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FUZZY REGRESSION APPROACH FOR MEASURING POVERTY IN PAKISTAN

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Abstract. Poverty is perceived to be vague in terms of (i) making judgement about who is to be considered as poor, (ii) selecting the relevant dimensions and indictors. Any benchmark selection to identify poor remains somewhat arbitrary and the vagueness exists irrespective of whether the conventional or non-conventional poverty measure is used. To address the issue of vagueness, this study employs fuzzy regression as a natural alternative to the conventional approach. The fuzzy logic assigns degree of membership to a set of poor people on a scale from 0 to 1 instead of the rigid dichotomization. To cater the second issue, we measure the welfare level of individuals using Engel curve method as it gives a lot of information regarding the consumption behavior of consumers at different levels of total expenditures and for various family compositions. Pakistan Social and Living Standards Measurement (PSLM) survey 2015-16 is used to estimate poverty. Findings reveal that poverty estimates vary significantly across the provinces and regions. Overall, highest incidence of poverty is observed in Balochistan followed by Sindh, KPK is the least poor province. Poverty is not only a rural but a provincial phenomenon as well in Pakistan.

Keywords: Engel Curves; Dichotomization; Identification; Unidimensional Poverty

JEL Classification: C02, C18, I32

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I. INTRODUCTION

The first sustainable development goal of the World is to end extreme poverty in its all forms by 2030 (United Nations, 2015). Since 1990s, sustained decline in global poverty is observed till 2017, but the deceleration in the poverty reduction rate is reported in 2018 making it hard to reach 2030 target of 3 percent global poverty. Almost 10 percent of the world is still living in extreme poverty (World Bank, 2020). As per the national poverty line, 24.3 percent of the population in Pakistan lives in poverty (ADB, 2020). Poverty eradication strategies rely on the estimation of poverty trends at national and sub-national levels. Therefore, several methods are devised to measure poverty. On balance, we may categorize the poverty measurement methods into unidimensional and multidimensional approaches. Unidimensional or conventional approach to the measurement of poverty stresses only on one variable, usually the income while multidimensional approach utilizes several indicators to obtain more exhaustive and useful measure (Costa, 2003). Both the conventional and multidimensional poverty measures evolved from distribution insensitive (unidimensional: head count index, poverty gap, income gap; multidimensional: Alkaire & Foster MPI) to sensitive measures (unidimensional: square poverty gap, Watt index, average exit time; multidimensional: Datt's MDPI). The distribution sensitivity implies that the poverty measure is convex in deprivations, i.e., the value of the measure increases if transfer is made from relatively more to a less poor person (Najam, 2020). Not only the evolution of these measures but the debate regarding poverty threshold and selecting the welfare dimensions and indicators is also same.

For unidimensional or conventional poverty measures, the debate is on setting the benchmark value, i.e., poverty line. In the same way, for multidimensional poverty measures debate is on setting the poverty cut offs at dimensional and indicator levels [see, e.g., Naveed & Islam (2012), Salahuddin, & Zaman (2012), and Bourguignon & Chakravarty (2019)]. Therefore, the analysis of poverty requires to dichotomize the population into poor and non-poor based on a benchmark value (Betti et. al, 2006). This use of a stringent cut-off not only results in loss of information but also removes the fuzziness that exists between the two extremes of welfare. Poverty is perceived to be vague in terms of (i) making judgement about who is to be considered as poor, (ii) selecting

the relevant dimensions and indictors (Neff, 2013). Any benchmark selection in this regard remains somewhat arbitrary and the vagueness exists irrespective of whether the conventional or non-conventional poverty measure is used. Conventional regression models require crisp data and relationship between the dependent and independent variables. However, in case of poverty which is a vague phenomenon in terms of identification of poor and evaluating parameters (indicators), fuzzified regression seems to be a more natural alternative to the conventional approach (Chukhrova & Johannssen, 2019).

Poverty is not an attribute that is simply present or absent for individuals or households instead it should be considered as a matter of degree. Cerioli & Zani (1990) introduce this concept for measuring poverty based on their inspiration from the theory of Fuzzy Sets. Many researchers utilize this concept for the measurement of poverty [see, e.g., Chakravarty (2006), Oyekale (2009), Montrone (2011) and Betti (2017)] as it allows to tackle the direct criticism on the conventional measures of poverty in terms of dichotomization into poor and non-poor (for details see, Chukhrova & Johannssen, 2019). The fuzzy logic assigns degree of membership to a set of poor people on a scale from 0 to 1, where 1 means full membership and 0 means full non-membership to the set of poor. For example, the official poverty line for Pakistan based on HIES 2013-14 is PKR 3030 per adult equivalent per month. As per official poverty line, a person who earns more than PKR 3030 is non-poor but fuzzy logic assigns him a slightly lower degree of membership to the set of the poor people. The advantage of the fuzzy logic is that it allows us to tackle the problem of identification and vagueness as it does not require a single precise poverty line (e.g., PKR 3030). This fuzzy membership function has the ability to accommodate non-distinctive cases as described above and does not require the stringent cut-off line dividing the poor and non-poor (Neff, 2013).

For measuring poverty, another pertinent issue is the choice of the indicator for the identification of poor. Conventional methods of poverty measurement based on income or expenditure and its distribution are indirect way of studying poverty because the economic access to the private goods like food is determined not only by the individual's income but also determined by the government and non-government

organizations. Therefore, the personal income is not the main determinant of poverty (Kumar et. al., 2008). To measure the welfare level of individuals, we use Engel curve method as it gives a lot of information regarding the consumption behavior of consumers at different levels of total expenditures and for various family compositions. Food shares are good indicators of welfare across different household sizes as budget share for food falls if expenditure increases and there is a positive relationship between food share and household size. Engel curves have widespread empirical applications. Many demand systems like linear expenditure system, almost ideal demand system (Deaton and Muellbauer, 1980) and quadratic almost ideal demand system (Banks, Blundell, & Lewbel 1997, Chen & Chen; 2016) are based on Engel curve. Rao (1981) utilizes the conventional property of the Engle Curve of a necessity to explore that the proportion of food expenditure increases, reaches a maximum point and then declines. Rao suggests that one and half time of the maximum point should be taken as poverty line and the maxima as a threshold for acute poverty.

A handful of studies on Pakistan explore the consumption patterns by computing the Engel curves (see, e.g., Burney, & Khan, 1992; Shamim & Ahmad, 2007; Ahmad & Arshad, 2007 and Kiani, 2013) however, the use of Engle curves and Fuzzy logic in measuring poverty is relatively a nascent idea in local literature. This study contributes to the scarce literature by utilizing Engle curve approach for estimating poverty in Pakistan through Fuzzy Regression.

II. METHODOLOGY AND DATA

Health, education, sanitation, and drinking water are the public goods provided by the government whereas there are some private goods like food provided by non-government agents through market mechanism (Kumar et. al., 2008). Therefore, the economic access to these public and private goods depends on an individual's and government and non-government organization's resources. Hence, personal income is not the main driver of poverty. To measure the welfare level of individuals, we use Engel curve method as it gives a lot of information regarding the consumption behavior of consumers at different levels of total expenditures and for various family compositions.

We prefer Engel method to other approaches due to its simplicity, low data requirements and clear theoretical foundations. There is one assertion and two regularities regarding this method. The assertion is that food shares are good indicators of welfare across different household sizes. The regularities are (i) the budget share for food falls if expenditure or income rises, (ii) holding total expenditure constant, household size and food share has positive relationship between them. The identifying assumption in this approach is that there should be a stable structural relationship between real income and food shares.

Regarding the estimation of Engle curve, we consider the Working (1943) and Lesser (1963) model (eq. 1) as its mathematical form is consistent with consumer behavior and fulfills the requirements of consumer demand theory and adding-up restriction. The following empirical model is the restricted form of Almost Ideal Demand System (Deaton and Muellbauer, 1980)

$$m_{ip} = \alpha + \beta ln y_{ip} + X_{ip}\phi + \sum_{p=1}^{3} D_{ip} + \varepsilon_{ip} \ldots\ldots (1)$$

where m_{ip} is the budget share for food (table 1) for the ith household in province p, y_{ip} is the per capita total expenditure, X_{ip} is a vector of household-specific control variables (age, gender & education of the household head, household size, region, and occupation), D_{ip} is a province-level dummy variable for province p and ε_{ip} is the error term. The provincial dummy variable, D_{ip}, takes the value one for the province 'i' and zero otherwise. Dummy variable, D_{ip}, is used to control for the latent affects including price differentials across the provinces. Budget share for food is used due to its several advantages over the other indicators. Firstly, budget share for food is more sensitive to changes in income, second, it is a nondurable good hence there is no lag between its expenditure and consumption unlike the durable goods. Lastly, unlike the GDP and CPI, food expenditure is not politically sensitive indicator (Hamilton, 2001). Per capita consumption expenditure, y_{ip}, is used instead of income due to (i) it is less volatile than income, (ii) strong

relationship between individual's wellbeing and consumption (Lewis, 2014), and (iii) people declare consumption expenditure more truly than income and it is more accurately measured at the lower quintile of income distribution.

To tackle the criticism on dichotomization of population into poor and non-poor, we utilize the concept introduced by Cerioli & Zani (1990) based on the theory of Fuzzy Sets initiated by Zadeh (1965).

For any given set X of elements $x \in X$, any fuzzy subset F of X is defined as follows: $F = \{x, \mu_F(x)\}$, where $\mu_F(x): X \rightarrow [0,1]$ is called the membership function (mf) in the fuzzy subset F. The $\mu_F(x)$ indicates the degree of membership of x in F. Hence, $\mu_F(x) = 0$ means x does not belong to F, $\mu_F(x) = 1$ means x belongs to F completely. Whereas, $0 < \mu_F(x) < 1$ means x partially belongs to F. Its degree of membership to F increases as $\mu_F(x) \rightarrow 1$.

In the conventional headcount ratio H, the mf may be perceived as $\mu(y_i) = 1 \; if \; y_i < z$, $\mu(y_i) = 0 \; if \; y_i > z$ and as $\mu(y_i) = {(z_1 - y_i)}/{(z_1 - z_2)} \; if \; z_2 < y_i < z_1$ where y_i is the consumption expenditure of household i and z is the expenditure poverty line. In order to avoid the poor and non-poor dichotomy, Cerioli & Zani (1990) introduce a transition zone $(z_1 - z_2)$ between the two extremes, the mf in this zone declines from 1 to 0 linearly. Cheli and Lemmi (1995) defines the mf as the distributional function of income linearly transformed such that the value equal to 1 represent poorest and 0 the richest person in the population. Membership function provides the possibility distribution therefore; the estimation approaches are referred to as "possibilistic regression analysis" and the data distribution as "possibilistic distribution".

This study considers the fuzzy regression approach proposed by Lee & Tanaka (1999) based on quadratic programing. This method constructs upper and lower approximation models which allow us to compute the membership function, $\mu(y_i)$. We consider this non-linear programing model as it tackles the criticism on linear regression models in terms of non-interactive possibilistic parameters. When the possibilistic

distribution is defined on minimum parameters, they often become crisp due to zero spread of fuzzy regression parameters (for details see, Chukhrova & Johannssen, 2019).

The data used to estimate eq. 1 come from Pakistan Social and Living Standards Measurement (PSLM) survey 2015-16. It covers all rural and urban areas of the four provinces namely Sindh, Punjab, Balochistan and Khyber Pakhtunkhwa excluding forces restricted military areas of these provinces. Table 1 provides the summary statistics for the important variables.

TABLE 1

Descriptive Statistics

Variables	n	Mean	SD	Min	Max
Food expenditure per capita	24,181	690.28	340.37	91.77	5407.92
Total expenditure per capita	24,181	9105.81	8031.43	1415.21	182517.1
Budget share for food per capita (%)	24,181	9.79	5.76	0.3239	49.02
Literacy	24,181	0.645	--	0	1
Household Size	24,181	4.33	1.94	1	40

This research used a detailed sample consisting of 8072 households from rural areas, 16109 households from urban area and a total of 24181 households that are covered by PSLM 2015-16. The control variables include the age, gender & education of the household head, household size, region (rural/urban), and occupation. It is pertinent to mention that we use OECD equivalence scale to construct the household size. The average household size is 4.33 with a standard deviation of 1.94. Total monthly expenditure per capita is PKR 9106 with standard deviation of PKR 8031 indicating enormous variation of expenditure within and across the regions. Per capital budget share of food (9.8 percent) with standard deviation of 5.8 percent further reiterate the fact that income distribution of the people in Pakistan varies according to region and groups. These findings motivate us to measure poverty at national and provincial level.

III. RESULTS AND DISCUSSION

One of the main goal of SDGs is to attain the level of 'no poverty' in its all dimensions by 2030. In line with this goal, the vision 2025 of Pakistan is also aiming to reduce poverty. To achieve the goal, first step is the measurement of poverty using a robust and reliable method. The rigid dichotomization of the population into poor and non-poor based on the poverty line results in loss of fuzziness that exist between poor and non-poor. As already discussed, poverty is perceived to be vague in terms of choosing the indicator(s) to evaluate and make judgement about who is to be considered as poor (Neff, 2013). Any benchmark selection in this regard remain somewhat arbitrary and the vagueness exists irrespective of whether the conventional or non-conventional poverty measure is used. Thus, fuzzified regression seems to be a more natural alternative to the conventional approach (Chukhrova & Johannssen, 2019). This study employs the fuzzy regression technique that treats poverty as a matter of degree rather than an attribute that is simply present or absent in individuals. Conventional methods of poverty measurement based on income or expenditure and its distribution are indirect way of studying poverty (Kumar et. al., 2008). Therefore, to measure the welfare level of individuals, we use Engel curve method as it gives a lot of information regarding the consumption behavior of consumers at different levels of total expenditures and for various family compositions.

Poverty estimates help in targeting the people who lacks in resources and essentials required to maintain a minimum standard of wellbeing. Social safety net programs like Benazir Income Support Program (BISP) are initiated in this regard. However, the coverage of these programs is limited due to financial constraints. Thus, a selected proportion of the vulnerable population can be targeted which requires the poverty estimation at different benchmarks. This study provides poverty estimates at different values of membership function (μ) and results are summarized in table 2. Suppose, government choses to target people with degree of membership to the set of poor equal to 0.88 (top 12 percent). At the chosen cut off, 30.34 percent people at national level are living in poverty with 48.59 percent of the households in rural whereas 21.20 percent in urban areas of Pakistan are poor. Poverty is a rural phenomenon in Pakistan (table 2 & 3) due to interregional differences in

consumption patterns of households as the income increases (Siddiqui, 1982 and Burney & Khan, 1991). Pearson Chi-square test is utilized to test the hypothesis of no association between poverty and provinces. We are unable to accept the hypothesis (p-value=0.0000) indicating that the poverty is not only a rural but a provincial phenomenon as well in Pakistan.

Pakistan is the sixth most populated country (World Population Prospects, 2019) and aiming to reduce poverty levels in all dimensions to half by 2025. With instable economy, Pakistan is struggling to make progress in human development, food security, health, education, and unemployment (Padda & Hameed, 2018). Figure 1 is clearly indicating the need of education and creating employment opportunities to reduce poverty in Pakistan. Pakistan has launched different social safety net programs to reduce poverty in different regimes such as Cash Transfer, Benazir Income Support Program, Worker's Welfare Fund, Ehsas etc., however in all programs, education aspect is not given the due consideration. Figure. 2 clearly demonstrates the lack of educational opportunities to low income group as they are mostly located in rural areas. Access to education and health facilities in rural areas of Pakistan should be the priority of the government. Employment opportunities are limited due to instable economic and political conditions in the country. Besides, more than 50 percent of literate persons are under employed in Pakistan (Labor Force Survey, 2017-18) and they are looking for alternate work.

Figure 1

Poverty Vs Literacy and Unemployment

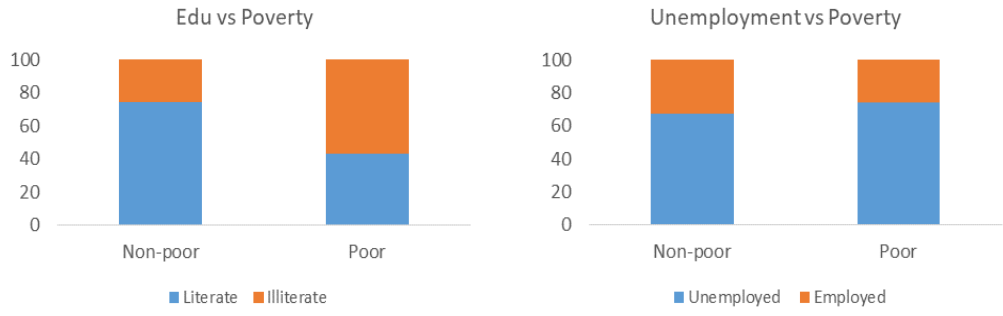


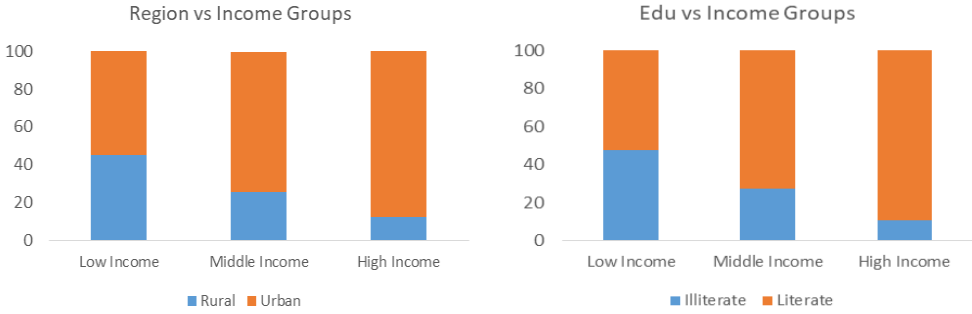
TABLE 2

Poverty Estimates at National Level

μ	National	Rural	Urban
0.85	46.92%	67.54%	36.58%
0.86	41.81%	62.17%	31.62%
0.87	36.24%	55.33%	26.67%
0.88	30.34%	48.59%	21.20%
0.89	24.40%	40.80%	16.18%
0.90	18.44%	32.45%	11.42%

Figure 2

Income Distribution Vs Education and Region



Highest incidence of poverty is in Balochistan province where 41.54 percent of the population is living below the poverty threshold with degree of membership equal to 0.88. More than half of the rural Balochistan (57.19%) is living in poverty whereas 36.65 percent of urban population is also poor. Our results corroborate with the findings in Saleem, Shabbir & Khan (2019). Despite its richness in natural resources, the province is considered as "lagging region" where social indicators and living standards are lowest in the country. Balochistan has witnessed the slowest growth over the decades due to weak fiscal base and worst infrastructure. Due to shortage of water and inappropriate arid conditions, only 6% of the land is capable of being farmed productively. Furthermore, typical feudal system hinders the growth and induces

poverty in the region as they can easily exploit more than 49 percent of the population belonging to low income group (Figure. 3).

Second highest incidence of poverty is observed in Sindh where approximately 28.0 percent population is suffering from poverty with 45.81 rural and 19.72 percent urban population is deprived. Our findings corroborate with the results in Khan *et. al*., (2014) and Padda & Hameed, (2018). The rural population of Sindh is dependent on agriculture, animal rearing and fishing for their livelihood. Crop yield is low due to soil erosion, water shortage/mismanagement, water logging, over irrigation and high cost of input materials. Employment opportunities are scarce for rural residents and mechanized farming has closed the door for new entrants from the local workforce. Although, unemployment is relatively less in Sindh however, more than 57 percent of the population in Sindh belong to low income group and only 9.85 percent are rich (Figure. 3).

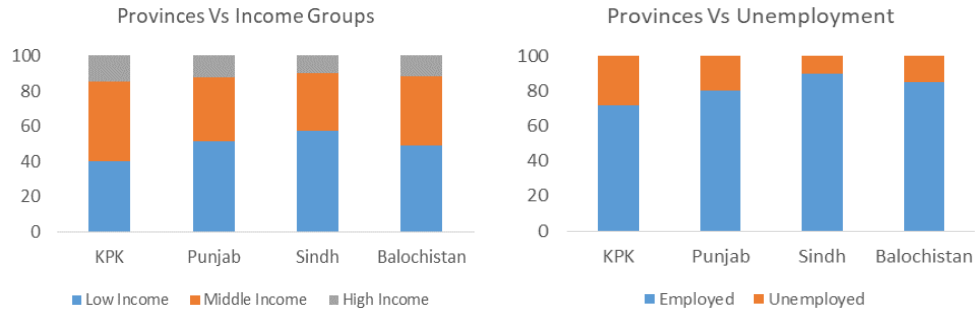
TABLE 2

Poverty Estimates at Provincial Level

Province	Overall	Rural	Urban
KPK	24.26%	32.29%	19.69%
Punjab	27.98%	45.81%	19.72%
Sindh	35.07%	63.95%	18.00%
Balochistan	41.54%	57.19%	36.65%

Figure 3

Provincial comparison



Punjab and KPK provinces are at the third and fourth place with overall poverty rates of 27.98% & 24.26% respectively. Both provinces share almost the same rate of poverty (20.0%) for urban region however the poverty rate for Punjab's rural population is much higher (45.81%) than KPK (32.29%). The contributing factors to high rate of poverty in rural Punjab includes low productivity in farms, bigger size of households, lower prices of output, absence of infrastructure, high dependency ratio and illiteracy. Poverty in KPK province has declined over the last decade due to increase in remittances (Jamal, 2016) and improved law and order situation after the successful operation against the anti-state agents in KPK which allowed the local business and tourism industry to flourish.

IV. CONCLUSION

Although in the previous decade a lot of progress has been made in eliminating poverty, still this is an alarming problem and needs to be resolved. The key purpose of this research is to analyze consumption patterns of households through which we can calculate poverty in Pakistan. This study estimates Engle curves by utilizing the budget share of food through the fuzzy regression.

On balance, 30.34 percent people at national level are living in poverty with 48.59 percent of the households in rural whereas 21.20 percent in urban areas of Pakistan are poor. Highest incidence of poverty at provincial level is recorded in Balochistan where 41.54 percent of the population is poor. Majority of the poor are living in rural areas (57.19%) and the urban poverty in Balochistan is also high (36.65%) relative to other provinces. Rural Sindh is the most deprived area in Pakistan with highest incidence of poverty (63.95%) and the urban areas of Sindh are least deprived in the country with 18.0 percent of population living in poverty. Almost 20 percent of the urban population of both Punjab and KPK is below poverty however, the rural population in Punjab is more deprived (48.81%) as compared to the rural population of KPK (32.29%).

Illiteracy and lack of employment opportunities in Pakistan are the main drivers of poverty. Political unrest and crumbling economy are the hindrances in foreign investment resulting in scarce employment opportunities. Access to education and health facilities in rural areas of

Pakistan should be the priority of the government. Instead of cash transfer programs, vocational training programs should be launched on large scale by the government of Pakistan.

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