis is rejected under homoscedasticity and is accepted under heteroscedasticity then we can say the series does not follow random walk due to heteroscedasticity. In contrast, if the rejection of the random walk hypothesis is consistent under homoscedasticity and heteroscedasticity tests statistic, then the series does not follow random walk due to autocorrelations of increments.

rupee/British pound, Pak rupee/Deutsche mark, and Pak rupee/French franc). The weekly currency returns are derived from daily currency exchange rate. The weekly return of each exchange rate pair is computed as the return from Wednesday's closing exchange rate to the following Wednesday's close. If the following Wednesday's exchange rate is missing, then Thursday's exchange rate is used. If Thursday's exchange rate is missing the return for that week is reported as missing. The data on exchange rates are obtained from various issues of the Monthly Statistical Bulletin of the State Bank of Pakistan.

IV. EMPIRICAL RESULTS AND ANALYSIS

This section presents all the empirical results of the tests that had been conducted to investigate the random walk in exchange rate series. The variance-ratio tests performed at different interval length for entire sample period and for two sub-periods will be reported and analyses in section 1. The section 2 will report and analyze the coefficients of autocorrelation, the Box-pierce Q-statistic, and Ljung-Box Q-statistic at different lag lengths.

1. Results from Variance-Ratio Tests

Using a base observation period of one week, the variance ratios and homoscedasticity test statistic $Z(q)$, and the variance ratios and heteroscedasticity test statistic $Z^*(q)$ are reported in Table 1a and 1b respectively. The actual variance ratios are presented in the main rows in each table, the *Z*- and *Z**-statistics are given in parentheses in Table 1a and 1b, respectively. Each row presents results for one exchange rate series.

Table 1a indicates that under the assumption of homoscedasticity the four out of the five nominal exchange rate series follow random walk over the entire sample period. The null hypothesis of random walk, however, is rejected (at one percent significance level) for the French franc for all four values q examined. Moreover, there is positive autocorrelation only in the case of the dollar and the Japanese yen exchange rate series and the remainder three exchange rate series contain negative autocorrelation.

However, there is no evidence to reject the null hypothesis of random walk for the French franc when the test statistic is corrected for heteroscedasticity (*see* Table 1b). It is indicating that the French franc does not follow random walk due to heteroscedasticity (*i.e.*, variance of increments changes overtime). It is found that these evidences are strongly consistent (however, now the variance ratio for the dollar is less than unity) for the sub-period from January 1, 1995 to May 27, 1998 as reported in Table 2a and 2b.

TABLE 1a

Estimates of Variance-Ratios VR(q) and
Variance-Ratio Test Statistics $Z(q)$

Variance-ratio test of the random walk hypothesis for five weekly exchange rate series, for the sample period from January 1, 1995, to July 31, 2004. One-week is taken as a base observation interval. The variance ratios $1+\overline{M}_r(q)$ are repotted in the main rows, with the homoscedasticity test statistic $Z(q)$ given in parentheses immediately below each main row. Under the random walk null hypothesis the value of the variance ratio is 1 and the test statistics have a standard normal distribution (asymptotically). Test statistics marked with one asterisks and with two asterisks indicate that the corresponding variance ratios are statistically different from 1 at the one percent and 5 percent levels of significance, respectively.

Currency	Number nq of base observation	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
Pak Rupee/US Dollar	420	1.04 (0.74)	0.99 (–0.06)	1.09 (0.61)	1.24 (1.08)
Pak Rupee/British Pound	420	0.99 (–0.03)	0.97 (–0.35)	0.94 (–0.41)	0.85 (–0.65)
Pak Rupee/Japanese Yen	420	1.02 (0.35)	1.01 (1.04)	1.05 (0.33)	0.91 (–0.39)
Pak Rupee/Deutsche Mark	420	0.98 (–0.24)	1.01 (–0.13)	1.05 (0.36)	0.99 (–0.04)
Pak Rupee/French Franc	420	0.68 (–6.30)*	0.49 (–5.32)*	0.31 (–4.54)*	0.23 (–3.42)*

TABLE 1b

Estimates of Variance-Ratios VR(q) and
Heteroscedasticity-Robust Variance-Ratio Test Statistics $Z^*(q)$

Variance-ratio test of the random walk hypothesis for five weekly exchange rate series, for the sample period from January 1, 1995, to July 31, 2004. One-week is taken as a base observation interval. The variance ratios $1+\overline{M}_r(q)$ are repotted in the main rows, with the heteroscedasticity-robust test statistic $Z^*(q)$ given in parentheses immediately below each main row. Under the random walk null hypothesis the value of the variance ratio is 1 and the test statistics have a standard normal distribution (asymptotically). Test statistics marked with one asterisks and with two asterisks indicate that the corresponding variance ratios are statistically different from 1 at the one percent and 5 percent levels of significance, respectively.

Currency	Number nq of base observation	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
Pak Rupee/US Dollar	420	1.04 (0.58)	0.99 (–0.06)	1.09 (0.60)	1.24 (1.17)
Pak Rupee/British Pound	420	0.99 (–0.02)	0.97 (–0.31)	0.94 (–0.39)	0.85 (–0.66)
Pak Rupee/Japanese Yen	420	1.02 (0.53)	1.01 (1.19)	1.05 (0.23)	0.91 (–0.26)
Pak Rupee/Deutsche Mark	420	0.98 (–0.23)	1.01 (–0.13)	1.05 (0.37)	0.99 (–0.04)
Pak Rupee/French Franc	420	0.68 (–1.30)	0.49 (–1.38)	0.31 (–1.56)	0.23 (–1.59)

TABLE 2a

Estimates of Variance-Ratios VR(q) & Variance-Ratio Test Statistics Z(q) for Subperiod Weekly: 01-01-1995 to 27-05-1998

Variance-ratio test of the random walk hypothesis for five weekly exchange rate series. One-week is taken as a base observation interval. The variance ratios $1+\overline{M}_r(q)$ are repotted in the main rows, with the homoscedasticity test statistic $Z(q)$ given in parentheses immediately below each main row. Under the random walk null hypothesis the value of the variance ratio is 1 and the test statistics have a standard normal distribution (asymptotically). Test statistics marked with one asterisks and with two asterisks indicate that the corresponding variance ratios are statistically different from 1 at the one percent and 5 percent levels of significance, respectively.

Currency	Number nq of base observation	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
Pak Rupee/US Dollar	174	0.99 (–0.05)	0.97 (–0.23)	0.98 (–0.09)	0.97 (–0.09)
Pak Rupee/British Pound	174	0.99 (–0.05)	1.07 (0.51)	1.57 (0.69)	1.00 (0.003)
Pak Rupee/Japanese Yen	174	0.97 (–0.42)	1.04 (0.29)	1.23 (0.57)	0.86 (–0.42)
Pak Rupee/Deutsche Mark	174	0.94 (–0.82)	1.00 (0.07)	1.08 (0.37)	0.94 (–0.19)
Pak Rupee/French Franc	174	0.58 (–5.51)*	0.33 (–4.70)*	0.20 (–3.58)*	0.12 (–2.64)*

Table 2b

Estimates of Variance-Ratios VR(q) and Heteroscedasticity-Robust Variance-Ratio Test Statistics $Z^*(q)$ for Subperiod Weekly: 01.01.1995 to 27.05.1998

Variance-ratio test of the random walk hypothesis for five weekly exchange rate series. One-week is taken as a base observation interval. The variance ratios $1+\overline{M}_r(q)$ are repotted in the main rows, with the heteroscedasticity-robust test statistic $Z^*(q)$ given in parentheses immediately below each main row. Under the random walk null hypothesis the value of the variance ratio is 1 and the test statistics have a standard normal distribution (asymptotically). Test statistics marked with one asterisks and with two asterisks indicate that the corresponding variance ratios are statistically different from 1 at the one percent and 5 percent levels of significance, respectively.

Currency	Number nq of base observation	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
Pak Rupee/US Dollar	174	0.99 (–0.19)	0.97 (–0.90)	0.98 (–0.23)	0.97 (–0.17)
Pak Rupee/British Pound	174	0.99 (–0.07)	1.07 (0.65)	1.57 (0.85)	1.00 (0.004)
Pak Rupee/Japanese Yen	174	0.97 (–0.41)	1.04 (0.30)	1.23 (0.58)	0.86 (–0.43)
Pak Rupee/Deutsche Mark	174	0.94 (–0.85)	1.00 (0.07)	1.08 (0.39)	0.94 (–0.21)
Pak Rupee/French Franc	174	0.58 (–1.21)	0.33 (–1.28)	0.20 (–1.30)	0.12 (–1.33)

TABLE 3a

Estimates of Variance-Ratios VR(q) and Variance-Ratio Test Statistics $Z(q)$ for Subperiod Weekly: 27.05.1998 to 31.07.2004

Variance-ratio test of the random walk hypothesis for five weekly exchange rate series. One-week is taken as a base observation interval. The variance ratios $1+\overline{M}_r(q)$ are repotted in the main rows, with the homoscedasticity test statistic $Z(q)$ given in parentheses immediately below each main row. Under the random walk null hypothesis the value of the variance ratio is 1 and the test statistics have a standard normal distribution (asymptotically). Test statistics marked with one asterisks and with two asterisks indicate that the corresponding variance ratios are statistically different from 1 at the one percent and 5 percent levels of significance, respectively.

Currency	Number nq of base observation	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
Pak Rupee/US Dollar	246	1.07 (0.98)	1.03 (0.27)	1.19 (0.91)	1.53 (1.72)
Pak Rupee/British Pound	246	1.00 (0.11)	0.92 (–0.59)	0.83 (–0.83)	0.84 (–0.51)
Pak Rupee/Japanese Yen	246	1.03 (0.49)	1.12 (0.96)	1.05 (0.25)	0.94 (–0.18)
Pak Rupee/Deutsche Mark	246	1.04 (0.54)	1.05 (0.36)	1.09 (0.48)	1.15 (0.48)
Pak Rupee/French Franc	246	0.91 (–1.25)	0.88 (–0.84)	0.61 (–1.88)	0.43 (–1.86)

Table 3b

Estimates of Variance-Ratios VR(q) and Heteroscedasticity-Robust Variance-Ratio Test Statistics $Z^*(q)$ for Subperiod Weekly: 27.05.1998 to 31.07.2004

Variance-ratio test of the random walk hypothesis for five weekly exchange rate series. One-week is taken as a base observation interval. The variance ratios $1+\bar{M}_r(q)$ are repotted in the main rows, with the heteroscedasticity-robust test statistic $Z^*(q)$ given in parentheses immediately below each main row. Under the random walk null hypothesis the value of the variance ratio is 1 and the test statistics have a standard normal distribution (asymptotically). Test statistics marked with one asterisks and with two asterisks indicate that the corresponding variance ratios are statistically different from 1 at the one percent and 5 percent levels of significance, respectively.

Currency	Number nq of base observation	Number q of base observations aggregated to form variance ratio			
		2	4	8	16
Pak Rupee/US Dollar	246	1.07 (0.69)	1.03 (0.22)	1.19 (0.82)	1.53 (1.74)
Pak Rupee/British Pound	246	1.00 (0.08)	0.92 (–0.49)	0.83 (–0.76)	0.84 (–0.50)
Pak Rupee/Japanese Yen	246	1.03 (0.95)	1.12 (1.29)	1.05 (0.21)	0.94 (–0.12)
Pak Rupee/Deutsche Mark	246	1.04 (0.48)	1.05 (0.34)	1.09 (0.49)	1.15 (0.49)
Pak Rupee/French Franc	246	0.91 (–0.89)	0.88 (–0.72)	0.61 (–1.54)	0.43 (–1.19)

The results, presented in Tables 3a and 3b, for the subperiod May 28, 1998 to July 31, 2004, are slightly different. Both the test statistics, homoscedasticity and corrected for heteroscedasticity, indicate that the random walk null hypothesis is accepted at 5% level of significance for all the five nominal exchange rate series. The estimated variance ratio for the dollar, the Japanese yen, and the Deutsche mark is grater than one when $q = 2$, 4, 8, and 16, implying that there is positive autocorrelation of weekly increments in the exchange rate series. For dollar, the autocorrelation is 7 percent, and 3 percent and 4 percent for the Japanese yen and for the Deutsche mark, respectively.

2. Results from Box-Pierce and Ljung-Box Q-Statistics

To provide further evidence about testing the random walk hypothesis for the Pakistani foreign exchange market, the standard Box-Pierce Q(q) statistic and Ljung-Box Q(q) statistic also applied to the same data set used in this analysis. To generate a test which is comparable with our previous variance-ratio tests, the Box-Pierce Q(q) and Ljung-Box Q(q) statistics are estimated at lags 5, 10 , and 15.[14]

The first 15 autocorrelation coefficients, the Box-Pierce and Ljung-Box Q(q) statistics (when $q = 5$, 10, and 15) for the first order differences of weekly nominal exchange rate series over the period from January 1, 1995 to July 31, 2004 are given in Table 4. The null hypothesis of random walk is rejected for the France franc at all values q examined not only based on the estimated standard Box-Pierce Q statistic but also based on the estimation of the Ljung-Box Q statistic. The random walk null hypothesis, however, for the Japanese yen is rejected by the Box-Pierce test only when $q = 5$ and by the Ljung-Box test when $q = 5$, and 10. Both the Box-Pierce test and the Ljung-Box test do not provide any rejection for any remainder exchange rate series.

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[14]Since the variance-ratio computed with an aggregate value q is approximately a linear combination of the first $q - 1$ autocorrelation coefficients, it is comparable with the box-Pierce Q(q) statistic of order $q - 1$, which is a linear combination of the first $q - 1$ squared autocorrelations. If the series increments exhibit small autocorrelations of the same sign for many lags, it is more likely that the Box-pierce Q(q) statistic does not reject the null hypothesis while the variance-ratio test does. However, if the autocorrelations are large and of varying signs, then the Box-Pierce Q(q) statistic my be more powerful than the variance-ratio test.

TABLE 4

Autocorrelation Coefficients and Box-Pierce and Ljung-Box Q(q) Statistics

Autocorrelation coefficients, Box-Pierce and Lung-Box Q(q) statistics for the first order difference of weekly nominal exchange rate series, for the sample period from January 1, 1995 to July 31, 2004 are reported in each column.

Lag	US Dollar	British Pound	Japanese Yen	France Franc	Deutsche Mark
1	0.032	–0.007	0.014	–0.326	–0.016
2	–0.026	–0.023	0.033	0.034	0.037
3	–0.080	–0.027	0.068	0.015	–0.016
4	0.076	0.034	0.030	–0.069	0.024
5	0.042	–0.009	–0.176	–0.022	–0.020
6	0.081	–0.043	0.053	0.018	0.036
7	–0.019	–0.051	–0.031	–0.015	–0.042
8	0.022	0.011	–0.039	0.007	0.001
9	0.006	0.021	0.009	–0.013	0.000
10	–0.040	0.007	–0.062	0.036	–0.029
11	0.034	0.014	0.012	–0.034	0.065
12	0.029	–0.081	–0.028	0.010	–0.091
13	0.018	–0.011	0.011	–0.018	–0.064
14	–0.052	–0.055	0.054	0.000	0.040
15	0.027	0.015	–0.018	0.010	–0.005
Box-Pierce Q(5)	5.990	0.974	14.471**	43.241**	1.094
Box-Pierce Q(10)	9.453	2.912	18.002	44.031**	2.588
Box-Pierce Q(15)	11.656	6.791	19.644	44.675**	9.569
Ljung-Box Q(5)	6.062	0.969	14.661**	43.43**	1.106
Ljung-Box Q(10)	9.608	2.955	18.256**	44.235**	2.634
Ljung-Box Q(15)	11.885	6.997	19.998	44.907**	9.870

**Significant at the five percent level of significance.

V. ECONOMIC IMPLICATIONS

One main advantage of the variance-ratio test is to report the average level of autocorrelations of increments. Since the variance-ratio minus one is approximately $q - 1$ times the weighted sum of the first $q - 1$ autocorrelation coefficients. There are a number of alternative explanations for autocorrelation, including an exchange rate undershooting or overshooting phenomenon, risk aversion, and official interventions in foreign exchange markets. With government interventions, the increments in exchange rates may be positively or negatively correlated, depending on the objective of the intervention policy.[15] Below the possibility of overshooting or undershooting is briefly defined.

Greater than one magnitude of the estimated variance-ratio suggests a positive serial correlation in the return series.[16] Due to the positive serial correlation, the rejection of the random walk hypothesis may attribute to the exchange rate undershooting phenomenon. In contrast, if the estimated variance-ratio is less than unity then the rejections of the null hypothesis of random walk may characteristic to the exchange rate overshooting phenomenon.

We find positive, however, statistically insignificant autocorrelations for the US dollar and for the Japanese yen and negative autocorrelations for the British pound over the entire sample period. However, statistically significant less than unity variance-ratio obtained for the French franc, which suggests negative serial correlation. Thus, the rejection of the null hypothesis of random walk is the evidence of overshooting exchange rate phenomenon.

VI. CONCLUSION AND RECOMMENDATIONS

This study contributes to the growing literature on random walk in exchange rate by examining the foreign exchange market for Pakistan. The random walk hypothesis is tested by adopting a variance-ratio test developed by Lo and Mackinlay (1988). The study uses weekly data for five pairs of nominal exchange rate series (the US dollar, the French franc, the Japanese yen, the Deutsch mark, and the British pound) over the span from January 1, 1995 to July 31, 2004. Furthermore, the same methodology is employed for the two sub-periods from January 1, 1995 to May 27, 1998, and from May 28, 1998 to July 31, 2004.

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[15] Pakistan pursued a floating exchange rate policy over the period from January 1982 to July 21, 1998. In order to minimize the adverse effects of economic sanctions, government imposed a dual exchange rate system in July 22, 1998. The dual exchange rate system was replaced with market-based unitary exchange rate system in May 19, 1999. In July 21, 2000, however, the market-based unified exchange rate system was also replaced with free-floating exchange rate regime.

[16] The variance ratio equal to unity, however, implies there is no autocorrelation in return series.

The random walk null hypothesis is rejected only for one (the French franc) out of the five currencies tested at all aggregation levels (2, 4, 8, and 16). However, there is no evidence to reject the null hypothesis of random walk for the French franc when the test statistic is corrected for heteroscedasticity. It is indicating that the French franc does not follow random walk due to heteroscedasticity. The analysis found that these evidences are strongly consistent for the first sub-period. For the second sub-period, the random walk null hypothesis is accepted at 5% level of significance for all the examined series.

To provide comparisons, the Box-Pierce and Ljung-Box Q tests are also applied to the same data set employed in this study. The variance-ratio test, the Box-pierce Q test, and the Ljung-Box Q test agree with one another under the assumption of homoscedasticity for the entire sample period. The study thus concludes that nominal exchange rates follow random walk for the entire sample period as well as for both the sub-periods. The presence of a random walk in the Pakistani forex market suggests that economic models of exchange rate determination are useless for explaining exchange rates fluctuations. It implies that today's spot price of currency is the best predictor of the future spot price of that currency.

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